Quiz 2: (MSO 205) ①

Q1. → From Practice problem set

Marginal p.d.f. of X_1 is

$$f_{X_1}(x_1) = \int_{-\infty}^{\infty} f_X(x_1, x_2)\, dx_2$$

1 mark

1.5 marks

$$= \begin{cases} 0, & \text{if } x_1 \le 0 \text{ or } x_1 \geq 1 \\ \int_{-x_1}^{x_1} 1\, dx_2 & \text{if } 0 < x_1 < 1 \end{cases}$$

1.5 marks.

$$= 0 \text{ if } x_1 \le 0 \text{ or } x_1 \geq 1$$

$$= 2x_1 \quad \text{if } 0 < x_1 < 1$$

Marginal p.d.f. of X_2 is

$$f_{X_2}(x_2) = \int_{-\infty}^{\infty} f_X(x_1, x_2)\, dx_1\, dx_2$$

1.5 marks

$$= \begin{cases} 0 & \text{if } x_2 \le -1 \text{ or } x_2 \geq 1 \text{ or } x_2 = 0 \\ \int_{|x_2|}^{1} 1\, dx_1, & \text{if } 0 < |x_2| < 1 \end{cases}$$

1.5 marks

$$= 0 \text{ if } x_2 \le -1 \text{ or } x_2 \geq 1 \text{ or } x_2 = 0$$

$$= 1 - |x_2| \text{ if } 0 < |x_2| < 1$$

~~Objective~~ (2)

No, Here X_1 & X_2 are NOT independent
mark
since
$f_X\left(\frac{1}{2},\frac{1}{2}\right)=1$ but $f_{X_1}\left(\frac{1}{2}\right)\times f_{X_2}\left(\frac{1}{2}\right)$
marks
$$=\left(2\times\frac{1}{2}\right)\times\frac{1}{2}=\frac{1}{2}$$

So, $X_1 \not\perp\!\!\!\perp X_2$.

Q 2:- (From the solⁿ of Practice problem)

mark Yes, they have the same sampling distribution.

Reason:- Note that $P[Y=0]=0$.

marks. Then for $\zeta\in\mathbb{R}$, we have

$$\mathbb{P}\left[\frac{X}{Y}\leq\zeta\right]=\mathbb{P}\left[\frac{X}{Y}\leq\zeta,\ Y>0\right]+\mathbb{P}\left[\frac{X}{Y}\leq\zeta,\ Y<0\right]$$
$$=\mathbb{P}\left[\frac{X}{|Y|}\leq\zeta,\ Y>0\right]+\underbrace{\mathbb{P}\left[-\frac{X}{|Y|}\leq\zeta,\ Y<0\right]}_{①}$$

Next, further observe that

$$\left\{\begin{array}{l}(X,Y) \stackrel{d}{=} (-X,Y), \\ \Rightarrow \mathbb{P}\left[-\frac{X}{|Y|} \le z,\ Y<0\right] = \mathbb{P}\left[\frac{X}{|Y|} \le z,\ Y<0\right] \quad \text{--- (2)}\end{array}\right.$$

[mark

Using (2) in (1), we have

$$F_{\frac{X}{Y}}(z) = \mathbb{P}\left[\frac{X}{Y} \le z\right] \stackrel{\text{Using (1)}}{=} \mathbb{P}\left[\frac{X}{|Y|} \le z,\ Y>0\right] + \mathbb{P}\left[-\frac{X}{|Y|} \le z,\ Y<0\right]$$

$$\stackrel{\text{Using (2)}}{=} \mathbb{P}\left[\frac{X}{|Y|} \le z,\ Y>0\right] + \mathbb{P}\left[\frac{X}{|Y|} \le z,\ Y<0\right]$$

marks.

$$= \mathbb{P}\left[\frac{X}{|Y|} \le z\right] \quad \forall z \in \mathbb{R}.$$

$$:= F_{\frac{X}{|Y|}}(z)$$

Hence,

$$\frac{X}{Y} \stackrel{d}{=} \frac{X}{|Y|}.$$ [It can be proved alternative ways al

Q3:-

1 mark.

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{\mathrm{Var}(Y)}}$$

$$= \frac{E(XY) - E(X)E(Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{\mathrm{Var}(Y)}} \quad \text{—— } (*)$$

Now, observe that

mark.

$$E(XY) - E(X)E(Y)$$

$$= \underbrace{E(X^{2m+1})}_{=0} - \underbrace{E(X)}_{=0}\, E(Y^{2m})$$

odd moments

$E(Y^{2m}) < \infty$ since $N(0,1)$ has all finite moments

$$= 0. \quad \text{—— } (1)$$

1 mark

$$\left.\begin{aligned} \mathrm{Var}(X) &= 1 \neq 0 \\ \mathrm{Var}(X^{2m+1}) &\neq 0 \quad \forall\, m \in \mathbb{N} \end{aligned}\right\} \text{—— } (2)$$

Using 1 & 2 in (*), we have,

$$\mathrm{corr}\big(X, \underset{\substack{\| \\ X^{2m}}}{Y}\big) = 0 \quad \forall\, m \in \mathbb{N}.$$