NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

# CCDS24105 - Uncertainty Quantification over Financial Time Series with Conformalized Risk Modelling


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## Motivation

Financial markets, particularly the cryptocurrency domain, exhibit high volatility, posing challenges in return prediction. The lack of uncertainty quantification of predictive models is a major barrier to the adoption of powerful machine learning methods, but at the same time, probabilistic forecasts are only valid asymptotically or upon strong assumptions on the data (Zaffran et al., 2022).

This research paper explores a recent technique that works by building confidence intervals — known formally as **conformal prediction** and uses techniques to improve to further enhance its coverage, which can provide market analysts with a reliable tool for risk-aware forecasting.

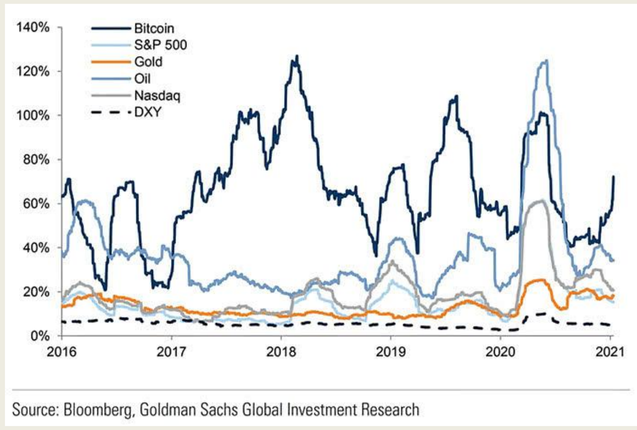


*Figure 1. Highly volatile nature of financial markets, especially Bitcoin*

## Methodology

### Conformal prediction

Originally introduced for **classification[2]** problems in the early 2000s. It was later adapted for Time Series Regression.

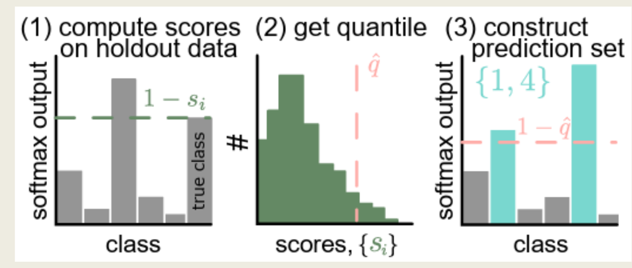


*Figure 2. Conformal Prediction for Classification* (Angelopoulos, 2022).

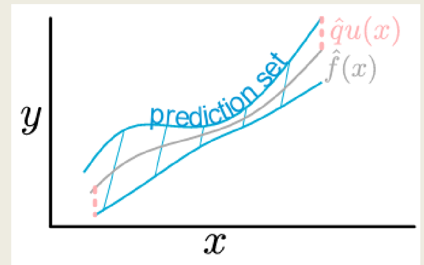


*Figure 3. Conformal Prediction for Regression* (Angelopoulos, 2022).

### Understanding the Basics

1. Split dataset into train, calibrate, and test data
2. After training, create set s by applying a suitable **error function** to calculate (conformal) errors on the calibrated data.
3. Calculate the $1-\alpha$ quantile based on all the error scores and use it to construct a test prediction set C:
$$C(X_{test}) = \{y : s(X_{test}, y) \leq \hat{q}\}$$
4. When done correctly, we are guaranteed coverage under the chosen alpha.
$$P(Y_{test} \in C(X_{test})) \geq 1-\alpha$$

### Incorporating into Time Series

1. As such, for our **time series regression[3]**, we compute a test prediction set using $|y_i - \hat{\mu}(\boldsymbol{X}_i)|$ as our **error function**:
$$\widehat{C}_n^{\alpha}(\boldsymbol{Z}_{t+1}) = \hat{\mu}(\boldsymbol{X}_{t+1}) \pm \mathrm{Q}_{1-\alpha}\left(\left\{|y_i - \hat{\mu}(\boldsymbol{X}_i)|\right\}_{i=1}^{n}\right).$$
2. Because asset return series frequently exhibit non-stationarity, with volatility changing over time in ways that standard models may fail to capture, a novel approach to achieving adaptive confidence intervals is employing **distribution shift** (Gibbs et al., 2021), which continuously updates the alpha involved in generating the quantile:
$$\alpha_{t+1} = \alpha_t + \gamma\left(\alpha - \sum_{s=1}^{t} w_s \,\mathrm{err}_s\right)$$

### Our Proposed Method

1. For our research, we propose a **different error function** $\frac{|y_i - \hat{\mu}(\boldsymbol{X}_i)|}{\sigma(\boldsymbol{X}_i)}$ to construct test prediction set C such that **return AND volatility** are considered, which we hope will outperform/complement distribution shift:
$$\widehat{C}_n^{\alpha}(\boldsymbol{Z}_{t+1}) = \hat{\mu}(\boldsymbol{X}_{t+1}) \pm \sigma(\boldsymbol{X}_{t+1}) \times \mathrm{Q}_{1-\alpha}\left(\left\{\frac{|y_i - \hat{\mu}(\boldsymbol{X}_i)|}{\sigma(\boldsymbol{X}_i)}\right\}_{i=1}^{n}\right)$$

### Choice of Models

Implementing our technique requires us to train a 2nd model to predict volatility, which is represented by σ.

1. Volatility (σ) Model[4]: We decided to use a Transformer model as its self-attention mechanism captures correlations between different cryptocurrencies.
2. Return (μ) Model: Catboost and LightGBM were chosen as our baseline return models due to their robust performance on tabular data.

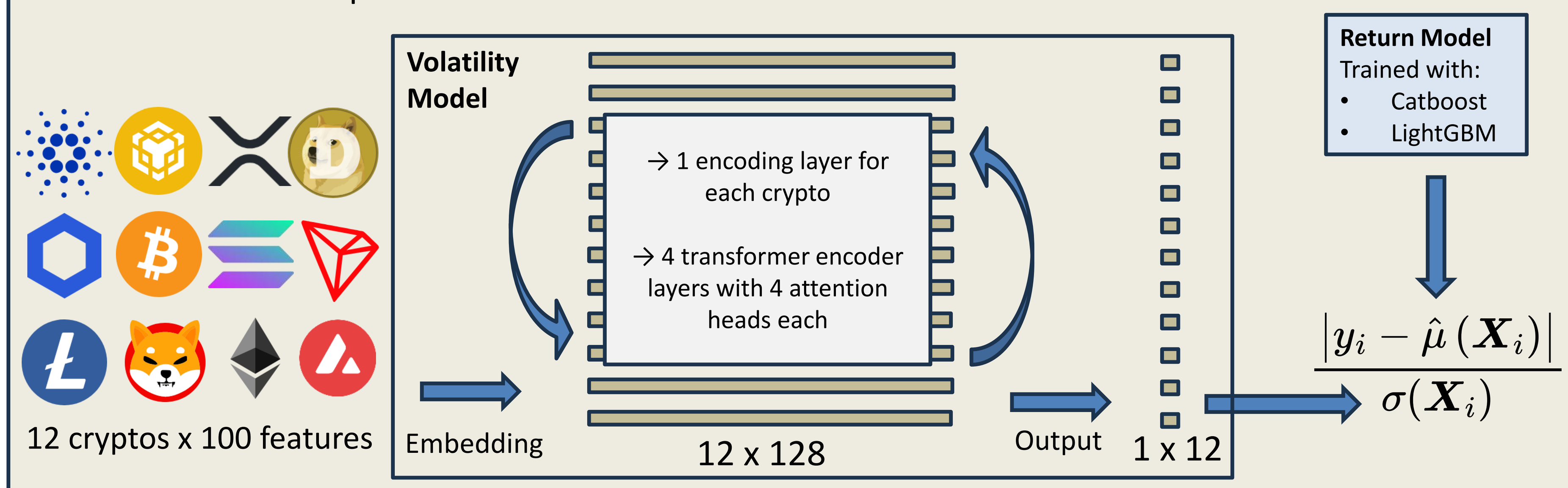


*Figure 4. Implementation of volatility transformer model with a baseline return model*

## Results

We tested our CP under volatility based on 2 metrics commonly used in conformal prediction.

- **Coverage:** Proportion of true values that fall inside the predicted confidence interval.
- **Average Size:** Mean width of the predicted confidence intervals.

Together, they reflect how **reliable** (coverage) and **precise** (size) your prediction intervals are.

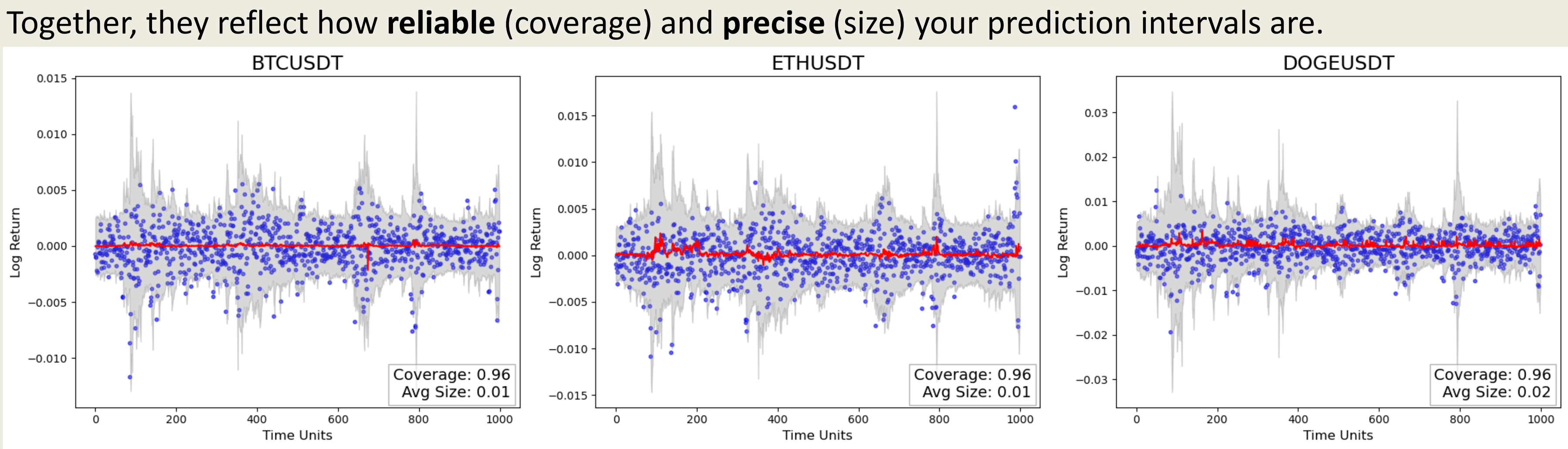


**Figure 5.** Results from the volatility Transformer model trained with the CatBoost return model at α = 0.06. The observed coverage slightly exceeds the target level, indicating conservative interval estimates.

The results below are shown for **α = 0.06**, which is representative of the performances of other α values.

### Coverage

| Symbol | Return Model Used | Original CP | CP under distribution shift | CP under volatility |
|---|---|---|---|---|
| BTCUSDT | CatBoost | 0.9796 | 0.9298 | 0.9627 |
| | LightGBM | 0.9799 | 0.9359 | 0.9617 |
| ETHUSDT | CatBoost | 0.9743 | 0.9298 | 0.9555 |
| | LightGBM | 0.9741 | 0.9362 | 0.9553 |
| DOGEUSDT | CatBoost | 0.9850 | 0.9304 | 0.9649 |
| | LightGBM | 0.9846 | 0.9357 | 0.9636 |

*Table 1:* ***CP under distribution shift*** *is the closest match to the desired confidence level and target coverage (1-0.06)*

### Average Size

| Symbol | Return Model Used | Original CP | CP under distribution shift | CP under volatility |
|---|---|---|---|---|
| BTCUSDT | CatBoost | 0.0105 | 0.0108 | 0.0077 |
| | LightGBM | 0.0107 | 0.0112 | 0.0078 |
| ETHUSDT | CatBoost | 0.0122 | 0.0125 | 0.0090 |
| | Light | 0.0121 | 0.0131 | 0.0091 |
| DOGEUSDT | CatBoost | 0.0226 | 0.0215 | 0.0158 |
| | LightGBM | 0.0227 | 0.0210 | 0.0157 |

*Table 2:* ***CP under volatility*** *provides the smallest average size, ie. has the highest precision.*

### Plot of coverage rate against α for Catboost

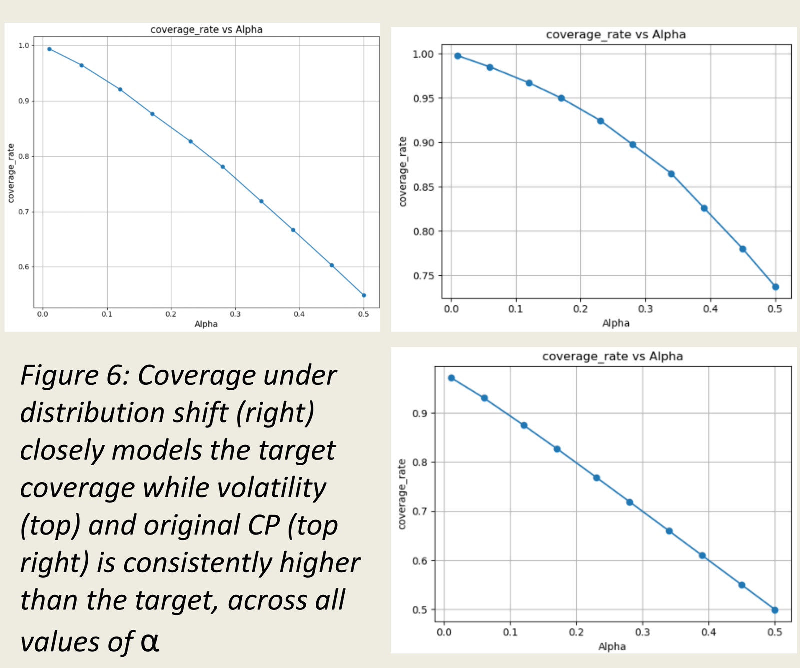


*Figure 6: Coverage under distribution shift (right) closely models the target coverage while volatility (top) and original CP (top right) is consistently higher than the target, across all values of α*

### Plot of average size against α for Catboost

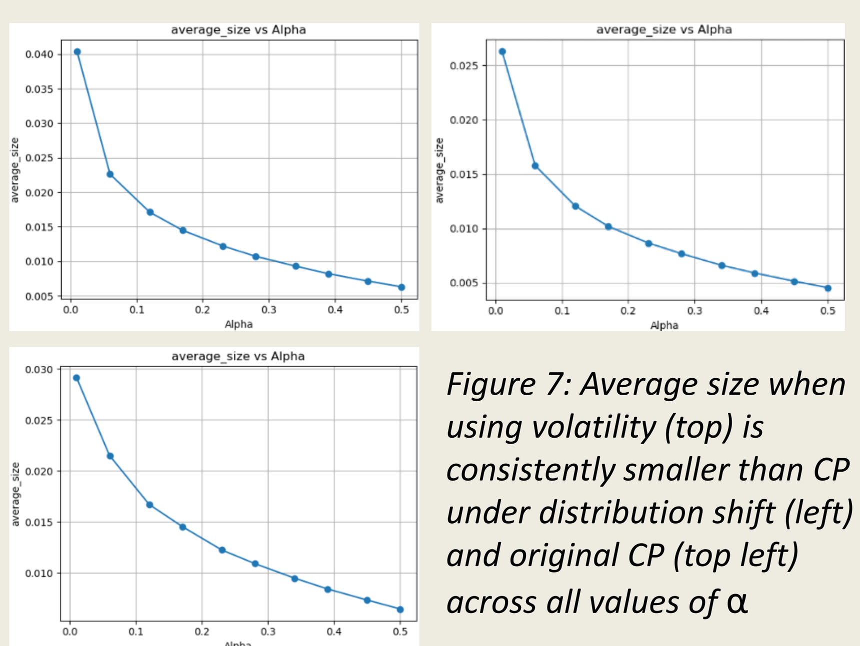


*Figure 7: Average size when using volatility (top) is consistently smaller than CP under distribution shift (left) and original CP (top left) across all values of α*

## Conclusion and Analysis

In financial markets, it is safe to assume that Conformal Prediction under distribution shift and under volatility will always outperform the original Conformal Prediction technique because of intervals being adaptive to sudden temporal changes.

Whether our technique outperforms distribution shift remains an open question, as it currently exhibits over-coverage while simultaneously achieving a smaller average interval size, which is not intuitive. Generally, we want models that match the desired coverage as closely as possible for conformal prediction. More research is required to determine whether our approach outperforms current industry techniques, and whether we can further take advantage of this tradeoff to reduce the average interval size while achieving desired coverage.

In terms of computational trade-offs, our volatility-based approach incurs higher space complexity due to training two models but offers significantly faster inference by eliminating the need for frequent alpha recalibration required in distribution shift methods.

## References


- Zaffran, M., Féron, O., Goude, Y., Josse, J., & Dieuleveut, A. (2022). Adaptive conformal predictions for time series. Proceedings of the 39th International Conference on Machine Learning, 162, 25834–25866. https://doi.org/10.48550/arXiv.2202.07282
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- Angelopoulos, A. N., & Bates, S. (2022). A gentle introduction to conformal prediction and distribution-free uncertainty quantification. arXiv preprint arXiv:2107.07511. https://doi.org/10.48550/arXiv.2107.07511