Introduction to Reinforcement Learning

https://github.com/racousin/rl_introduction

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Course Objectives

- The keys to go by yourself in RL
- Practice coding
- General culture

What do you already know about RL?

Why reinforcement learning?

Figure 1: In 2017, AlphaGo defeated Ke Jie, the world's top-ranked Go player.

RL vs. Other Machine Learning Paradigms

- Supervised Learning: Learning from labeled data to predict outcomes.
- Unsupervised Learning: Finding patterns in data without explicit labels.
- Reinforcement Learning: Learning decision-making by interacting with an environment to achieve goals.

In contrast to the other two, RL is focused on learning from the consequences of actions.

The RL Framework

- Absence of explicit "correct" actions.
- Learning is guided by rewards that represent the objectives.

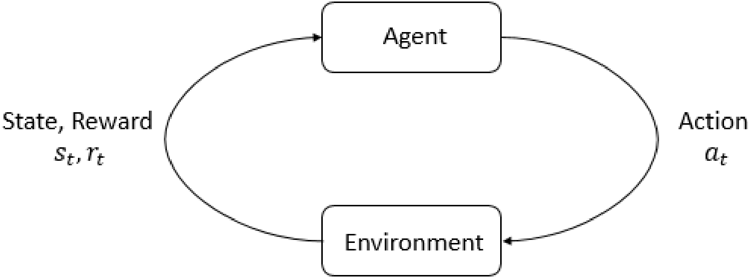


Figure 2: The agent-environment interaction in RL.

Applications of RL

RL has been successfully applied in various fields, including:

- Autonomous vehicles/Robotics
- Control systems
- Chat bots policy
- Marketing/Trading strategies
- Game playing and beyond

Figure 3: RL in video games.

Course Outline

- I) Understanding Markov Decision Processes (MDPs)
- II) Exploring Model-Free Reinforcement Learning
- III) Diving into Deep Reinforcement Learning

We'll start with the basics and gradually move to more complex concepts.

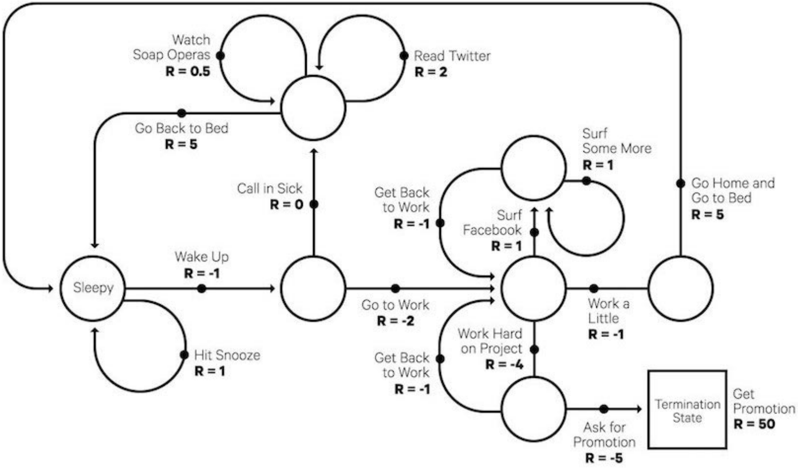


Figure 4: Illustrative example of an MDP, showcasing state transitions, actions, and rewards.

First Glossary of MDPs

- State Space (S)
- Action Space (A)
- Transition Model (P)
- Reward function (R)
- Policy (π)
- Trajectory (τ)
- Return (G)

Simple Grid World Problem

Our environment is a 4x4 grid where an agent aims to reach a goal.

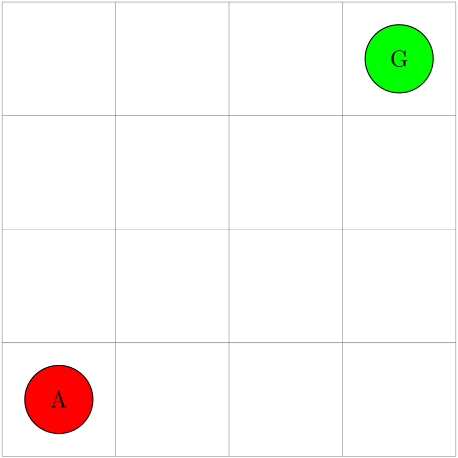


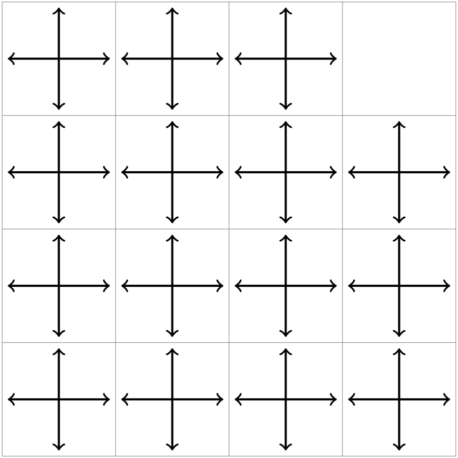
Figure 5: A: Agent, G: Goal

State Space (S)

16 discrete states.

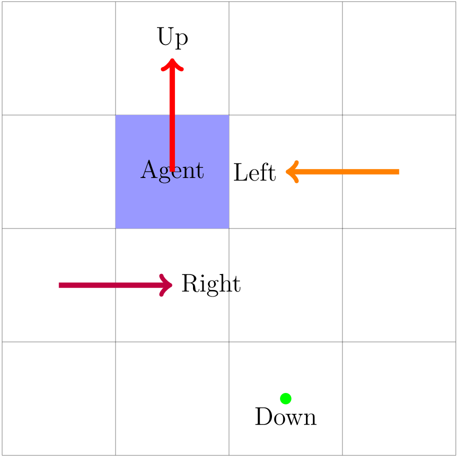
$S_{0,3}$	$S_{1,3}$	$S_{2,3}$	$S_{3,3}$
$S_{0,2}$	$S_{1,2}$	$S_{2,2}$	$S_{3,2}$
$S_{0,1}$	$S_{1,1}$	$S_{2,1}$	$S_{3,1}$
$S_{0,0}$	$S_{1,0}$	$S_{2,0}$	$S_{3,0}$

4 discrete actions (Up, Down, Left, Right).



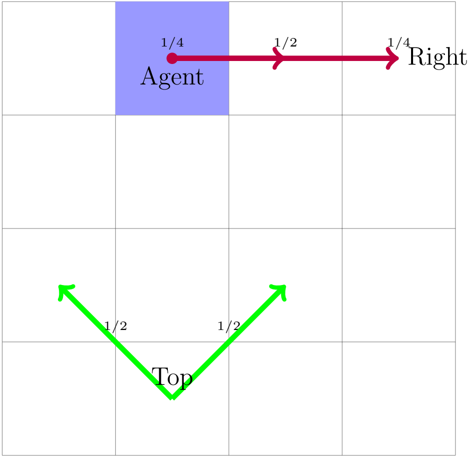
Transition Model: $P_{ss'}^{a} = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]$

Deterministic environment.



Transition Model: $P^a_{ss'} = \mathbb{P}[S_{t+1} = s' | S_t = s, A_t = a]$

Stochastic environment.



Reward function: $r = R(s, a) = r(s')$

Simple goal reward.

0	0	0	+1
0	0	0	0
0	0	0	0
0	0	0	0

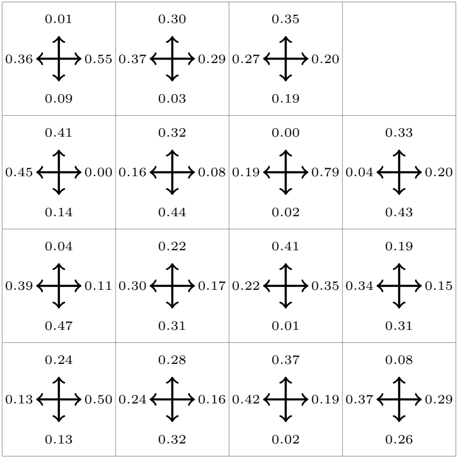
Reward function: $r = R(s, a) = r(s')$

Other example of environment reward function.

-77	-46	-28	+1
-31	-64	-23	-9
-86	-50	-82	-35
-58	-40	-15	-85

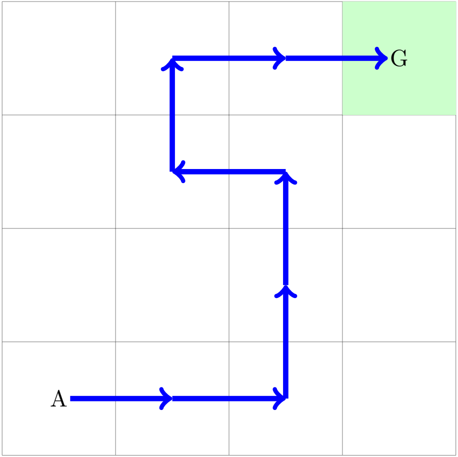
Policy: $(\pi : S \rightarrow A)$

Agent action in a state defined by its policy
deterministic/stochastic



Trajectory: $\tau_\pi = (s_0, a_0, s_1, a_1, ...)$

$(s_{0,0},\ \rightarrow,\ 0,\ s_{1,0},\ \rightarrow,\ 0,\ s_{2,0}, \uparrow,\ 0,\ s_{2,1}, \uparrow,\ 0,\ s_{2,2},\ \leftarrow,\ 0,\ s_{1,2}, \uparrow,\ 0,\ s_{1,3},\ \rightarrow,\ 0,\ s_{2,3},\ \rightarrow,\ 1)$



Return: $G_t = \sum_{k=1}^{T} \gamma^k r_{t+k}$

Cumulative rewards

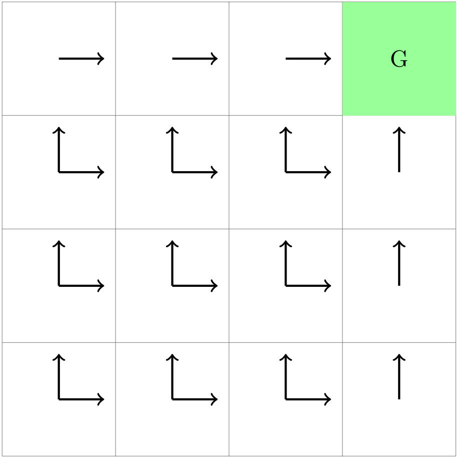
	t=6 1	t=7 1	G
	t=5 1	t=4 1	
		t=3 1	
t=0 1	t=1 1	t=2 1	

Discounted rewards (0.95)

	t=6 0.90	t=7 0.95	G
	t=5 0.86	t=4 0.81	
		t=3 0.77	
t=0 0.66	t=1 0.7	t=2 0.74	

Objective: Find best Policy $\pi^* = \arg\max_\pi E_{\tau\sim\pi}[G(\tau)]$

Optimal policy in the grid world environment.



Let's Code: Environment and Agent Interaction

Second Glossary of MDPs

- Value Function (V)
- Action Value Function (Q)
- Bellman Equations
- Dynamic Programming

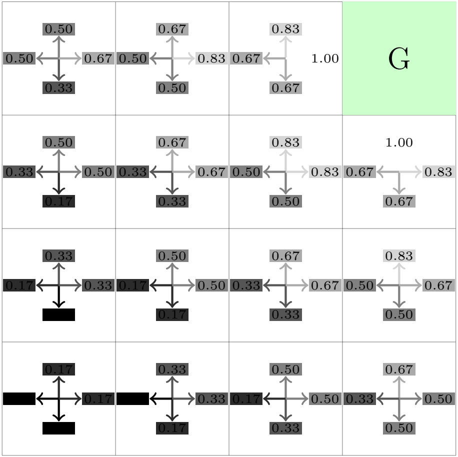
Value Function: $V^\pi(s) = E_{\tau\sim\pi}[G_t \,|S_t = s]$

Expected Return for State following π

0.61	0.78	0.94	G
0.44	0.61	0.78	0.94
0.28	0.44	0.61	0.78
0.11	0.28	0.44	0.61

Action Value Function: $Q^{\pi}(s,a) = E_{\tau\sim\pi}[G_t \,|S_t = s, A_t = a]$

Expected Return for State-Action following π



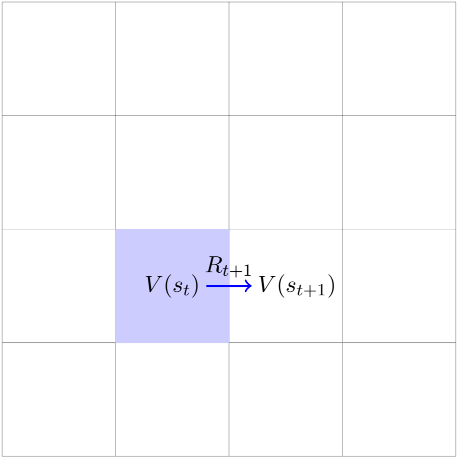
- **Idea :** The value of your starting point is the reward you expect to get from being there, plus the value of wherever you land next.

$$
\begin{aligned}
V(s) &= \mathbb{E}[G_t|S_t = s] \\
&= \mathbb{E}[R_{t+1} + \gamma G_{t+1}|S_t = s] \\
&= \mathbb{E}[R_{t+1} + \gamma V(S_{t+1})|S_t = s]
\end{aligned}
$$

$$
Q(s,a) = \mathbb{E}[R_{t+1} + \gamma \mathbb{E}_{a\sim\pi} Q(S_{t+1}, a) \mid S_t = s, A_t = a]
$$

Value Function Decomposition: $V^\pi(s)$

Value Function: $V^\pi(s) = E[R_{t+1} + \gamma V^\pi(S_{t+1}) | S_t = s]$



Bellman Equations development

$$
\begin{aligned}
V_\pi(s) &= \sum_{a \in \mathcal{A}} \pi(a|s) Q_\pi(s,a) \\
Q_\pi(s,a) &= R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} V_\pi(s') \\
V_\pi(s) &= \sum_{a \in \mathcal{A}} \pi(a|s) \big(R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} V_\pi(s') \big) \\
Q_\pi(s,a) &= R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} \sum_{a' \in \mathcal{A}} \pi(a'|s') Q_\pi(s',a')
\end{aligned}
$$

The MDP Solution

Dynamic Programming allows to resolve the MDP optimization problem ($\pi^* = \arg\max_\pi E_{\tau \sim \pi}[G(\tau)]$). It is an iterative process:

- Policy initialization
- Policy evaluation
- Policy improvement

Policy evaluation

Policy Evaluation: compute the state-value V_π for a given policy π:
We initialize V_0 arbitrarily. And we update it using:

$$\begin{aligned} \mathrm{V}_{k+1}(s) &= \mathbb{E}_\pi[r + \gamma V_k(s_{t+1}) | S_t = s] \\ &= \sum_a \pi(a|s) \sum_{s',r} P(s', r|s, a)(r + \gamma V_\pi(s'))\ (1) \end{aligned}$$

$V_\pi(s)$ is a fix point for (1), so if $(V_k)_{k\in\mathbb{N}}$ converges, it converges to V_π.

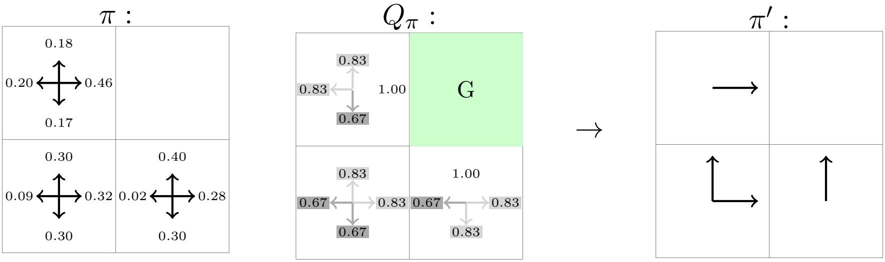
Policy Improvement

Policy Improvement: generates a better policy $\pi' \geq \pi$ by acting greedily. Compute Q from V $(\forall a, s)$:

$$
\begin{aligned}
Q_\pi(s,a) &= \mathbb{E}[R_{t+1} + \gamma V_\pi(S_{t+1}) | S_t = s, A_t = a] \\
&= \sum_{s',r} P(s', r | s, a)(r + \gamma V_\pi(s'))
\end{aligned}
$$

Update greedily: $\pi'(s) = \arg\max_{a \in \mathcal{A}} Q_\pi(s,a)$ $(\forall s)$

Policy improvement: $\pi'(s) = \arg\max_{a \in A} Q_\pi(s, a)$



Dynamic Programming

Policy Iteration: iterative procedure to improve the policy when combining policy evaluation and improvement.

$$\pi_0 \xrightarrow{\text{evaluation}} V_{\pi_0} \xrightarrow{\text{improve}} \pi_1 \xrightarrow{\text{evaluation}} \dots \xrightarrow{\text{improve}} \pi_* \xrightarrow{\text{evaluation}} V_* \quad (1)$$

Bellman Equations Optimality

Bellman equations for the optimal value functions

$$V_*(s) = \max_{a \in \mathcal{A}} Q_*(s,a)$$

$$Q_*(s,a) = R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} V_*(s')$$

$$V_*(s) = \max_{a \in \mathcal{A}} \big(R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} V_*(s') \big)$$

$$Q_*(s,a) = R(s,a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} \max_{a' \in \mathcal{A}} Q_*(s',a')$$

Take home message

Initialize $\pi(s), \forall s$

1. Evaluate $V_\pi(s), \forall s$ (using $\mathbb{P}^a_{ss'}$)
2. Compute $Q_\pi(s,a), \forall s, a$ (using $\mathbb{P}^a_{ss'}$)
3. Update $\pi'(s) = \max_a Q_\pi(s,a), \forall s$
4. While $\pi'(s) \neq \pi(s)$ do $\pi(s) = \pi'(s)$ and iterate

Result : $\pi = \arg\max_\pi E[G]$

Coding session
Dynamic Programming