Vehicle Kinematics Modeling

ECE 495/595 Lecture Slides

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Summary and Quick Links

These slides contain the following concepts:

What is a Vehicle Model?

- A vehicle model describes how controlling the vehicle's actuators will affect the *state* of the vehicle.
- The inputs to a vehicle model are typically the control signals that drive the system.
- The output of a vehicle model is usually the state of the vehicle, which includes any quantities that are needed for higher-level algorithms to function.

Kinematics vs. Dynamics

- Kinematic and dynamics models are the two broad types of models that can be used to describe a vehicle system.
- Kinematic models relate control variables to state variables through analysis of system geometry and basic properties of motion.
- Dynamic models also take Newton's three laws into account, typically relating how the forces generated by the actuators, along with forces external to the system, affect the acceleration of the vehicle as a whole.

Kinematics vs. Dynamics

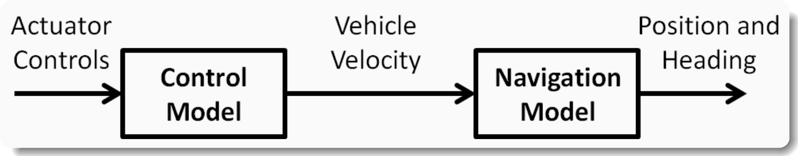
- In general, kinematic models are much simpler and require less knowledge of system parameters that can be difficult to measure or calibrate.

- On the other hand, dynamic models generally represent the real-world system more realistically, which could be a necessity depending on the application.

- For the purposes of this class, only kinematic models will be discussed. The subject of vehicle dynamics modeling easily fills an entire course, if not an entire degree program!

Control and Navigation Models

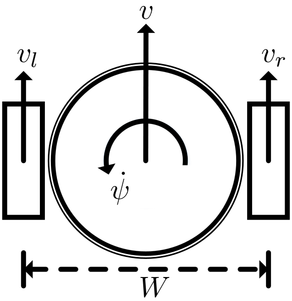
- Sometimes, it is helpful to split the complete vehicle model into two parts; one for control and one for navigation.
- The control model relates the actuator controls to the velocity of the vehicle.
- The navigation model relates the velocity of the vehicle to its position and heading in a global reference frame.



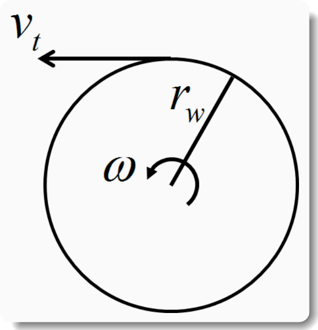
Differential-Drive Vehicle Kinematics

- A differential-drive vehicle is controlled by applying power independently to two wheels.
- The average speed of the two wheels governs the overall speed of the vehicle.
- Steering is controlled by varying the relative velocity of each wheel.

Differential-Drive Vehicle Kinematics



Differential-Drive Vehicle Kinematics



- The speed of a wheel at a tangent point v_t is kinematically related to the rotation speed of the wheel ω by the wheel's radius r_w:

Wheel Kinematics

$$v_t = r_w \omega$$

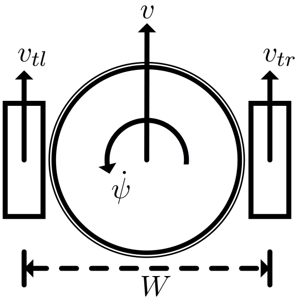
Differential-Drive Vehicle Kinematics

- The average speed controls the overall vehicle speed v:

$$v = \frac{1}{2}\left(v_{tr} + v_{tl}\right)$$

- After plugging in the wheel kinematics equations, this can be expressed in terms of wheel rotation speeds:

$$v = \frac{r_w}{2}\left(\omega_r + \omega_l\right)$$



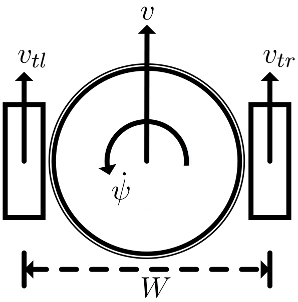
Differential-Drive Vehicle Kinematics

- The right wheel produces a counterclockwise (positive) rotation.
- The left wheel produces a clockwise (negative) rotation.
- Combined:

$$\dot{\psi} = \frac{v_{tr}}{W} - \frac{v_{tl}}{W}$$

- Plugging in wheel kinematics:

$$\dot{\psi} = \frac{r_w}{W}\left(\omega_r - \omega_l\right)$$



Differential-Drive Vehicle Kinematics

- Computing the velocity values v and $\dot{\psi}$ in terms of the wheel speeds is known as the <u>*forward kinematics*</u> of the differential-drive vehicle.

Forward Kinematics

$$\begin{cases} v = \frac{r_w}{2}(\omega_r + \omega_l) \\ \dot{\psi} = \frac{r_w}{W}(\omega_r - \omega_l) \end{cases}$$

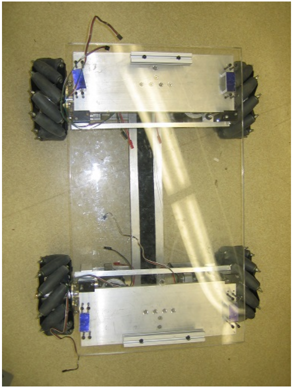
- Solving the same equations for the wheel speeds in terms of the velocity values produces the differential-drive <u>*inverse kinematics*</u>.

Inverse Kinematics

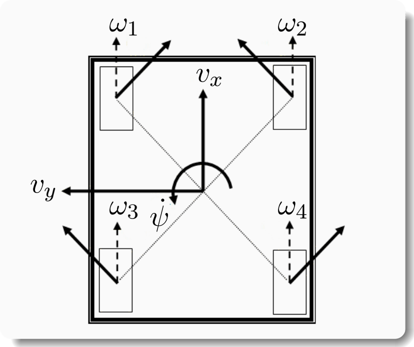
$$\begin{cases} \omega_l = \frac{1}{r_w}\left(v - \frac{W\dot{\psi}}{2}\right) \\ \omega_r = \frac{1}{r_w}\left(v + \frac{W\dot{\psi}}{2}\right) \end{cases}$$

Mecanum Omnidirectional Vehicle Kinematics

- Mecanum wheel ODVs have independent control of forward, lateral and yaw velocity.
- The wheels apply driving force at a 45 degree angle relative to the vehicle's body.
- These forces can be combined to generate motion in any direction, as well as rotation.
- Baxter mobility base at Dataspeed Inc:

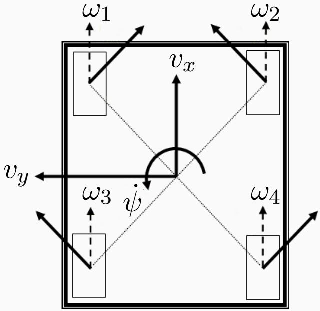


Mecanum ODV Kinematics



Mecanum ODV Kinematics

- The kinematics of a Mecanum ODV can be described by a linear transformation.
- The parameters a, b and c are constants that depend on the radius of the wheels and the geometry of the wheelbase.



Kinematic Equations

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{bmatrix} = \begin{bmatrix} a & -b & -c\\ a & b & c\\ a & b & -c\\ a & -b & c\end{bmatrix} \begin{bmatrix} v_x\\ v_y\\ \dot{\psi}\end{bmatrix}$$

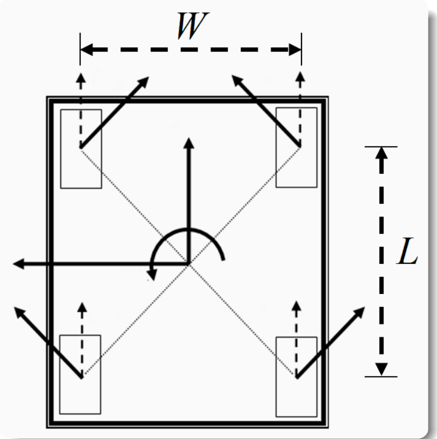
Mecanum ODV Kinematics

- The a and b parameters come directly from the same wheel kinematics as before:

$$a = b = \frac{1}{r_w}$$

- After some derivations, the c parameter is given by:

$$c = \frac{1}{2}\,\frac{W + L}{r_w}$$

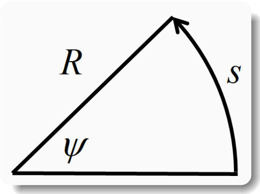


Car Kinematics

- A car's kinematics relates the wheel speeds and steering angles to the forward speed and yaw rate.
- A model of the dynamics of the vehicle would be required to actuate the pedals to control acceleration.

Car Kinematics

- The relationship between vehicle speed, yaw rate and turning radius can be seen from the simple circular arc length formula:



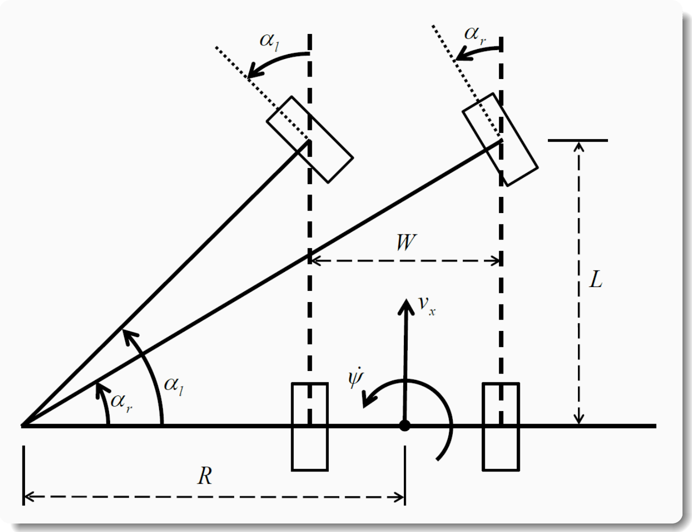
Circular Arc Length

$$s = R\psi$$

Time Derivative

$$v = R\dot{\psi}$$

Car Kinematics



Car Kinematics

- The turning radius is related to the left and right steering angles using some simple trigonometric analysis.

$$\alpha_l = \tan^{-1}\left(\frac{L}{R - \frac{W}{2}}\right)$$

$$\alpha_r = \tan^{-1}\left(\frac{L}{R + \frac{W}{2}}\right)$$

Car Kinematics

- Plugging in the circular arc segment equations, the steering angles can be computed from speed and yaw rate by:

Left Steering Angle

$$\alpha_l = \tan^{-1}\left(\frac{L\dot{\psi}}{v - \frac{W}{2}\dot{\psi}}\right)$$

Right Steering Angle

$$\alpha_r = \tan^{-1}\left(\frac{L\dot{\psi}}{v + \frac{W}{2}\dot{\psi}}\right)$$

Car Kinematics

- Similarly, the left and right wheel speeds will be different when the vehicle is turning.
- This can be seen by computing the speed of each wheel using the circular arc formula:

Left Wheel Speed

$$\begin{aligned} v_l &= \left(R - \tfrac{W}{2}\right)\dot{\psi} \\ &= R\dot{\psi} - \tfrac{W}{2}\dot{\psi} \\ &= v - \tfrac{W}{2}\dot{\psi} \end{aligned}$$

$$\Rightarrow \omega_l = \frac{v_l}{r_w}$$

Right Wheel Speed

$$\begin{aligned} v_r &= \left(R + \tfrac{W}{2}\right)\dot{\psi} \\ &= R\dot{\psi} + \tfrac{W}{2}\dot{\psi} \\ &= v + \tfrac{W}{2}\dot{\psi} \end{aligned}$$

$$\Rightarrow \omega_r = \frac{v_r}{r_w}$$

Car Kinematics

- DISCLAIMER: This simple kinematics model is only valid at low speed, where tire slip is negligible.

Forward Kinematics

$$\begin{cases} v &= \dfrac{r_w}{2}\left(\omega_l + \omega_r\right) \\ \dot{\psi} &= \dfrac{r_w}{2}\dfrac{(\omega_l + \omega_r)\tan\alpha_l}{L + \frac{W}{2}\tan\alpha_l} \\ & \text{or} \\ \dot{\psi} &= \dfrac{r_w}{2}\dfrac{(\omega_l + \omega_r)\tan\alpha_r}{L - \frac{W}{2}\tan\alpha_r} \end{cases}$$

Inverse Kinematics

$$\begin{cases} \omega_l &= \dfrac{1}{r_w}\left(v - \frac{W}{2}\dot{\psi}\right) \\ \omega_r &= \dfrac{1}{r_w}\left(v + \frac{W}{2}\dot{\psi}\right) \\ \alpha_l &= \tan^{-1}\left(\dfrac{L\dot{\psi}}{v - \frac{W}{2}\dot{\psi}}\right) \\ \alpha_r &= \tan^{-1}\left(\dfrac{L\dot{\psi}}{v + \frac{W}{2}\dot{\psi}}\right) \end{cases}$$

Car Kinematics

- The steering wheel of a car is usually linked to the front wheels with a mechanical gear ratio.
- Therefore, to get a certain tire steering angle, the steering wheel needs to rotate to some multiple of that angle.
- Based on this, a simple model of the steering wheel can be constructed:

$$\boxed{\alpha_s = \gamma\alpha}$$

- α is the equivalent bicycle steering angle (ignoring Ackermann kinematics), and α_s is the steering wheel angle.
- γ is the ratio between steering wheel angle and tire angle.

Car Kinematics

- Tire angle is related to turning radius by:

$$\alpha = \tan^{-1}\left(\frac{L}{R}\right)$$

- This means the steering wheel angle is related to turning radius by:

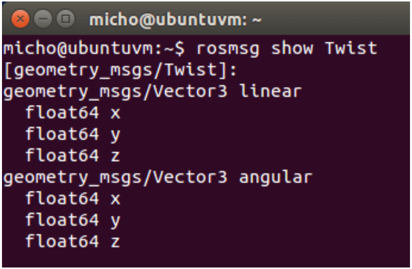
$$\alpha_s = \gamma \tan^{-1}\left(\frac{L}{R}\right)$$

- Plugging in the relationship between turning radius, speed, and yaw rate:

$$\alpha_s = \gamma \tan^{-1}\left(\frac{L\dot{\psi}}{v}\right)$$

The Twist Message

- A vehicle's velocity command is typically represented in a *geometry_msgs/Twist* message.



- The Twist message contains both linear and angular velocity values.

- For ground robots, only the *linear.x*, *linear.y* and *angular.z* values are typically used.

The Twist Message

- A twist message will be used to control simulated versions of each of the three vehicles described in these notes.
- The individual velocity values correspond to particular elements in the *geometry_msgs/Twist* message.

Differential Drive

$$\begin{aligned} v &\equiv \text{twist\_msg.linear.x} \\ \dot{\psi} &\equiv \text{twist\_msg.angular.z} \end{aligned}$$

Mecanum ODV

$$\begin{aligned} v_x &\equiv \text{twist\_msg.linear.x} \\ v_y &\equiv \text{twist\_msg.linear.y} \\ \dot{\psi} &\equiv \text{twist\_msg.angular.z} \end{aligned}$$

Ackermann

$$\begin{aligned} v &\equiv \text{twist\_msg.linear.x} \\ \dot{\psi} &\equiv \text{twist\_msg.angular.z} \end{aligned}$$