**MTH 301 - ANALYSIS - I**
**IIT KANPUR**
Instructor: Indranil Chowdhury
Odd Semester, 2025-26

---

Quiz 2 | Max Marks: 20 | 12:00pm - 13:15pm

---

1. Let $f : (M, d) \to (N, \rho)$ be an isomorphic (i.e. $\rho(f(x), f(y)) = d(x, y)$) and onto map. Show that $f$ is a homeomorphism. [5]

2. Let $(\ell_2, \|\cdot\|_2)$ be the complete metric space where $\ell_2 = \{(x_1, x_2, \dots) : \sum_{n=1}^{\infty} |x_n|^2 < \infty\}$ and the norm is given by $\|x\|_2 = \left(\sum_{n=1}^{\infty} |x_n|^2\right)^{\frac{1}{2}}$ for $x = (x_1, x_2, \dots, x_n, \dots)$.
   (a) Show that the set $A = \{x \in \ell_2 : |x_n| \leq 1/n \text{ for } n \in \mathbb{N}\}$ is a closed set in $\ell_2$.
   (b) Is $B = \{x \in \ell_2 : |x_n| < 1/n \text{ for } n \in \mathbb{N}\}$ an open set? Justify.
   (c) Is $A$ complete where $A$ is defined in part (a)? [2+2+1]

3. $(M, d)$ is a connected metric space and has at-least two elements. Show that $M$ is uncountable. [3]

4. Let $(M, d)$ be a metric space and $A \subseteq M$. Prove that if $A$ is totally bounded then (the closure) $\overline{A}$ is totally bounded. [3]

5. Let $(M, d)$ be a complete metric space. A subset $A \subseteq M$ is complete if and only if $A$ is a closed set. [4]