COMPUTER SCIENCE E-20, SPRING 2014
Homework Problems
Pigeonhole, Proofs

Due Thursday, February 5, 2015 before 9PM EST. Upload a PDF of your answers at https://canvas.harvard.edu/courses/1815/assignments/17263

1. What is the minimum number of unique integers that you have to be pick from $\{1, 2, ..., 16\}$ to ensure that there is at least one pair whose sum is equal to 17?

   The pigeons in this case are the numbers from $\{1, 2, ..., 16\}$ The pigeonholes is the the pair whose sum is 17 The maximum number that can be chosen is 16, and the minimum number is 1. Each of these are less than 17, but their sum is 17. Since no single number is equal to 17, the minimum number of integers needed is 2.

2. Every day a ketchup factory produces a positive whole number of gallons of ketchup. Show, using the pigeonhole principle, that within the next two months there will be a period of some number of consecutive days, in which the total production will fit exactly into one or more 50-gallon containers.

   Solution:

   - Assume that on each day over the 60 day period the factory will produce 1 gallon of ketchup each day.
   - So over the 60 days, a total 60 gallons will be produced.
   - By the pigeonhole principle,60 gallons will at least fill 1 50 gallon container.
   - Hence there are will be a period of some consecutive days where the total production will fit into one or more 50-gallon containers.

3. Prove by contradiction that if $17n + 2$ is odd then n is *odd*.

   Solution:

   - Assume that $\exists n$ such that $17n + 2$ is odd and n is *even*
   - By definition of even $n = 2k$ where k is an integer
   - By substitution we have $17n + 2 = 17(2k) + 2$
   - $17(2k) + 2 = 2(17k + 1)$
   - which implies that $17n + 2$ is even as it is equal to $2 * (17k + 1)$
   - This contradicts the assumption that $17n + 2$ is odd
   - Therefore the assumption that n is *even* is wrong. Hence n is *odd*
   - This proves that if $17n + 2$ is odd then n is *odd*