COMPUTER SCIENCE E-20, SPRING 2015
Homework Problems
Functions and Relations, Countability, Recursive Definitions, Structural Induction

Due Thursday, March 12, 2015 before 9PM EDT. Upload a PDF of your answers at https://canvas.harvard.edu/courses/1815/assignments/20343

1. (a) Give an example of an injective relation that is not a function (or even a partial function).

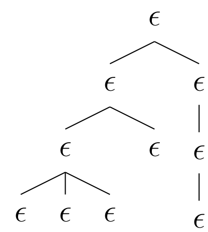
   (b) The *inverse* of a relation R on $X \times Y$ is the following relation R^{-1} on $Y \times X$:
   $$R^{-1}(y,x) \iff R(x,y)$$
   Fill in the blanks in the following table

R is...	iff R^{-1} is...
injective	
surjective	
bijective	
total	
a (possibly partial) function	

2. Prove or disprove the following statement: If f is a function from X to Y and g a function from Y to Z such that g and $g \circ f$ are surjective, then f is surjective.

3. Trees Let T be the set of rooted trees.

   - Base Case:
     $$\epsilon \in T$$
     This represents a node with no edges.
   - Constructor Case: If $t_1, ..., t_n \in T$, then
     $$(\epsilon, t_1, ..., t_n) \in T$$
     Here, ϵ represents the root node and t_i represent the subtrees of our new tree. We say that there is an $e$dge between ϵ and each of the root nodes of the subtrees.
   - Example: $(\epsilon, (\epsilon, (\epsilon, \epsilon, \epsilon, \epsilon), \epsilon), (\epsilon, (\epsilon, \epsilon)))$ represents the following tree:



For all $n \geq 1$, prove that any tree with n nodes has $n-1$ edges.

4. Show that the set of real numbers between 0 and 1 is uncountable. (Hint: Can you represent the numbers as strings using some alphabet?)