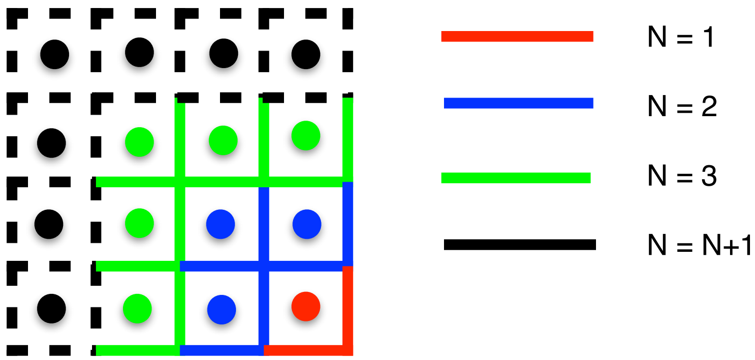
Show that N × N is countably infinite by providing a bijection with N. Use this to conclude that the set of positive rational numbers is countably infinite.

The grid N x N:



For every value of N, the grid NxN will be N^2. This is represented in the bijection:

N	NxN
1	1
2	4
3	9
N+1	$(N+1)^2$

Each value of N is mapped to exactly one value of NxN, and vice versa. As we have shown that NxN has a one-to-one correspondence with the natural number set, N, it fulfills the definition for countably infinite.

This is also visually demonstrated in the grid above: NxN is infinite as it holds for all N = N+1, and we can show that it is countable by our ability to physically place a dot in each known (f(N) = 1 - 3) and projected (f(N+1) = N^2) unit.

We can use this result to prove that the set of positive rational numbers, Q, is countably infinite:

Any rational number can be expressed as p/q, where p and q are positive integers that are relatively prime. In other words, all positive rational numbers can be expressed as pairs of p and q, where both p and q are positive integers. We can therefore associate each fraction p/q with the tuple (p, q) in NxN. This injection creates a one-to-one relationship between Q and the set of natural numbers in NxN. We have already shown that NxN is countably infinite, and by the establishment of a one-to-one relationship, it follows that Q is as well.