③ $$\begin{bmatrix} 0.8 & -0.4 & 0 \\ -0.4 & 0.8 & -0.4 \\ 0 & -0.4 & 0.8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 41 \\ 25 \\ 105 \end{Bmatrix}$$

a) This problem is divergent, because iterations are showing different values for a, b, c

c) Primera Iteracion:

$$x_1^{(1)} = \frac{1}{0.8}(41 + 0.4x_2(0) = \frac{1}{0.8}(41) = 51.25$$

$$x_2^{(1)} = \frac{1}{0.8}(25 + 0.4x_1^{(1)} + 0.4x_3^{(0)})$$

$$= \frac{1}{0.8}(25 + 0.4(51.25) + 0)$$

$$= 56.875$$

$$x_3^{(1)} = \frac{1}{0.8}(105 + 0.4x_2^{(1)}$$

$$= \frac{1}{0.8}(105 + 0.4(56.875)$$

$$= 159.6875$$

Segunda Iteracion:

$$x_1^{(1)} = \frac{1}{0.8}(41 + 0.4x_2^{(1)}$$

$$= \frac{1}{0.8}(41 + 0.4(56.8751)$$

$$= 79.6875$$

$$x_1^{(2)} = \frac{1}{0.8}\left(25 + 0.4x_2^{(1)} + 0.4x_3^{(1)}\right)$$

$$= \frac{1}{0.8}\left(25 + 0.4(79.6875) + 0.4(159.6875)\right)$$

$$= 150.9375$$

$$x_3^{(2)} = \frac{1}{0.8}\left(105 + 0.4x_2^{(2)}\right)$$

$$= \frac{1}{0.8}\left(41 + 0.4(150.9375)\right)$$

$$= 206.71875$$

Tercera Iteracion

$$x_1^{(3)} = \frac{1}{0.8}\left(41 + 0.4x_2^{(2)}\right)$$

$$= \frac{1}{0.8}\left(41 + 0.4(150.9375)\right)$$

$$= 126.71875$$

$$x_2^{(3)} = \frac{1}{0.8}\left(25 + 0.4x_1^{(3)} + 0.4x_3^{(2)}\right)$$

$$= \frac{1}{0.8}\left(25 + 0.4(126.71875) + 0.4(206.71875)\right)$$

$$= 197.96875$$

$$x_3^{(3)} = \frac{1}{0.8}\left(105 + 0.4(197.96875)\right)$$

$$= 230.23475$$

b) $|0.8| > |0.4| + |0|$

$|0.8| = |-0.4| + |0.4|$

$= |0.8|$

$$L = \begin{bmatrix} 0.8 & 0 & 0 \\ -0.4 & 0.8 & 0 \\ 0 & -0.4 & 0.8 \end{bmatrix} \qquad U = \begin{bmatrix} 0 & -0.4 & 0 \\ 0 & 0 & -0.4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L^{-1} = \begin{bmatrix} 1.25 & 0 & 0 \\ 0.625 & 1.25 & 0 \\ 0.3125 & 0.625 & 1.25 \end{bmatrix}$$

$$L^{-1}U = \begin{bmatrix} 1.25 & 0 & 0 \\ 0.625 & 1.25 & 0 \\ 0.3125 & 0.625 & 1.25 \end{bmatrix} \times \begin{bmatrix} 0 & -0.4 & 0 \\ 0 & 0 & -0.4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & 0 \\ 0 & 0.25 & 0.5 \\ 0 & 0.125 & 0.25 \end{bmatrix}$$

Calculando C

$$\begin{bmatrix} 1.25 & 0 & 0 \\ 0.625 & 1.25 & 0 \\ 0.3125 & 0.625 & 1.25 \end{bmatrix} \times \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} = \begin{bmatrix} 51.25 \\ 56.875 \\ 159.6875 \end{bmatrix}$$

$$D = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix} \qquad C = \begin{bmatrix} 1.25 & 0 & 0 \\ 0.625 & 1.25 & 0 \\ 0.3125 & 0.625 & 1.25 \end{bmatrix}$$

d) $a_{k+1} = \frac{1}{0.8}(41 + 0.4b_k - 0c_k)$

$b_{k+1} = \frac{1}{0.8}(25 + 0.4a_{k+1} + 0.4c_k)$

$c_{k+1} = \frac{1}{0.8}(105 - 0.a_{k+1} + 0.4b_{k+1})$

$a_{k+1} = (1-\lambda)a_k + \lambda \cdot \frac{1}{0.8}(41 + 0.4b_k - 0c_k)$

$b_{k+1} = (1-\lambda)b_k + \lambda \cdot \frac{1}{0.8}(25 + 0.4a_{k+1} + 0.4c_k)$

$c_{k+1} = (1-\lambda)c_k + \lambda \cdot \frac{1}{0.8}(105 - 0a_{k+1} + 0.4b_{k+1})$

First Aprox:

$a_1 = (1-1.5).0 + 1.5\left(\frac{1}{0.8}\right)[41 + 0.4(0) - 0(0)] = 76.88$

$b_1 = (1-1.5).0 + \frac{1}{0.8}[25 + 0.4(76.88) + 0.4(0)] = 104.53$

$c_1 = (1-1.3).0 + (1.5)\frac{1}{0.8}[105 - 0(76.88) + 0.4(104.53)] = 275.27$