In-class Problems - Rachna Sha
28.2

An ambitious preschool, whose aim is to prepare toddlers for Harvard, has 8 students, 4 boys and 4 girls. Following the example of Harvard, it divides its students randomly into two groups of 4, which it names Lowell House and Eliot House.

1. What is the probability P4 that all four boys end up in Eliot House?

   P4 = Number of event outcomes/Total number of outcomes

   Total Number of outcomes = choosing 4 students out of 8 can be done in $\binom{8}{4}$ ways

   Total Number of event outcomes = choosing all 4 boys = $\binom{4}{4}$ ways

   P4 = $\frac{\binom{4}{4}}{\binom{8}{4}} = \frac{1}{70}$

2. What is the probability P3 that three boys and one girl end up in Eliot House, with the other boy and three girls in Lowell House?

   P3 = Number of event outcomes/Total number of outcomes

   Total Number of outcomes = choosing 4 students out of 8 can be done in $\binom{8}{4}$ ways

   Total Number of event outcomes = choosing all 3 boys out of 4 and choosing 1 girl out of 4 = $\binom{4}{3} * \binom{4}{1}$ ways

   P3 = $\frac{\binom{4}{3}*\binom{4}{1}}{\binom{8}{4}} = \frac{16}{70}$

3. What is the probability P2 that optimal diversity is achieved, with two boys in Eliot House and two in Lowell House?

   P2 = Number of event outcomes/Total number of outcomes

   Total Number of outcomes = choosing 4 students out of 8 can be done in $\binom{8}{4}$ ways

   Total Number of event outcomes = choosing all 2 boys out of 4 for and choosing other 2 girls of out 4 for Elliot = $\binom{4}{2} * \binom{4}{2}$ ways

   P2 = $\frac{\binom{4}{2}*\binom{4}{2}}{\binom{8}{4}} = \frac{36}{70}$

4. Verify that the sum of probabilities for all the ways of dividing the class between the houses is correct.

   probability of all boys in Elliot + probability of all girls in Lowell + three boys and one girl end up in Eliot House + one boy and three girls end up in Lowell House + probability of two boys in Eliot House and two in Lowell House

$= \frac{1}{70} + \frac{1}{70} + \frac{16}{70} + \frac{16}{70} + \frac{36}{70} = 1$

5. A newly hired teacher meets one student, chosen at random, from Eliot House. The student is a boy. Calculate the conditional probabilities, given this event, that there are 4, 3, 2, 1, or 0 boys in Eliot House. (These conditional probabilities should also sum to 1.)

   Given 1 Boy at Eliot house $P(0B|1B) = 0$

   $P(1B|1B) =$ if there is one boy , then other 3 are girls $= \frac{\binom{4}{3}}{\binom{7}{3}}$

   $P(2B|1B) =$ choose 1 boy from 3 and 2 girls from 4 $= \frac{\binom{3}{1}*\binom{4}{2}}{\binom{7}{3}}$

   $P(3B|1B) =$ choose 2 boy from 3 and 1 girl from 4 $= \frac{\binom{3}{2}*\binom{4}{1}}{\binom{7}{3}}$

   $P(4B|1B)=$ choose 3 boy from 3. This will total to 4 students, so no girls need to be choosen $= \frac{\binom{3}{3}}{\binom{7}{3}}$

6. Next the teacher meets a randomly chosen student from Lowell House, who turns out to be a girl. Calculate the conditional probabilities,given both this event and the previous one, that there are 4, 3, 2, 1, or 0 boys in Eliot House.

   Given 1 Boy at Eliot and 1 Girl at Lowell

   $P(0B|1B1G) = 0$

   $P(1B|1B1G) =$ if there is one boy , then other 3 are girls $= \frac{\binom{3}{3}}{\binom{6}{3}}$

   $P(2B|1B1G) =$ choose 1 boy from 3 and 2 girls from 3 $= \frac{\binom{3}{1}*\binom{3}{2}}{\binom{6}{3}}$

   $P(3B|1B1G) =$ choose 2 boy from 3 and 1 girl from 3 $= \frac{\binom{3}{2}*\binom{3}{1}}{\binom{6}{3}}$

   $P(4B|1B1G)=$ choose 3 boy from 3. This will total to 4 students, so no girls need to be choosen $= \frac{\binom{3}{3}}{\binom{6}{3}}$