Considered the following 3x3 matrix:

$$A = \begin{bmatrix} 8 & 5 & -4 \\ -9 & 1 & 3 \\ 1 & -1 & -6 \end{bmatrix}$$

(a) Find the following norms of the matrix

i. $\|A\|_\infty$

ii. $\|A\|_{Frob}$

iii. $\|A\|_2$

(b) Find the eigenvalues of the matrix A

(c) What is the condition number of this matrix? Use any convenient matrix norms in this calculation.

(d) Is the system ill-condition? Explain your answer.

Ai) $\sum_{j=1}^{3} |a_{ij}| = |8| + |5| + |-4| = 17$

$\sum_{j=1}^{3} |a_{2j}| = |-9| + |1| + |3| = 13$

$\sum_{j=1}^{3} |a_{3j}| = |1| + |-1| + |-6| = 8$

$\|A\|_\infty = \max(17, 13, 8)$
$= 17$

Aii) $\|A\|_{frob} = \sqrt{\sum_{i=1}^{3} \sum_{j=1}^{3} |a_{ij}|}$

$= \sqrt{8^2 + 5^2 + 4^2 + 9^2 + 1^2 + 3^2 + 1^2 + 1^2 + 6}$
$= \sqrt{64 + 25 + 16 + 81 + 1 + 9 + 1 + 1 + 36}$
$= \sqrt{234}$
$= 15.29$

Aiii) $\|A_2\| = \max_i (\lambda_i)$

$\text{Trace}(A) = 8 + 1 + (-6)$
$= 3$

$|A| = 3$

$\lambda^2 - (\text{trace of } A)\lambda + |A| = 0$
$\lambda^2 - 3\lambda + 3 = 0$
$(\lambda - 3)(\lambda + 3) = 0$
$\lambda_1 = 3 \ ; \ \lambda_2 = 3$

$\|A_2\| = \max(3, 3)$
$= 3$

b) $A = \begin{bmatrix} 8 & 5 & -4 \\ -9 & 1 & 3 \\ 1 & -1 & -6 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 8 & 5 & -4 \\ -9 & 1 & 3 \\ 1 & -1 & -6 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 8-\lambda & 5 & -4 \\ -9 & 1-\lambda & 3 \\ 1 & -1 & -6-\lambda \end{vmatrix} = 0$$

$$(8-\lambda)\left[(1-\lambda)(-6-\lambda) - 2\right] - 5\left((-6-\lambda) - 3\right) + 4\left(9 - (1-\lambda)\right) = 0$$

c) The condition of matrix $= \|A\| \, \|A^{-1}\|$

$= 3$

d) Since condition norm < 100 so it is not ill condition