

COMPUTER SCIENCE E-20, SPRING 2014

In-class Problems - Rachna Sha

9.1

1. Convert the following English sentences into logical formulas. You may use expressions like  $x = y$  or  $x \neq y$  to indicate whether or not the variables  $x$  and  $y$  denote different people. The domain of discourse is all people. Let the predicate  $H(x)$  mean that  $x$  is happy, and let the predicate  $L(x, y)$  mean that  $x$  loves  $y$ .

- At least one person is happy :  $\exists x H(x)$
- At least two people are happy:  $\exists x \exists y (H(x) \wedge H(y) \wedge x \neq y)$
- At least one person is unhappy:  $\exists x \neg H(x)$
- Exactly one person is happy :  $\exists x H(x) \wedge (\forall y H(y) \implies y = x)$
- Not everyone loves someone else. (Write this in 3 different ways).  
 $\neg(\forall x \forall y (L(x, y)) \wedge (x \neq y))$   
 $\exists x \exists y \neg(L(x, y)) \wedge (x \neq y)$   
 $\exists x \exists y (\neg L(x, y)) \wedge (x \neq y)$