COMPUTER SCIENCE E-20, SPRING 2014
Homework Problems
Induction I, Strong Induction

Due Thursday, February 12, 2015 before 9PM EST. Upload a PDF of your answers at https://canvas.harvard.edu/courses/1815/assignments/17757

1. Prove that for all nonnegative integers n

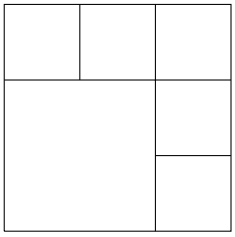
$$\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$$

Hint: the following identity may be useful

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

2. Consider the sequence $a_1 = 1, a_2 = 3, ..., a_n = a_{n-1} + a_{n-2}$. Using strong induction prove that $a_n \leq \left(\frac{7}{4}\right)^n$ for all positive integers n.

3. Prove using strong induction that any square can be subdivided into n smaller squares, where $n > 5$. For example, the large square below has been subdivided into 6 squares.
Hint: first show that any square subdivided into k squares can easily be subdivided into $k + 3$ squares, then think how many base cases you need show are true (it is not just the case of n=6).



4. The Fibonacci numbers are defined by $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$. Prove using strong induction that for all $n \geq 0$, F_{3n} is even.