2

UNIT

Steady-State Analysis of 1ϕ AC Circuits

PART-1

Representation of Sinusoidal Waveforms – Average and Effective Values, Form and Peak Factor, Concept of Phasors, Phasor Representation of Sinusoidally Varying Voltage and Current.

CONCEPT OUTLINE : PART-1

- An alternating quantity that varies sinusoidally is called sinusoidal quantity.
- Average value
$$V_{av} = \frac{1}{T}\int_0^T v\,dt$$
- RMS value or effective value
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$
- $$\text{Form factor} = \frac{\text{RMS value of current}}{\text{Average value of current}}$$
- $$\text{Peak factor} = \frac{\text{Maximum value of current}}{\text{RMS value of current}}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **Derive expression for average value and rms value of a sinusoidally varying AC voltage.**

AKTU 2017-18(Sem-2), Marks 07

OR

Derive expression for average and rms value of a sinusoidally varying AC voltage. Also write form factor and peak factor.

Answer

A. Average value for sinusoidal current or voltage :

1. The average value of a sine wave over a complete cycle is zero. Therefore, the half cycle average value is intended.
2. Instantaneous value of sinusoidal current is given by
$$i = I_{max} \sin \omega t$$
Consider first half cycle, *i.e.*, when ωt varies from 0 to π, we get

$v(t) = 10t$

2. Average value, $V_{av} = \frac{1}{1}\int_0^1 v(t)\,dt = \int_0^1 10t\,dt = \frac{10\,[t^2]_0^1}{2} = \frac{10}{2}[1] = 5\text{ V}$

3. rms value, $V_{rms} = \left[\frac{1}{1}\int_0^1 v^2(t)\,dt\right]^{1/2} = \left[\int_0^1 (10t)^2\,dt\right]^{1/2}$

$$= \left[\left(\frac{100\,t^3}{3}\right)_0^1\right]^{1/2} = \sqrt{\frac{100}{3}} = \frac{10}{\sqrt{3}}\text{ V}$$

4. Form factor, $K_f = \frac{V_{rms}}{V_{av}} = \frac{10/\sqrt{3}}{5} = \frac{2}{\sqrt{3}} = 1.154$

5. Peak factor, $K_p = \frac{V_{max}}{V_{rms}} = \frac{10}{10/\sqrt{3}} = \sqrt{3} = 1.73$

imp

Que 2.4. **Find average and rms values of following voltage waveform.**

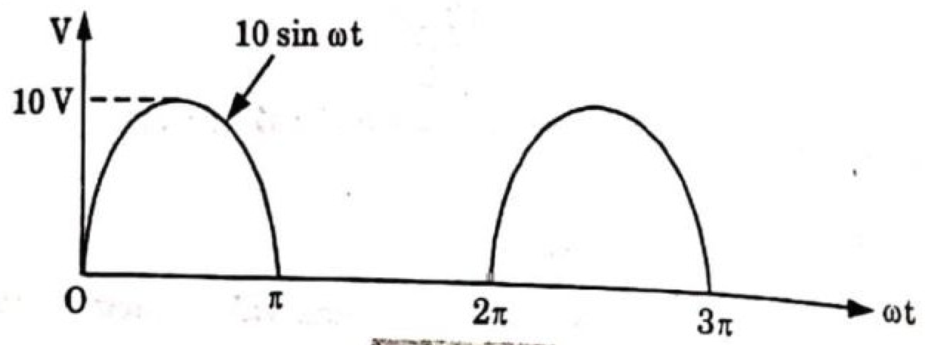


Fig. 2.4.1.

AKTU 2015-16(Sem-2), Marks 10

Answer

1. $V_{avg} = \frac{1}{T}\int_0^T V\,dt = \frac{1}{2\pi}\int_0^{2\pi} V\,d\omega t$

$$= \frac{1}{2\pi}\int_0^{\pi} V\,d\omega t + \frac{1}{2\pi}\int_{\pi}^{2\pi} 0\,d\omega t = \frac{1}{2\pi}\int_0^{\pi} V\,d\omega t$$

$$= \frac{1}{2\pi}\int_0^{\pi} 10\sin\omega t\,d\omega t = \frac{1}{2\pi}[-10\cos\omega t]_0^{\pi}$$

$$= \frac{10}{2\pi}\times 2 = \frac{10}{\pi} = 3.18\text{ V}$$

2. Also, $V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2\,d\omega t}$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi}(10\sin\omega t)^2\,d\omega t + \frac{1}{2\pi}\int_{\pi}^{2\pi} 0^2\,d\omega t}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{\pi}\sin^2\omega t\,d\omega t} = \sqrt{\frac{50}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t}$$

$$= \sqrt{\frac{25}{\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}} = \sqrt{\frac{25}{\pi}[\pi - 0 - 0 + 0]} = 5\text{ V}$$

I.mp

Que 2.5. **Find form factor and peak factor for given waveform in Fig. 2.5.1.**

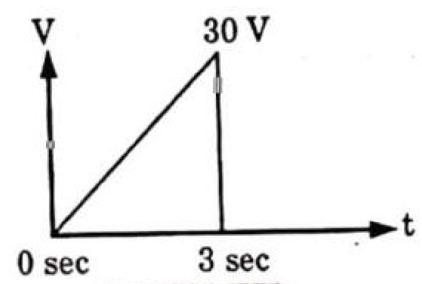


Fig. 2.5.1.

AKTU 2014-15(Sem-2), Marks 10

Answer

1. $V_{rms} = \sqrt{\frac{1}{3}\int_0^3 (10\,t)^2\,dt} = 17.32\text{ V}$

2. $V_{avg} = \frac{1}{3}\int_0^3 10\,t\,dt = \frac{10}{3}\left[\frac{t^2}{2}\right]_0^3 = \frac{10}{3}\left[\frac{9}{2}\right] = 15\text{ V}$

3. $V_{max} = \sqrt{2}\,V_{rms} = \sqrt{2}\times 17.32 = 24.49\text{ V}$

4. Form factor $= \frac{V_{rms}}{V_{avg}} = \frac{17.32}{15} = 1.1547$

5. Peak factor $= \frac{V_{max}}{V_{rms}} = \frac{24.49}{17.32} = 1.4139$

Que 2.6. **Discuss the concept of phasors.**

Answer

1. A phasor is a physical quantity which has a magnitude as well as direction.
2. Such phasor quantities are completely known when particulars of their direction, magnitude and the sense in which they act are given. They are graphically represented by straight lines.

3. OP is such a phasor which represents maximum value of voltage and its angle with x axis gives its phase.
4. It will be seen that the projection of OP and y axis at any instant gives instantaneous value of voltage (V).

$$OM = OP \sin \omega t$$

$\therefore$ $$v = V_{max} \sin \omega t$$

$\therefore$ $$v = V_{max} \sin \omega t$$

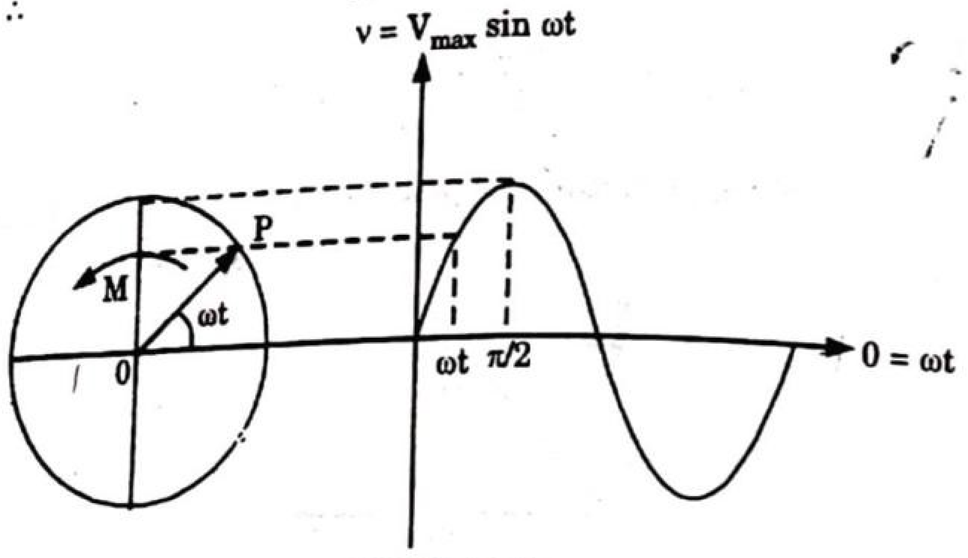


Fig. 2.6.1.

Imp

Que 2.7. **Draw the phasor diagram for the following voltages. Calculate the resultant voltage. Also find the rms voltage.**

$$v_1 = 100 \sin 500t$$
$$v_2 = 200 \sin (500t + \pi/3)$$
$$v_3 = -50 \cos 500t$$
$$v_4 = 150 \sin (500t - \pi/4)$$

AKTU 2016-17(Sem-2), Marks 07

Answer

1. **Phasor diagram :**

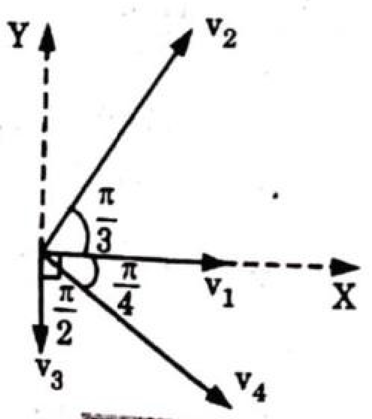


Fig. 2.7.1.

2. Algebraic sum of X-components

$$= 100 \cos 0° + 200 \cos \pi/3 + 50 \cos (-\pi/2) + 150 \cos (-\pi/4)$$

$$= 100 + (200 \times 0.5) + 0 + \frac{150}{\sqrt{2}} = 306 \text{ V}$$

3. Algebraic sum of Y-components

$$= 100 \sin 0° + 200 \sin \pi/3 + 50 \sin (-\pi/2) + 150 \sin (-\pi/4)$$
$$= 0 + (200 \times 0.866) - 50 - 106 = 17.2 \text{ V}$$

4. $$V_{r\,max} = \sqrt{(306)^2 + (17.2)^2} = 306.5 \text{ V}$$

5. Phase angle, $$\phi_r = \tan^{-1} \frac{Y\text{-component}}{X\text{-component}}$$

$$= \tan^{-1} \frac{17.2}{306} = 3.22° = 0.018\pi$$

6. Resultant voltage, $v_r = 306.5 \sin (500t + 0.018\pi)$

7. $$V_{r\,rms} = \frac{306.5}{\sqrt{2}} = 216.73 \text{ V}$$

Imp

Que 2.8. **Draw a phasor diagram showing the following voltage :**

$$v_1 = 100 \sin 500\,t$$
$$v_2 = 200 \sin (500\,t + 45°)$$
$$v_3 = \cos 500\,t$$

Find rms value of resultant voltage.

AKTU 2013-14(Sem-2), Marks 10

Answer

The procedure is same as Q. 2.7, Page 2–7D, Unit-2.

(Ans : V_{rms} = 198.2 V)

Que 2.9. **If two alternating currents represented by $i_1 = 7 \sin \omega t$ and $i_2 = 10 \sin (\omega t + \pi/3)$ are fed into a common conductor, then find equation for resultant current and its rms value.**

AKTU 2013-14(Sem-1), Marks 05

Answer

1. Resultant current, $i_r = i_1 + i_2$

$$= 7 \sin \omega t + 10 \sin \left(\omega t + \frac{\pi}{3}\right)$$
$$= 7 \sin \omega t + 10 \sin \omega t \cos \frac{\pi}{3} + 10 \cos \omega t \sin \frac{\pi}{3}$$
$$= 7 \sin \omega t + 10 \sin \omega t \times 0.5 + 10 \cos \omega t \times 0.866$$
$$i_r = 12 \sin \omega t + 8.66 \cos \omega t \quad ...(2.9.1)$$

$\therefore$ $$I_{r\,max} = \sqrt{(12)^2 + (8.66)^2} = 14.8$$

$\therefore$ Multiplying and dividing eq. (2.9.1) by 14.8, we get

$$i_r = 14.8\left[\sin \omega t \times \frac{12}{14.8} + \cos \omega t \times \frac{8.66}{14.8}\right]$$

As $$i_r = 14.8\left[\sin \omega t \cos \phi_r + \cos \omega t \sin \phi_r\right] \quad ...(2.9.2)$$

where $\cos \phi_r = \frac{12}{14.8}$ and $\sin \phi_r = \frac{8.66}{14.8}$

$\therefore$ $$\phi_r = \tan^{-1}\frac{8.66}{12} = 0.199\,\pi \text{ radians}$$

Thus, eq. (2.9.2) can be written as

$$i_r = 14.8 \sin(\omega t + \phi_r) = 14.8 \sin(\omega t + 0.199\,\pi)$$

2. Peak value of resultant current, $I_{r\,max} = 14.8$ A
3. rms value of resultant current, $I_{r\,rms} = \frac{14.8}{\sqrt{2}} = 10.46$ A

PART-2

Single Phase AC Consisting of R, L, C, RL, RC, RLC Combinations (Series and Parallel).

CONCEPT OUTLINE : PART-2

- In series RL circuit,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \text{ and } \phi = \tan^{-1}\frac{\omega L}{R}$$

- In series RC circuit,

$$I = \frac{V}{\sqrt{R^2 + 1/\omega^2 C^2}} \text{ and } \phi = \tan^{-1}\frac{1}{\omega RC}$$

- In series RLC circuit,

$$I = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \text{ and } \phi = \tan^{-1}\frac{\omega L - 1/\omega C}{R}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.10. **Show that the instantaneous power consumed in a pure resistive circuit is not constant but it is fluctuating.**

OR

Derive phasor relationship between voltage and current phasor for purely resistive circuit.

Answer

1. Consider an AC circuit containing resistance of R ohms connected across a sinusoidal voltage represented by

$$v = V_{max} \sin \omega t \quad ...(2.10.1)$$

2. $$iR = v$$

$$i = \frac{v}{R} = \frac{V_{max}}{R} \sin \omega t$$

$\therefore$ $$I_{max} = \frac{V_{max}}{R} \text{ A}$$

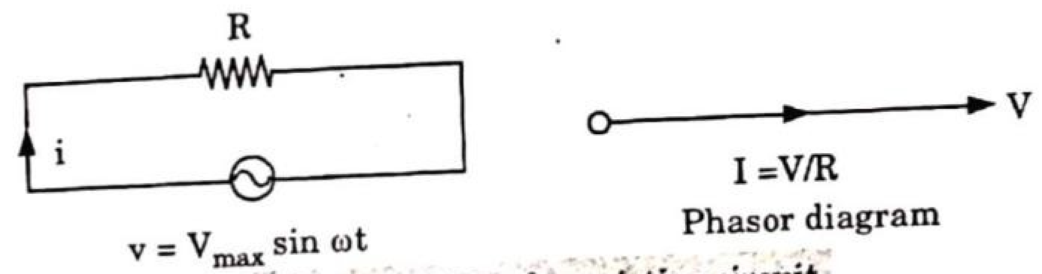


Fig. 2.10.1. Purely resistive circuit.

3. Instantaneous current may be expressed as

$$i = i_{max} \sin \omega t \quad ...(2.10.2)$$

4. From eq. (2.10.1) and eq. (2.10.2), it can be said that the applied voltage and current are in phase with each other.
5. The instantaneous power delivered to the circuit

$$p = vi = V_{max} \sin \omega t\, I_{max} \sin \omega t = V_{max} I_{max} \sin^2 \omega t$$

$$= \frac{V_{max} I_{max}}{2}(1 - \cos 2\omega t)$$

$$= \frac{V_{max} I_{max}}{2} - \frac{V_{max} I_{max}}{2} \cos 2\omega t \quad ...(2.10.3)$$

From eq. (2.10.3), it is clear that power consumed in a purely resistive circuit is not constant, it is fluctuating.

6. Average power, $$P = \text{Avg of } \frac{V_{max} I_{max}}{2} - \text{Avg of } \frac{V_{max} I_{max}}{2} \cos 2\omega t$$

$$P = \frac{V_{max} I_{max}}{2} = \frac{V_{max}}{\sqrt{2}}\frac{I_{max}}{\sqrt{2}} = VI \text{ watts}$$

where V and I are the rms values of applied voltage and current respectively.

Que 2.11. **In a purely inductive circuit, prove that the current lags behind applied voltage by quarter of a cycle and also show that average power demand is zero.**

Answer

1. Consider an AC circuit containing inductance of L H connected across a sinusoidal voltage
$$v = V_{max} \sin \omega t \quad ...(2.11.1)$$
2. Self induced emf in the coil,
$$e_L = -L\frac{di}{dt}$$
$$\therefore \quad V_{max} \sin \omega t = -\left(-L\frac{di}{dt}\right)$$
$$di = \frac{V_{max}}{L} \sin \omega t \, dt$$
$$\int di = \frac{V_{max}}{L} \int \sin \omega t \, dt$$

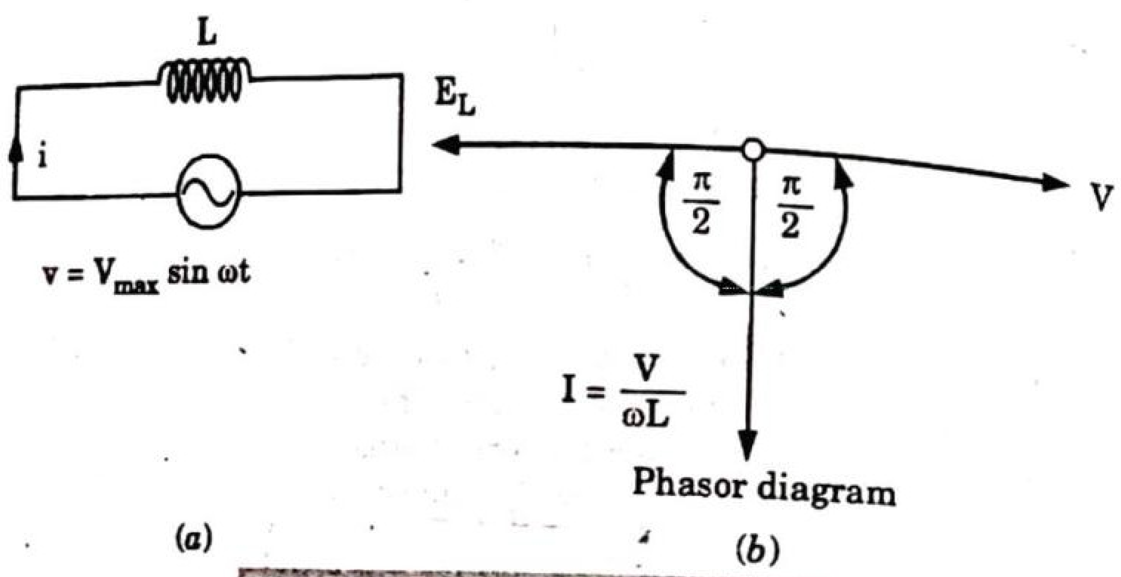


Fig. 2.11.1. Purely inductive circuit.

3. Integrating both sides, we get
$$i = \frac{V_{max}}{\omega L}(-\cos \omega t) + C$$
where, C is a constant which is found to be zero from initial condition.
$$I_{max} = \frac{V_{max}}{\omega L}$$
$$\therefore \quad i = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \quad ...(2.11.2)$$
4. From eq. (2.11.1) and (2.11.2) it is observed that current lags behind the applied voltage by $\pi/2$, *i.e.*, quarter of a cycle.
5. Instantaneous power, $p = v \times i = V_{max} \sin \omega t \, I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$
$$p = -V_{max} I_{max} \sin \omega t \cos \omega t$$
$$= -\frac{V_{max} I_{max}}{2} \sin 2\omega t \quad ...(2.11.3)$$

6. The power measured by wattmeter is the average value of p which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Hence in a purely inductive circuit power absorbed is zero.

Que 2.12. **Explain the concept of phasors. Derive the phasor relationship between voltage and current phasors for purely inductive, purely capacitive and purely resistive circuits.**

AKTU 2017-18(Sem-1), Marks 07

Answer

A. **Concept of phasors :** Refer Q. 2.6, Page 2–6D, Unit-2.
B. **Purely resistive circuits :** Refer Q. 2.10, Page 2–9D, Unit-2.
C. **Purely inductive circuits :** Refer Q. 2.11, Page 2–10D, Unit-2.
D. **Purely capacitive circuits :**

1. Let an alternating voltage represented by $v = V_{max} \sin \omega t$ be applied across a capacitor of capacitance C farads.
Instantaneous applied voltage,
$$v = V_{max} \sin \omega t \quad ...(2.12.1)$$
2. The expression for instantaneous charge is given as
$$q = C V_{max} \sin \omega t$$
$$i = \frac{dq}{dt} = \frac{d}{dt}[C V_{max} \sin \omega t]$$
$$= \omega C V_{max} \cos \omega t = \frac{V_{max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$
3. Current is maximum when $t = 0$
$$\therefore \quad I_{max} = \frac{V_{max}}{1/\omega C}$$
4. Instantaneous current, $i = I_{max} \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(2.12.2)$

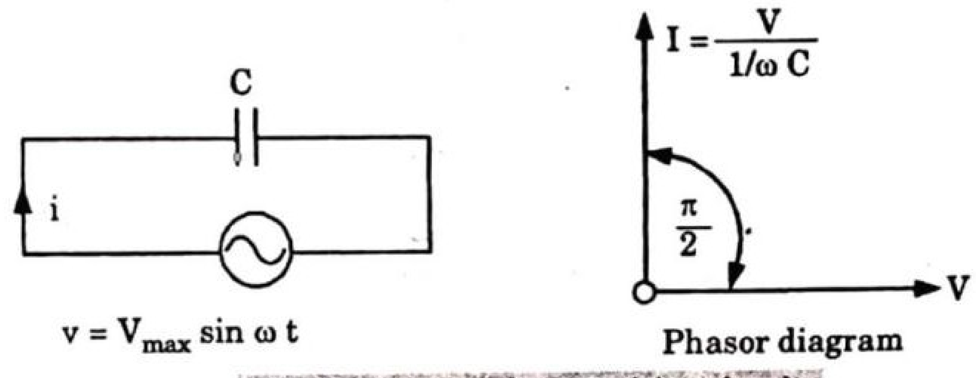


Fig. 2.12.1. Purely capacitive circuit.

3. From eq. (2.12.1) and (2.12.2), it is observed that current leads the applied voltage by $\pi/2$.

4. Instantaneous power, $p = v \times i = V_{max} \sin \omega t \, I_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$

$$p = V_{max} I_{max} \sin \omega t \cos \omega t$$

$$= \frac{V_{max} I_{max}}{2} \sin 2\omega t$$

5. Average power, $P = \frac{V_{max} I_{max}}{2} \times$ Average of $\sin 2\omega t$ over complete cycle $= 0$

Que 2.13. Deduce an expression for the average power in a single phase series RL circuit.

Answer

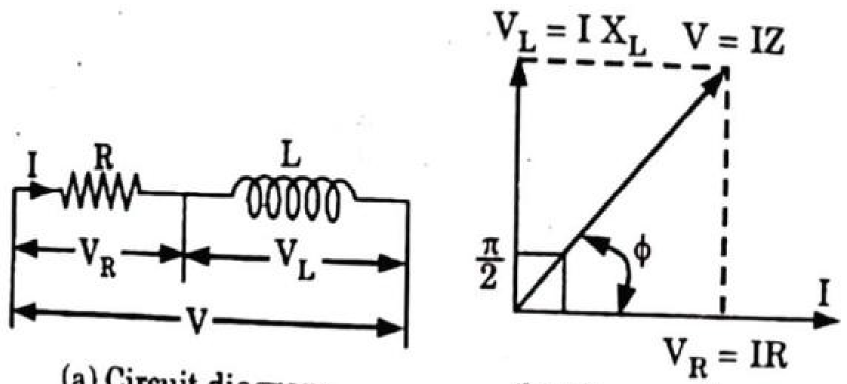
1. Consider an AC circuit consisting of resistance of R ohms and inductance of L henrys connected in series, as shown in Fig. 2.13.1(*a*).
2. Voltage drop across resistance, $V_R = IR$ in phase with the current. Voltage drop across inductance, $V_L = IX_L = I\omega L$ leading I by $\pi/2$ radians.
3. The applied voltage is equal to the phasor sum of V_R and V_L, given by

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (I\,X_L)^2}$$

$$= \sqrt{R^2 + X_L^2} = IZ$$

where, $X_L = \omega L = 2\,\pi f l$

Quantity $\sqrt{R^2 + X_L^2}$ is known as impedance, denoted by Z and is expressed in ohms.



(a) Circuit diagram (b) Phasor diagram

Fig. 2.13.1.

4. From phasor diagram shown in Fig. 2.13.1(*b*) the current lags behind the applied voltage V by angle ϕ, which is given by

$$\tan \phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\phi = \tan^{-1} \frac{X_L}{R}$$

5. If the applied voltage $v = V_{max} \sin \omega t$, then expression for the circuit current will be

$$i = I_{max} \sin(\omega t - \phi)$$

where, $I_{max} = \frac{V_{max}}{Z}$ and $\phi = \tan^{-1} \frac{X_L}{R}$

6. Instantaneous power, $p = v\,i = V_{max} \sin \omega t \times I_{max} \sin(\omega t - \phi)$

$$= \frac{1}{2} V_{max} I_{max} \cos \phi - \frac{1}{2} V_{max} I_{max} \cos(2\,\omega t - \phi)$$

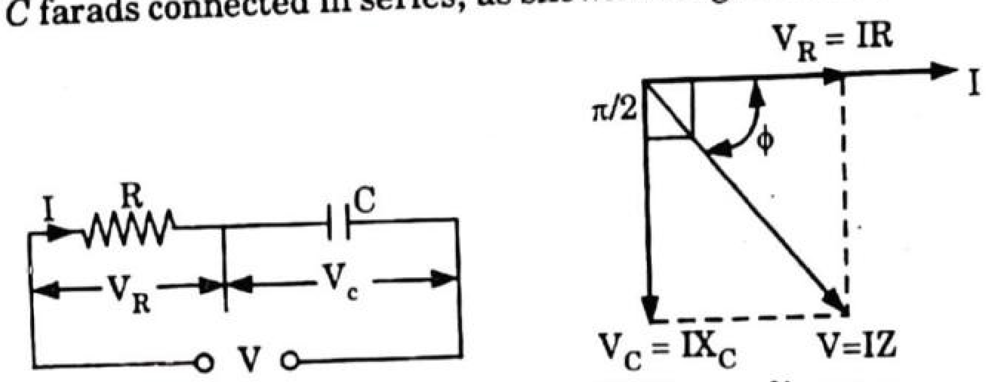
7. $$P_{avg} = \frac{V_{max}}{\sqrt{2}} \frac{I_{max}}{\sqrt{2}} \cos \phi = V\,I \cos \phi$$

where V and I are the rms values of voltage and current respectively and ϕ is the phase angle between applied voltage V and circuit current I.

Que 2.14. How do you analyse series RC circuit ? Draw its phasor diagram.

Answer

1. Consider an AC circuit consisting of resistance of R ohms and capacitance of C farads connected in series, as shown in Fig. 2.14.1(*a*).



(a) Circuit diagram (b) Phasor diagram

Fig. 2.14.1.

2. Voltage drop across resistance,

$$V_R = IR \text{ in phase with current.}$$

Voltage drop across capacitance, $V_C = IX_C$ lagging behind I by $\pi/2$ radians or 90°.

3. The applied voltage is equal to phasor sum of V_R of V_C

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2} = IZ \qquad \text{where } Z^2 = R^2 + X_C^2$$

4. The applied voltage lags behind the current by an angle ϕ

where, $\tan \phi = \frac{V_C}{V_R} = \frac{I\,X_C}{I_R} = \frac{X_C}{R} = \frac{1}{\omega RC}$

$$\phi = \tan^{-1} \frac{1}{\omega RC}$$

Power factor, $\cos \phi = \frac{R}{Z}$

5. If instantaneous voltage is represented by

$$v = V_{max} \sin \omega t$$

then instantaneous current will be expressed as

$$i = I_{max} \sin (\omega t + \phi)$$

6. Power consumed by the circuit is given by

$$P = VI \cos \phi$$

Que 2.15. **Derive expression for impedance, current, and power factor for an RLC series circuit when applied with AC voltage. Also draw vector diagram.** **AKTU 2017-18(Sem-2), Marks 07**

Answer

1. Consider an AC circuit containing resistance of R ohms, inductance of L H and capacitance of C F connected in series, as shown in Fig. 2.15.1.
2. Let the current flowing through the circuit be of I amperes and supply frequency be ϕ Hz.
3. Voltage drop across resistance, $V_R = IR$ in phase with I.

   Voltage drop across inductance, $V_L = I\omega L$ leading I by $\pi/2$ radians or 90°.

   Voltage drop across capacitance, $V_C = \frac{I}{\omega C}$ or IX_C lagging behind I by $\pi/2$ radians or 90°.
4. The circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant.

Case I : When V_L is greater than V_C.

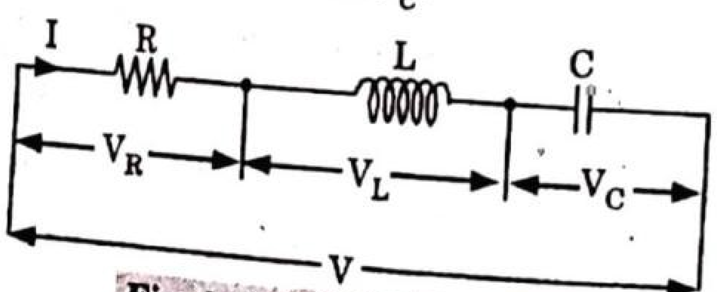


Fig. 2.15.1. Circuit Diagram.

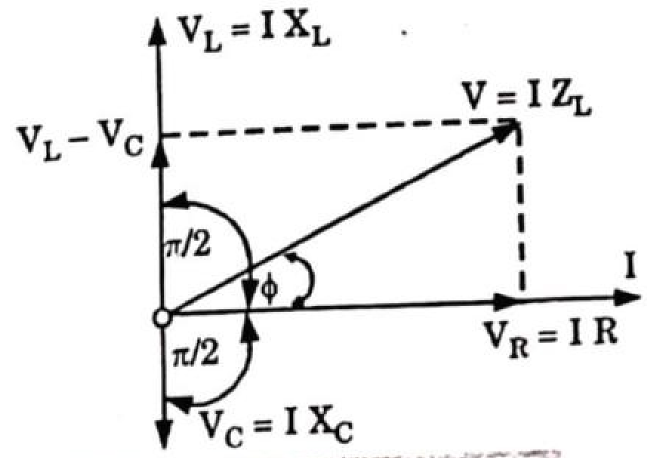


Fig. 2.15.2. Phasor Diagram.

1. $V = \sqrt{(V_R)^2 + (V_L - V_C)} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2}$

   The term $\sqrt{R^2 + (X_L - X_C)^2}$ is known as impedance of the circuit and is represented by Z. Its unit is ohm.
2. Phase angle ϕ between voltage and current is given by

   $$\phi = \tan^{-1} \frac{V_L - V_C}{V_R} = \tan^{-1} \frac{IX_L - IX_C}{IR} = \tan^{-1} \frac{X_L - X_C}{R} = \tan^{-1} \frac{X}{R}$$

   ϕ will be positive, *i.e.*, applied voltage will lead the current if $X_L > X_C$ and ϕ will be negative, *i.e.*, applied voltage will be behind the current if $X_L < X_C$.
3. Power factor, $\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$
4. Power consumed, $P = VI \cos \phi$.
5. Magnitude of effective current,

   $$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Case II : When $\omega L > \frac{1}{\omega C}$, the RLC circuit behaves like inductive circuit and current phasor lags the voltage phasor.

Case III : When $\omega L < \frac{1}{\omega C}$, the circuit behaves as capacitive circuit and current phasor leads the voltage phasor.

I.M.P

Que 2.16. **A metal filament lamp, rated at 750 W, 100 V, is to be connected in series with a capacitor across 230 V, 50 Hz supply. Calculate the value of capacitor.**

AKTU 2013-14(Sem-1), Marks 05

Answer

1. Let pure capacitor C farads be connected in series with the lamp, as shown in Fig. 2.16.1.

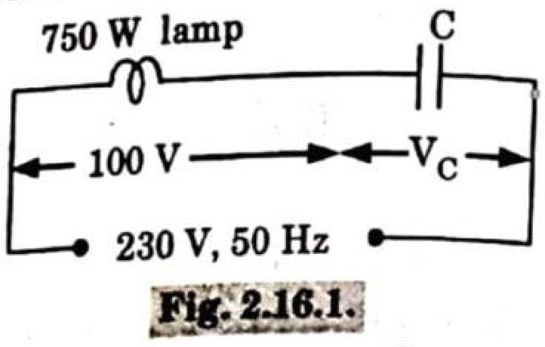


Fig. 2.16.1.

2. Since in this case voltage drop across the capacitor lags behind the current by 90° while that across the lamp will be in phase with it.

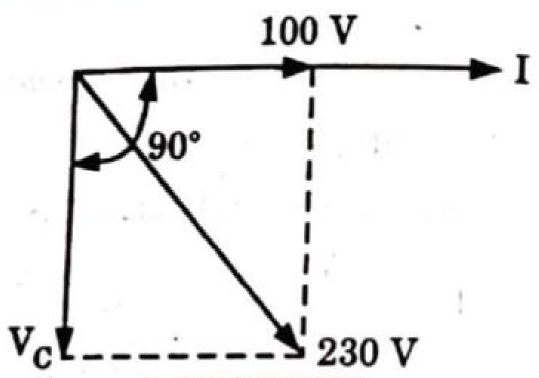


Fig. 2.16.2.

3. $$V_C = \sqrt{(230)^2 - (100)^2} = 207 \text{ V}$$

4. $$I = \frac{P}{V} = \frac{750}{100} = 7.5 \text{ A}$$

5. $$X_C = \frac{V_C}{I} = \frac{207}{7.5} = 27.6\ \Omega$$

6. $$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 27.6} = 115.33\ \mu\text{F}$$

Que 2.17. **A resistance and inductance are connected in series with voltage $v = 283 \sin 314\, t$. The current expression is found to be $i = 4 \sin (314\, t - 45°)$. Find the value of resistance, inductance and power factor.**

AKTU 2013-14(Sem-2), Marks 10

Answer

$$v = 283 \sin 314\, t \quad ...(2.17.1)$$

$$i = 4 \sin (314\, t - 45°) \quad ...(2.17.2)$$

1. From eq. (2.17.1) and (2.17.2) it is seen that current I lags behind the applied voltage v by $\pi/4$ radian or 45°, so phase angle = 45° (lagging).
2. Power factor of the circuit, $\cos \phi = \cos 45° = 0.707$ (lagging)

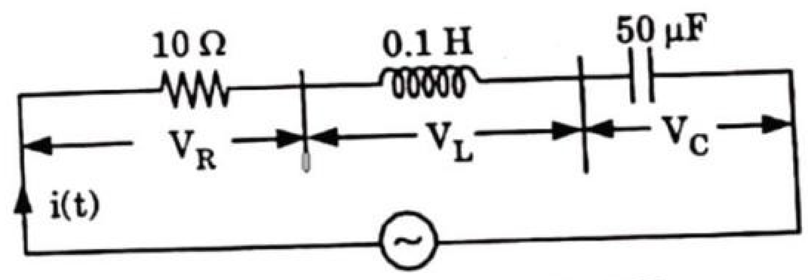
3. Circuit impedance, $Z = \dfrac{V_{max}}{I_{max}} = \dfrac{283}{4} = 70.75\ \Omega$
4. Circuit resistance, $Z = Z \cos \phi = 70.75 \times 0.707 = 50\ \Omega$
5. Circuit reactance, $X_L = Z \sin \phi = 70.75 \times \sin 45° = 50\ \Omega$
6. Frequency, $f = \dfrac{314}{2\pi} = 50$ Hz
7. Inductance, $L = \dfrac{X_L}{2\pi f} = \dfrac{50}{2\pi \times 50} = 0.159$ H

Que 2.18. **A series RLC circuit is composed of 10 Ω resistance, 0.1 H inductance and 50 μF capacitance. A voltage $v(t) = 141.4 \cos (100\, \pi t)$ V is impressed upon the circuit. Determine**

i. The expression for instantaneous current.
ii. The voltage drops V_R, V_L and V_C across resistor, capacitor and inductor.
iii. Draw the phasor diagram using all the voltage relations.

AKTU 2014-15(Sem-1), Marks 10

Answer



$v(t) = 141.4 \cos (100\pi t)$ V

Fig. 2.18.1.

1. $$X_L = \omega L = 100\pi \times 0.1 = 31.416\ \Omega$$

2. $$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 50 \times 10^{-6}} = 63.662\ \Omega$$

3. $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{10^2 + (31.416 - 63.662)^2} = 33.76\ \Omega$$

4. Instantaneous current, $i(t) = \dfrac{v(t)}{Z}$
$$= \frac{141.4}{33.76} \cos (100\pi t + \phi) = 4.188 \cos (100\pi t + \phi)$$

where $$\phi = \tan^{-1} \frac{X_L - X_C}{R}$$

$$= \tan^{-1}\left(\frac{31.416 - 63.662}{10}\right) = -72.77°$$

Thus

$$i(t) = 4.188 \cos(100\pi t - 72.77°) \text{ A}$$

5. $V_R = IR = 4.188 \times 10 = 41.88$ V
   $V_L = IX_L = 4.188 \times 31.416 = 131.57$ V
   $V_C = IX_C = 4.188 \times 63.662 = 266.62$ V
6. **Phasor diagram :**

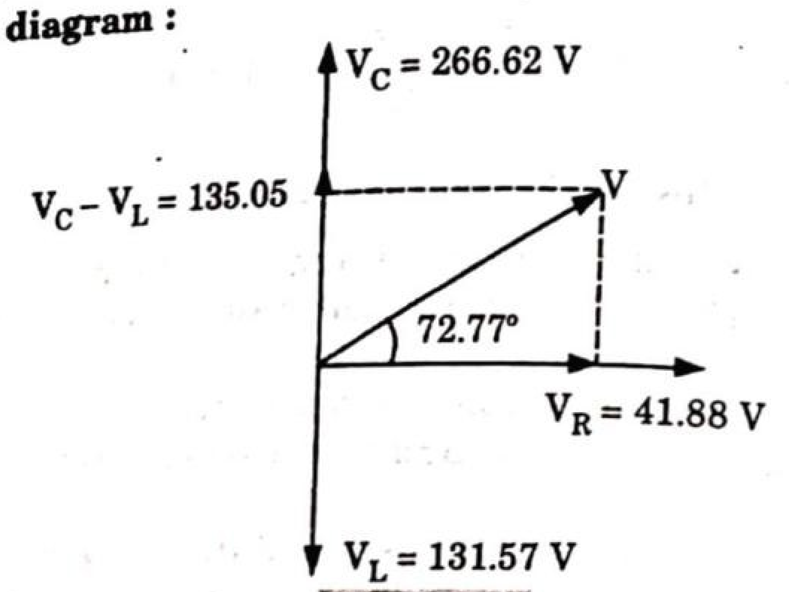


Fig. 2.18.2.

Que 2.19. **A non-inductive resistance of 10 ohm is connected in series with an inductive coil across 200 V, 50 Hz AC supply. The current drawn by the series combination is 10 amp. The resistance of coil is 2 ohms. Determine :**

i. Inductance of the coil
ii. Power factor
iii. Voltage across the coil.

AKTU 2017-18(Sem-1), Marks 07

Answer

1. Total resistance of the circuit,
   R = Non-inductive resistance + Resistance of coil
   $= 10 + 2 = 12\ \Omega$
2. Voltage drop across the resistance of whole circuit,
   $V_R = IR = 10 \times 12 = 120$ V

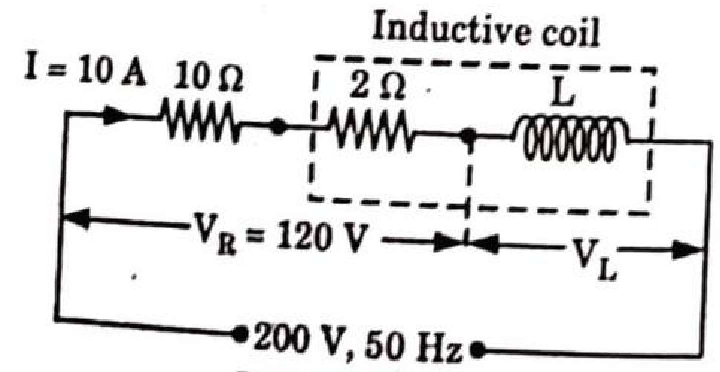


Fig. 2.19.1.

3. Let the voltage drop across the inductance of the coil be V_L volts.

Supply voltage, $V_S = \sqrt{V_R^2 + V_L^2}$

$$V_L = \sqrt{V_S^2 - V_R^2} = \sqrt{200^2 - 120^2} = 160 \text{ V}$$

4. Inductive reactance of the coil, $X_L = \dfrac{V_L}{I} = \dfrac{160}{10} = 16\ \Omega$
5. Inductance of coil, $L = \dfrac{X_L}{2\pi f} = \dfrac{16}{2\pi \times 50} = 0.051 \text{ H} = 51 \text{ mH}$
6. Power factor of the coil $= \cos\left(\tan^{-1}\dfrac{16}{2}\right) = 0.124$ (lagging)
   Power factor of the circuit, $\cos\phi = \dfrac{R}{Z} = \dfrac{12}{\sqrt{12^2 + 16^2}} = 0.6$ (lagging)
7. Voltage across the coil, $V_C = I\sqrt{2^2 + 16^2} = 10 \times \sqrt{260} = 161.245$ V

Que 2.20. **A 46 mH inductive coil has a resistance of 10 ohm. How much current will it draw, if connected across 100 V, 50 Hz source ? Also determine the value of capacitance that must be connected across the coil to make the power factor of the circuit to be unity.**

AKTU 2016-17(Sem-2), Marks 07

Answer

1. R and L are connected in series

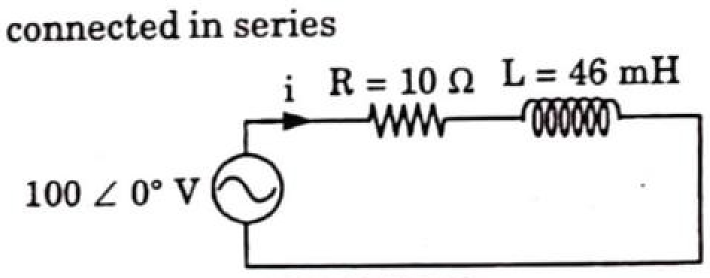


Fig. 2.20.1.

2. $X_L = 2\pi f L = 2 \times \pi \times 50 \times 46 \times 10^{-3}$
   $= 14.44\ \Omega$
3. $Z = R + jX_L$
   $= 10 + j14.44 = 17.56 \angle 55.30°\ \Omega$
4. $i = \dfrac{V}{Z} = \dfrac{100 \angle 0°}{17.56 \angle 55.30°} = 5.7 \angle -55.30°$ A
5. Let C is connected in series with R and L.

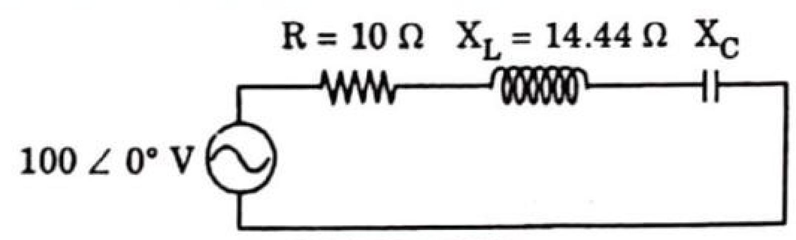


Fig. 2.20.2.

$$Z = R + j(X_L - X_C)$$

$$\therefore \quad |Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

3. Power factor = 1 $\therefore$ $\cos\phi = 1$

$$\frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = 1$$

$$R^2 = R^2 + (X_L - X_C)^2$$

$$X_C = X_L$$

$$\frac{1}{2\pi f C} = 14.44 \quad \therefore C = 220\ \mu\text{F}$$

Que 2.21. **A series AC circuit has a resistance of 15 Ω and inductive reactance of 10 Ω. Calculate the value of a capacitor which is connected across this series combination so that system has unit power factor. The frequency of AC supply is 50 Hz.**

AKTU 2016-17(Sem-1), Marks 07

Answer

The procedure is same as Q. 2.20, Page 2–20D, Unit-2.

(**Ans.** $C = 318\ \mu\text{F}$)

Que 2.22. **Discuss the parallel RLC circuit with its phasor diagram.**

Answer

1. In the parallel RLC circuit, the supply voltage, V_s is common to all three components, the supply current I_s consists of three parts.
2. Current flowing through the resistor = I_R
   Current flowing through the inductor = I_L
   Current through the capacitor = I_C

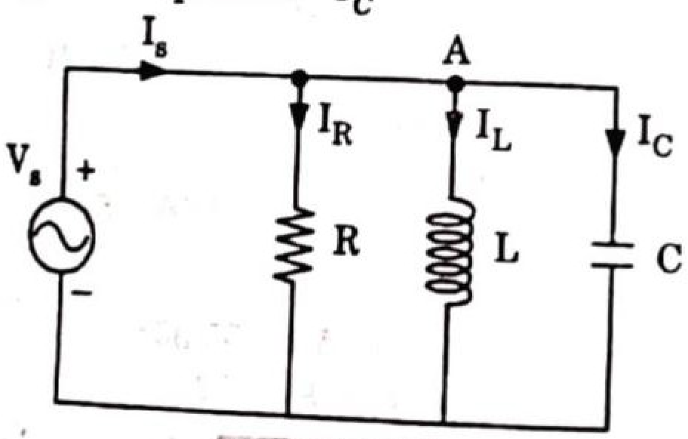


Fig. 2.22.1.

3. Impedance of a parallel RLC circuit

$$R = \frac{V}{I_R};\ X_L = \frac{V}{I_L};\ X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

$$\therefore \quad \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

4. Current for a parallel RLC circuit

$$I_s^2 = I_R^2 + (I_L - I_C)^2$$

$$I_s = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\therefore \quad I_s = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

Phasor diagram :

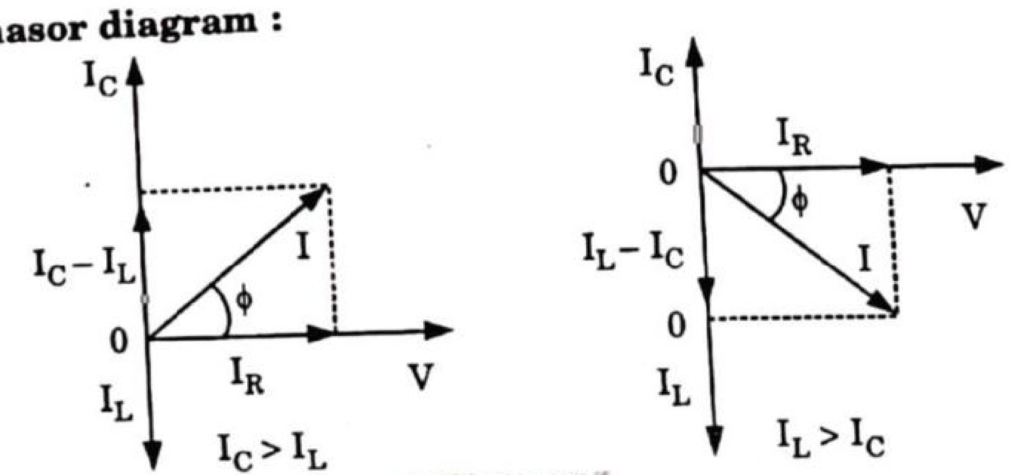


Fig. 2.22.2.

Que 2.23. **In the given parallel RLC circuit, determine $i_R(t)$, $i_L(t)$, $i_C(t)$ and $i_{CL}(t)$. Determine the phasor diagram showing all currents and voltage.**

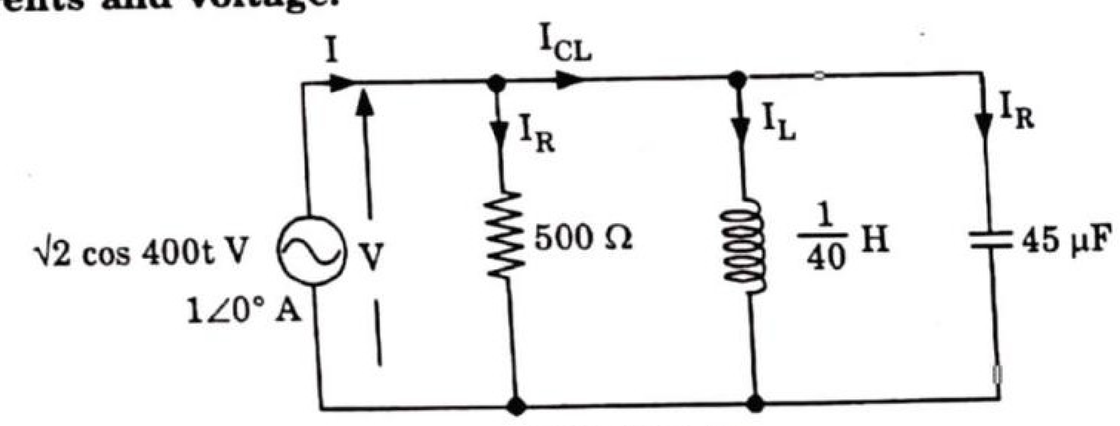


Fig. 2.23.1.

AKTU 2014-15(Sem-1), Marks 10

Answer

1. Frequency, $\omega_o = 400$ rad/sec

2. $$\omega L = 400 \times \frac{1}{40} = 10\ \Omega$$

3. $$\frac{1}{\omega C} = \frac{10^6}{400 \times 250} = 10\ \Omega$$

4. As $\omega L = \dfrac{1}{\omega C}$, ω is the resonant frequency

$\omega = \omega_o = 400$ rad/sec

5. Therefore $I_{CL} = 0$ as $I_L + I_C = 0$ (at resonance)

6. Thus $I_R = I = 1\angle 0°$ A

7. $i_R(t) = \sqrt{2}\cos 400t$ A

8. $v = 500\, I_R = 500\angle 0°$.

9. $v(t) = 500\sqrt{2}\cos 400t$ V

10. Then $I_L = \dfrac{V}{j\omega L} = \dfrac{-j500}{10} = 50\angle -90°$ A

11. $i_L(t) = 50\sqrt{2}\cos(400\,t - 90°)$

12. $I_C = j\omega C,\ V = 50\angle 90°$ A

13. $i_C(t) = 50\sqrt{2}\cos(400t + 90°)$

14. $I_{CL} = I_L + I_C = -j50 + j50 = 0$ A

Therefore circulating current = 50 A

15. Phasor diagram is shown in Fig. 2.23.2.

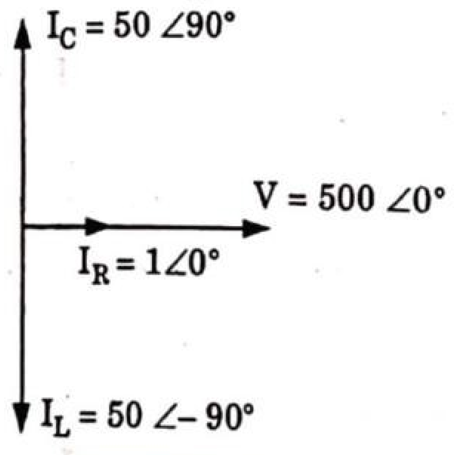


Fig. 2.23.2.

PART-3

Apparent, Active and Reactive Power, Power Factor, Power Factor Improvement.

CONCEPT OUTLINE : PART-3

- **Power factor :** It may be defined as
1. Cosine of the phase angle between voltage and current or,
2. The ratio of the resistance to the impedance or,
3. The ratio of true power to the apparent power.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

VAP

Que 2.24. **Define the following :**
1. **Apparent power**
2. **True power**
3. **Reactive power**
4. **Power factor.**

Answer

1. **Apparent power :** The product of rms values of current and voltage, *VI* is called the apparent power and is measured in volt-amperes or kilo-volt amperes (kVA).
2. **True power :** The true power in an AC circuit is obtained by multiplying the apparent power by the power factor and is expressed in watts or kilo-watts (kW).
3. **Reactive power :** The product of apparent power, *VI* and the sine of the angle between voltage and current, sin ϕ is called the reactive power. This is also known as wattless power and is expressed in reactive volt-amperes or kilo-volt amperes reactive (kVAR).

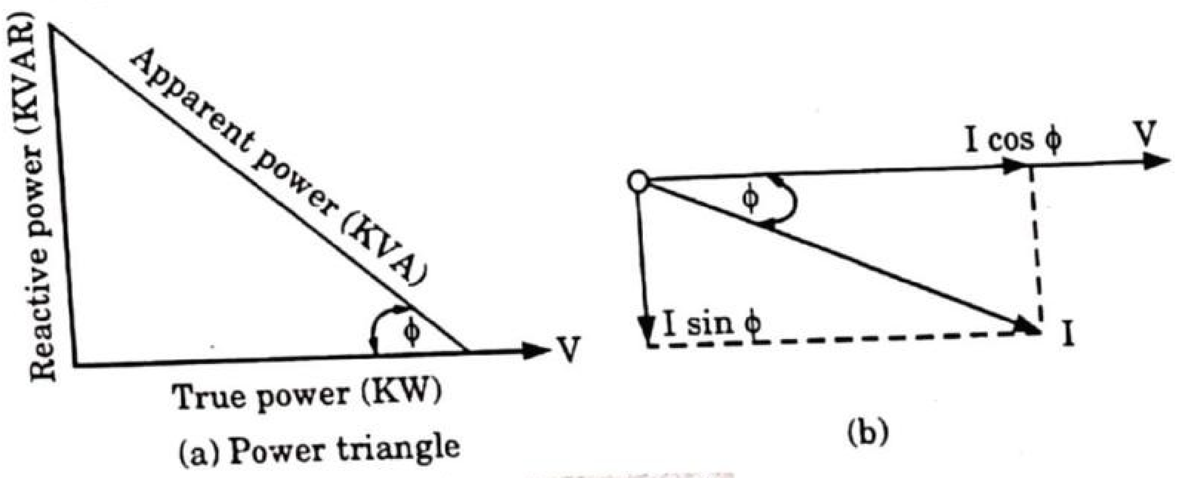


Fig. 2.24.1.

4. **Power factor** may be defined as
   i. Cosine of the phase angle between voltage and current or
   ii. The ratio of the resistance to impedance or
   iii. The ratio of true power to apparent power.

Imp **Que 2.25.** **Two impedances $Z_1 = 5 + j10\ \Omega$ and $Z_2 = 10 - j15\ \Omega$ are connected in parallel. If total current is 20 A, then find :**
1. **Current taken by each branch**
2. **Power factor**
3. **Power consumed in each branch.**

AKTU 2013-14(Sem-1), Marks 05

Answer

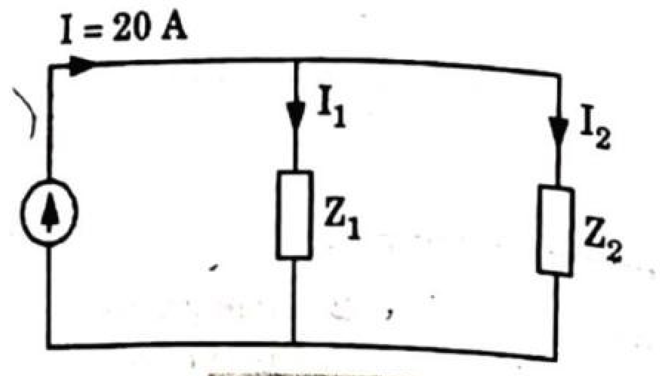


Fig. 2.25.1.

1. Equivalent impedance of parallel combination

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(5 + j10)(10 - j15)}{(5 + j10) + (10 - j15)}$$

$$Z = \frac{(5 + j10)(10 - j15)}{(15 - 5j)}$$

$$I = 20 + j0$$

2. **Current taken by each branch :**

$$I_1 = \frac{IZ}{Z_1} = (20 + j0)\frac{(5 + j10)(10 - j15)}{(15 - j5)(5 + j10)}$$

$$= \frac{20(10 - j15)}{(15 - j5)} = \frac{360.4 \angle -56.31°}{15.81 \angle -18.43°}$$

$$= 22.79 \angle -37.88° \text{ A}$$

$$I_2 = \frac{IZ}{Z_2} = (20 + j0)\left[\frac{(5 + j10)(10 - j15)}{(15 - j5)(10 - j15)}\right]$$

$$= 20\frac{(5 + j10)}{15 - j5} = \frac{223.6 \angle 63.43}{15.81 \angle -18.43}$$

$$= 14.14 \angle 81.86° \text{ A}$$

3. **Power factor :**

Circuit current, $I = 20 + j0$

Phase angle, $\phi = 0$

Power factor, $\cos \phi = \cos (0) = 1.$

4. **Power consumed in each branch :**

Power consumed in branch 1, $P_1 = I_1^2 R_1 = (22.79)^2 \times 5 = 2596.9$ W

Power consumed in branch 2, $P_2 = I_2^2 R_2 = (14.14)^2 \times 10 = 1999.3$ W

Imp

Que 2.26. **A coil having a resistance of 30 Ω and inductance of 0.05 H is connected in series with a capacitor of 100 μF. The whole circuit has been connected to a single phase 230 V, 50 Hz supply. Calculate impedance, current, power factor, power and apparent power of the circuit.**

AKTU 2014-15(Sem-2), Marks 10

Answer

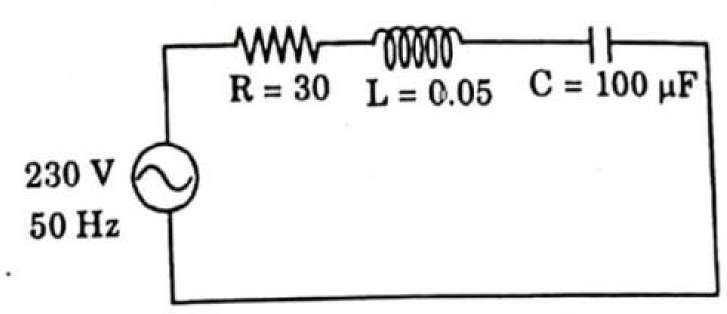


Fig. 2.26.1.

1. Inductive reactance of the circuit,

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$$

2. Capacitive reactance of the circuit,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$$

3. Magnitude of impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(30)^2 + (15.7 - 31.83)^2} = 34.061\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{15.7 - 31.83}{30}\right)$$

$$= -28.26°$$

Hence impedance, $Z = 34.061 \angle -28.26°\ \Omega$.

4. Current, $I = \dfrac{V}{Z} = \dfrac{230 \angle 0°}{34.061 \angle -28.26°} = 6.75 \angle 28.26°$ A

5. Power factor, $\cos \phi = \cos (-28.26°) = 0.8808$ (leading)

6. Power, $VI \cos \phi = 230 \times 6.75 \times 0.88 = 1367.45$ W

7. Apparent power, $VI = \dfrac{230}{\sqrt{2}} \times \dfrac{6.75}{\sqrt{2}}$

$$= 776.25 \text{ VA.}$$

V.V.Imp

Que 2.27. **What are the causes of low power factor in supply system? Discuss its effect and how power factor is improved ?**

AKTU 2015-16(Sem-1), Marks 10

OR

Define power factor. Discuss reasons for poor power factor. How can power factor be improved ? **AKTU 2016-17(Sem-2), Marks 07**

OR

Explain the causes of low power factor. How can it be improved?

AKTU 2017-18(Sem-1), Marks

OR

What are causes and disadvantages of low power factor ?

AKTU 2013-14(Sem-1), Marks

Answer

A. **Power factor :** Refer Q. 2.24, Page 2–24D, Unit-2.

B. **Causes of low power factor :**

1. All AC motor and transformers operate at low power factor.
2. Arc lamps operate at low power factor (lagging) due to typical characteristic of arc.
3. Industrial heating furnaces and induction furnace operates at low power factor.
4. With increase in supply voltage usually occurs at lunch hour, night hour etc., the magnetising current of inductive reactance increases and the power factor of the plant as whole becomes lower.

C. **Power factor can be improved by :**

1. Using induction motor with phase advancers.
2. Connecting the static capacitors in parallel with the equipment operating at lagging power factor such as induction motors, fluorescent tubes etc.

D. **Problems (Disadvantages) of low power factor :**

1. Low power factor results in large voltage drop in generator, transmission lines, transformer and distributors which results in poor regulation. Hence, extra equipments are required to make voltage permissible.
2. For the same power to be transmitted at low power factor, the transmission cable has to carry more current. Thus it requires more conductor material for cable to deliver the load at low power factor.
3. Low power factor increases the capital cost for transformers, transmission lines, cables and distributors etc.

PART-4

Concept of Resonance in Series and Parallel Circuits, Bandwidth and Quality Factor.

CONCEPT OUTLINE : PART-4

- Resonance is the term employed for describing the steady state operation of a circuit or system at that frequency for which the resultant response is in time phase with the source function despite the presence of energy-storing elements.
- An RLC series circuit is said to be in electrical resonance when $X_L = X_C$, *i.e.*, the net reactance $X = 0$
- A parallel AC circuit is said to be in resonance when it draws no reactive current.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.28. **Explain resonance in a series RLC circuit with the help of impedance v/s frequency diagram and derive an expression for resonant frequency. Write properties of series resonance circuit.**

AKTU 2015-16(Sem-2), Marks 10

OR

Derive the condition for resonance in series RLC circuit. What are the different applications of resonance ?

AKTU 2013-14(Sem-2), Marks 10

Answer

A.

1. Consider an AC circuit containing a resistance R, inductance L and a capacitance C connected in series, as shown in Fig. 2.28.1.
2. Impedance of the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$
3. If resonant frequency is denoted by f_r, then

$$X_L = \omega L = 2\pi f_r L$$

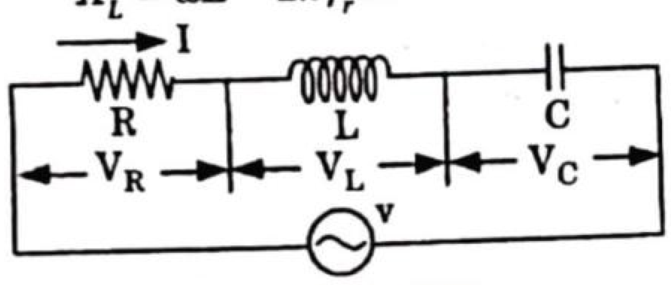


Fig. 2.28.1.

and $X_C = \dfrac{1}{2\pi f_r C}$

4. Since for resonance $X_L = X_C$

$\therefore$ $2\pi f_r L = \dfrac{1}{2\pi f_r C}$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_r = \sqrt{1/LC} \qquad ...(2.2$$

5. From eq. (2.28.1) it is obvious that the value of resonance freque depends on the parameters of the two energy-storing elements.

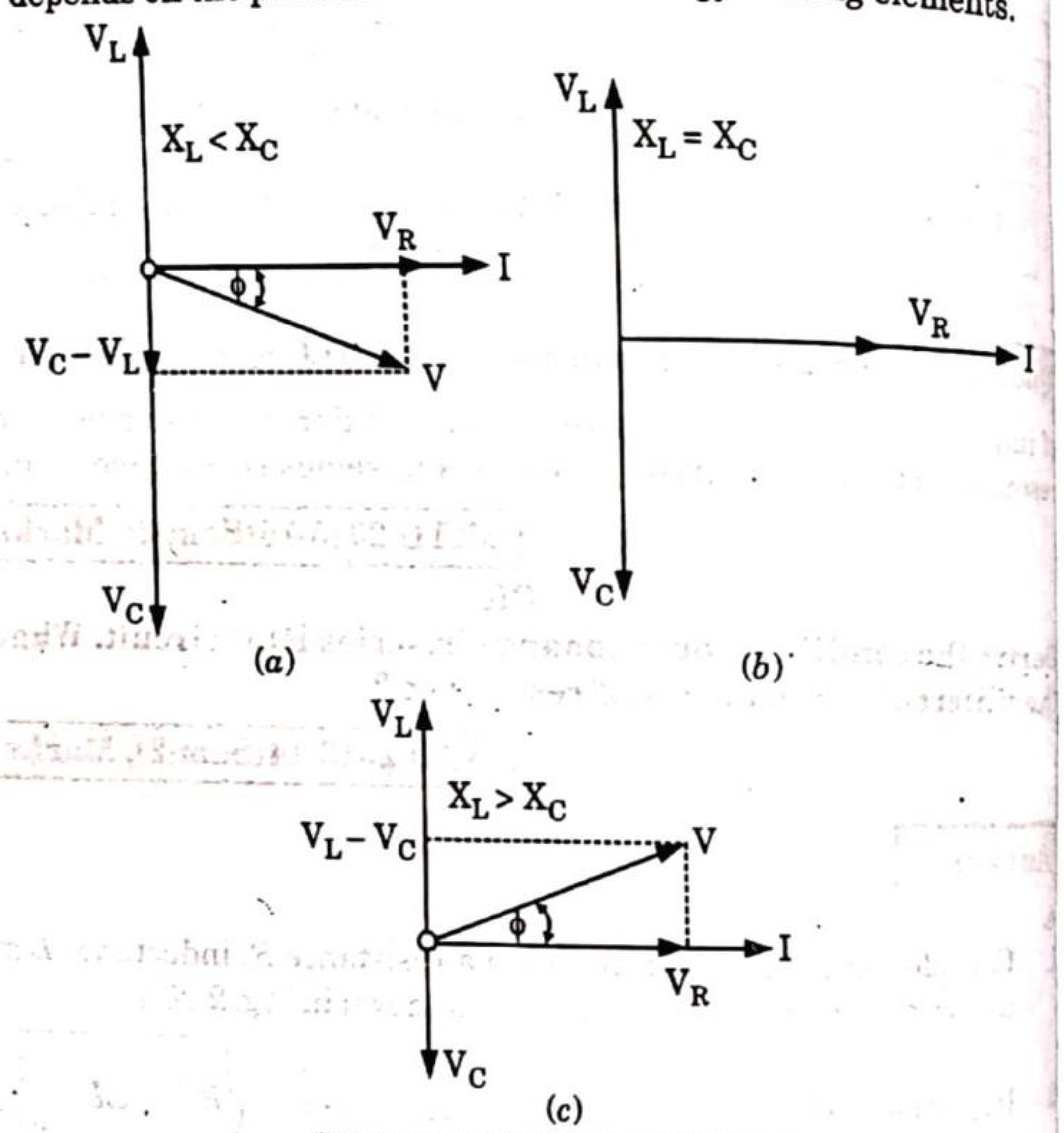


Fig. 2.28.2. Phasor diagram.

B. Property : At resonance,

1. Net reactance is zero, *i.e.*, $X = 0$.
2. Impedance of the circuit, $Z = R$.
3. The current flowing through the circuit is maximum and in phase wi the applied voltage. The magnitude of the current will be equal to V/
4. The voltage drop across the inductance is equal to the voltage dr across capacitance.
5. The power factor is unity.

C. Impedance *v/s* frequency diagram :

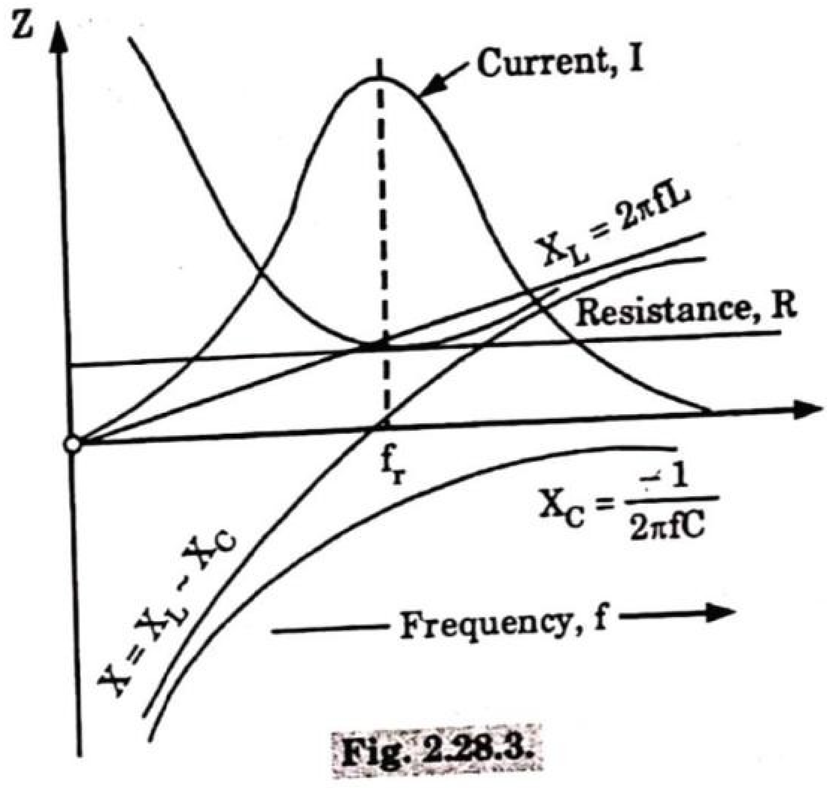


Fig. 2.28.3.

D. Applications of resonance :

1. Resonance circuits are used in tuning applications for radio and TV.
2. These circuits are also used in oscillators.

Que 2.29. **Derive the expression of half power frequencies in terms of resonant frequency f_r.**

OR

Derive bandwidth for series resonance.

AKTU 2013-14(Sem-1), Marks 05

OR

Derive resonance conditions in series RLC circuit. Also derive the expression for bandwidth. **AKTU 2014-15(Sem-2), Marks 10**

Answer

A. Resonance condition : Refer Q. 2.28, Page 2–28D, Unit-2.

B. Derivation of half power frequencies and bandwidth :

1. Consider resonance curve as shown in Fig. 2.29.1.

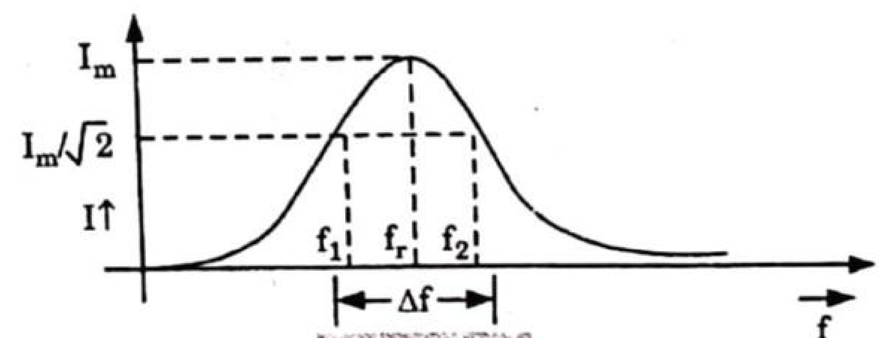


Fig. 2.29.1.

2. Cut off frequency or half power frequency is the frequency where current in the circuit is $1/\sqrt{2}$ times to maximum value of the current.

$\therefore$ At $f_1, X_L < X_C$ whereas at $f_2, X_C < X_L$

3. Impedance at resonance (f_r) frequency

$$Z = R = \frac{V}{I_m}$$

At f_1 impedance, $$Z_1 = \frac{V}{I_1} = \frac{V}{I_m/\sqrt{2}} = \sqrt{2}\,\frac{V}{I_m}$$

$$Z_1 = \sqrt{2}\,R \quad ...(2.29.1)$$

Similarly at f_2, $$Z_2 = \frac{V}{I_2} = \frac{V}{I_m/\sqrt{2}} = \sqrt{2}\,\frac{V}{I_m}$$

$$Z_2 = \sqrt{2}\,R \quad ...(2.29.2)$$

4. At f_1, impedance

$$Z_1 = \sqrt{R^2 + (X_L - X_C)^2}$$

5. Let $X_L - X_C = X$

$$\therefore \quad Z_1 = \sqrt{R^2 + X^2} \quad ...(2.29.3)$$

6. From eq. (2.29.1) and (2.29.3),

$$\therefore \quad \sqrt{2}\,R = \sqrt{R^2 + X^2}$$

$$\therefore \quad R = X$$

7. But since $X_L < X_C$, X is negative.

$$\therefore \quad R = -X$$

$$-R = X_L - X_C$$

$$-R = \omega_1 L - \frac{1}{\omega_1 C}$$

$$\therefore \quad \omega_1^2 LC - 1 = -RC\omega_1$$

$$\omega_1^2 LC + RC\omega_1 - 1 = 0$$

$$\omega_1^2 + \frac{RC}{LC}\,\omega_1 - \frac{1}{LC} = 0$$

$$\omega_1^2 + \left(\frac{R}{L}\right)\omega_1 - \frac{1}{LC} = 0 \quad ...(2.29.4)$$

$$\therefore \quad \omega_1 = \frac{-\dfrac{R}{L} \pm \sqrt{\dfrac{R^2}{L^2} + \dfrac{4}{LC}}}{2}$$

i.e., $$\omega_1 = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

8. Let $$\frac{R}{2L} = \alpha \quad ...(2.29.5)$$

And $$\frac{1}{\sqrt{LC}} = \omega_r \quad ...(2.29.6)$$

$$\therefore \quad \omega_1 = -\alpha \pm \sqrt{\alpha^2 + \omega_r^2} \quad ...(2.29.7)$$

9. Similarly at f_2,

$$R = X$$

$$X_L - X_C = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\omega_2^2 LC - 1 - RC\omega_2 = 0$$

$$\omega_2^2 - \frac{R}{L}\,\omega_2 - \frac{1}{LC} = 0$$

$$\omega_2 = \frac{\dfrac{R}{L} \pm \sqrt{\dfrac{R^2}{L^2} - 4\left(-\dfrac{1}{LC}\right)}}{2}$$

$$\omega_2 = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

From eq. (2.29.5) and (2.29.6)

$$\omega_2 = \alpha \pm \sqrt{\alpha^2 + \omega_r^2} \quad ...(2.29.8)$$

10. $$\Delta\omega = \omega_2 - \omega_1 = 2\alpha = 2\left(\frac{R}{2L}\right) = \frac{R}{L}$$

11. $\therefore$ **Bandwidth,** $$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{R}{2\pi L}$$

$\therefore$ From Fig. 2.29.1 $$f_1 = f_r - \frac{\Delta f}{2} \quad ...(2.29.9)$$

and $$f_2 = f_r + \frac{\Delta f}{2} \quad ...(2.29.10)$$

eq. (2.29.9) and (2.29.10) are the expressions for upper and lower cut off frequencies, respectively.

Que 2.30. **Explain series resonance in RLC circuit. What are the bandwidth and quality factor of the circuit ? Derive expressions for lower and upper half power frequencies for a series RLC circuit.**

AKTU 2016-17(Sem-2), Marks 07

OR

Derive the expression of Bandwidth of a series RLC circuit. Explain the relationship between bandwidth and quality factor.

AKTU 2017-18(Sem-1), Marks 07

Answer

A. Resonance condition in series RLC circuit : Refer Q. 2.28, Page 2–28D, Unit-2.

B. Bandwidth of RLC series circuit : Refer Q. 2.29, Page 2–30D, Unit-2.

C. Lower and upper half power frequencies for series RLC circuit : Refer Q. 2.29, Page 2–30D, Unit-2.

D. Quality factor :

1. The Q-factor of an RLC series circuit is the voltage magnification that the circuit produces at resonance.
2. Since current at resonance is maximum, supply voltage, $V = I_{max} R$
3. Voltage across inductance or capacitance $= I_{max} X_L = I_{max} X_C$
4. Voltage magnification $= \dfrac{I_{max} X_L}{I_{max} R} = \dfrac{X_L}{R}$

$$= \frac{\omega_r L}{R}$$

Q factor at resonance,

$$Q_r = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \frac{2\pi L}{R} \times \frac{1}{2\pi\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\because \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

E. Relation between bandwidth and Quality Factor :

Q factor is also defined as the ratio of resonant frequency to bandwidth, *i.e.*,

$$Q_r = \frac{f_r}{\Delta f} = \frac{\frac{1}{2\pi\sqrt{LC}}}{R/2\pi L} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

where, $\Delta f = \dfrac{R}{2\pi L}$ = Bandwidth.

Que 2.31. **Voltages across R, L, C connected in series are 5, 8 and 10 volt respectively. Calculate the value of supply voltage at 50 Hz. Also find the frequency at which this circuit would resonate.**

Answer

1. The supply voltage is equal to the phasor sum of V_R, V_L and V_C and is given by

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{5^2 + (8-10)^2}$$

$$V = \sqrt{25+4}$$

$$V = \sqrt{29} = 5.385 \text{ V}$$

2. $V_L = I\omega L = 8$ ...(2.31.1)

3. $V_C = \dfrac{I}{\omega C} = 10$ ...(2.31.2)

4. Dividing eq. (2.31.2) by eq. (2.31.1)

$$\frac{1}{\omega C} \times \frac{1}{\omega L} = \frac{10}{8}$$

$$\frac{1}{\omega^2 LC} = \frac{10}{8}$$

$$\frac{1}{LC} = \frac{10}{8} \times (2\pi f)^2$$

Putting f = 50 Hz, we get

$$\frac{1}{LC} = 123370.05$$

$$\frac{1}{\sqrt{LC}} = 351.240$$

5. Since resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$, we get

$$\frac{1}{2\pi\sqrt{LC}} = \frac{351.240}{2\pi} = 55.901 \text{ Hz}$$

Que 2.32. **Derive as expression for parallel resonance and mention its salient features.** **AKTU 2017-18(Sem-2), Marks 07**

Answer

A. Derivation :

1. Consider a coil in parallel with a condenser, as shown in Fig. 2.32.1.
2. Let the coil be of resistance R ohms and inductance L henrys and the condenser of resistance R ohms and capacitance C farads.

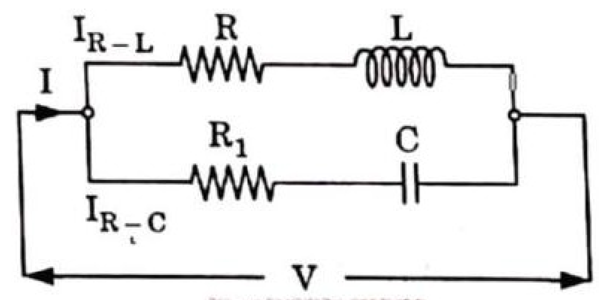


Fig. 2.32.1.

3. Such a circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequency at which this happens is known as resonant frequency.

4. Circuit will be in electrical resonance if reactive component of R-L branch current, $I_{R-L} \sin \phi_{R-L}$ = Reactive component of R-C branch current, $I_{R-C} \sin \phi_{R-C}$.

5. Now since $I_{R-L} = \dfrac{V}{\sqrt{R^2 + (\omega_r L)^2}}$

$$\text{and } \sin \phi_{R-L} = \frac{X_L}{Z_{R-L}} = \frac{\omega_r L}{\sqrt{R^2 + (\omega_r L)^2}}$$

$$I_{R-C} = \frac{V}{Z_{R-C}} = \frac{V}{\sqrt{R_1^2 + \left(\dfrac{1}{\omega_r C}\right)^2}}$$

and
$$\sin \phi_{R-C} = \frac{X_C}{Z_{R-C}} = \frac{1/\omega_r C}{\sqrt{R_1^2 + \left(\dfrac{1}{\omega_r C}\right)^2}}$$

$$\therefore \frac{V}{\sqrt{R^2 + (\omega_r L)^2}} \times \frac{\omega_r L}{\sqrt{R^2 + (\omega_r L)^2}} = \frac{V}{\sqrt{R_1^2 + \left(\dfrac{1}{\omega_r C}\right)^2}} \times \frac{1/\omega_r C}{\sqrt{R_1^2 + \left(\dfrac{1}{\omega_r C}\right)^2}}$$

$$\frac{\omega_r L}{R^2 + (\omega_r L)^2} = \frac{1/\omega_r C}{R_1^2 + (1/\omega_r C)^2}$$

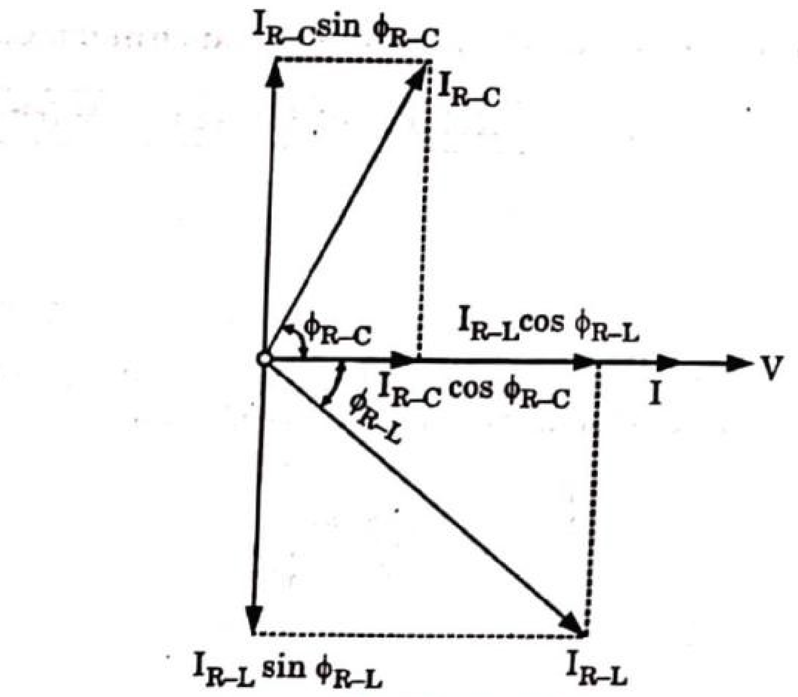


Fig. 2.32.2.

$$\frac{\omega_r L}{R^2 + \omega_r^2 L^2} = \frac{\omega_r C}{\omega_r^2 R_1^2 C^2 + 1}$$

$$L(\omega_r^2 R_1^2 C^2 + 1) = C(R^2 + \omega_r^2 L^2)$$

$$\omega_r^2 L C(R_1^2 C - L) = CR^2 - L$$

$$\omega_r = \frac{1}{\sqrt{LC}} \sqrt{\frac{CR^2 - L}{CR_1^2 - L}}$$

$$\text{Resonant frequency, } f_r = \frac{1}{2\pi} \omega_r$$

$$= \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{CR^2 - L}{CR_1^2 - L}} \quad \text{...(2.32.1)}$$

6. If resistance of capacitor is negligible, *i.e.*, $R_1 = 0$, as is usually the case,

$$\text{Resonant frequency, } f_r = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{CR^2}{L}} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{...(2.32.2)}$$

7. If resistance of coil R is zero

$$\text{Resonant frequency, } f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \quad \text{...(2.32.3)}$$

B. Features of current or parallel resonance :

1. Net susceptance is zero.
2. The admittance is equal to conductance.
3. Reactive or wattless component of line current is zero, hence circuit power factor is unity.
4. Line current is minimum and is equal to $\dfrac{V}{L/CR}$ in magnitude and is in phase with the applied voltage.
5. Frequency is equal to $\dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$ Hz.

Que 2.33. **Derive the quality factor of the parallel RLC circuit at resonance.**

AKTU 2014-15(Sem-1), Marks 10

Answer

1. Consider a current excited RLC parallel network

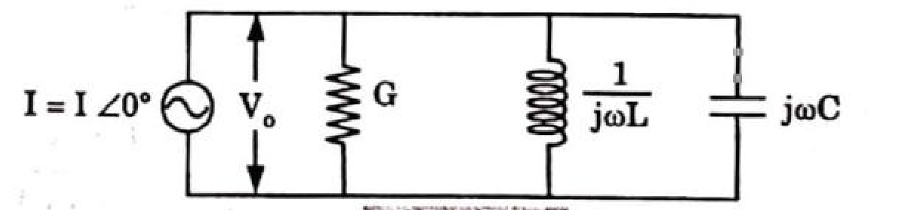


Fig. 2.33.1.

2. Let, $i(t) = I_m \cos \omega_o t$
3. At resonance condition, the currents in inductance and capacitance cancel themselves out and the circuit current I flow in the conductance.

4. The corresponding voltage is (at resonance)

$$v(t) = \frac{i(t)}{G} = \frac{I_m}{G} \cos \omega_o t$$

5. The instantaneous energy stored in the capacitance is

$$w_C(t) = \frac{1}{2} Cv^2 = \frac{I_m^2 C}{2G^2} \cos^2 \omega_o t$$

6. The instantaneous energy stored in the inductor is

$$w_L(t) = \frac{1}{2} Li^2 = \frac{1}{2} L\left(\frac{1}{L}\int_0^L v dt\right)^2$$

$$= \frac{I_m^2 C}{2G^2} \sin^2 \omega_o t$$

7. Total instantaneous energy stored in C and L is

$$w(t) = w_C(t) + w_L(t) = \frac{I_m^2 C}{2G^2} \cos^2 \omega_o t + \frac{I_m^2 C}{2G^2} \sin^2 \omega_o t$$

$$= \frac{I_m^2 C}{2G^2}$$

8. Average power dissipated by the conductance

$$P_G = \frac{I_m^2 C}{2G^2}$$

9. Energy dissipated in one cycle

$$P_{GT} = \frac{1}{f_0} \frac{I_m^2}{2G} = \frac{2\pi}{\omega_o} \frac{I_m^2}{2G}$$

10. Quality factor, $Q_0 = 2\pi \left[\frac{\text{Maximum energy stored per period}}{\text{Total energy lost per period}}\right]$

$$Q_0 = 2\pi \left[\left(\frac{I_m^2 C}{2G^2}\right) \div \left(\frac{2\pi}{\omega_o} \frac{I_m^2}{2G}\right)\right]$$

$$Q_0 = \frac{\omega_o C}{G} = \omega_o RC$$

Que 2.34. **Explain parallel resonance. A circuit of a resistance of 20 Ω, and inductance of 0.3 H and a variable capacitance in series across a 220 V, 50 Hz supply. Calculate :**

A. The value of capacitance to produce resonance.

B. The voltage across the capacitance and inductance.

C. The Q-factor of the circuit. **AKTU 2014-15(Sem-2), Marks 10**

Answer

A. **Parallel Resonance :** Refer Q. 2.32, Page 2–34D, Unit-2.

B. **Numerical :**

1. For resonance, $X_L = X_C$

$$2\pi fL = \frac{1}{2\pi fC}$$

$$(2\pi f)^2 L = \frac{1}{C}$$

$$C = 3.38 \times 10^{-5}\ \text{F}$$

2. $$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{20}\sqrt{\frac{0.3}{3.38\times10^{-5}}} = 4.7105$$

3. $$I = \frac{V}{R}$$

$$I = \frac{220}{20}$$

$$I = 11\ \text{A}$$

4. Voltage across inductor $(V_L) = I\omega L$

$$= 11 \times 2 \times 3.14 \times 50 \times 0.3 = 1036.2\ \text{V}$$

5. Voltage across capacitor $(V_C) = \frac{I}{\omega C}$

$$= \frac{11}{2\times3.14\times50\times3.38\times10^{-5}}$$

$$= 1036.44\ \text{V}$$

Que 2.35. **What do you understand by "series resonance" and "parallel resonance" ? What are the applications of tank circuits ?** **AKTU 2016-17(Sem-1), Marks 07**

Answer

A. **Series resonance :** Refer Q. 2.28, Page 2–28D, Unit-2.

A. **Parallel resonance :** Refer Q. 2.32, Page 2–34D, Unit-2.

C. **Applications of tank circuits :**

1. A tank circuit can be used for stabilize the electrical frequency of an AC oscillator circuit.
2. The frequency set by the tank circuit is solely dependent upon the value of L and C and not on the magnitudes of voltage or current present in the oscillations.

Que 2.36. **Derive the expression of resonant frequency of parallel RLC circuit. In series-parallel circuit A and B are in series with C. The impedances are :**

$Z_A = 4 + j3\ \Omega$, $Z_B = 4 - j5\ \Omega$, $Z_C = 2 + j8\ \Omega$. If the current $I_C = (25 + j0)$, calculate :

i. Branch voltage
ii. Branch current
iii. Total power
iv. Phasor diagram.

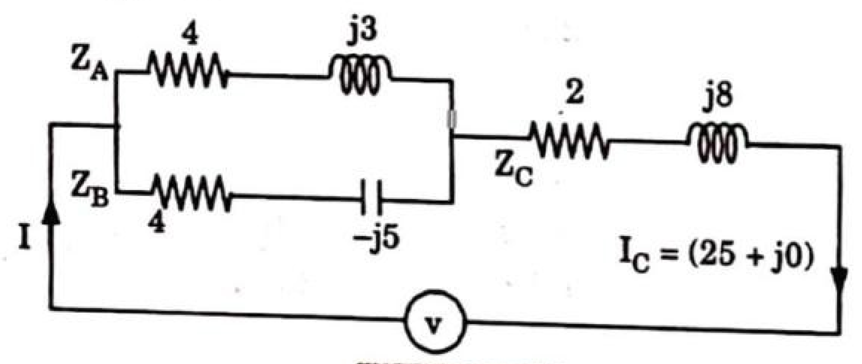


Fig. 2.36.1.

AKTU 2015-16(Sem-1), Marks 15

Answer

A. Parallel RLC circuit : Refer Q. 2.32, Page 2–34D, Unit-2.

B. Numerical :

1. The circuit is shown in Fig. 2.36.1.

$$Z_A = (4 + j3) = 5\angle 36.87°$$
$$Z_B = (4 - j5) = 6.4\angle -51.34°$$
$$Z_C = (2 + j8) = 8.25\angle 76°$$
$$I_C = (25 + j0) = 25\angle 0°$$

2. $$V_C = I_C Z_C = 206\angle 76°$$

3. $$Z_{AB} = \frac{(4 + j3)(4 - j5)}{(8 - j2)} = \frac{31 - j8}{8 - j2} = 3.88 - j0.029$$

4. $$V_{AB} = I_C Z_{AB} = 25\angle 0 \times 3.88\angle -0.43° = 97\angle -0.428°$$

5. $$Z = Z_C + Z_{AB} = (2 + j8) + 3.88 - j0.02$$
$$= 5.88 + j7.98 = 9.91\angle 53.61°$$

6. $$V = I_C Z = 25\angle 0° \times 9.91\angle 53.61°$$
$$= 247.75\angle 53.61°$$

7. $$I_A = \frac{V_{AB}}{Z_A} = \frac{97\angle -0.43°}{5\angle 36.52°} = 19.4\angle -37.3°$$

8. $$I_B = \frac{V_{AB}}{Z_B} = \frac{97\angle -0.43°}{6.4\angle -51.34°} = 15.15\angle 50.91°$$

9. Various voltages and currents are shown in Fig. 2.36.2. Powers would be calculated by using voltage conjugates.

10. Power for whole circuit is

$$P = VI_C = 247.75\angle 53.61° \times 25\angle 0°$$
$$= 6193.75\angle 53.61°$$

$$= 6193.75\,(\cos 53.61° - j \sin 53.61°)$$
$$= 3674.62 - j4985.95$$

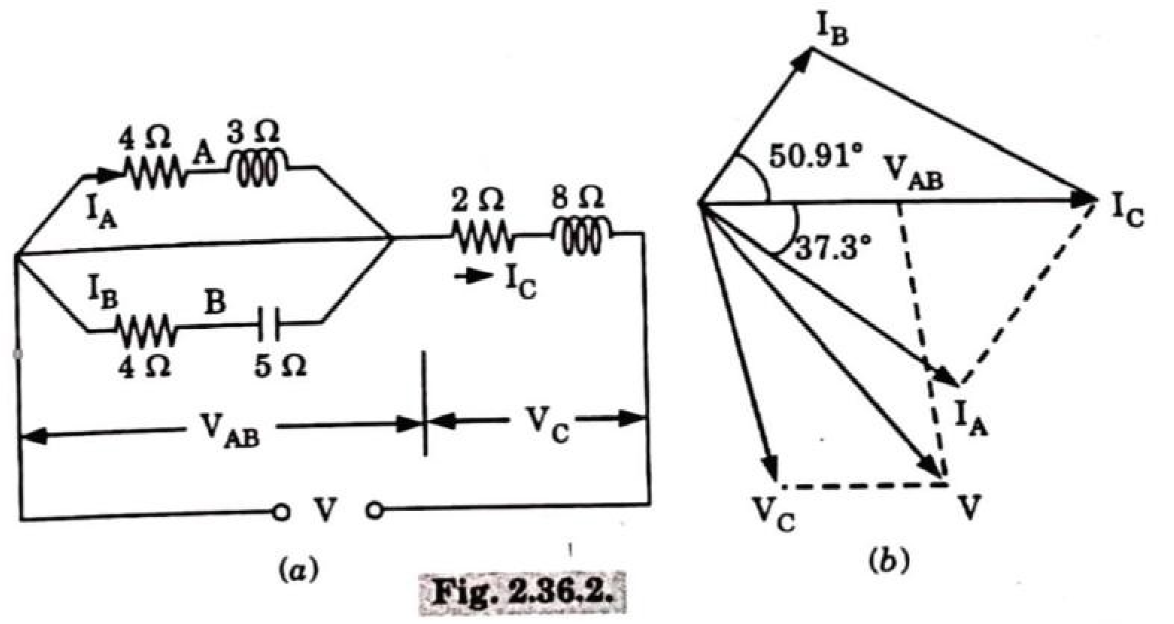


Fig. 2.36.2.

PART-5

Three Phase Balanced Circuits, Voltage and Current Relations in Star and Delta Connections.

CONCEPT OUTLINE : PART-5

i. **Star-connected system :** It is obtained by joining together similar ends, either the start or the finish, the other ends are joined to the line wires. The common point at which similar ends are connected is called the neutral or star point.

ii. **Delta-connected system :** It is obtained when the starting end of one coil is joined to the finishing end of another coil.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.37. **Prove the voltage and current relations in *Y*-connected system.**

Answer

1. Consider a balanced load Z_{ph} connected in *Y*-connection. The supply to this load is assumed to be balanced.
2. Phase to neutral voltage, *i.e.*, phase voltages are

$$V_{ph} = V_{RN} = V_{YN} = V_{BN} \quad ...(2.37.1)$$

And line to line voltages *i.e.*, line voltages are

$$V_L = V_{RY} = V_{YB} = V_{BR} \quad ...(2.37.2)$$

3. Now, line voltage $\vec{V}_{RY} = \vec{V}_{RN} - \vec{V}_{YN}$

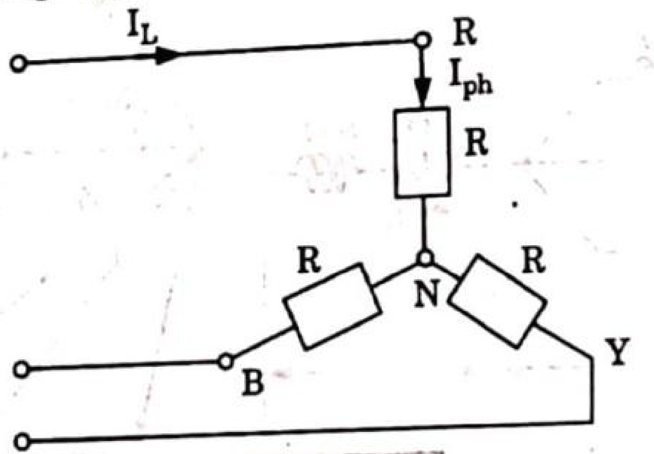


Fig. 2.37.1.

4. Magnitude of $\vec{V}_{RY}$

$$|\vec{V}_{RY}| = \sqrt{V_{RN}^2 + V_{YN}^2 + 2V_{RN} V_{YN} \cos 60°} \quad ...(2.37.3)$$

From eq. (2.37.1), (2.37.2) and (2.37.3)

$$V_L = \sqrt{3V_{ph}^2}$$

$$V_L = \sqrt{3}\ V_{ph}$$

5. From the Fig. 2.37.1 of star - connected load it is clear that $I_L = I_{ph}$.

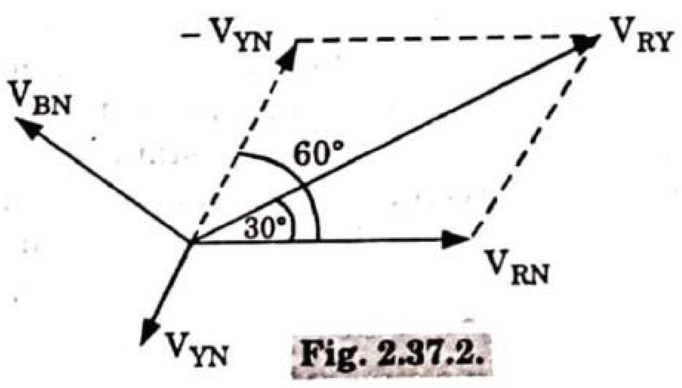


Fig. 2.37.2.

Que 2.38. **Derive the relationship between line current and phase current for delta connected 3-phase load when supplied from 3-phase balanced supply.**

AKTU 2017-18(Sem-2), Marks 07

Answer

1. Consider a balanced delta connected load as shown in Fig. 2.38.1.
2. Currents I_R, I_Y and I_B are known as line currents.

And $I_{RY} = I_{YB} = I_{BR} = I_{ph}$, *i.e.*, phase currents ...(2.38.1)

$V_L = V_{ph}$ for delta connection. ...(2.38.2)

3. Now, applying KCL at node R

$$\vec{I}_R + \vec{I}_{BR} = \vec{I}_{RY}$$

$$\vec{I}_R = \vec{I}_{RY} - \vec{I}_{BR} \quad ...(2.38.3)$$

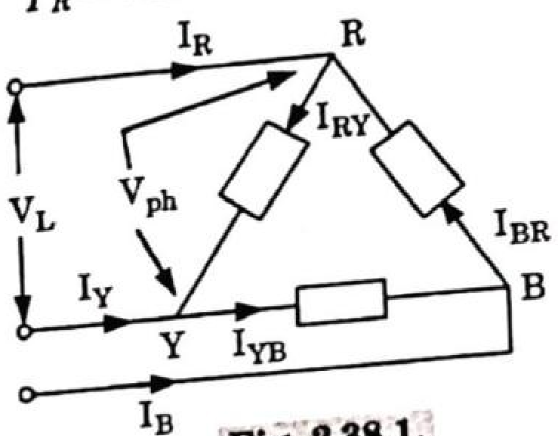


Fig. 2.38.1.

4. Magnitude of I_R can be found out as

$$I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2I_{RY} I_{BR} \cos 60°}$$

$I_R = I_L$ and $I_{RY} = I_{BR} = I_{ph}$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}^2 \times 0.5}$$

$$I_L = \sqrt{3}\ I_{ph}$$

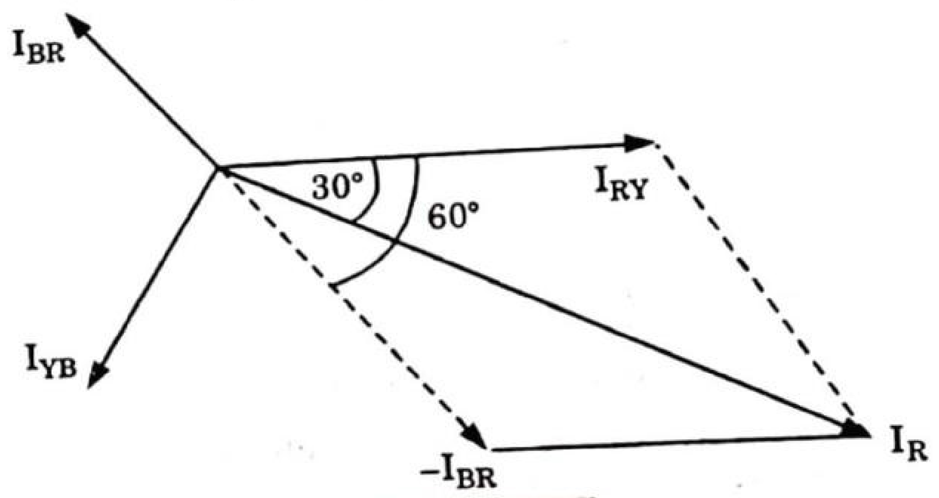


Fig. 2.38.2.

Que 2.39. **Show that power in 3-phase, balanced system is constant at every instant is given by 3 $V_p I_p \cos \phi$, where V_p, I_p and ϕ have usual meanings.**

AKTU 2013-14(Sem-2), Marks 10

Answer

1. **Power for star connection :**

$\because \quad V_L = \sqrt{3}\ V_{ph}$ and $I_L = I_{ph}$

$\therefore \quad P = 3\ V_{ph} I_{ph} \cos \phi = 3 \left(\dfrac{V_L}{\sqrt{3}}\right) I_L \cos \phi$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

2. **Power for delta connection :**

$$V_L = V_{ph},\quad I_L = I_{ph} \times \sqrt{3}$$

$$P = 3V_L \frac{I_L}{\sqrt{3}} \cos\phi = \sqrt{3}\, V_L\, I_L \cos\phi$$

Thus power is same in both star as well as delta connection.

3. **3-Phasor power :** Phase voltages and currents in a balanced 3ϕ circuit (star or delta) may be written in the instantaneous form as,

$$V_R = \sqrt{2}\, V_{ph} \sin\omega t$$
$$V_Y = \sqrt{2}\, V_{ph} \sin(\omega t - 120°)$$
$$V_B = \sqrt{2}\, V_{ph} \sin(\omega t + 120°)$$

And

$$I_R = \sqrt{2}\, I_{ph} \sin(\omega t - \phi)$$
$$I_Y = \sqrt{2}\, I_{ph} \sin(\omega t - \phi - 120°)$$
$$I_B = \sqrt{2}\, I_{ph} \sin(\omega t - \phi + 120°)$$

where ϕ is the phase angle between phase voltage and phase current.

$$P = V_R I_R + V_B I_B + V_Y I_Y$$
$$= V_{ph} I_{ph}\,[2 \sin\omega t \sin(\omega t - \phi)] + V_{ph} I_{ph}\,[2 \sin(\omega t - 120°) \sin(\omega t - \phi - 120°)] + V_{ph} I_{ph}\,[2 \sin(\omega t + 120°) \sin(\omega t - \phi + 120°)]$$
$$= V_{ph} I_{ph}\,[\cos\phi - \cos(2\omega t - \phi) + \cos\phi - \cos(2\omega t - \phi - 240°) + \cos\phi - \cos(2\omega t - \phi + 240°)]$$

Average power, $P = 3\, V_{ph} I_{ph} \cos\phi$

Que 2.40. **A 3-phase load consisting of resistance 25 Ω, inductance of 0.15 H and capacitor of 100 μF is connected to 400 V, 50 Hz. Calculate line current, power factor, total power**

i. When connected in star

ii. When connected in delta. **AKTU 2013-14(Sem-1), Marks 05**

Answer

1. Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.12\ \Omega$
2. Capacitive reactance, $X_C = \dfrac{1}{2\pi f C} = 1(2\pi \times 50 \times 100 \times 10^{-6}) = 31.83\ \Omega$
3. Net reactance, $X = X_L - X_C = 15.29\ \Omega$
4. Impedance, $Z = \sqrt{R^2 + X^2} = \sqrt{(25)^2 + (15.29)^2} = 29.305\ \Omega$
5. Phase angle $\phi = \tan^{-1}\dfrac{X}{R} = \tan^{-1}\dfrac{15.29}{25} = 31.440$

A. **When connected in star :**

1. Line current = Phase current $= I_P = \dfrac{V_L/\sqrt{3}}{Z} = \dfrac{V_P}{Z}$

$$= \frac{400/\sqrt{3}}{29.305} = 7.88\text{ A} \qquad [\because\ (V_P = V_L/\sqrt{3})]$$

2. Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{25}{29.305} = 0.853$ (lagging)
3. Total power $= \sqrt{3}\, V_L I_L$

$$= \sqrt{3} \times 400 \times 7.88 = 5459.42\text{ W}$$

B. **When connected in delta :**

1. Phase current, $I_P = \dfrac{V_P}{Z} = \dfrac{400}{29.305} = 13.65$ A
2. Line current, $I_L = \sqrt{3}\, I_p = \sqrt{3} \times 13.65 = 23.64$ A
3. Total power $= \sqrt{3}\, V_L I_L = \sqrt{3} \times 400 \times 23.64 \times = 16378.27$ W

Que 2.41. **A balanced delta connected load impedance 16 + *j*12 Ω/phase is connected to a three-phase 400 V supply. Find the phase current, line power factor, active power, reactive power and total power. Also draw the phasor diagram.**

AKTU 2014-15(Sem-1), Marks 10

Answer

1. $V_p = V_L = 400$ V
2. Phase current, $I_p = \dfrac{400\angle 0°}{16 + j12} = 20\angle -36.9°\text{ A} = 20\angle -37°$
3. Power factor, $\cos\phi = \cos 36.9° = 0.8$ (lagging)
4. $I_L = \sqrt{3} \times 20 = 34.64$ A
5. Active power, $P = \sqrt{3} \times 400 \times 34.64 \times 0.8 = 19.2$ kW
6. Reactive power, $Q = \sqrt{3} \times 400 \times 36.64 \times \sin 36.9° = 14.4$ kVAR
7. Total power, $S = [P^2 + Q^2]^{1/2} = 24$ kVA
8. Phasor diagram is shown in Fig. 2.41.1.

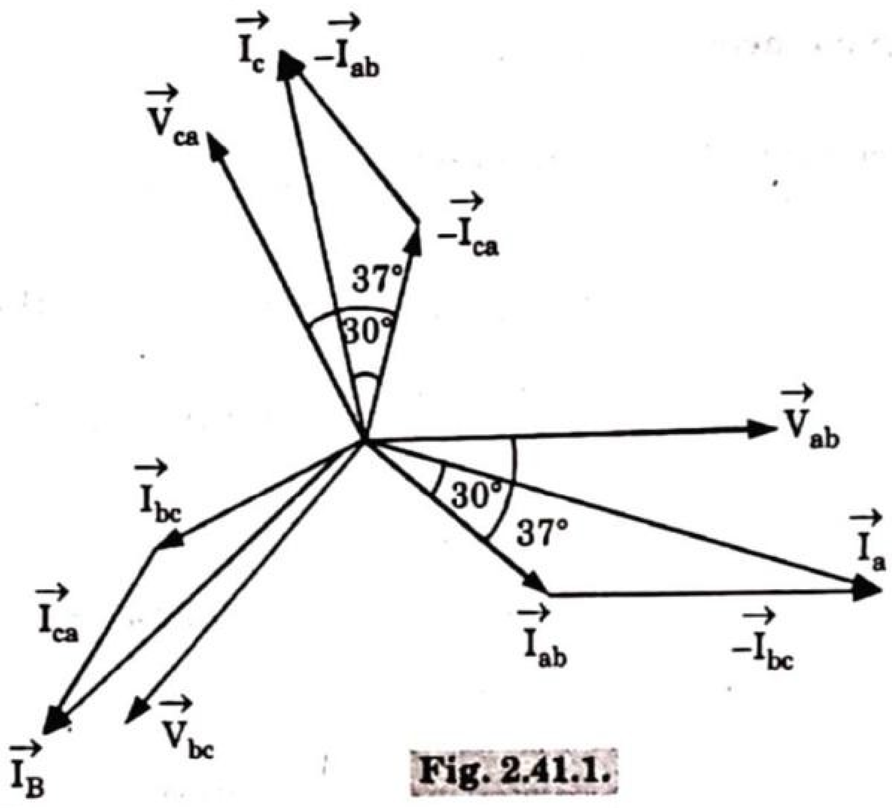


Fig. 2.41.1.

Que 2.42. **Derive the relation between line and phase voltage and current for a delta connected 3-phase balanced system. A balanced delta-connected load of impedance, $Z = 30 \angle 60° \ \Omega$ is connected to line voltage of 440 V. Obtain the current and power supplied to load.**

AKTU 2014-15(Sem-2), Marks 10

Answer

A. **Relation between line and phase voltage and current in delta connection :** Refer Q. 2.38, Page 2–41D, Unit-2.

B. **Numerical :**

The procedure is same as Q. 2.41, Page 2–44D, Unit-2.

(**Ans.** Line current = 25.46 A ; Power = 3241.35 W)

Que 2.43. **Obtain the relation between line and phase voltages in balanced star connected load system. Also draw its phasor diagram. A 3-phase, star connected balanced load is supplied by 400 V, 50 Hz. The load takes a leading current of $100\sqrt{3}$ A and power 20 kW. Calculate power factor of load and resistance and inductance per phase.**

AKTU 2015-16(Sem-1), Marks 15

Answer

A. **Relation between line and phase voltage in balanced star connected system :** Refer Q. 2.37, Page 2–40D, Unit-2.

B. **Numerical :**

1. Power, $P = \sqrt{3}\ V_L I_L \cos\phi$

2. Power factor, $\cos\phi = \dfrac{P}{\sqrt{3}\,V_L\,I_L} = \dfrac{20 \times 10^3}{\sqrt{3} \times 400 \times 100\sqrt{3}} = 0.167$ (lead)

3. Phase voltage, $V_{ph} = V_L / \sqrt{3} = 400 / \sqrt{3} = 230.94$ V

4. Phase current, $I_{ph} = I_L = 100\sqrt{3}$ A

5. Phase impedance, $Z_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{230.94}{100\sqrt{3}} = 1.33\ \Omega$

6. Phase resistance, $R_{ph} = Z_{ph}\cos\phi = 1.33 \times 0.167 = 0.22\ \Omega$

7. Phase reactance, $X_{ph} = \sqrt{Z_{ph}^2 - R_{ph}^2} = \sqrt{(1.33)^2 - (0.22)^2} = 1.31\ \Omega$

8. Phase inductance, $L_{ph} = \dfrac{X_{ph}}{2\pi f} = \dfrac{1.31}{2 \times 3.14 \times 50} = 4.17 \times 10^{-3}$ H

Que 2.44. **Derive relation between line and phase values in delta connected 3-phase balance system. A 3-phase voltage source has a phase voltage of 120 V and supplies star connected load having impedance of $(24 + j36)\ \Omega$ per phase. Calculate**

i. **Line voltage**
ii. **Line current**
iii. **Total 3-phase power supplied to the load.**

AKTU 2016-17(Sem-1), Marks 07

Answer

A. **Relation between line and phase values in delta connection :** Refer Q. 2.38, Page 2–41D, Unit-2.

B. **Numerical :**

1. Line voltage : $V_L = \sqrt{3}\ V_p$

$V_L = \sqrt{3} \times 120 = 207.84$ V

Line current : $Z_p = \sqrt{(24)^2 + (36)^2} = 43.269\ \Omega$

$I_L = I_p = \dfrac{V_p}{Z_p} = \dfrac{120}{43.269}$

$I_L = 2.77$ A