MTH208: Quiz 2

Instructions

- Download `q2_template.R` and rename it `q2_<ROLL>.R` (e.g., `q2_123456.R`).
- Fill in the blanks without changing required function names or I/O behavior.
- Allowed packages: `imager`, `rbenchmark`, base R.
- Your script must run from a fresh R session without errors.
- Deadline: 10:00 am. Submit `q2_<ROLL>.R` on helloIITK only.

Problem 1

Implement

```
add_border(img, border, save_path = NULL)
```

that:

- Adds a black border if `img` is grayscale and a dark navy border (`c(0,0,128/255)`) if `img` is RGB.
- Returns the bordered image as an `imager::cimg`.
- If `save_path` is not `NULL`, saves the image to that path.

Problem 2

Let $X \in \mathbb{R}^{n \times p}$. The Euclidean distance between rows i and j is

$$d(i,j) = \|x_i - x_j\|_2 = \sqrt{\sum_{k=1}^{p} (x_{i,k} - x_{j,k})^2}.$$

The full pairwise distance matrix $D \in \mathbb{R}^{n \times n}$ has entries $D[i,j] = d(i,j)$.

Task. Implement three methods to compute D **:**

- A. Nested `for` loops.

- B. `dist()` from base R (converted to full matrix).

- C. Usin the identity:

$$\|x_i - x_j\|_2 = \sqrt{\|x_i\|_2^2 + \|x_j\|_2^2 - 2(x_i \cdot x_j)}.$$

Then:

1. Verify the three outputs are numerically equal on a small problem.
2. For $n \in \{200, 400, 800\}$ and $p = 50$, compare the three methods using `rbenchmark::benchmark()` with:

   - `replications = 5`
   - `columns = c("test","replications","elapsed","relative")`
   - `order = "elapsed"`

   Name the three expressions **`loop`**, **`dist`**, and **`xprod`**.