



ECE 495/595 Lecture Slides

Winter 2017

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Summary and Quick Links

These slides contain the following concepts:

- ▷ Unicycle observer state space model (Slide [3](#))
- ▷ Measurement model (Slide [6](#))

Unicycle Observer Model

- ▷ The eventual goal of the Kalman filter will be to combine various sensor data and produce a complete state estimate.
- ▷ Specifically, the filter will be designed to use the following information:
 - > GPS receiver (x and y position)
 - > Wheel encoders (forward speed v)
 - > Gyroscope (yaw rate $\dot{\psi}$)
- ▷ A compass can be very hard to calibrate properly, so it won't be included.
- ▷ Based on this, define the observer state space model.

Unicycle Observer Model

- ▷ Discrete state equation:

$$\mathbf{X}_{k+1} = f(\mathbf{X}_k) \Rightarrow \begin{cases} x_{k+1} &= x_k + T_s v_k \cos \psi_k \\ y_{k+1} &= y_k + T_s v_k \sin \psi_k \\ \psi_{k+1} &= \psi_k + T_s \dot{\psi}_k \\ v_{k+1} &= v_k \\ \dot{\psi}_{k+1} &= \dot{\psi}_k \end{cases}$$

- ▷ Observation equation:

$$\mathbf{Z}_k = h(\mathbf{X}_k) \Rightarrow \begin{cases} \text{GPS}_x &= x_k \\ \text{GPS}_y &= y_k \\ \text{Enc.} &= v_k \\ \text{Gyro} &= \dot{\psi}_k \end{cases}$$

Unicycle Observer Model

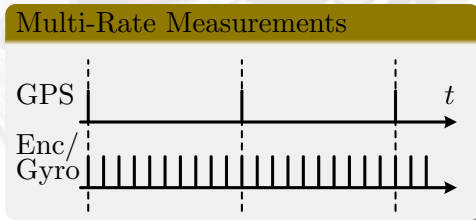
- ▷ Since the Kalman filter operates on a linear state space model, the state space must be linearized:

$$\mathbf{A}_k = \left. \frac{\partial f}{\partial \mathbf{X}} \right|_k = \begin{bmatrix} 1 & 0 & -T_s v_k \sin \psi_k & T_s \cos \psi_k & 0 \\ 0 & 1 & T_s v_k \cos \psi_k & T_s \sin \psi_k & 0 \\ 0 & 0 & 1 & 0 & T_s \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}_k = \left. \frac{\partial h}{\partial \mathbf{X}} \right|_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Measurement Model

- ▷ Most GPS receivers update between 1 Hz and 10 Hz, while encoders and inertial sensors update much faster, usually at least 50 Hz:



- ▷ This means not all measurements are present during every iteration of the Kalman filter.

Measurement Model

- ▷ Therefore, the measurement model behind the Kalman filter must be constructed piece-wise.
- ▷ Each measurement will have a separate:
 - > Standard deviation r
 - > Measurement equation $h = f(\mathbf{X}_k)$
 - > Measurement Jacobian row \mathbf{C}_i
- ▷ The complete measurement model is then constructed from all present measurements each time the filter is iterated.
- ▷ The filter sample time should be the same as the fastest sensor.

Measurement Model

- ▷ GPS East measurement:
 - > Standard deviation r_{GPS_x}
 - > Measurement equation $h_{\text{GPS}_x} = x_k$
 - > Jacobian row:

$$C_{\text{GPS}_x} = \frac{\partial h_{\text{GPS}_x}}{\partial \mathbf{X}} = [1 \quad 0 \quad 0 \quad 0 \quad 0]$$

- ▷ GPS North measurement:
 - > Standard deviation r_{GPS_y}
 - > Measurement equation $h_{\text{GPS}_y} = y_k$
 - > Jacobian row:

$$C_{\text{GPS}_y} = \frac{\partial h_{\text{GPS}_y}}{\partial \mathbf{X}} = [0 \quad 1 \quad 0 \quad 0 \quad 0]$$

Measurement Model

▷ Forward speed measurement:

- > Standard deviation $r_{\text{Enc.}}$
- > Measurement equation $h_{\text{Enc.}} = v_k$
- > Jacobian row:

$$C_{\text{Enc.}} = \frac{\partial h_{\text{Enc.}}}{\partial \mathbf{X}} = [0 \quad 0 \quad 0 \quad 1 \quad 0]$$

▷ Gyro measurement:

- > Standard deviation r_{Gyro}
- > Measurement equation $h_{\text{Gyro}} = \dot{\psi}_k$
- > Jacobian row:

$$C_{\text{Gyro}} = \frac{\partial h_{\text{Gyro}}}{\partial \mathbf{X}} = [0 \quad 0 \quad 0 \quad 0 \quad 1]$$

Measurement Model

- ▷ For example, on iterations where only encoder and gyro readings are available, the measurement model will be:

$$\mathbf{R} = \begin{bmatrix} r_{\text{Enc.}}^2 & 0 \\ 0 & r_{\text{Gyro}}^2 \end{bmatrix}, \quad h(\mathbf{X}_k) = \begin{cases} h_{\text{Enc.}} & = v_k \\ h_{\text{Gyro}} & = \dot{\psi}_k \end{cases}$$

$$\mathbf{C}_k = \begin{bmatrix} C_{\text{Enc.}} \\ C_{\text{Gyro}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Measurement Model

- ▷ On iterations where GPS measurements are also available, the complete measurement model is used:

$$\mathbf{R} = \begin{bmatrix} r_{\text{GPS}_x}^2 & 0 & 0 & 0 \\ 0 & r_{\text{GPS}_y}^2 & 0 & 0 \\ 0 & 0 & r_{\text{Enc.}}^2 & 0 \\ 0 & 0 & 0 & r_{\text{Gyro}}^2 \end{bmatrix}$$

$$h(\mathbf{X}_k) = \begin{cases} h_{\text{GPS}_x} & = & x_k \\ h_{\text{GPS}_y} & = & y_k \\ h_{\text{Enc.}} & = & v_k \\ h_{\text{Gyro}} & = & \dot{\psi}_k \end{cases}$$

Measurement Model

$$\mathbf{C}_k = \begin{bmatrix} C_{\text{GPS}_x} \\ C_{\text{GPS}_y} \\ C_{\text{Enc.}} \\ C_{\text{Gyro}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$