In-class Problems - Rachna Sha
30.3

1. You are going to play 2 games of chess with an opponent whom you have never played against before. Your opponent is equally likely to be a beginner, intermediate, or a master. If your opponent is a beginner, you have an 80% chance of winning, if your opponent is an intermediate, you have a 50% chance of winning, and if your opponent is a master, you have a 20% chance of winning.
(a) What is the probability that you win the first game?

Solution:
Let $Pr(W1)$ = Probability of Winning the first game
Let $Pr(B)$= Winning when opponent is beginner = 0.8
Let $Pr(I)$= Winning when opponent is intermediate= 0.5
Let $Pr(M)$= Winning game 1 when opponent is master = 0.2

$Pr(W1) = (1/3 * 0.8 + 1/3 * 0.5 + 1/3 * 0.2) = 0.5$

(b) Congratulations, you won the first game! Given this information, what is the probability that you will also win the second game? Assume that, given the skill level of your opponent, the outcomes of the games are independent.

Solution:
Let $Pr(W2|W1)$ = Conditional Probability of Winning game 2 giving you have won game 1

$Pr(W2|W1) = Pr(W2 \cap W1)/\ Pr(W1)$

Since the two games are independent : $Pr(W2 \cap W1) = Pr(W2) * Pr(W1)$

$Pr(W2|W1) = Pr(W2) = 0.5$