

MTH 301 - ANALYSIS - I

IIT KANPUR

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Odd Semester, 2025-26

Quiz 1

Max Marks: 20

6:45pm - 8:00pm

1. (a) Can we find a metric space (M, d) having exactly 2025 open sets? Justify your answer. [2.5]
(b) Let $f(x) = \log\left(\frac{23+\sin(2\pi x)^7}{1+e^{2x}}\right)$ and $x_n = \frac{n+\sin n}{n}$ be a sequence for $n \in \mathbb{N}$. Calculate $\limsup f(x_n)$ and $\liminf f(x_n)$. [2.5]
2. Let (\mathbb{R}, d) be the usual metric space and $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuous. Show that the set $U = \{x \in \mathbb{R} : f(x) > a\}$ for some $a \in \mathbb{R}$ is an open set. Check whether U will be an open set if f is monotone but not continuous. [5]
3. Let (M, d) be a metric space. A point $x \in M$ is said to be a **Boundary Point** of a set A iff $B_\epsilon(x) \cap A \neq \emptyset$ and $B_\epsilon(x) \cap A^C \neq \emptyset$ for every $\epsilon > 0$.
If F is a nonempty bounded subset of \mathbb{R} with usual metric, show that $\sup F$ is a boundary point of F . [5]
4. Consider $M = [0, 2)$ and define $d_1(x, y) = \min\{|x - y|, 0.1\}$.
 - Determine all open balls in (M, d_1) .
 - Are $\{0\}$ and $\{1\}$ an open set or a closed set in (M, d_1) ? Justify.
 - Is $(0, 0.1]$ an open set in (M, d_1) ? Justify. [2 + 2 + 1]