COMPUTER SCIENCE 20, SPRING 2014
Module #7 (Equivalences and Normal Forms)

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Executive Summary

1. Forms of propositional formulas

   - Disjunctive form: ORs of AND-terms, where each AND-term consists of variables (or negations of variables). E.g.: $(p\wedge q)\vee(p\wedge r)\vee(q\wedge\neg r\wedge t)$
   - Disjunctive *normal* form: like the disjunctive form, but with the restriction that every AND-term must consists of *all* variables (or negations of variables). E.g.: $(p\wedge q\wedge r)\vee(\neg p\wedge q\wedge r)\vee(\neg p\wedge\neg q\wedge r)$ There will be one factor for each "true" line in the truth table.
   - Conjunctive form: ANDs of OR-terms, where each OR-term consists of variables (or negations of variables). E.g.: $(p\vee\neg q)\wedge(p\vee r)\wedge(q\vee\neg r\vee t)$
   - Conjunctive *normal* form: like the conjunctive form, but with the restriction that every OR-term must consist of *all* variables (or negations of variables). E.g.: $(p\vee q\vee r)\wedge(p\vee\neg q\vee r)\wedge(\neg p\vee q\vee r)$ There will be one factor for each "false" line in the truth table.

2. DeMorgan's Laws and Double Negation:

   - $\text{Not}(A \text{ And } B) \longleftrightarrow \overline{A} \text{ Or } \overline{B}$
   - $\text{Not}(A \text{ Or } B) \longleftrightarrow \overline{A} \text{ And } \overline{B}$
   - $\text{Not}(\overline{A}) \longleftrightarrow A$

3. Distributive Laws

   - $A \text{ And } (B \text{ Or } C) \longleftrightarrow (A \text{ And } B) \text{ Or } (A \text{ And } C)$
   - $A \text{ Or } (B \text{ And } C) \longleftrightarrow (A \text{ Or } B) \text{ And } (A \text{ Or } C)$

4. Tautologies and Satisfiability

   - Tautology: Formula that is true under all possible truth assignments.
   - Satisfiable: Formula that is true for at least one truth assignment.
   - Unsatisfiable: Formula whose negation is a tautology.

Check-In problems

1. Which of the following is in conjunctive form?

   (a) $(\neg p \wedge q \vee r) \vee (\neg r \wedge p) \vee (p \wedge \neg q \wedge \neg r)$

   (b) $(\neg p \vee q) \wedge (p \vee r) \wedge (p \vee \neg q \vee r)$

   (c) $(\neg r \vee q) \wedge (p \wedge q)$

   (d) $(p \wedge q) \vee (\neg q \wedge p)$

In-class problems

1. (a) Put the following formula in disjunctive *normal* form: $(p \leftrightarrow q) \wedge r$

   (b) Put the following formula in disjunctive form: $(\neg p \wedge \neg q) \rightarrow (r \vee q)$

2. (a) Put the following formula in conjunctive *normal* form: $(\neg(p \oplus q)) \rightarrow r$

   Hint: Recall that $p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$

   (b) Put the following formula in conjunctive form: $(p \rightarrow q) \wedge p$

3. The following formula is satisfiable. *Without using a truth table*, find a satisfying assignment:

$$(((p \vee q \vee r) \wedge (\neg r \wedge s)) \vee \neg p) \rightarrow ((\neg s \vee t) \oplus \neg q)$$

4. (a) Find an easy way to check whether a formula in disjunctive normal form is satisfiable.

   (b) Sam wakes up one night thinking that he has solved the P=?NP question: P = NP! Given a logical formula, just put the formula into disjuntive normal form, then use the method in part (a) to check to see if it is satisfiable. Why won't Sam win the million dollar prize for settling the question?

5. (a) Let $\circ$ be a new binary logical operator such that $p \circ q$ is equivalent to $p \wedge \neg q$. Show that any formula using $\wedge, \vee, \neg$ can be rewritten using $\circ$ and the constant T alone, even without using the constant F.

   (b) Show that on the other hand there are formulas that cannot be written if the only available operators are $\wedge, \vee$, and the constants T and F.