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Electrical and Computer Engineering program
COMP 411
Numerical Methods with Programming
Second Assignment

Problem 1: Given the system of equations:

$$x1 - 3x2 + 7x3 = 4 \quad ①$$
$$x1 + 2x2 - x3 = 0 \quad ②$$
$$5x1 - 2x2 = 3 \quad ③$$

(a) Compute the determinant. (b) Use Cramer's rule to solve for the x's.
(c) Use Gauss elimination with partial pivoting to solve for the x's. As part of the computation, calculate the determinant in order to verify the value computed in (a).
(d) Substitute your results back into the original equations

a) $A = \begin{bmatrix} 0 & -3 & 7 \\ 1 & 2 & -1 \\ 5 & -2 & 0 \end{bmatrix}$

$\det(A) = \begin{vmatrix} 0 & -3 & 7 \\ 1 & 2 & -1 \\ 5 & -2 & 0 \end{vmatrix}$

$= 0 \begin{vmatrix} 2 & -1 \\ -2 & 0 \end{vmatrix} - (-3) \begin{vmatrix} 1 & -1 \\ 5 & 0 \end{vmatrix} + 7 \begin{vmatrix} 1 & 2 \\ 5 & -2 \end{vmatrix}$

$= 0 + 3\,(0 - (-5)) + 7\,(-2 - 10)$

$= 3(5) + 7(-12)$

$= 15 - 84$

$\det(A) = -69$

b) $D_1 = \begin{vmatrix} 2 & -3 & 7 \\ 3 & 2 & -1 \\ 2 & -2 & 0 \end{vmatrix}$

$= 2(2 \times 0 - (-1) \times (-2)) + 3(3 \times 0 - 2 \times (-1)) + 7(3 \times (-2) - 2 \times 2)$

$= -4 + 6 - 70$

$= -68$

$$D_2 = \begin{vmatrix} 0 & 2 & 7 \\ 1 & 3 & -1 \\ 5 & 2 & 0 \end{vmatrix}$$

$$= 0(3\times 0-(-1)\times 2) - 2(1\times 0 - 5\times(-1)) + 7(2\times 1 - 3\times 5)$$

$$= -10 - 91$$

$$= -101$$

$$D_3 = \begin{vmatrix} 0 & -3 & 2 \\ 1 & 2 & 3 \\ 5 & -2 & 2 \end{vmatrix}$$

$$= 0(2\times 2 - 3\times(-2)) + 3(2\times 1 - 3\times 5) + 2(1\times(-2) - 5\times 2)$$

$$= -39 - 24$$

$$= -63$$

$$X_1 = \frac{D_1}{D}$$

$$= \frac{68}{69}$$

$$X_2 = \frac{D_2}{D}$$

$$= \frac{101}{69}$$

$$X_3 = \frac{D_3}{D}$$

$$= \frac{21}{23}$$

c) $0x_1 - 3x_2 + 7x_3 = 2$

$x_1 + 2x_2 - x_3 = 3$

$5x_1 - 2x_2 + 0x_3 = 2$

$$\begin{bmatrix} 0 & -3 & 7 \\ 1 & 2 & -1 \\ 5 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

Pivot element.

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 5 & -2 & 0 \\ 1 & 2 & -1 \\ 0 & -3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

Forward Elimination:

① $R_2 \rightarrow 5R_2 - R_1$

$$\begin{bmatrix} 5 & -2 & 0 \\ 0 & 12 & -5 \\ 0 & -3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \\ 2 \end{bmatrix}$$

② $R_3 \rightarrow 4R_3 + R_2$

$$\begin{bmatrix} 5 & -2 & 0 \\ 0 & 12 & -5 \\ 0 & 0 & 23 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \\ 21 \end{bmatrix}$$

Ultima fila implica:

$23x_3 = 21$

$x_3 = \frac{21}{23}$

$12x_2 - 5x_3 = 13$

$12x_2 = 13 + 5\left(\frac{21}{23}\right)$

$x_2 = \frac{101}{69}$

$5x_1 - 2x_2 + x_3 = 2$

$5x_1 = 2 + 2\left(\frac{101}{69}\right)$

$x_1 = \frac{68}{69}$

d) Verificamos con substitucion: $x_1 = \frac{68}{69}$, $x_2 = \frac{101}{69}$, $x_3 = \frac{21}{23}$

① $0x_1 - 3x_2 + 7x_3 = 2$

$$0 - 3\left(\frac{101}{69}\right) + 7\left(\frac{21}{23}\right) = 2$$

$$-\frac{101}{23} + \frac{147}{23} = 2$$

$$\frac{46}{23} = 2$$

$$2 = 2$$

② $x_1 + 2x_2 - x_3 = 3$

$$\left(\frac{68}{69}\right) + 2\left(\frac{101}{69}\right) - \left(\frac{21}{23}\right) = 3$$

$$\frac{68}{69} + \frac{202}{69} - \frac{63}{69} = 3$$

$$3 = 3$$

③ $5x_1 - 2x_2 + x_3 = 2$

$$5\left(\frac{68}{69}\right) - 2\left(\frac{101}{69}\right) + 0 = 2$$

$$\frac{138}{69} = 2$$

$$2 = 2$$

Problem 2: An electrical engineer supervises the production of three types of electrical components. Three kinds of material-metal, plastic, and rubber-are required for production. The amounts needed to produce each component are:

Component	Metal (g/ component)	Plastic (g/ component)	Rubber (g/ component)
1	15	0.30	1.0
2	17	0.40	1.2
3	19	0.55	1.5

If totals of 3.89, 0.095, and 0.282 kg of metal, plastic, and rubber, respectively, are available each day, how many components can be produced by per day?

Metal constrain: $15x + 17y + 19z = 3.89 \times 10^3$ ① X

Plastic constrain: $0.3X + 0.4Y + 0.55z = 0.095 \times 10^3$ ② Y

Rubber constrain: $1X + 1.2y + 1.5z = 0.282 \times 10^3$ ③ Z

Crammer's Rules:

$$\begin{bmatrix} 15 & 17 & 19 \\ 0.3 & 0.4 & 0.55 \\ 1 & 1.2 & 1.5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3890 \\ 95 \\ 282 \end{bmatrix}$$

Deteminante de $|A| = 0.04$

- $x = \frac{1}{0.04} \begin{vmatrix} 3890 & 17 & 19 \\ 95 & 0.4 & 0.55 \\ 282 & 1.2 & 1.5 \end{vmatrix}$

$= \frac{3.6}{0.04}$

$= 90$

- $y = \frac{1}{0.04} \begin{vmatrix} 15 & 3890 & 19 \\ 0.3 & 95 & 0.55 \\ 1 & 282 & 1.5 \end{vmatrix}$

$= \frac{2.4}{0.04}$

$= 60$

- $z = \frac{3.89 \times 10^3 - 15x - 17y}{19}$

$= \frac{3890 - 15 \times 90 - 17 \times 60}{19}$

$= 80$

Problem 3: (a) Solve the following system of equations using LU Factorization with partial pivoting:

$$2x1 - 6x2 - x3 = -38$$
$$-3x1 - x2 + 7x3 = -34$$
$$-8x1 + x2 - 2x3 = -40$$

(b) verify your results → los resultados satisfacen todas las equaciones

$$\begin{bmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -38 \\ -34 \\ -40 \end{bmatrix}$$

① $R_2 \to R_2 + \left(\frac{3}{2}\right) R_1, \; R_3 \to R_3 + 4R_1$

$$\begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 5.5 \\ 0 & -23 & -6 \end{bmatrix} \quad R_3 \to R_3 - (2.3)R_2$$

$$\sim \begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 5.5 \\ 0 & 0 & -18.65 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 5.5 \\ 0 & 0 & -18.65 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1.5 & 1 & 0 \\ -4 & 2.3 & 1 \end{bmatrix}$$

$Ax = b$

$A = LU \Rightarrow LUx = B$

$Ux = y \Rightarrow Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1.5 & 1 & 0 \\ -4 & 2.3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -38 \\ -34 \\ -40 \end{bmatrix}$$

- $y_1 = -38$

$-1.5y_1 + y_2 = -34$

$-4y_1 + 2.3y_2 + y_3 = 40$

- $y_1 = -38, \; y_2 = -91, \; y_3 = -17.3$

$Ux = y$

- $x_1 = 6.305, \; x_2 = 8.58, \; x_3 = -0.9276$

Problem 4: The following system of equations is designed to determine concentrations (the c's in g/m3) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day):

15c1 − 3c2 − c3 = 4000
−3c1 + 18c2 − 6c3 = 1200
−4c1 − c2 + 12c3 = 2350

(a) Determine the matrix inverse.

(b) Use the inverse to determine the solution.

(c) Determine how much the rate of mass input to reactor 3 must be increased to induce a 10 g/m3 rise in the concentration of reactor 1.

(d) How much will the concentration in reactor 3 be reduced if the rate of mass input to reactors 1 and 2 is reduced by 500 and 250 g/day, respectively?

a) $|A| = \begin{vmatrix} 15 & -3 & -1 \\ -3 & 18 & 6 \\ -4 & -1 & 12 \end{vmatrix}$

$= 15(18\times 12+6) + 3(-36-24) - 1(3+72)$

$= 15(216-6) + 3 \times -60 - 75$

$= 210 \times 15 - 18 - 75$

$= 3150 \mp 180 - 75$

$= 297 - 75$

$= 2895$

• Minor of A

$M_{11} = \begin{vmatrix} 18 & -6 \\ -1 & 12 \end{vmatrix} = 216 - 6 = 210$

$M_{12} = \begin{vmatrix} -3 & -6 \\ -4 & 12 \end{vmatrix} = -36 - 24 = -60$

$M_{13} = \begin{vmatrix} -3 & 18 \\ -4 & 1 \end{vmatrix} = 3 + 72 = 75$

$M_{21} = \begin{vmatrix} -3 & -1 \\ -1 & 12 \end{vmatrix} = -36 - 1 = -37$

$M_{22} = \begin{vmatrix} 15 & -1 \\ -4 & 12 \end{vmatrix} = 180 - 4 = 176$

$M_{23} = \begin{vmatrix} 15 & -3 \\ -4 & -1 \end{vmatrix} = -15 - 12 = -27$

$M_{31} = \begin{vmatrix} -3 & -1 \\ 18 & -6 \end{vmatrix} = 18 + 18 = 36$

$M_{32} = \begin{vmatrix} 15 & -1 \\ -3 & -6 \end{vmatrix} = -90 - 3 = -93$

$M_{33} = \begin{vmatrix} 15 & -3 \\ -3 & 18 \end{vmatrix} = 270 - 9 = 261$

• Cofactor de A

$A_{11} = (-1)^{1+1}\, 210 = 210$ $\quad A_{12} = -(-60) = 60$

$A_{21} = -(-37) = 37$ $\quad A_{22} = 176$

$A_{31} = 36$ $\quad A_{33} = -(-93) = 93$

$A_{13} = 75$ $\quad A_{23} = -(-27) = 27$

$A_{33} = 261$

$$A^{-1} = \frac{1}{|A|} \text{ads } A = \frac{1}{2895} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \frac{1}{2895} \begin{bmatrix} 210 & 60 & 75 \\ 37 & 176 & 27 \\ 36 & 43 & 261 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2895} \begin{bmatrix} 210 & 37 & 36 \\ 66 & 176 & 93 \\ 75 & 27 & 261 \end{bmatrix}$$

b) $AC = B$

$C = A^{-1} B$

$$= \frac{1}{2895} \begin{bmatrix} 210 & 37 & 36 \\ 60 & 176 & 93 \\ 75 & 27 & 261 \end{bmatrix} \begin{bmatrix} 4080 \\ 1200 \\ 2350 \end{bmatrix}$$

$$= \frac{1}{2895} \begin{bmatrix} 840000 + 44400 + 84600 \\ 240000 + 211200 + 218556 \\ 300000 + 32400 + 613350 \end{bmatrix}$$

$$= \frac{1}{2895} \begin{bmatrix} 969000 \\ 669750 \\ 945750 \end{bmatrix}$$

$$= \begin{bmatrix} 334.72 \\ 231.35 \\ 326.68 \end{bmatrix}$$

c) c1 by $c_1 + 10$

$$C = \begin{bmatrix} c_1 + 10 \\ c_2 \\ c_3 \end{bmatrix}$$

Mass input: $\begin{bmatrix} 15 & -3 & 1 \\ -3 & 18 & -6 \\ -4 & -1 & 12 \end{bmatrix} \begin{bmatrix} 334.72 + 10 \\ 231\ 35 \\ 326.68 \end{bmatrix}$

$$= \begin{bmatrix} 15 & -3 & 1 \\ -3 & 18 & -6 \\ -4 & -1 & 12 \end{bmatrix} \begin{bmatrix} 340.72 \\ 231\ 35 \\ 326.68 \end{bmatrix}$$

$$= \begin{bmatrix} 5110.8 - 694.05 + 326.68 \\ -1022.16 + 4164.3 - 1960.08 \\ -1362.88 - 231.35 + 3920.16 \end{bmatrix}$$

$$= \begin{bmatrix} 4743.43 \\ 1182\ 06 \\ 2325.93 \end{bmatrix}$$

Mass input in 3rd reactor is decreased by = 24.07

$(2325.93 - 2350)$

d) to find decrease in reactor 3

new value of matrix B is $B^{*} = \begin{bmatrix} 4000 - 500 \\ 1200 - 250 \\ 2350 \end{bmatrix}$

$$B^{*} = \begin{bmatrix} 3500 \\ 950 \\ 2350 \end{bmatrix}$$

so $C^{*} = A^{*} B^{*}$

$$C = \frac{1}{2895} = \begin{bmatrix} 210 & 37 & 36 \\ 60 & 176 & 93 \\ 75 & 37 & 281 \end{bmatrix} \begin{bmatrix} 3500 \\ 950 \\ 2350 \end{bmatrix}$$

$$= \frac{1}{2895} \begin{bmatrix} 735000 + 35150 + 84600 \\ 210000 + 167200 + 218550 \\ 262500 + 25650 + 813550 \end{bmatrix}$$

$$= \frac{1}{2895} \begin{bmatrix} 854750 \\ 595750 \\ 901500 \end{bmatrix} = \begin{bmatrix} 295.25 \\ 205.79 \\ 311\ 400 \end{bmatrix}$$

which means reactor 3 is reduced by $= 15.28\ g/cm^3$

$(326.68 - 311\ 40)$

Problem 5: Use the system in Problem 4 to find the all norms of the matrix A, the eigen values, and the condition number. Is the system illcondition? Justify your answer.

(Hint: In all problems, rewrite the system in the form Ax=b)

$15C_1 - 3C_2 - C_3 = 4000$

$C_1 + 18C_2 - 6C_3 = 1200$

$-C_2 + 12C_3 = 2950$

Matrix

$Ax = b$

$$\begin{bmatrix} 15 & -3 & -1 \\ 1 & 18 & -6 \\ 0 & -1 & 12 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 4000 \\ 1200 \\ 2350 \end{bmatrix}$$

we find eigen value of A

$$\det(A - dI) = \det \begin{vmatrix} 15-d & -3 & -1 \\ 1 & 18-d & -6 \\ 0 & -1 & 12-d \end{vmatrix}$$

$$= d^3 + 45d^2 - 663d + 3187$$

After solving this we will get three eigen values

$d = 11.24 \quad d_2 = 15.69 \quad d_3 = 18.06$

three of matrix A is

$$A^{-1} = \begin{bmatrix} 210/3187 & 37/3187 & 36/3187 \\ -12/3187 & 140/3187 & 89/3187 \\ -1/3187 & 15/3187 & 273/3187 \end{bmatrix}$$

Row - sum norm of $A = \|A\| = \max\{16, 22, 19\} = 22$

Row sum norm of $A^{-1} = \|A^{-1}\| = \max\{0.069, 0.072, 0.12\} = 0.12$

Condition number $= 0.12 \times 22 = 2.64$

System is not ill condition