COMPUTER SCIENCE E-20, SPRING 2014
In-class Problems - Group 5
3.2

1. Prove by induction that the decimal representation of every power of 3 ends
   in one of the digits 1, 3, 7, or 9.

   Solution:

   - Proof: By induction on n that decimal representation of every power
     of 3 ends in one of the digits 1, 3 ,7, or 9.

   - The Induction hypothesis P(n), is: 3^n ends in 1, 3, 7, or 9 $\forall$ n ≥ 0

   - Base Case (n = 0) : $3^0 = 1$. This satisfies the base case.

   - Inductive Step : Assume P(n) is true $\forall$ n ≥ 0, and prove
     P(n+1) : $3^{n+1} = 3^n * 3^1$
     From the inductive step :
     3^n is a number that ends in 1, 3, 7, or 9 and
     $3^1 = 3$
     substituting 3^n with 1, 3, 7, or 9

       - 3^n ends in $1 = 1 * 3 = 3$
       - 3^n ends in $3 = 3 * 3 = 9$
       - 3^n ends in $7 = 7 * 3 = 21$
       - 3^n ends in $9 = 9 * 3 = 27$

     This proves P(n+1), completing the proof by induction.