COMPUTER SCIENCE 20, SPRING 2015

Module #1 (Pigeonhole Principle)

Last modified: January 25, 2015

Executive Summary

- Let A be the set of your pigeons, and let B be the set of pigeonholes in which they live.
- Let $f : A \to B$ be the function that assigns to each pigeon the pigeonhole in which it lives.
- The pigeonhole principle states that if $|A| > |B|$, then there exists a pigeonhole in which more than one pigeon lives.
- The generalized pigeonhole principle states that if $|A| > k|B|$, then there exists a pigeonhole in which more than k pigeons live.
- A more formal statement of the pigeonhole principle might be:

  Let A and B be finite sets, with $|A| > |B|$. Then no function $f : A \to B$ can be injective.

Check-in questions

1. You awake in your tent in pitch darkness and need to get dressed. You know that your knapsack contains white, brown, blue, and black socks. Use the pigeonhole principle(even if you have not known it by that name) to determine the minimum number of socks that you must carry to the nearby campfire in order to be sure of having a pair with matching colors.

   (a) 5

   (b) 6

   (c) 7

   (d) 8

In-class problems

- The key step for each problem is to identify the appropriate "pigeons" and "pigeonholes."

1. Choose any five points inside the unit square. Prove that there is at least one pair for which the distance between the points is less than $\frac{1}{\sqrt{2}}$. Hint: Split the square into equal-sized regions.

2. Show in the case of the 5×5 grid problem done in the minilecture that some square winds up empty.