EKF Case Study 1 – Vehicle Sensor Fusion

ECE 495/595 Lecture Slides

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Summary and Quick Links

These slides contain the following concepts:

- ▷ Unicycle observer state space model (Slide 3)
- ▷ Measurement model (Slide 6)

Unicycle Observer Model

- The eventual goal of the Kalman filter will be to combine various sensor data and produce a complete state estimate.
- Specifically, the filter will be designed to use the following information:
  - GPS receiver (x and y position)
  - Wheel encoders (forward speed v)
  - Gyroscope (yaw rate $\dot{\psi}$)
- A compass can be very hard to calibrate properly, so it won't be included.
- Based on this, define the observer state space model.

Unicycle Observer Model

- Discrete state equation:

$$\mathbf{X}_{k+1} = f(\mathbf{X}_k) \Rightarrow \begin{cases} x_{k+1} & = & x_k + T_s v_k \cos\psi_k \\ y_{k+1} & = & y_k + T_s v_k \sin\psi_k \\ \psi_{k+1} & = & \psi_k + T_s \dot{\psi}_k \\ v_{k+1} & = & v_k \\ \dot{\psi}_{k+1} & = & \dot{\psi}_k \end{cases}$$

- Observation equation:

$$\mathbf{Z}_k = h(\mathbf{X}_k) \Rightarrow \begin{cases} \mathrm{GPS}_x & = & x_k \\ \mathrm{GPS}_y & = & y_k \\ \mathrm{Enc.} & = & v_k \\ \mathrm{Gyro} & = & \dot{\psi}_k \end{cases}$$

Unicycle Observer Model

- Since the Kalman filter operates on a linear state space model, the state space must be linearized:

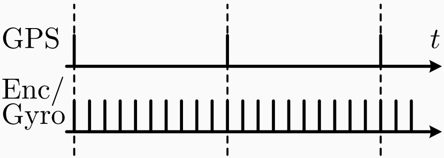
$$\mathbf{A}_k = \frac{\partial f}{\partial \mathbf{X}}\bigg|_k = \begin{bmatrix} 1 & 0 & -T_s v_k \sin\psi_k & T_s \cos\psi_k & 0 \\ 0 & 1 & T_s v_k \cos\psi_k & T_s \sin\psi_k & 0 \\ 0 & 0 & 1 & 0 & T_s \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_k = \frac{\partial h}{\partial \mathbf{X}}\bigg|_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Measurement Model

- Most GPS receivers update between 1 Hz and 10 Hz, while encoders and inertial sensors update much faster, usually at least 50 Hz:

Multi-Rate Measurements



- This means not all measurements are present during every iteration of the Kalman filter.

Measurement Model

- Therefore, the measurement model behind the Kalman filter must be constructed piece-wise.
- Each measurement will have a separate:
  - Standard deviation r
  - Measurement equation $h = f(\mathbf{X}_k)$
  - Measurement Jacobian row $\mathbf{C}_i$
- The complete measurement model is then constructed from all present measurements each time the filter is iterated.
- The filter sample time should be the same as the fastest sensor.

Measurement Model

- GPS East measurement:
  - Standard deviation r_{GPS_x}
  - Measurement equation $h_{\mathrm{GPS}_x} = x_k$
  - Jacobian row:

$$C_{\mathrm{GPS}_x} = \frac{\partial h_{\mathrm{GPS}_x}}{\partial \mathbf{X}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- GPS North measurement:
  - Standard deviation r_{GPS_y}
  - Measurement equation $h_{\mathrm{GPS}_y} = y_k$
  - Jacobian row:

$$C_{\mathrm{GPS}_y} = \frac{\partial h_{\mathrm{GPS}_y}}{\partial \mathbf{X}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Measurement Model

- Forward speed measurement:
  - Standard deviation $r_{\text{Enc.}}$
  - Measurement equation $h_{\text{Enc.}} = v_k$
  - Jacobian row:

$$C_{\text{Enc.}} = \frac{\partial h_{\text{Enc.}}}{\partial \mathbf{X}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- Gyro measurement:
  - Standard deviation r_{Gyro}
  - Measurement equation $h_{\text{Gyro}} = \dot{\psi}_k$
  - Jacobian row:

$$C_{\text{Gyro}} = \frac{\partial h_{\text{Gyro}}}{\partial \mathbf{X}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Measurement Model

- For example, on iterations where only encoder and gyro readings are available, the measurement model will be:

$$\mathbf{R} = \begin{bmatrix} r^2_{\text{Enc.}} & 0 \\ 0 & r^2_{\text{Gyro}} \end{bmatrix}, \qquad h(\mathbf{X}_k) = \begin{cases} h_{\text{Enc.}} & = & v_k \\ h_{\text{Gyro}} & = & \dot{\psi}_k \end{cases}$$

$$\mathbf{C}_k = \begin{bmatrix} C_{\text{Enc.}} \\ C_{\text{Gyro}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Measurement Model

- On iterations where GPS measurements are also available, the complete measurement model is used:

$$\mathbf{R} = \begin{bmatrix} r^2_{\text{GPS}_x} & 0 & 0 & 0 \\ 0 & r^2_{\text{GPS}_y} & 0 & 0 \\ 0 & 0 & r^2_{\text{Enc.}} & 0 \\ 0 & 0 & 0 & r^2_{\text{Gyro}} \end{bmatrix}$$

$$h(\mathbf{X}_k) = \begin{cases} h_{\text{GPS}_x} & = & x_k \\ h_{\text{GPS}_y} & = & y_k \\ h_{\text{Enc.}} & = & v_k \\ h_{\text{Gyro}} & = & \dot{\psi}_k \end{cases}$$

Measurement Model

$$\mathbf{C}_k = \begin{bmatrix} C_{\mathrm{GPS}_x} \\ C_{\mathrm{GPS}_y} \\ C_{\mathrm{Enc.}} \\ C_{\mathrm{Gyro}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$