

COMPUTER SCIENCE E-20, SPRING 2014

Homework Problems

Pigeonhole, Proofs

Due Thursday, February 5, 2015 before 9PM EST. Upload a PDF of your answers at <https://canvas.harvard.edu/courses/1815/assignments/17263>

1. What is the minimum number of unique integers that you have to be pick from $\{1, 2, \dots, 16\}$ to ensure that there is at least one pair whose sum is equal to 17?

Solution:

- The given set has 16 numbers each of which are < 17 .
 - In order to pick a pair whose sum is 17, we need to pick a number i such that $1 \leq i \leq 8$ and a number j such that $8 \leq j \leq 16$
 - So we need to pick at least 9 integers to ensure we choose a pair whose sum is 17
2. Every day a ketchup factory produces a positive whole number of gallons of ketchup. Show, using the pigeonhole principle, that within the next two months there will be a period of some number of consecutive days, in which the total production will fit exactly into one or more 50-gallon containers.

Solution:

- Assume that on each day over the 60 day period the factory will produce 1 gallon of ketchup each day.
 - So over the 60 days, a total 60 gallons will be produced.
 - By the pigeonhole principle, 60 gallons will at least fill 1 50 gallon container.
 - Hence there will be a period of some consecutive days where the total production will fit into one or more 50-gallon containers.
3. Prove by contradiction that if $17n + 2$ is odd then n is *odd*.

Solution:

- Assume that $\exists n$ such that $17n + 2$ is odd and n is *even*
- By definition of even $n = 2k$ where k is an integer
- By substitution we have $17n + 2 = 17(2k) + 2$
- $17(2k) + 2 = 2(17k + 1)$
- which implies that $17n + 2$ is even as it is equal to $2 * (17k + 1)$

- This contradicts the assumption that $17n + 2$ is odd
- Therefore the assumption that n is *even* is wrong. Hence n is *odd*
- This proves that if $17n + 2$ is odd then n is *odd*