

In-class Problems - Rachna Sha  
17.1

1. Prove that any finite DAG has at least one vertex with in-degree 0.

Proof:

Suppose a DAG contains no vertex with in-degree 0. This implies that each of the vertices have an in-degree count of at least 1. Consider a DAG  $G$ , with a set of three vertices  $(1,2,3)$  as shown below - where each vertex has a in-degree count of 1.

The Graph ' $G$ ' has a walk that starts at vertex 1 and finishes at 1, with no other repeated vertices. This walk is a cycle. This contradicts that fact that ' $G$ ' is a DAG.

This proves (by contradiction) that a DAG must have at least one vertex with in-degree 0.

$G$  :

