MTH208: Quiz 5

Instructions

- Download **q5_template.R** and rename it **q5_<ROLL>.R** (e.g., **q5_123456.R**).
- Copy and paste all relevant code in the renamed **q5_template.R** file provided. You MUST only submit relevant code. In the event of any error, you will receive 0 marks.
- Your script must run from a fresh R session without errors.
- You are **not allowed** to use any built-in "numerically safe" function.
- Test the correctness before submitting your code.
- Deadline: 3:00 pm. Submit **q5_<ROLL>.R** on helloIITK only.

Problem 1

Complete the function **logprob** that takes any two real numbers **a** and **b** as input and returns the numerically stable value

$$\log\left(\frac{e^a}{e^a + e^b}\right).$$

Direct computation using **exp(a)** and **exp(b)** can lead to overflow (for large positive inputs) or underflow (for large negative inputs).

You are **not allowed** to use **log1p()** or any other built-in "numerically safe" function. Your implementation must therefore use algebraic manipulation to ensure numerical stability.

```
logprob <- function(a,b) {
...
...
}
```

Problem 2

You are provided with an **q5_data.csv** file that that contains a numeric vector **z_vec** of length n. Mathematically, denote $z = (z_1, z_2, \ldots, z_n)$.

Your task is to compute the vector $p = (p_1, \ldots, , p_n)$ where

$$p_i = \frac{\exp(z_i)}{\sum_{j=1}^{n} \exp(z_j)} \qquad \text{for } i = 1, \ldots, n\,,$$

with numerical stability, that is, your code should not overflow or underflow when some z_i values are large in magnitude.

Save the resulting numeric vector as `prob_vec` where the code block should have the following structure:

```
...
...

prob_vec <- ...
```