Problem Set 3: MTH207M

Problem 1. Answer the following questions with reference to the linear model $\mathbb{E}(y_1) = \beta_1 + \beta_2$, $\mathbb{E}(y_2) = 2\beta_1 - \beta_2$, $\mathbb{E}(y_3) = \beta_1 - \beta_2$, where y_1, y_2, y_3 are uncorrelated with a common variance σ^2.

1. Find two different linear functions of y_1, y_2, y_3 that are unbiased for β_1. Determine their variances and the covariance between the two.

2. Find two linear functions that are both unbiased for β_2 and are uncorrelated.

3. Write the model in terms of the new parameters $\theta_1 = \beta_1 + 2\beta_2$, $\theta_2 = \beta_1 - 2\beta_2$.

Problem 2. Consider the model $\mathbb{E}(y_1) = 2\beta_1 - \beta_2 - \beta_3$, $\mathbb{E}(y_2) = \beta_2 - \beta_4$, $\mathbb{E}(y_3) = \beta_2 + \beta_3 - 2\beta_4$ with the usual assumptions. Describe the estimable functions.

Problem 3. COnsider the model $\mathbb{E}(y_1) = \beta_1 + \beta_2$, $\mathbb{E}(y_2) = \beta_1 - \beta_2$, $\mathbb{E}(y_3) = \beta_1 + 2\beta_2$ with the usual assumptions. Obtain the BLUE of $2\beta_1 + \beta_2$ and find its variance.

Problem 4. Consider the model $\mathbb{E}(y_1) = 2\beta_1 + \beta_2$, $\mathbb{E}(y_2) = \beta_1 - \beta_2$, $\mathbb{E}(y_3) = \beta_1 + \alpha\beta_2$ with the usual assumptions. Determine α such that the BLUE of β_1 and β_2 are uncorrelated.