EKF Case Study 2 – IMU Attitude Filter

ECE 495/595 Lecture Slides

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Summary and Quick Links

These slides contain the following concepts:

- ▷ Observer state space model (Slide 3)
- ▷ Measurement model (Slide 7)

State Model

- Using an Inertial Measurement Unit (IMU) consisting of a 3-axis accelerometer and a 3-axis gyroscope, a Kalman filter can be used to estimate the attitude of the sensor.
- Attitude is not the same as orientation. Attitude refers to just roll and pitch, and ignores yaw.
- Without incorporating a magnetometer, it isn't possible to observably estimate a yaw component.

State Model

- Since the filter isn't estimating yaw, the z parameter of the quaternion can be assumed to be zero.

- Therefore, the state vector for the filter is:

State Vector

$$\mathbf{X} = \begin{bmatrix} w \\ x \\ y \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

State Model

- Using the time derivative equations of each quaternion parameter and plugging in $z = 0$, the non-linear state space equations are:

State Equations

$$\mathbf{X}_{k+1} = f(\mathbf{X}_k) \Rightarrow \begin{cases} w_{k+1} & = & w_k - \frac{1}{2}T_s\left(x\omega_x + y\omega_y\right) \\ x_{k+1} & = & x_k + \frac{1}{2}T_s\left(w\omega_x + y\omega_z\right) \\ y_{k+1} & = & y_k + \frac{1}{2}T_s\left(w\omega_y - x\omega_z\right) \\ \omega_{x|k+1} & = & \omega_{x|k} \\ \omega_{y|k+1} & = & \omega_{y|k} \\ \omega_{z|k+1} & = & \omega_{z|k} \end{cases}$$

State Model

- The linearization of the state equations is obtained by computing the Jacobian:

$$\mathbf{A} = \frac{\partial f}{\partial \mathbf{X}} = \mathbf{I}_6 + \frac{1}{2} T_s \begin{bmatrix} 0 & -\omega_x & -\omega_y & -x & -y & 0 \\ \omega_x & 0 & \omega_z & w & 0 & y \\ \omega_y & -\omega_z & 0 & 0 & w & -x \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Measurement Model

- The accelerometer measures the orientation of the gravity vector in the sensor frame.
- The measurement model for the accelerometer can be constructed from the equivalent rotation matrix of a quaternion.

$$R_{q_B^A} = \begin{bmatrix} w^2 + x^2 - y^2 - z^2 & 2(xy - wz) & 2(xz + wy) \\ 2(xy + wz) & w^2 - x^2 + y^2 - z^2 & 2(-wx + yz) \\ 2(xz - wy) & 2(wx + yz) & w^2 - x^2 - y^2 + z^2 \end{bmatrix}$$

Measurement Model

- Assuming the z parameter of the quaternion is zero, and the gravity vector is aligned with the negative z axis, the accelerometer measurement model is constructed from:

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = \begin{bmatrix} w^2 + x^2 - y^2 & 2xy & 2wy \\ 2xy & w^2 - x^2 + y^2 & -2wx \\ -2wy & 2wx & w^2 - x^2 - y^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = \begin{bmatrix} -2wy \\ 2wx \\ x^2 + y^2 - w^2 \end{bmatrix}$$

Measurement Model

- The gyro measurement model is much simpler, as the measurement values directly correspond to a state variable:

$$\begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Measurement Model

- The **C** matrix is then obtained by computing the Jacobian of the combined accelerometer and gyro measurement functions:

$$\mathbf{C} = \frac{\partial h}{\partial X} = \begin{bmatrix} -2y & 0 & -2w & 0 & 0 & 0 \\ 2x & 2w & 0 & 0 & 0 & 0 \\ -2w & 2x & 2y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- With a closed-form expression for the linearized state space model, it is now possible to run an EKF algorithm to estimate the attitude of the sensor.