(2) Find the solution of $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ of the system $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$.

i. Find L and U
ii. Use Forward substitution to solve Ly=b for y
iii. Use Ux=y to solve for x

i) $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$

$R_2 + -3R_1 \to R_2 \quad \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 2 & 6 & 13 \end{bmatrix}$

$R_3 + -2R_1 \to R_3 \quad \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{bmatrix}$

$R_3 + -1R_2 \to R_3 \quad \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

$U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

ii) $Ly = b$ for y

$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix} \Rightarrow$

$y_1 = 3$

$3y_1 + y_2 = 13;\ 3(3) + y_2 = 13;\ y_2 = 4$

$2y_1 + y_2 + y_3 = 4;\ 2(3) + 4 + y_3;\ y_3 = -6$

$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

iii) $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$

$x_1 + 2x_2 + 4x_3 = 3$ $\quad$ $x_1 + 2x_2 + 4x_3 = 3$; $x_1 + 2(4) + 4(-2) = 3$; $x_1 = 3$

$2x_2 + 2x_3 = 4$ $\quad$ $2x_2 + 2x_3 = 4$; $2x_2 + 2(-2) = 4$; $x_2 = 4$

$3x_3 = 6$ $\quad$ $3x_3 = -6$; $x_3 = -2$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$