Introduction to Deep Reinforcement Learning

https://github.com/racousin/rl_introduction

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Course Objective

- The keys to go by yourself in RL
- Practice coding
- General culture

What do you already know about RL?

0.1)Intro

Figure 1: 2017, AlphaGo beat Ke Jie, the number one ranked player in the world

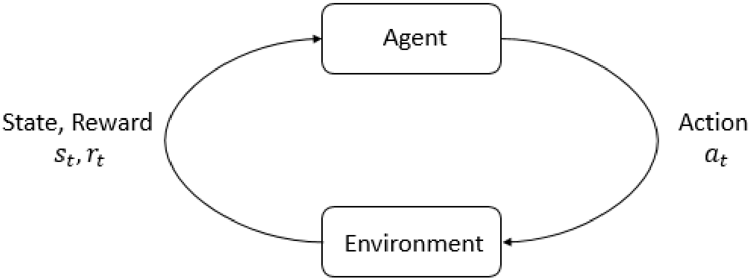
0.2)RL!=ML?

- Supervised learning: $(X, Y) \to \hat{f}(X) = Y$
- Unsupervised learning: $X \to \hat{f}(X|\hat{Y})$
- Reinforcement learning (RL): environment/goal $\to$ optimally interact with the environment to achieve the goal

0.3)Agent takes action in an Environment

RL framework:

- Unknown “correct” actions → no explicit supervision for a learning algorithm to try to mimic.
- Provide our algorithms only a reward function → indicates to the learning agent when it is doing well, and when it is doing poorly.



0.4)Application

Applications: Autonomous vehicle, robot legged locomotion, cell-phone network routing, marketing strategy, factory control, efficient web-page indexing, video games, Neural Architecture Search, ...

Plan

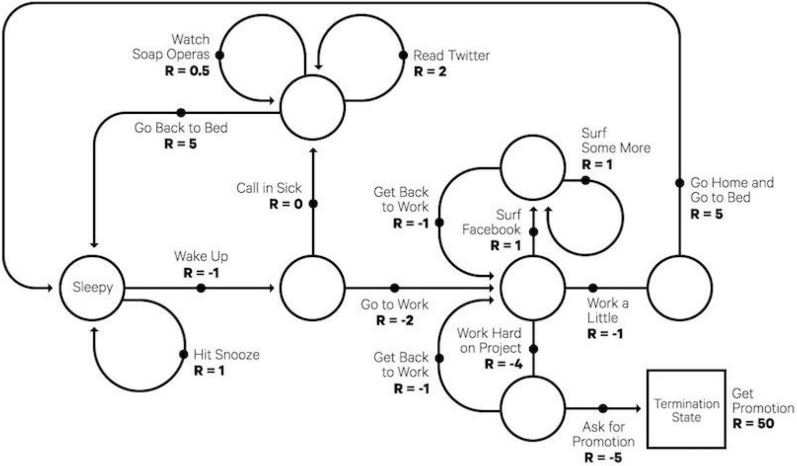
- I) MDP - The RL framework
- II) RL - Model free
- III) Deep RL - Model free

I) MDP and dynamic Programming

- State space: S
- Action space: A
- Transition model: $P_{ss'}^{a} = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]$
- Immediate reward $R_a(s, s') = R_{t+1} \in \mathbb{R}$
- Policy function $\pi(s) \in A$

$\forall s, s \in S, a \in A, t \in \mathbb{N}$

I.2)Example: MDP long term reward



I.3)The MDP Objective

- Find the optimal policy:
  $\pi^* = \arg\max_\pi E[G]$
- Finite-horizon return: $G_t = \sum_{k=0}^{T} R_{t+k+1}$.
- Infinite-horizon discounted return: $G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$

$\gamma \in]0, 1[, t \in \mathbb{N}$

I.4)Trajectories

- Trajectories: $\tau = (s_0, a_0, s_1, a_1, ...)$
- States follow:
  $s_0 \sim \rho_0(\cdot)$
  $s_{t+1} \sim P(\cdot|s_t, a_t, s_{t-1}, a_{t-1}, ..., s_0, a_0) = P(\cdot|s_t, a_t)$
- Actions follow: $s_t \sim \pi(s_t)$
- So probability of a T -step trajectory is:
  $\mathbb{P}(\tau|\pi) = \rho_0(s_0) \prod_{t=0}^{T-1} P(s_{t+1}|s_t, a_t)\pi(a_t|s_t)$
- $\pi^* = \arg\max_\pi E_{\tau\sim\pi}[G(\tau)] = \int_\tau \mathbb{P}(\tau|\pi)G(\tau)$

Coding session
Environment and Agent

I.5)Value and Action-Value Functions

- Value Function: $V^{\pi}(s) = E_{\tau\sim\pi}[G_t \,|s_t = s]$
- Action-Value Function:
  $Q^{\pi}(s,a) = E_{\tau\sim\pi}[G_t \,|s_t = s, a_t = a]$
- The Optimal Value Function:
  $V^{*}(s) = \max_{\pi} E_{\tau\sim\pi}[G_t \,|s_t = s] = E_{\pi^*}[G_t \,|s_t = s]$
- Optimal Action-Value Function:
  $Q^{*}(s,a) = \max_{\pi} E_{\tau\sim\pi}[G_t \,|s_t = s, a_t = a] = E_{\pi^*}[G_t \,|s_t = s, a_t = a]$

- **Idea :** The value of your starting point is the reward you expect to get from being there, plus the value of wherever you land next.

$$
\begin{aligned}
V(s) &= \mathbb{E}[G_t|S_t = s] \\
&= \mathbb{E}[R_{t+1} + \gamma G_{t+1}|S_t = s] \\
&= \mathbb{E}[R_{t+1} + \gamma V(S_{t+1})|S_t = s]
\end{aligned}
$$

$$
Q(s,a) = \mathbb{E}[R_{t+1} + \gamma \mathbb{E}_{a\sim\pi} Q(S_{t+1}, a) \mid S_t = s, A_t = a]
$$

I.7)Bellman Equations development

$$
\begin{aligned}
V_\pi(s) &= \sum_{a \in \mathcal{A}} \pi(a|s) Q_\pi(s,a) \\
Q_\pi(s,a) &= R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} V_\pi(s') \\
V_\pi(s) &= \sum_{a \in \mathcal{A}} \pi(a|s) \big(R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} V_\pi(s') \big) \\
Q_\pi(s,a) &= R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} \sum_{a' \in \mathcal{A}} \pi(a'|s') Q_\pi(s',a')
\end{aligned}
$$

I.8)Bellman Equations Optimality

Bellman equations for the optimal value functions

$$
\begin{aligned}
V_*(s) &= \max_{a \in \mathcal{A}} Q_*(s,a) \\
Q_*(s,a) &= R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P_{ss'}^{a} V_*(s') \\
V_*(s) &= \max_{a \in \mathcal{A}} \big(R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P_{ss'}^{a} V_*(s') \big) \\
Q_*(s,a) &= R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P_{ss'}^{a} \max_{a' \in \mathcal{A}} Q_*(s',a')
\end{aligned}
$$

I.9)The MDP Solution

Dynamic Programming allows to resolve the MDP optimization problem ($\pi^* = \arg\max_\pi E_{\tau\sim\pi}[G(\tau)]$). It is an iterative process:

- Policy initialization
- Policy evaluation
- Policy improvement

I.10) Policy evaluation

Policy Evaluation: compute the state-value V_π for a given policy π:
We initialize V_0 arbitrarily. And we update it using:

$$\begin{aligned}\mathrm{V}_{k+1}(s) &= \mathbb{E}_\pi[r + \gamma V_k(s_{t+1})|S_t = s] \\ &= \sum_a \pi(a|s) \sum_{s',r} P(s', r|s, a)(r + \gamma V_\pi(s'))\ (1)\end{aligned}$$

$V_\pi(s)$ is a fix point for (1), so if $(V_k)_{k\in\mathbb{N}}$ converges, it converges to V_π.

I.11) Policy Improvement

Policy Improvement: generates a better policy $\pi' \geq \pi$ by acting greedily. Compute Q from V $(\forall a, s)$:

$$\begin{aligned} Q_\pi(s,a) &= \mathbb{E}[R_{t+1} + \gamma V_\pi(S_{t+1}) | S_t = s, A_t = a] \\ &= \sum_{s',r} P(s', r | s, a)(r + \gamma V_\pi(s')) \end{aligned}$$

Update greedily: $\pi'(s) = \arg\max_{a \in \mathcal{A}} Q_\pi(s,a)$ $(\forall s)$

I.12) Dynamic Programming

Policy Iteration: iterative procedure to improve the policy when combining policy evaluation and improvement.

$$\pi_0 \xrightarrow{\text{evaluation}} V_{\pi_0} \xrightarrow{\text{improve}} \pi_1 \xrightarrow{\text{evaluation}} \dots \xrightarrow{\text{improve}} \pi_* \xrightarrow{\text{evaluation}} V_* \quad (1)$$

Take home message

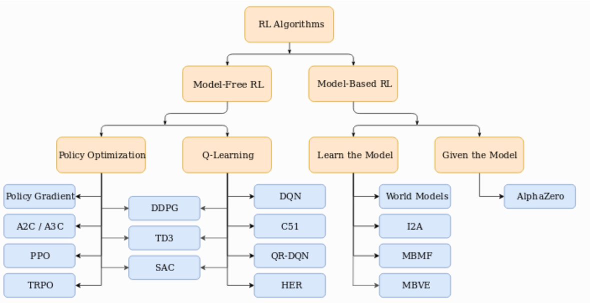
Initialize $\pi(s), \forall s$

1. Evaluate $V_\pi(s), \forall s$ (using $\mathbb{P}^a_{ss'}$)
2. Compute $Q_\pi(s,a), \forall s, a$ (using $\mathbb{P}^a_{ss'}$)
3. Update $\pi'(s) = \max_a Q_\pi(s,a), \forall s$
4. While $\pi'(s) \neq \pi(s)$ do $\pi(s) = \pi'(s)$ and iterate

Result : $\pi = \arg\max_\pi E[G]$

Coding session
Dynamic Programming

RL - model free



To simplify:

- **Model free:** large data + low complexity
- **Model based:** less data + high complexity

The objective is the same as MDP:

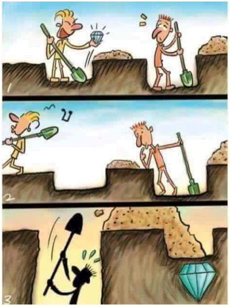
- $J(\pi) = \int_\tau \mathbb{P}(\tau|\pi)G(\tau) = E_{\tau\sim\pi}[G(\tau)]$
- The optimization problem:
  $\pi^* = \arg\max_\pi J(\pi)$

But we don't know the transition model P. The constraint is therefore to interact intelligently with the environment to obtain the information needed to solve the problem.

- **Q-learning:** learn the action value function Q
  $(\pi'(s) = \arg\max_{a\in\mathcal{A}} Q_\pi(s,a))$
- **Policy Optimization:** learn directly the policy π

II.2)Exploration-Exploitation

Knowledge of the environment comes from interaction. There are trade-offs to be made between using what we know and further exploration.



Naive solution: force exploration with random action (control by an ϵ factor)

II.3)Definition

- **On-policy:** Use the deterministic outcomes or samples from the target policy to train the algorithm.
- **Off-policy:** Training on a distribution of transitions or episodes produced by a different behavior policy rather than that produced by the target policy.

II.4) Monte-Carlo

- To evaluate $V_\pi(s) = E_{\tau \sim \pi}[G_t \,|s_t = s]$
- Generate an episode with the policy π $S_1, A_1, R_2, \ldots, S_T$ to compute $G_t = \sum_{k=0}^{T-t-1} \gamma^k R_{t+k+1}$.
- The empirical value function is : $V_\pi(s) = \frac{\sum_{t=1}^{T} \mathbb{1}[S_t = s] G_t}{\sum_{t=1}^{T} \mathbb{1}[S_t = s]}$
- As, well, the empirical action-value function is :
- $Q_\pi(s, a) = \frac{\sum_{t=1}^{T} \mathbb{1}[S_t = s, A_t = a] G_t}{\sum_{t=1}^{T} \mathbb{1}[S_t = s, A_t = a]}$

II.5) Monte-Carlo Algorithm

Initialize Q $Q(s,a) \forall s, a$.

1. Generate an episode with the policy π (extract from Q ϵ-greedy)
2. Evaluate Q using the episode:
   $q_\pi(s,a) = \frac{\sum_{t=1}^{T} \left(\mathbb{1}[S_t=s, A_t=a] \sum_{k=0}^{T-t-1} \gamma^k R_{t+k+1} \right)}{\sum_{t=1}^{T} \mathbb{1}[S_t=s, A_t=a]}$
3. Improve the policy greedily: $\pi(s) = \arg\max_{a \in \mathcal{A}} Q(s,a)$
4. Iterate

II.6) Temporal difference/bootstrapping

- Remember $V(S_t) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s]$
- So $R_{t+1} + \gamma V(S_{t+1})$ is an unbiased estimate for $V(S_t)$
- As well $R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})$is an unbiased estimate for $Q(S_t, A_t)$
- $R_{t+1} + \gamma V(S_{t+1})$ is called the TD target.
- α improvement:
  $V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$

This observation motivates the following algorithm.

II.7) SARSA Algorithm

Initialize Q function $Q(s, a) \forall s, a$

$S_t =$ initial state, act with π to get A_t, R_{t+1}, S_{t+1}

1. Act with π to get $A_{t+1}, R_{t+2}, S_{t+2}$
2. Evaluate Q using the observation step:
   $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t))$
3. Improve the policy greedily: $\pi(s) = \arg\max_{a \in \mathcal{A}} Q(s, a)$
4. $A_t = A_{t+1}, R_{t+1} = R_{t+2}, S_{t+1} = S_{t+2}$. Iterate

II.8) Q-learning

- Remember
  $Q_{\pi^*}(S_t, A_t) = \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q_*(S_{t+1}, a') | S_t = s, A_t = a]$
- So $R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a')$ is an unbiased estimate for $Q(S_t, A_t)$
- $R_{t+1} + \gamma \max_{a'} Q(S_{t+1}, a')$ is called the Q target.
- α improvement:
  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(R_{t+1} + \gamma \max_{a \in \mathcal{A}} Q(S_{t+1}, a) - Q(S_t, A_t))$

This observation motivates the following algorithm.

II.9) Q-learning Algorithm

Initialize Q function $Q(s,a) \forall s, a$
$S_t =$ initial state

1. $Q : S \times A \rightarrow \mathbb{R}$
2. $\forall s, a Q(s,a) = \mathbb{E}_{\pi}[G_t | S_t = s, A_t = a]$
3. Improve the policy greedily: $\pi(s) = \arg\max_{a \in \mathcal{A}} Q(s,a)$
4. Iterate

Coding session
Temporal Difference

II.10)Policy Optimization

- Parametrization of policy, π_θ.
- We aim to maximize the expected return $J(\pi_\theta) = E_{\tau \sim \pi_\theta}[G(\tau)]$.
- Gradient ascent:
  $\theta_{k+1} = \theta_k + \alpha \, \nabla_\theta J(\pi_\theta)|_{\theta_k}$.
- We can proof that:
  $\nabla_\theta J(\pi_\theta) = E_{\tau \sim \pi_\theta}[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) G(\tau)]$
  `https://lilianweng.github.io/lil-log/2018/04/08/policy-gradient-algorithms.html`

Deep RL

I.4)Introduction - Q Learning limitations

- S high dimension $\rightarrow$ low convergence
- It doesn't work for continuous action space

II.1) Deep Q Learning

Parametrize Q with θ, initialize $\theta \in \mathbb{R}^d$

- $Q_\theta : S \times A \to \mathbb{R}$
- Objective find $\theta^* \in \mathbb{R}^d$ $\forall s, a$ $Q_{\theta^*}(s,a) = \mathbb{E}_\pi^*[G_t | S_t = s, A_t = a]$
- $Q_{\pi^*}(S_t, A_t) = \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q_*(S_{t+1}, a') | S_t = s, A_t = a]$
- $y = R_{t+1} + \gamma \max_{a'} Q_\theta(S_{t+1}, a')$ is called the Q target.
- Loss (eg MSE): $L(\theta) = \mathbb{E}_{s,a \sim Q}[(y - Q(s,a,\theta))^2]$

II.2) Deep Q Learning Algorithm

Initialize $\theta \in \mathbb{R}^d$, get initial state S_t

1. $\forall a$ predict/compute $Q_\theta(S_t, a)$
2. Act greedily $A_t = \arg\max([Q_\theta(s, a_0), Q_\theta(s, a_1), ...Q_\theta(s, a_{dim(A)}])$ and get R_{t+1}, S_{t+1}
3. $\forall a$ predicts $Q_\theta(S_{t+1}, a)$
   Compute the target $y = R_{t+1} + \gamma \max_{a \in A} Q_\theta(S_{t+1}, a)$
4. Evaluate + improve Q by minimizing the loss
   $L(\theta) = (y - Q(S_t, A_t, \theta))^2$
5. Iterate

II.3) Convergence and stability improvements

Experience replay

- At every step t, we get $S_t, A_t, R_{t+1}, S_{t+1} \rightarrow$ memory
- New loss at each step
  $L_i(\theta_i) = \mathbb{E}_{(s,a,r,s')\sim U(D)}\left[\left(r + \gamma \max_{a'} Q(s', a'; \theta_i^-) - Q(s, a; \theta_i)\right)^2\right]$
- Other improvements: Epsilon decay, Clipping, Double Q learning

Coding session
Deep Q - learning

III.1) Policy Optimization

- Parametrize the policy, π_θ.
- We aim to maximize the expected return $J(\pi_\theta) = E_{\tau\sim\pi_\theta}[G(\tau)]$.
- Gradient ascent:
  $\theta_{k+1} = \theta_k + \alpha\ \nabla_\theta J(\pi_\theta)|_{\theta_k}$.
- We can proof that:
  $\nabla_\theta J(\pi_\theta) = E_{\tau\sim\pi_\theta}[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) G(\tau)]$
  `https://lilianweng.github.io/lil-log/2018/04/08/policy-gradient-algorithms.html`

III.2) Reinforce/VPG algorithm

Initialize policy π_θ

1. Generate episodes $\mathcal{D} = \{\tau_i\}_{i=1,\dots,N}$ with the policy π_θ
2. Compute gradient approximation
   $\hat{\nabla} = \frac{1}{|\mathcal{D}|} \sum_{\tau \in \mathcal{D}} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) G_t$
3. Update policy (apply gradient ascent) $\theta \leftarrow \theta + \alpha \hat{\nabla}$
4. Iterate

Coding session
Policy gradient - reinforce

III.3) Improvement Actor-Critic

We can rewrite the policy gradient
$\nabla_\theta J(\pi_\theta) = E_{\tau\sim\pi_\theta}[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\Phi_t]$,
whith Φ_t could be any of

- $\Phi_t = G_t$
- $\Phi_t = \sum_{t'=t}^{T} R_{t+1} - V(s_t)$
- $\Phi_t = \sum_{t'=t}^{T} R_{t+1} - Q(s_t, a_t)$

For the last 2 cases we need to estimate V or Q (the critics).

III.4) Actor-Critic Algorithm

Initialize $\theta, \in \mathbb{R}^{d_1}, \phi \in \mathbb{R}^{d_2}$ Get start State S_t

1. Generate A_t R_{t+1} following π_{θ_t}
2. Update actor (apply gradient ascent)
   $\theta \leftarrow \theta + \alpha \hat{\nabla} = \theta + Q_\phi(a_t, s_t) \nabla_\theta \log \pi_\theta(a_t|s_t)$
3. Compute critic target $y = R_{t+1} + \gamma\ Q_\phi(S_{t+1}, a')$
4. Evaluate/Improve Q_ϕ by minimizing the loss
   $L(\phi) = (y - Q(S_t, A_t, \phi))^2$
5. Iterate

References

Course:
http://incompleteideas.net/book/the-book-2nd.html
https://lilianweng.github.io/lil-log/2018/02/19/a-long-peek-into-reinforcement-learning.html
http://rail.eecs.berkeley.edu/deeprlcourse-fa17/f17docs/
https://github.com/kengz/awesome-deep-rl
Framework:
https://spinningup.openai.com/en/latest/
https://gym.openai.com/envs/#atari
https://github.com/deepmind/bsuite

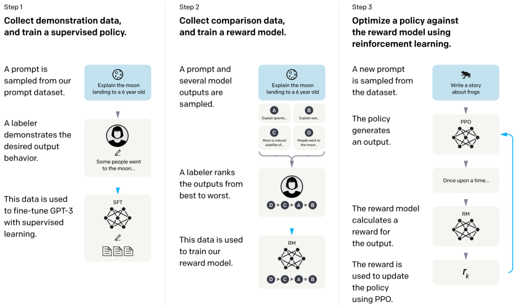


Figure 2: instructgpt-chart-openai