COMPUTER SCIENCE 20, SPRING 2014
Module #3 (Induction I)

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Executive Summary

1. Let $P(x)$ be a predicate and m, n nonnegative integers. If $P(m)$ is true and $P(n) \Rightarrow P(n+1)$ for all $n \geq m$, then $P(n)$ is true for all $n \geq m$.

2. One way to think about how to approach the proofs is to climb a ladder. You describe how to get onto one of the lower rungs of the ladder, and then you describe how to get from one rung to the next one. You can follow this general procedure as many times as you'd like to climb a ladder of any height.

3. To write a proof by induction, first you need to identify the proposition to be proven. Then prove the base case, prove the inductive step (how you can get from the proposition holding for n to the proposition holding for $n+1$), and the conclusion. Be sure you identify properly the thing being inducted upon.

Check-in questions

1. Suppose that we want to prove using induction that the statement S is true for all $N > k$. Which of the following approaches is correct:

   (a) We assume that the statement S is true for k, and show that S is true for k+1

   (b) We assume that the statement S is true for N, and show that S is true for N+1

   (c) We assume that the statement S is true for k, and show that S is true for k-1

   (d) We assume that the statement S is true for N, and show that S is true for N-1

In-class problems

1. What is wrong with this proof that all cars are of the same color?

   Let $P(n) =$ "every group of n cars contains only cars of the same color."

   Base case $P(1)$: There is only one car in the group, thus clearly all cars in the group have the same color.
   Inductive step: Assume $P(n)$ is true. We will show that it follows that $P(n+1)$ is true.
   Consider a group of $n+1$ cars. The first n cars must have the same color by the inductive assumption, and likewise the last n cars must also have the same color by the inductive assumption. Because the two groups overlap in the middle, the cars in the first group must have the same color as the cars in the seconds group. Therefore, all $n+1$ cars must have the same color. Thus we have shown that $P(n+1)$ is true, and our proof is complete.

2. Prove by induction that the decimal representation of every power of 3 ends in one of the digits 1, 3 ,7, or 9.

3. Use induction to prove that for all nonnegative integers n:

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$