Homework #4
Dominant Eigenvalues & Dominant Eigenvectors
Numerical Analysis I - Math 4340

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1 Problems

Use the power method to calculate the dominant eigenvalue and the associated eigenvector for the following matrices. Use 6 significant digits.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 5 & 1 \\ -2 & -1 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & -3 & 1 & 5 \\ 3 & 1 & 6 & -2 \\ 4 & 5 & -2 & -1 \end{bmatrix}$$

1.1 Matrix A

In this case, I implemented the following function:

```
function [lambda, x] = DomEig(A)

lenA = max(size(A));
x = ones(lenA,1);
xOld = x * 0;

while norm(x -xOld, 2) > 10^(-10),
    xOld = x;
    x = A*x/norm(A*x,2);
end

lambda = norm(A*x,2)/norm(x,2);

end
```

Running the program, each step gives the following output:
[lambda, x, ns] = DomEig(A)

1st loop
0.000000000000
0.7592566023653
0.650791373456
Eigenvalue = 6.807435381301

2nd loop
-0.111533427765
0.6532672197665

0.7488673007079
Eigenvalue = 7.4588657886103

3rd loop
-0.1025357834832
0.5233597281957
0.8459202137367
Eigenvalue = 8.0560231787904

4th loop
-0.0776928638099
0.4171019616824
0.9055328665895
Eigenvalue = 8.4234332827714

If we continue in the same fashion, we arrive at the 29th iteration, where the values are as follows:

$$\textbf{Dominant Eigenvector} = \begin{bmatrix} -0.0314851015576 \\ 0.2459344183513 \\ 0.9687749740007 \end{bmatrix}$$

Dominant Eigenvalue = 8.811138511406
Now, rounded to 6 significant digits:

$$\text{Dominant Eigenvector} = \begin{bmatrix} -0.0314851 \\ 0.245934 \\ 0.968775 \end{bmatrix}$$

Dominant Eigenvalue = 8.81114

To confirm, we just work with the eigenvector. Let's create a matrix D with all zeros but the dominant eigenvalue is in the diagonal. Then create a matrix $E = A - D$, this way, the elements in the diagonal of the original matrix A are now set up with the dominant eigenvalue. Finally, we define a matrix F such that it has 3 rows and 1 column whose elements are the eigenvector of the dominant eigenvalue. If we multiply $E * F$, the result should be close to 0. In this case, it is up to 6 significant digits.

```
--> F
 F  =

  -0.0314851015576
   0.2459344183513
   0.9687749740007


--> D
 D  =

   8.811138511406   0.               0.
   0.               8.811138511406   0.
   0.               0.               8.811138511406


--> E = A - D
 E  =

  -7.811138511406  -1.               0.
   1.              -3.811138511406   1.
  -2.              -1.               0.188861488594


--> E*F
 ans  =

   0.000000070961
  -0.0000002606156
   0.0000000684664
```

1.2 Matrix B

Running the previous function, but with matrix $B = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & -3 & 1 & 5 \\ 3 & 1 & 6 & -2 \\ 4 & 5 & -2 & -1 \end{bmatrix}$ as the input, we obtain the following first four outputs:

1st loop
0.6804138174398
0.2721655269759
0.5443310539518
0.4082482904639
Eigenvalue = 7.7052170878159

2nd loop
0.6358003194112
0.3179001597056
0.6181391994276
0.3355612796893
Eigenvalue = 7.8152002507582

3rd loop
0.6124172589312
0.2531023357945
0.673433000596
0.3276771311625
Eigenvalue = 7.8714189843245

4th loop
0.6109374167061
0.275036675378
0.6956303386649
0.2592464695891
Eigenvalue = 7.9011200576295

Surprisingly, it took 1704 iterations to obtain the desired output. However, let us switch the sign of the output to obtain the real dominant eigenvalue with its corresponding eigenvector, since it was negative, a small correction had to be done to the program. 1704th loop

$$\textbf{Dominant Eigenvector} = \begin{bmatrix} 0.2634624228974 \\ 0.6590407285308 \\ -0.1996334906499 \\ -0.6755733411478 \end{bmatrix}$$

Dominant Eigenvalue = -8.0285783523965

Now, rounded to 6 significant digits:

$$\text{Dominant Eigenvector} = \begin{bmatrix} 0.263462 \\ 0.659041 \\ -0.199634 \\ -0.675573 \end{bmatrix}$$

Dominant Eigenvalue = -8.02858

To confirm, we just work with the eigenvector. Let's create a matrix D with all zeros but the dominant eigenvalue is in the diagonal. Then create a matrix $E = B - D$, this way, the elements in the diagonal of the original matrix B are now set up with the dominant eigenvalue. Finally, we define a matrix F such that it has 4 rows and 1 column whose elements are the eigenvector of the dominant eigenvalue. If we multiply $E * F$, the result should be close to 0. In this case, it is up to 6 significant digits.

```
 F  =

   0.2634624228974
   0.6590407285308
  -0.1996334906499
  -0.6755733411478


-->D
 D  =

  -8.0285783523965   0.                 0.                 0.
   0.               -8.0285783523965   0.                 0.
   0.                0.                -8.0285783523965   0.
   0.                0.                 0.                -8.0285783523965


-->E
 E  =

   10.028578352397   1.                 3.                 4.
   1.                5.0285783523965   1.                 5.
   3.                1.                14.028578352397   -2.
   4.                5.                -2.                7.0285783523965


-->E * F
 ans  =

   0.0000004429289
   0.0000001673466
   0.0000006141746
   0.0000001544961
```