COMPUTER SCIENCE 20, SPRING 2015
Module #2 (Proof)

Last modified: January 27, 2015
Author: Steve

Executive Summary

1. Some proof strategies (this list will grow as the course progresses)

   - Contraposition: to prove "$P \Rightarrow Q$", prove "not $Q \Rightarrow$ not P."
   - Contradiction: to prove that P is true, assume that P is false and show that the assumption leads to a contradiction.
   - If and only if: to prove that $P \Leftrightarrow Q$, prove $P \Rightarrow Q$ and $Q \Rightarrow P$.
   - Alternative: to prove that $P \Leftrightarrow Q$, prove $P \Rightarrow Q$ and not $P \Rightarrow$ not Q.

2. Some rookie mistakes (or veteran techniques for bogus proofs)

   - Canceling a factor that might be zero from both sides of an equation ("division by zero").
   - Multiplying both sides of an inequality by a number that might be negative.
   - Assuming that $(a^2 = b^2) \Rightarrow (a = b)$. Correct is $(a^2 = b^2) \Rightarrow (|a| = |b|)$.
   - Ignoring a special case; e.g. proving $P(n)$ by considering only $n > 0$ and $n < 0$.

Check-in questions

1. Which of the following statements is equivalent to "if A, then B"

   (a) if not B then not A
   (b) A if B
   (c) if not A then not B
   (d) if not B then A

In-class problems

1. Each of the following two claims and corresponding proofs is incorrect. For each one, explain why each proof is incorrect.

   (i) Claim: $1c = \$1$

   $$1c = \$0.01 = (\$0.1)^2 = (10c)^2 = 100c = \$1$$

   (ii) Claim: If $a = b$ then $a = 0$

   $$a = b$$
   $$a^2 = ab$$
   $$a^2 - b^2 = ab - b^2$$
   $$(a - b)(a + b) = (a - b)b$$
   $$a + b = b$$
   $$a = 0$$

2. Prove that in any group of six people, at least two of them know the same number of people. Note that you don't know yourself, and that if A knows B then B knows A (thus "knows" is a symmetric relation).

3. Prove by contradiction that $\sqrt[3]{4}$ is irrational.