Problem 1

The following data are measured precisely:

T	2	2.1	2.2	2.7	3	3.4
Z	6	7.752	10.256	36.576	66	125.168

a) Use Newton interpolating polynomials to determine z at t = 2.5. Make sure that you order your points to attain the most accurate results. What do your results tell you regarding the order of the polynomials used to generate the data?
b) Use a third-order Lagrange interpolating polynomial to determine y at 2.5.

Using Newton Interpolating Polynomial

Given our points T and Z, we start computing the first order divided difference:

Formula

$$f(x_i, x_j) = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

We now construct the Divided Difference table using this formula, starting with the First Order.

For T = 2, to T = 2.1:

$$\frac{7.752 - 6}{2.1 - 2} = \frac{1.752}{0.1} = 17.52$$

For T = 2.1 to T = 2.2:

$$\frac{10.256 - 7.752}{2.2 - 2.1} = \frac{2.504}{0.1} = 25.04$$

For T = 2.2 to T = 2.7:

$$\frac{36.576 - 10.256}{2.7 - 2.2} = \frac{26.32}{0.5} = 52.64$$

For T = 2.7 to T = 3:

$$\frac{66 - 36.576}{3 - 2.7} = \frac{29.424}{0.3} = 98.08$$

For T = 3 to T = 3.4:

$$\frac{125.168 - 66}{3.4 - 3} = \frac{59.168}{0.4} = 147.92$$

For the second order, we use the values gotten when finding the first order divided difference.

For T = 2 to T = 2.2:

$$\frac{25.04 - 17.52}{2.2 - 2} = \frac{7.52}{0.2} = 37.6$$

For T = 2.1 to T = 2.7:

$$\frac{52.64 - 25.04}{2.7 - 2.1} = \frac{27.6}{0.6} = 46$$

For T = 2.2 to T = 3:

$$\frac{98.08 - 52.64}{3 - 2.2} = \frac{45.44}{0.8} = 56.8$$

For T = 2.7 to T = 3.4:

$$\frac{147.92 - 98.08}{3.4 - 2.7} = \frac{49.84}{0.7} = 71.2$$

Now, we continue with the third order using the computer values from the second order.

For T = 2, T = 2.2 and T = 2.7 using deltas from second order: T = 2.1 and T = 2:

$$\frac{46 - 37.6}{2.7 - 2} = \frac{8.4}{0.7} = 12$$

For T = 2.1, T = 2.7 and T = 3 using deltas from second order: T = 2.2 and T = 2.1:

$$\frac{56.8 - 46}{3 - 2.1} = \frac{10.8}{0.9} = 12$$

For T = 2.2, T = 3 and T = 3.4 using deltas from second order: T = 2.7 and T = 2.2:

$$\frac{71.2 - 56.8}{3.4 - 2.2} = \frac{14.4}{1.2} = 12$$

Finally, we move to the fourth order, we again use the values from the previous order.

For T = 2, T = 2.2, T = 2.7 and T = 3, using deltas from third order: T = 2.1 and T = 2:

$$\frac{12-12}{3-2}=\frac{0}{1}=0$$

For T = 2.1, T = 2.7, T = 3 and T = 3.4 using deltas from third order: T = 2.2 and T = 2.1:

$$\frac{12-12}{3.4-2.1}=\frac{0}{1.3}=0$$

Divided Differences Table

We can now plug these values into the table liks so:

T	Z	Second Order (b_2)	Third Order (b_3)	Fourth Order (b_4)	Fifth Order (b_5)
2	**6**	**17.52**	**37.6**	**12**	**0**
2.1	**7.752**	**25.04**	**46**	**12**	**0**
2.2	**10.256**	**52.64**	**56.8**	**12**	
2.7	**36.576**	**98.08**	**71.2**		
3	**66**	**147.92**			
3.4	**125.168**				

The Polynomial

Now we attempt to construct the Newton Interpolating Polynomial using the following formula:

$$P(t)= b_0+b_1(x-x_1)+b_3(x-x_1)(x-x_2)+\cdots+b_n(x-x_1)(x-x_2)\ldots(x-x_{n-1})$$

Where:

$P(t)$ – is the value of the polynomial at time t

b_n – are the coefficients from the divided differences table

t_n- are the T values of our data points

We gather our coefficients:

$b_0=6$

$b_1=17.52$

$b_2=37.6$

$b_3=12$

We stop on the third order since our T = 2.5 falls just above it.

Now we substitute the values:

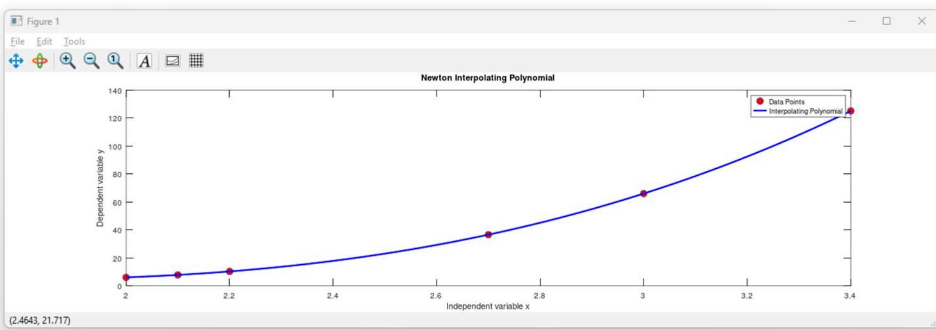
$$P(t) = 6 + 17.52(x - 2) + 37.6(x - 2)(x - 2.1) + 12(x - 2)(x - 2.1)(x - 2.2)$$

Solve for $t = 2.5$:

$$P(2.5) = 6 + 17.52(2.5 - 2) + 37.6(2.5 - 2)(2.5 - 2.1) + 12(2.5 - 2)(2.5 - 2.1)(2.5 - 2.2) = 23$$

By running the code in Listing [1], we get the following results:

```
>> T = [2 2.1 2.2 2.7 3 3.4]
T =

   2.0000   2.1000   2.2000   2.7000   3.0000   3.4000

>> Z = [6 7.752 10.256 36.576 66 125.168]
Z =

     6.0000     7.7520    10.2560    36.5760    66.0000   125.1680

>> xx = 0:0.1:4; yy = exp(xx);
>> [b, yint] = Newtint2(T, Z, xx)
b =

     6.0000    17.5200    37.6000    12.0000    -0.0000    -0.0000
     7.7520    25.0400    46.0000    12.0000    -0.0000          0
    10.2560    52.6400    56.8000    12.0000          0          0
    36.5760    98.0800    71.2000          0          0          0
    66.0000   147.9200          0          0          0          0
   125.1680          0          0          0          0          0

yint =

 Columns 1 through 21:

    18.0000    19.8320    20.9760    21.5040    21.4880    21.0000    20.1120    18.8960    17.4240    15.7680    14.0000    12.1920    10.4160     8.7440     7.2480     6.0000     5.0720     4.5360     4.4640     4.9280     6.0000

 Columns 22 through 41:

     7.7520    10.2560    13.5840    17.8080    23.0000    29.2320    36.5760    45.1040    54.8880    66.0000    78.5120    92.4960   108.0240   125.1680   144.0000   164.5920   187.0160   211.3440   237.6480   266.0000

>> |
```



The consistency of twelves in the 3rd order and the presence of Zero's in the fourth order suggests that the data was likely generated by a third-order polynomial (cubic polynomial). In Newton's method, if the *n*-th order divided differences are zero, it implies that the polynomial of best fit is of order *n – 1*. Since the fourth order divided differences are zero, the polynomial used to generate the data is of order 3, i.e., cubic polynomial.