COMPUTER SCIENCE E-20, SPRING 2015
Homework Problems
Sets
Solutions

Due Thursday, March 5, 2015 before 9PM EST. Upload a PDF of your answers at https://canvas.harvard.edu/courses/1815/assignments/19890

1. Let X and Y be sets. Provide a formula for the cardinality of the set $A = (X \times Y) - (Y \times X)$ in terms of the cardinalities of X, Y, $X - Y$, and $Y - X$.

   Solution : A is the set of all pairs (x, y) with $x \in X$, $y \in Y$ such that $y \notin X$ and/or $x \notin Y$. Thus $|A| = |X||Y - X| + |X - Y||Y| - |X - Y||Y - X|$.

   For example, if $X = \{1, 2\}$, and $Y = \{2, 3, 4\}$, then $X - Y = \{1\}$, $Y - X = \{3, 4\}$, $X \times Y = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\}$, and $Y \times X = \{(2, 1), (3, 1), (4, 1), (2, 2), (3, 2), (4, 2)\}$, so $A = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4)\}$. Checking our formula above, $|A| = 5$, $|X - Y| = 1$, $|Y| = 3$, $|X| = 2$, and $|Y - X| = 2$, so $|X||Y - X| + |X - Y||Y| - |X - Y||Y - X| = 2(2) + 1(3) - 1(2) = 5$.