Design and Analysis of Algorithms
Apr. 23, 2018

1. Given 2 matrices, **A** and **B** of dimension $p \times q$ and $q \times r$ respectively, what is the time complexity to compute $\mathbf{A} \times \mathbf{B}$. 8%
2. Show that building a max heap can be done in linear time. Describe the algorithm first then prove the algorithm takes linear time. 10%
3. Solve the recursion $T(n) = T(n/2) + 1$. 5%
4. Why any sorting algorithm using comparison as operator to sort needs $\Omega(n \log n)$ time. 8%
5. Given a computational problem A, by applying sorting, we can solve A, then A needs at least $\Omega(n \log n)$ time. Is this statement true? Why? 10%
6. Given a n nodes search tree, where each node has 3 children (3 pointers point to children), how many unused pointers in the tree? 8%
7. (a) Show that a binomial tree of depth k has at most degree k for every node in the tree. 7%
   (b) Describe the delete minimum algorithm for a min-binomial-heap of n nodes. Why the deletion can be done in $O(\log n)$ time? 7%
8. Show that radix sort **correctly** sorts n integers of length k in **linear time**, where k is a constant. You need to show that the algorithm correctly sorts n numbers, then argue that it takes linear time. 12%
9. (a) Build the max heap from the the tree in Figure 1. 6%
   (b) Then delete the maximum. 6%
10. Delete the node x from the the red-black tree shown in Figure 2. 8%
11. Multiplication of 4 matrices, $A_1A_2A_3A_4$, the dimensions are respectively 100×30, 30×70, 70×20, 20×50. Compute the sequence of multiplication that has the least number of multiplications. I need the value of the multiplications and the way you do the multiplication. 14%
12. Derive the recursion for computing the Longest Common Subsequence of two strings. 8%