ECE 495/595 Lecture Slides

Winter 2017

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Summary and Quick Links

These slides contain the following concepts:

Introduction
GPS Satellites

- The GPS network currently consists of 31 active satellites, which occupy 6 different orbital planes.

- Each of the orbital planes has an inclination of about 55 degrees, with each one having different longitudinal orientations.

- The orbits are designed to guarantee a minimum of 6 satellites being visible from any point on Earth.

- The orbital period of the orbits is 11 hours, 58 minutes, which makes sure the satellites pass over the same paths every day.

- Each satellite broadcasts signals at two different frequencies, named L1 (1575.42 MHz) and L2 (1227.6 MHz).
- For ranging, a pseudorandom code is modulated on each of these carrier frequencies, where the code is directly associated with the on-board atomic clocks.
- Additionally, the satellites broadcast details about their orbits so receivers can compute the position of the satellite, which is very important for positioning.
- These orbital parameters are known as ephemeris data.

Introduction

GPS Receivers

- A GPS receiver keeps its own time and generates the same pseudorandom codes as the satellites.
- As the GPS signals are received, the receiver correlates its internal code with the received code, and measures the time offset
- After multiplying this time offset by the speed of light, the range to the particular satellite is computed. This computed range is known as a *pseudorange*.
- If this were the whole story, things would be very simple!

Introduction
GPS Receivers

- A standard model for the measured pseudorange:

$$r = \rho + c(\tau - \tau_s) + I + T + m_p$$

  - r – measured pseudorange
  - ρ – true distance between receiver and satellite
  - c – speed of light
  - τ – receiver clock error
  - τ_s – satellite clock bias
  - $I,\ T,\ m_p$ – atmospheric and multi-path delays

- The receiver clock error is the most significant error source.

Introduction
GPS Receivers

- Since the correlation of the pseudorandom ranging codes is inherently inaccurate, using only pseudoranges, position accuracy is limited to about 10 meters.
- To achieve better accuracy, other signals are available, but require careful processing to use properly.
- GPS receivers also measure the phase of the received carrier wave, and also measure the Doppler shift of the incoming wave.

Introduction
Carrier Phase Measurements

- Carrier phase measurements are very accurate. Multiplying the phase angle accuracy by the wavelength, the carrier phase signal is accurate to about 1 mm.
- However, there is no way of telling how many wave cycles have occurred since the signal was transmitted.
- Lots of research has been done into estimating the number of cycles, known as carrier phase ambiguity.
- A technique called Precise Point Positioning (PPP) makes use of the carrier phase to achieve 10 cm accuracy. However, this level of accuracy can only usually be achieved when the receiver is left in one spot for extended periods of time.

- The Doppler shift measurement provides a fairly accurate indication of the relative velocity between the receiver and the satellite.
- GPS positioning algorithms use the Doppler shift measurement to filter the often-noisy pseudorange measurements, and detect large carrier phase measurement shifts.
- Doppler measurements are also immune to atmospheric delays and multipath, which can help to mitigate the subsequent errors.

Introduction
Differential GPS and RTK

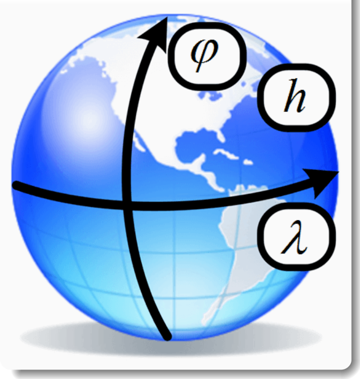
- It is possible to achieve down to about 2 cm accuracy using differential GPS and Real-Time Kinematic (RTK) algorithms.
- The main assumption of differential GPS is that a fixed base station with well-known coordinates is available, along with the raw pseudorange and carrier phase measurement data from it.
- If the base station is nearby (within about 20 km), then atmospheric and clock delay effects can be greatly mitigated by differencing the measurements.

GPS Coordinate Systems

- GPS receivers output their position estimates in latitude (ϕ), longitude (λ) and altitude (h), which are known as *geodetic* coordinates.

- The term geodetic means that the coordinates are relative to a *geoid*, which is an elliptical model of the Earth.

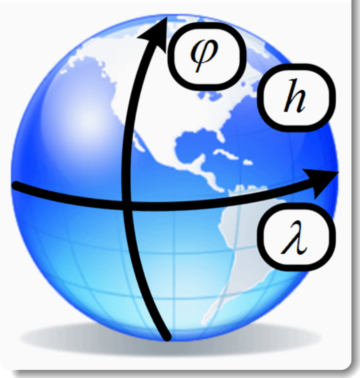
Geodetic Coordinates



GPS Coordinate Systems

- Latitude, longitude and altitude (geodetic) coordinates are not usually good for ground robot navigation.
- It is better to have a Cartesian coordinate system instead.
- There are a few different ways to linearize GPS coordinates.

Geodetic Coordinates



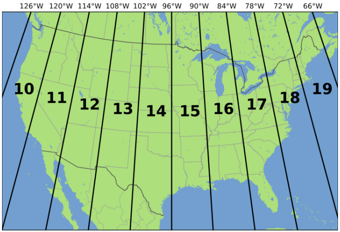
East-North-Up Coordinates

- Define a tangent plane to the Earth at a specific point.
- Project all geodetic coordinates onto this coordinate frame.
- The x axis is aligned with due East, y axis North, z axis up.
- More inaccurate as distance from tangent point increases.

ENU Coordinates

Universal Transverse Mercator Coordinates

- Universal Transverse Mercator (UTM) coordinates divides Earth into 60 zones, each spanning 6 degrees of longitude.
- The coordinates in each zone are projected to a Cartesian coordinate frame.
- Each zone has a different projection algorithm based on an elliptical model of the Earth.



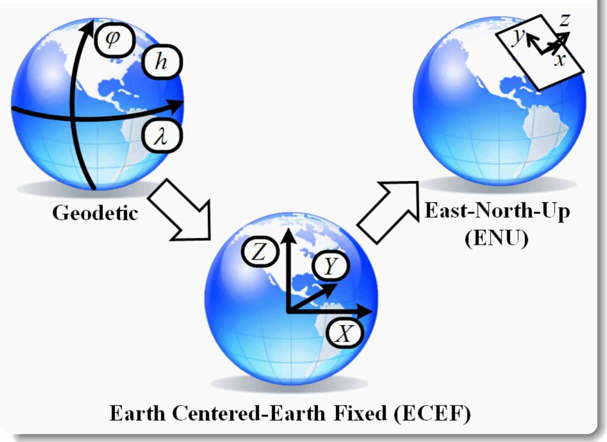
http://en.wikipedia.org/wiki/File:Utm-zones.svg

Universal Transverse Mercator Coordinates

- UTM projections are only defined from 80 degrees south to 84 degrees north. The polar regions require special handling.
- Pros:
  - Very accurate over wide areas.
  - Unique projection from geodetic to UTM position and zone.
- Cons:
  - Relatively complex equations to convert to and from geodetic coordinates.
  - Large jumps in the x and y coordinate values at zone boundaries.

Converting from Geodetic to ENU

Conversion Sequence



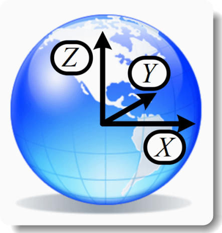
Converting from Geodetic to ENU

- Convert geodetic (ϕ, λ and h) into Earth-Centered Earth-Fixed (ECEF) (X, Y and Z):

Geodetic to ECEF

$$\begin{aligned} X &= [N(\phi) + h]\cos\phi\cos\lambda \\ Y &= [N(\phi) + h]\cos\phi\sin\lambda \\ Z &= \left[N(\phi)(1 - e^2) + h\right]\sin\phi \end{aligned}$$

$$N(\phi) = \frac{a}{\sqrt{1 - e^2\sin^2\phi}}$$



- $a = 6,378,137$m – Earth's semi-major axis length
- $e^2 = 6.6943799014 \times 10^{-3}$ – Earth's eccentricity, squared

Converting from Geodetic to ENU

- Select a reference point in geodetic coordinates (ϕ_r, λ_r, h_r).
- Convert reference point to ECEF (X_r, Y_r, Z_r) and store it.
- Convert any other geodetic points (ϕ, λ, h) into ENU by first converting to ECEF, then applying a regular coordinate transformation.

Converting from Geodetic to ENU

- The transformation is performed by:
  - Translating the ECEF origin to the ENU origin by the reference point's ECEF coordinates.
  - Applying the rotation matrix to align x due East and z normal to the surface (up).

Coordinate transformation from ECEF to ENU

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin\lambda_r & \cos\lambda_r & 0 \\ -\sin\phi_r\cos\lambda_r & -\sin\phi_r\sin\lambda_r & \cos\phi_r \\ \cos\phi_r\cos\lambda_r & \cos\phi_r\sin\lambda_r & \sin\phi_r \end{bmatrix} \begin{bmatrix} X - X_r \\ Y - Y_r \\ Z - Z_r \end{bmatrix}$$

Example

- Let's compute relative position to a reference point!
- Reference point's geodetic coordinates:

  $42^\circ\ 40'\ 14.68''$ N
  $83^\circ\ 13'\ 2.02''$ W
  282 m

- Our current geodetic coordinates:

  $42^\circ\ 40'\ 19.77''$ N
  $83^\circ\ 12'\ 54.01''$ W
  286 m

Example

- First, convert to decimal degrees
  - 60 arc-minutes in a degree
  - 60 arc-seconds in an arc-minute $\Rightarrow$ 3600 arc-seconds in a degree
  - North positive, South negative
  - East positive, West negative

- Reference:

$$42^\circ\ 40'\ 14.68''\ \mathrm{N} = 42 + \frac{40}{60} + \frac{14.68}{3600} = 42.6707444^\circ$$

$$83^\circ\ 13'\ 2.02''\ \mathrm{W} = -\left(83 + \frac{13}{60} + \frac{2.02}{3600}\right) = -83.2172277^\circ$$

Example

- Current coordinates:

$$42^\circ\ 40'\ 19.77''\ \mathrm{N} = 42 + \frac{40}{60} + \frac{19.77}{3600} = 42.6721583^\circ$$

$$83^\circ\ 12'\ 54.01''\ \mathrm{W} = -\left(83 + \frac{12}{60} + \frac{54.01}{3600}\right) = -83.2150027^\circ$$

Example
East-North-Up

- Compute reference ECEF coordinates from geodetic:

Reference Coordinates

$$\underbrace{42.6707444^\circ}_{\phi_r},\ \underbrace{-83.2172277^\circ}_{\lambda_r},\ \underbrace{282}_{h_r}\ \text{m}$$

Geodetic to ECEF

$$\begin{aligned}
X &= [N(\phi_r) + h_r] \cos\phi_r \cos\lambda_r &&= 554744.48 \text{ m} \\
Y &= [N(\phi_r) + h_r] \cos\phi_r \sin\lambda_r &&= -4664154.80 \text{ m} \\
Z &= \left[N(\phi_r)(1 - e^2) + h_r\right] \sin\phi_r &&= 4300870.75 \text{ m}
\end{aligned}$$

Example
East-North-Up

- Compute current position's ECEF coordinates from geodetic:

Current Coordinates

$$\underbrace{42.6721583^\circ}_{\phi},\ \underbrace{-83.2150027^\circ}_{\lambda},\ \underbrace{286}_{h}\ \text{m}$$

Geodetic to ECEF

$$\begin{aligned}
X &= [N(\phi) + h]\cos\phi\cos\lambda &&= 554913.37\ \text{m} \\
Y &= [N(\phi) + h]\cos\phi\sin\lambda &&= -4664030.46\ \text{m} \\
Z &= \left[N(\phi)(1 - e^2) + h\right]\sin\phi &&= 4300988.95\ \text{m}
\end{aligned}$$

Example
East-North-Up

▷ Finally, compute ENU from ECEF using reference geodetic and ECEF coordinates:

Coordinate transformation from ECEF to ENU

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin\lambda_r & \cos\lambda_r & 0 \\ -\sin\phi_r\cos\lambda_r & -\sin\phi_r\sin\lambda_r & \cos\phi_r \\ \cos\phi_r\cos\lambda_r & \cos\phi_r\sin\lambda_r & \sin\phi_r \end{bmatrix} \begin{bmatrix} X - X_r \\ Y - Y_r \\ Z - Z_r \end{bmatrix}$$

Coordinate transformation from ECEF to ENU

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.993 & 0.118 & 0 \\ -0.08 & 0.673 & 0.735 \\ 0.086 & -0.73 & 0.678 \end{bmatrix} \begin{bmatrix} 168.89 \\ 124.34 \\ 118.2 \end{bmatrix} = \begin{bmatrix} 182.4\text{ m} \\ 157.07\text{ m} \\ 3.995\text{ m} \end{bmatrix}$$

Example
UTM

- ▷ To compute the relative position in UTM coordinates, first compute the coordinates of reference and current points:

Reference UTM Coordinates

$$(x,\ y,\ z) = (318311.694\ \text{m}, \quad 4726635.938\ \text{m}, \quad 282.0\ \text{m})$$

$$\text{zone} = 17$$
$$\text{hemisphere} = 1\ \text{(northern)}$$

Current UTM Coordinates

$$(x,\ y,\ z) = (318498.143\ \text{m}, \quad 4726788.167\ \text{m}, \quad 286.0\ \text{m})$$

$$\text{zone} = 17$$
$$\text{hemisphere} = 1\ \text{(northern)}$$

Example
UTM

- The relative position is then obtained by simply subtracting the coordinates.
- This is only valid if they are in the same zone and hemisphere!

Relative Position

$$\underbrace{\begin{bmatrix} 318498.143 \text{ m} \\ 4726788.167 \text{ m} \\ 286.0 \text{ m} \end{bmatrix}}_{\text{current}} - \underbrace{\begin{bmatrix} 318311.694 \text{ m} \\ 4726635.938 \text{ m} \\ 282.0 \text{ m} \end{bmatrix}}_{\text{reference}} = \underbrace{\begin{bmatrix} 186.45 \text{ m} \\ 152.23 \text{ m} \\ 4.0 \text{ m} \end{bmatrix}}_{\text{relative position}}$$

Difference Between UTM and ENU

- The relative position vectors computed with ENU and UTM are quite different, even over the short distance of the example:

UTM

$$\begin{bmatrix} 186.45 \text{ m} \\ 152.23 \text{ m} \\ 4.0 \text{ m} \end{bmatrix}$$

ENU

$$\begin{bmatrix} 182.4 \text{ m} \\ 157.07 \text{ m} \\ 3.995 \text{ m} \end{bmatrix}$$

- However, the magnitude of the vectors are the same:

Magnitude

240.74 m

Difference Between UTM and ENU

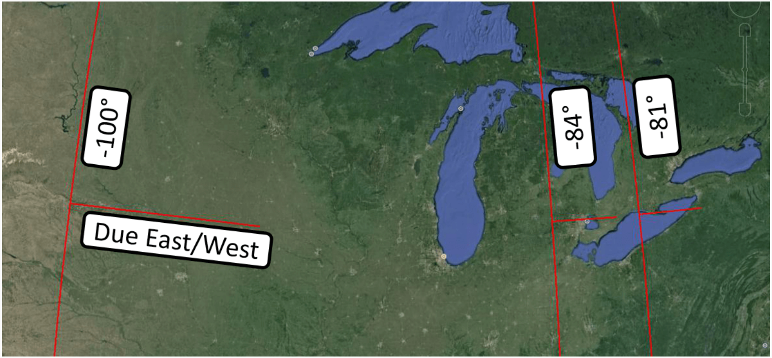
- This indicates that the UTM and ENU coordinate frames have an orientation difference between them.

$$\cos\theta = \frac{\mathbf{v}_{\text{ENU}} \cdot \mathbf{v}_{\text{UTM}}}{\|\mathbf{v}_{\text{ENU}}\| \cdot \|\mathbf{v}_{\text{UTM}}\|} = 0.99966 \quad \Rightarrow \theta = 0.026 \text{ rad} \approx 1.5^\circ$$

- The ENU coordinate frame is aligned to due East at the linearization point.
- The UTM coordinate frame is aligned to due East at the central meridian, which is the longitude of the center of the particular zone.

Difference Between UTM and ENU

Due East Lines at Different Longitude



Difference Between UTM and ENU

- This angle difference between grid North (UTM frame) and true North (ENU frame) is called the *convergence angle*.

Convergence Angle

$$\gamma = \tan^{-1}\left[\tan(\lambda - \lambda_0)\sin\phi\right]$$

  - ϕ, λ – Geodetic coordinates
  - λ_0 – Longitude of the central meridian of the UTM zone.

- The relationship between grid North and true North is then defined as:

$$\psi_{UTM} = \psi_{ENU} + \gamma$$

Difference Between UTM and ENU

- Going back to the example, the convergence at the reference coordinates is:

$$\gamma = \tan^{-1}\{\tan[-83.2172277^\circ - (-81^\circ)]\sin(42.6707444^\circ)\}$$
$$= -0.0262$$

- which matches the observation of the angle between the UTM and ENU relative position vectors.