Homework 9

This homework is designed to give you experience with:

- Markov chain Monte Carlo (MCMC) for generating random numbers from a posterior distribution in a Bayesian model.

The assignment has several parts.

Part 1

Let $x_1, \ldots, x_n$ be a random sample from the normal distribution with mean μ and variance 1. Therefore, the likelihood is $p(x_1, \ldots, x_n|\mu) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp \frac{-1}{2}(x_i - \mu)^2$. The mean μ is the parameter of interest and a Bayesian approach is taken. Suppose it is known that μ must be greater than 1.0 and less than 3.0 and consider a uniform prior of this range. That is, $p(\mu) = \frac{1}{2} I_{(1.0,3.0)}(\mu)$. Recall that, by Bayes theorem, the posterior distribution $p(\mu|x_1, \ldots, x_n)$ is proportional to the product of the likelihood $p(x_1, \ldots, x_n|\mu)$ and the prior $p(\mu)$.

The sample data is:
3.08, 0.68, 2.09, 0.87, -0.02, 0.25, 1.98, 1.47, 1.95, 0.99

Use the classes and method in mcmc-sampler.rb to draw from the posterior distribution via Markov chain Monte Carlo in Ruby. (You may want to refer to the logistic regression example from a recent lecture.) Using Monte Carlo integration (if you wish, you may use the 'dbd/broccoli' library), answer the following questions:

1. What is the mean of the posterior distribution?
2. What is the standard deviation of the posterior distribution?
3. What is the probability that μ is greater than 1.7?

Name your script 'mean-estimation.rb'.

Part 2

Modify the mcmc-sampler.rb script to accommodate non-symmetric proposal densities. That is, the script currently only implements the Metropolis algorithm and you are to modify it to implement the Metropolis-Hastings algorithm.

Part 3

Write a script called 'mean-estimation.R' which does Part 1 entirely in R instead of Ruby.

Print out and staple all of the files you generated for each part. In addition, e-mail the files to hw09@dahlgrapevine.org.