OI_Lab4

December 27, 2023

###Obrada informacija

#4. Laboratorijska vježba: Multivarijatni financijski vremenski nizovi

Prosinac 2023.

##Upute

U ovoj bilježnici dana je priprema sa svim uputama za 4. laboratorijsku vježbu iz predmeta Obrada informacija - uz bilježnicu su dostupni i podatci u datoteci `prices.csv`.

Vaš zadatak je u bilježnicu na odgovarajuća mjesta dopisati kod Vašeg rješenja,.

Riješenu bilježnicu potrebno je predati kao izvještaj u .pdf formatu na *Moodle* najkasnije do 14.1.2024. u 23:59h. Datoteka koju predajete se mora zvati *PrezimeImeJMBAG.pdf*.

0.1 Uvod

U laboratorijskoj vježbi razmatra se dinamika cijena vrijednosnica na financijskim tržištima. Dane su povijesne dnevne cijene 24 ETF-a (eng. *exchange traded fund*) koji prate određene dioničke, obvezničke ili druge indekse.

Oznaka	Naziv	Klasa imovine
SPY	SPDR S&P 500 ETF Trust	Equity: U.S. - Large Cap
IEFA	iShares Core MSCI EAFE ETF	Equity: Developed Markets Ex-U.S. - Total Market
VWO	Vanguard FTSE Emerging Markets ETF	Equity: Emerging Markets - Total Market
EWJ	iShares MSCI Japan ETF	Equity: Japan - Total Market
XLF	Financial Select Sector SPDR Fund	Equity: U.S. Financials
XLK	Technology Select Sector SPDR Fund	Equity: U.S. Technology
XLV	Health Care Select Sector SPDR Fund	Equity: U.S. Health Care
XLY	Consumer Discretionary Select Sector SPDR Fund	Equity: U.S. Consumer Cyclicals
XLP	Consumer Staples Select Sector SPDR Fund	Equity: U.S. Consumer Non-cyclicals

Oznaka	Naziv	Klasa imovine
XLU	Utilities Select Sector SPDR Fund	Equity: U.S. Utilities
XLI	Industrial Select Sector SPDR Fund	Equity: U.S. Industrials
XLE	Energy Select Sector SPDR Fund	Equity: U.S. Energy
XLC	Communication Services Select Sector SPDR Fund	Equity: U.S. Telecommunications
XLRE	Real Estate Select Sector SPDR Fund	Equity: U.S. Real Estate
XLB	Materials Select Sector SPDR Fund	Equity: U.S. Basic Materials
BND	Vanguard Total Bond Market ETF	Fixed Income: U.S. - Broad Market
LQD	iShares iBoxx USD Investment Grade Corporate Bond ETF	Fixed Income: U.S. - Corporate
BNDX	Vanguard Total International Bond ETF	Fixed Income: Global Ex-U.S. - Broad Market
SHV	iShares Short Treasury Bond ETF	Fixed Income: U.S. - Government
HYG	iShares iBoxx USD High Yield Corporate Bond ETF	Fixed Income: U.S. - Corporate
GLD	SPDR Gold Trust	Commodities: Precious Metals Gold
SLV	iShares Silver Trust	Commodities: Precious Metals Silver
PDBC	Invesco Optimum Yield Diversified Commodity Strategy No K-1 ETF	Commodities: Broad Market
USO	United States Oil Fund LP	Commodities: Energy Crude Oil

Pri modeliranju zajedničkog kretanja i rizika vrijednosnica, koristit ćemo aritmetičke povrate:

$R(t) = \frac{S(t)-S(t-1)}{S(t-1)}$,

gdje je $S(t)$ cijena vrijednosnice u danu t. U sklopu ove laboratorijske vježbe cilj je analizirati kretanje danih ETF-ova i izračunati glavne komponente (PCA) koje utječu na njihovu dinamiku. Laboratorijsku vježbu je potrebno riješiti unutar ove bilježnice i predati riješenu bilježnicu kao izvještaj.

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

```
prices = pd.read_csv('prices.csv')
prices.set_index('Time', inplace=True)
prices.index = pd.to_datetime(prices.index)

prices.head()
```

[1]:

Time	SPY	IEFA	VWO	EWJ	XLF	XLK	\
2019-01-02	231.492233	48.000053	33.417080	47.497765	21.776472	59.029892	
2019-01-03	225.968170	47.684727	32.893154	47.227894	21.286907	56.050797	
2019-01-04	233.537125	49.129986	33.932262	48.828522	21.994057	58.534962	
2019-01-07	235.378525	49.366474	34.002121	48.949490	22.021255	59.058445	
2019-01-08	237.589920	49.716839	34.194221	49.182148	22.039383	59.553371	

Time	XLV	XLY	XLP	XLU	…	XLB	\
2019-01-02	78.483353	95.259102	44.591763	44.784126	…	46.068901	
2019-01-03	76.889748	93.196533	44.335598	44.775520	…	44.762505	
2019-01-04	79.183434	96.280815	45.280781	45.438545	…	46.522518	
2019-01-07	79.487411	98.457954	45.218948	45.128563	…	46.685822	
2019-01-08	80.104607	99.546509	45.634129	45.688255	…	47.175724	

Time	BND	LQD	BNDX	SHV	HYG	\
2019-01-02	69.980576	97.039024	48.517841	101.234894	63.253788	
2019-01-03	70.253906	97.081886	48.517841	101.271584	63.285011	
2019-01-04	70.042305	97.021873	48.473194	101.299133	64.347198	
2019-01-07	69.936508	97.030441	48.357071	101.299133	64.971985	
2019-01-08	69.848312	97.244827	48.321350	101.289948	65.393723	

Time	GLD	SLV	PDBC	USO
2019-01-02	121.330002	14.56	8.545244	78.800003
2019-01-03	122.430000	14.75	8.607411	79.599998
2019-01-04	121.440002	14.73	8.731748	81.440002
2019-01-07	121.860001	14.67	8.799566	82.320000
2019-01-08	121.529999	14.69	8.873038	84.000000

[5 rows x 24 columns]

1 Zadatak 1 - Računanje korelacijske matrice i matrice kovarijance povrata

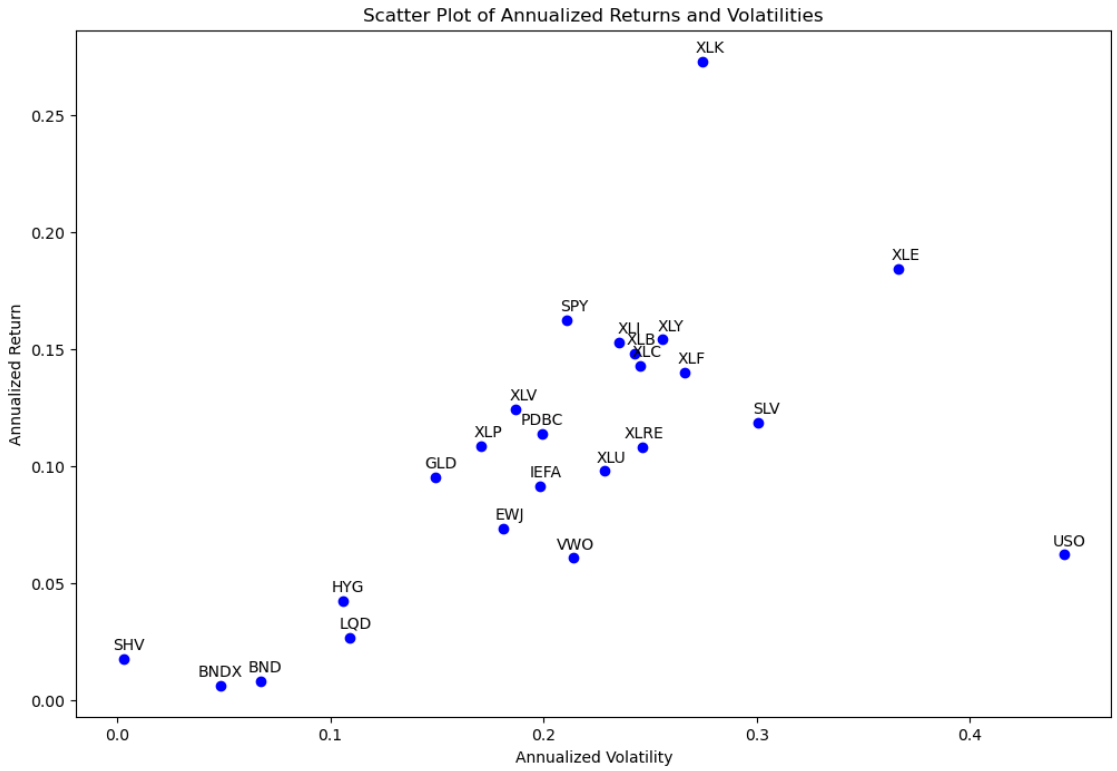
1.1. U prvom zadatku ove laboratorijske vježbe potrebno je prvo iz danih cijena (gore učitanih u Pandas DataFrame) izračunati dnevne povrate za sve pojedine vrijednosnice (prateći formulu danu u uvodu).

Izračunajte srednje povrate i volatilnost (standardnu devijaciju povrata) za svaku pojedinu vrijednosnicu. Pri analizi srednjih povrata i volatilnosti, te se brojke često *anualiziraju* - to znači da se srednji povrati pomnože s 252 (cca. broj trgovinskih dana u godini), a volatilnost s $\sqrt{252}$.

Izračunajte anualizirane srednje povrate i volatilnosti. Sve ETF-ove prikažite u dijagramu raspršenja s volatilnošću na x-osi i srednjim povratom na y-osi.

Razmislite - koji se ETF-ovi ističu po odnosu povrata i rizika (posebno dobri ili posebno loši kao investicije)?

[2]:
```
# calculate daily returns for every stock
ret = {}
for stock in prices:
    string = stock + ' return'
    ret[stock] = (prices[stock] - prices[stock].shift(1)) / prices[stock].
 ↪shift(1)
returns = pd.DataFrame(ret)

# calculate annualized mean return and volatility
annualized_returns = returns.mean() * 252
annualized_volatilities = returns.std() * np.sqrt(252)

# create a dataframe for annualized returns and volatilities
annualized_data = pd.DataFrame({
    'Annualized Return': annualized_returns,
    'Annualized Volatility': annualized_volatilities
})

# scatter plot
plt.figure(figsize=(12, 8))
plt.scatter(annualized_volatilities, annualized_returns, color='blue')
plt.title('Scatter Plot of Annualized Returns and Volatilities')
plt.xlabel('Annualized Volatility')
plt.ylabel('Annualized Return')

# adding labels for each ETF
for stock, return_value, volatility in zip(returns.columns, annualized_returns,␣
 ↪annualized_volatilities):
    plt.annotate(stock, (volatility, return_value), textcoords="offset points",␣
 ↪xytext=(14,6), ha='right')

plt.show()
```



1.2. Kovarijancu i korelaciju moguće je iz podataka izračunati koristeći Pandas, ali i NumPy ili neke druge biblioteke.

Koristeći dnevne povrate, izračunajte matricu kovarijance Σ i matricu korelacije C povrata svih ETF-ova. Matrice ispišite u konzolu ili vizualizirajte.

Proučite strukturu matrice i razmislite o tome koje zajedničke komponente u podatcima možete očekivati.

[3]:
```
# calculate covariance and correlation matrices
covariance_matrix = returns.cov()
correlation_matrix = returns.corr()

# print both covariance and correlation matrices
print("Covariance Matrix:")
print(covariance_matrix)
print("\nCorrelation Matrix:")
print(correlation_matrix)

# viisualize the correlation matrix using a heatmap
plt.figure(figsize=(12, 8))
sns.heatmap(correlation_matrix, annot=True, cmap='coolwarm', fmt='.2f',␣
 ↪linewidths=.5)
plt.title('Correlation Matrix of Daily Returns')
```

```
plt.show()
```

```
Covariance Matrix:
              SPY           IEFA           VWO           EWJ           XLF  \
SPY   1.761574e-04  1.457528e-04  1.376812e-04  1.170823e-04  1.918401e-04
IEFA  1.457528e-04  1.563030e-04  1.409613e-04  1.259424e-04  1.728926e-04
VWO   1.376812e-04  1.409613e-04  1.815550e-04  1.112717e-04  1.534720e-04
EWJ   1.170823e-04  1.259424e-04  1.112717e-04  1.299253e-04  1.357587e-04
XLF   1.918401e-04  1.728926e-04  1.534720e-04  1.357587e-04  2.808551e-04
XLK   2.158161e-04  1.701803e-04  1.706754e-04  1.387625e-04  2.036297e-04
XLV   1.318726e-04  1.061152e-04  9.604616e-05  8.357183e-05  1.406268e-04
XLY   1.934884e-04  1.614538e-04  1.574461e-04  1.310982e-04  1.994529e-04
XLP   1.107683e-04  8.944329e-05  7.605146e-05  7.151245e-05  1.225315e-04
XLU   1.274866e-04  1.049761e-04  8.575088e-05  8.290778e-05  1.461856e-04
XLI   1.753387e-04  1.551730e-04  1.373171e-04  1.224394e-04  2.245905e-04
XLE   1.907531e-04  1.835906e-04  1.738178e-04  1.417821e-04  2.810847e-04
XLC   1.796809e-04  1.441639e-04  1.424863e-04  1.176312e-04  1.748918e-04
XLRE  1.628937e-04  1.385278e-04  1.222450e-04  1.077890e-04  1.868533e-04
XLB   1.757895e-04  1.626840e-04  1.479333e-04  1.284975e-04  2.209945e-04
BND   1.032848e-05  1.106674e-05  9.632471e-06  9.707243e-06  5.550302e-06
LQD   3.073254e-05  3.035281e-05  2.706990e-05  2.332045e-05  2.590348e-05
BNDX  6.836683e-06  6.458734e-06  5.747600e-06  5.987623e-06  2.102074e-06
SHV  -1.782225e-07 -1.215006e-07 -1.699283e-07  2.821937e-08 -4.059010e-07
HYG   7.076129e-05  6.390899e-05  5.976081e-05  5.006845e-05  8.011112e-05
GLD   1.307499e-05  2.218935e-05  2.325321e-05  1.778782e-05 -1.998805e-06
SLV   6.410859e-05  7.962121e-05  8.086376e-05  5.718553e-05  5.123144e-05
PDBC  5.730408e-05  5.856160e-05  6.037230e-05  4.461771e-05  7.280871e-05
USO   1.171177e-04  1.116622e-04  1.196180e-04  7.435808e-05  1.564442e-04

              XLK           XLV           XLY           XLP           XLU  \
SPY   2.158161e-04  1.318726e-04  1.934884e-04  1.107683e-04  1.274866e-04
IEFA  1.701803e-04  1.061152e-04  1.614538e-04  8.944329e-05  1.049761e-04
VWO   1.706754e-04  9.604616e-05  1.574461e-04  7.605146e-05  8.575088e-05
EWJ   1.387625e-04  8.357183e-05  1.310982e-04  7.151245e-05  8.290778e-05
XLF   2.036297e-04  1.406268e-04  1.994529e-04  1.225315e-04  1.461856e-04
XLK   2.996448e-04  1.510928e-04  2.403027e-04  1.222616e-04  1.316509e-04
XLV   1.510928e-04  1.385254e-04  1.271010e-04  9.811664e-05  1.172870e-04
XLY   2.403027e-04  1.271010e-04  2.591421e-04  1.078307e-04  1.199366e-04
XLP   1.222616e-04  9.811664e-05  1.078307e-04  1.154083e-04  1.193987e-04
XLU   1.316509e-04  1.172870e-04  1.199366e-04  1.193987e-04  2.069732e-04
XLI   1.910086e-04  1.299066e-04  1.859044e-04  1.148908e-04  1.390192e-04
XLE   1.834210e-04  1.329837e-04  1.816440e-04  1.091994e-04  1.356845e-04
XLC   2.298729e-04  1.227429e-04  2.057247e-04  1.007367e-04  1.094027e-04
XLRE  1.805967e-04  1.317512e-04  1.722832e-04  1.208262e-04  1.756801e-04
XLB   1.937529e-04  1.310538e-04  1.861967e-04  1.143999e-04  1.386152e-04
BND   1.330583e-05  7.160052e-06  1.404303e-05  8.633379e-06  1.322661e-05
LQD   3.825942e-05  2.096342e-05  3.828408e-05  1.900692e-05  3.080993e-05
BNDX  8.851570e-06  5.695455e-06  9.282111e-06  5.575668e-06  1.193018e-05
```

```
SHV  -1.452970e-07 -9.666214e-08 -1.939843e-07 -4.617104e-08  4.848319e-08
HYG   8.294049e-05  5.245450e-05  7.940941e-05  4.330333e-05  5.912695e-05
GLD   1.719533e-05  1.085658e-05  1.310897e-05  1.405772e-05  2.878039e-05
SLV   7.808044e-05  4.390190e-05  7.235107e-05  4.033467e-05  6.391738e-05
PDBC  5.823096e-05  3.552949e-05  5.444496e-05  2.980149e-05  3.136934e-05
USO   1.189163e-04  7.579423e-05  1.034852e-04  6.289437e-05  5.387372e-05

      …            XLB           BND           LQD           BNDX  \
SPY   …  1.757895e-04  1.032848e-05  3.073254e-05  6.836683e-06
IEFA  …  1.626840e-04  1.106674e-05  3.035281e-05  6.458734e-06
VWO   …  1.479333e-04  9.632471e-06  2.706990e-05  5.747600e-06
EWJ   …  1.284975e-04  9.707243e-06  2.332045e-05  5.987623e-06
XLF   …  2.209945e-04  5.550302e-06  2.590348e-05  2.102074e-06
XLK   …  1.937529e-04  1.330583e-05  3.825942e-05  8.851570e-06
XLV   …  1.310538e-04  7.160052e-06  2.096342e-05  5.695455e-06
XLY   …  1.861967e-04  1.404303e-05  3.828408e-05  9.282111e-06
XLP   …  1.143999e-04  8.633379e-06  1.900692e-05  5.575668e-06
XLU   …  1.386152e-04  1.322661e-05  3.080993e-05  1.193018e-05
XLI   …  2.047923e-04  7.666242e-06  2.831092e-05  5.608365e-06
XLE   …  2.420647e-04  1.351107e-06  2.017134e-05 -8.716908e-07
XLC   …  1.612733e-04  1.230099e-05  3.346059e-05  7.892247e-06
XLRE  …  1.730627e-04  1.730387e-05  4.088237e-05  1.311175e-05
XLB   …  2.339992e-04  8.019766e-06  2.815773e-05  5.579961e-06
BND   …  8.019766e-06  1.804064e-05  2.506508e-05  9.805009e-06
LQD   …  2.815773e-05  2.506508e-05  4.741936e-05  1.392387e-05
BNDX  …  5.579961e-06  9.805009e-06  1.392387e-05  9.331558e-06
SHV   … -2.004156e-07  1.783662e-07  1.881198e-07  1.431022e-07
HYG   …  7.358275e-05  1.137993e-05  2.675037e-05  7.607734e-06
GLD   …  2.226311e-05  1.504004e-05  2.333186e-05  1.010811e-05
SLV   …  8.964691e-05  1.956764e-05  3.791384e-05  1.520044e-05
PDBC  …  7.391043e-05  9.697108e-07  9.178127e-06 -1.070061e-06
USO   …  1.377812e-04 -1.954240e-06  7.810558e-06 -3.598700e-06

                SHV           HYG           GLD           SLV           PDBC  \
SPY  -1.782225e-07  7.076129e-05  1.307499e-05  6.410859e-05  5.730408e-05
IEFA -1.215006e-07  6.390899e-05  2.218935e-05  7.962121e-05  5.856160e-05
VWO  -1.699283e-07  5.976081e-05  2.325321e-05  8.086376e-05  6.037230e-05
EWJ   2.821937e-08  5.006845e-05  1.778782e-05  5.718553e-05  4.461771e-05
XLF  -4.059010e-07  8.011112e-05 -1.998805e-06  5.123144e-05  7.280871e-05
XLK  -1.452970e-07  8.294049e-05  1.719533e-05  7.808044e-05  5.823096e-05
XLV  -9.666214e-08  5.245450e-05  1.085658e-05  4.390190e-05  3.552949e-05
XLY  -1.939843e-07  7.940941e-05  1.310897e-05  7.235107e-05  5.444496e-05
XLP  -4.617104e-08  4.330333e-05  1.405772e-05  4.033467e-05  2.980149e-05
XLU   4.848319e-08  5.912695e-05  2.878039e-05  6.391738e-05  3.136934e-05
XLI  -2.654281e-07  7.304679e-05  7.898453e-06  6.205533e-05  6.888892e-05
XLE  -6.132212e-07  7.971625e-05  1.550908e-05  9.618807e-05  1.797461e-04
XLC  -1.412257e-07  7.091742e-05  1.449226e-05  6.546367e-05  4.964330e-05
XLRE -4.647225e-08  7.650127e-05  2.367132e-05  7.822725e-05  4.649864e-05
```

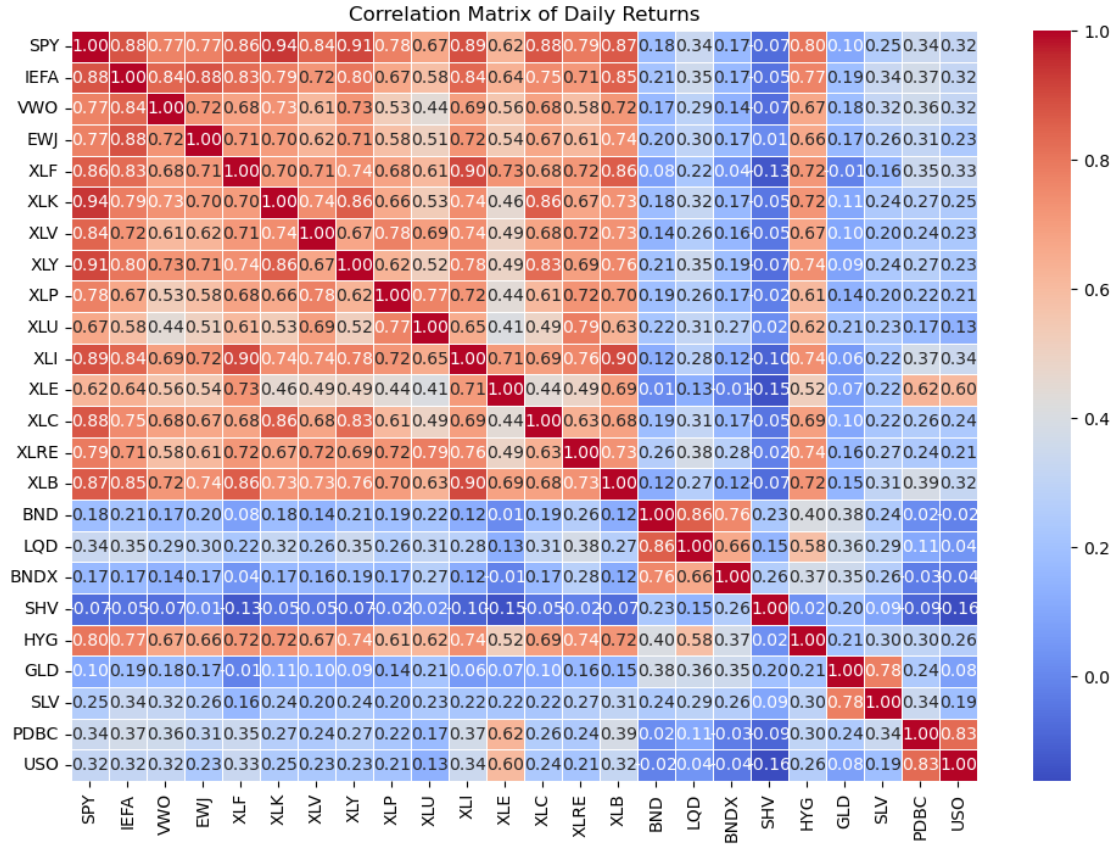
```
XLB  -2.004156e-07  7.358275e-05  2.226311e-05  8.964691e-05  7.391043e-05
BND   1.783662e-07  1.137993e-05  1.504004e-05  1.956764e-05  9.697108e-07
LQD   1.881198e-07  2.675037e-05  2.333186e-05  3.791384e-05  9.178127e-06
BNDX  1.431022e-07  7.607734e-06  1.010811e-05  1.520044e-05 -1.070061e-06
SHV   3.290195e-08  3.012192e-08  3.364304e-07  3.191481e-07 -2.125596e-07
HYG   3.012192e-08  4.433736e-05  1.293071e-05  3.729614e-05  2.542381e-05
GLD   3.364304e-07  1.293071e-05  8.868473e-05  1.389859e-04  2.891826e-05
SLV   3.191481e-07  3.729614e-05  1.389859e-04  3.588115e-04  8.087417e-05
PDBC -2.125596e-07  2.542381e-05  2.891826e-05  8.087417e-05  1.573913e-04
USO  -8.205055e-07  4.886216e-05  2.067511e-05  1.013434e-04  2.905185e-04

              USO
SPY   1.171177e-04
IEFA  1.116622e-04
VWO   1.196180e-04
EWJ   7.435808e-05
XLF   1.564442e-04
XLK   1.189163e-04
XLV   7.579423e-05
XLY   1.034852e-04
XLP   6.289437e-05
XLU   5.387372e-05
XLI   1.424165e-04
XLE   3.886086e-04
XLC   1.016578e-04
XLRE  9.185401e-05
XLB   1.377812e-04
BND  -1.954240e-06
LQD   7.810558e-06
BNDX -3.598700e-06
SHV  -8.205055e-07
HYG   4.886216e-05
GLD   2.067511e-05
SLV   1.013434e-04
PDBC  2.905185e-04
USO   7.829798e-04

[24 rows x 24 columns]

Correlation Matrix:
          SPY      IEFA       VWO       EWJ       XLF       XLK       XLV  \
SPY   1.000000  0.878381  0.769875  0.773916  0.862477  0.939356  0.844188
IEFA  0.878381  1.000000  0.836782  0.883774  0.825186  0.786362  0.721156
VWO   0.769875  0.836782  1.000000  0.724492  0.679648  0.731751  0.605635
EWJ   0.773916  0.883774  0.724492  1.000000  0.710689  0.703270  0.622943
XLF   0.862477  0.825186  0.679648  0.710689  1.000000  0.701934  0.712955
XLK   0.939356  0.786362  0.731751  0.703270  0.701934  1.000000  0.741610
XLV   0.844188  0.721156  0.605635  0.622943  0.712955  0.741610  1.000000
```

```
XLY   0.905598  0.802224  0.725870  0.714465  0.739316  0.862356  0.670835
XLP   0.776867  0.665956  0.525394  0.584005  0.680594  0.657458  0.775996
XLU   0.667663  0.583646  0.442362  0.505582  0.606326  0.528644  0.692673
XLI   0.890781  0.836904  0.687169  0.724299  0.903636  0.744034  0.744235
XLE   0.622738  0.636284  0.558951  0.538962  0.726742  0.459124  0.489573
XLC   0.876503  0.746577  0.684653  0.668155  0.675663  0.859777  0.675201
XLRE  0.790579  0.713748  0.584411  0.609142  0.718209  0.672044  0.721077
XLB   0.865835  0.850656  0.717719  0.736954  0.862051  0.731708  0.727910
BND   0.183215  0.208406  0.168309  0.200504  0.077974  0.180972  0.143227
LQD   0.336256  0.352564  0.291746  0.297107  0.224460  0.320965  0.258654
BNDX  0.168623  0.169117  0.139639  0.171961  0.041061  0.167394  0.158411
SHV  -0.074029 -0.053578 -0.069527  0.013649 -0.133527 -0.046275 -0.045277
HYG   0.800683  0.767703  0.666082  0.659678  0.717905  0.719579  0.669319
GLD   0.104608  0.188468  0.183254  0.165711 -0.012665  0.105483  0.097950
SLV   0.254996  0.336211  0.316823  0.264854  0.161385  0.238125  0.196918
PDBC  0.344148  0.373370  0.357144  0.312011  0.346300  0.268139  0.240621
USO   0.315353  0.319189  0.317261  0.233134  0.333613  0.245506  0.230142
```

```
           XLY       XLP       XLU  …       XLB       BND       LQD  \
SPY   0.905598  0.776867  0.667663  …  0.865835  0.183215  0.336256
IEFA  0.802224  0.665956  0.583646  …  0.850656  0.208406  0.352564
VWO   0.725870  0.525394  0.442362  …  0.717719  0.168309  0.291746
EWJ   0.714465  0.584005  0.505582  …  0.736954  0.200504  0.297107
XLF   0.739316  0.680594  0.606326  …  0.862051  0.077974  0.224460
XLK   0.862356  0.657458  0.528644  …  0.731708  0.180972  0.320965
XLV   0.670835  0.775996  0.692673  …  0.727910  0.143227  0.258654
XLY   1.000000  0.623527  0.517876  …  0.756129  0.205384  0.345359
XLP   0.623527  1.000000  0.772545  …  0.696145  0.189207  0.256930
XLU   0.517876  0.772545  1.000000  …  0.629864  0.216454  0.310997
XLI   0.778689  0.721124  0.651570  …  0.902713  0.121703  0.277217
XLE   0.488918  0.440439  0.408655  …  0.685659  0.013783  0.126923
XLC   0.827407  0.607115  0.492349  …  0.682585  0.187506  0.314599
XLRE  0.689391  0.724493  0.786606  …  0.728765  0.262427  0.382428
XLB   0.756129  0.696145  0.629864  …  1.000000  0.123432  0.267308
BND   0.205384  0.189207  0.216454  …  0.123432  1.000000  0.856969
LQD   0.345359  0.256930  0.310997  …  0.267308  0.856969  1.000000
BNDX  0.188756  0.169903  0.271464  …  0.119412  0.755692  0.661919
SHV  -0.066433 -0.023694  0.018579  … -0.072229  0.231513  0.150607
HYG   0.740829  0.605366  0.617225  …  0.722410  0.402373  0.583401
GLD   0.086472  0.138954  0.212430  …  0.154545  0.376009  0.359788
SLV   0.237270  0.198211  0.234546  …  0.309382  0.243209  0.290661
PDBC  0.269587  0.221121  0.173803  …  0.385131  0.018198  0.106239
USO   0.229739  0.209227  0.133827  …  0.321890 -0.016443  0.040535
```

```
          BNDX       SHV       HYG       GLD       SLV       PDBC       USO
SPY   0.168623 -0.074029  0.800683  0.104608  0.254996  0.344148  0.315353
IEFA  0.169117 -0.053578  0.767703  0.188468  0.336211  0.373370  0.319189
VWO   0.139639 -0.069527  0.666082  0.183254  0.316823  0.357144  0.317261
```

```
EWJ    0.171961  0.013649  0.659678  0.165711  0.264854  0.312011  0.233134
XLF    0.041061 -0.133527  0.717905 -0.012665  0.161385  0.346300  0.333613
XLK    0.167394 -0.046275  0.719579  0.105483  0.238125  0.268139  0.245506
XLV    0.158411 -0.045277  0.669319  0.097950  0.196918  0.240621  0.230142
XLY    0.188756 -0.066433  0.740829  0.086472  0.237270  0.269587  0.229739
XLP    0.169903 -0.023694  0.605366  0.138954  0.198211  0.221121  0.209227
XLU    0.271464  0.018579  0.617225  0.212430  0.234546  0.173803  0.133827
XLI    0.123795 -0.098669  0.739707  0.056554  0.220897  0.370256  0.343185
XLE   -0.012364 -0.146484  0.518735  0.071358  0.220025  0.620802  0.601757
XLC    0.167273 -0.050409  0.689556  0.099635  0.223753  0.256196  0.235216
XLRE   0.276487 -0.016503  0.740074  0.161916  0.266021  0.238749  0.211453
XLB    0.119412 -0.072229  0.722410  0.154545  0.309382  0.385131  0.321890
BND    0.755692  0.231513  0.402373  0.376009  0.243209  0.018198 -0.016443
LQD    0.661919  0.150607  0.583401  0.359788  0.290661  0.106239  0.040535
BNDX   1.000000  0.258261  0.374019  0.351373  0.262691 -0.027922 -0.042101
SHV    0.258261  1.000000  0.024939  0.196952  0.092886 -0.093407 -0.161657
HYG    0.374019  0.024939  1.000000  0.206212  0.295696  0.304344  0.262248
GLD    0.351373  0.196952  0.206212  1.000000  0.779136  0.244770  0.078460
SLV    0.262691  0.092886  0.295696  0.779136  1.000000  0.340319  0.191200
PDBC  -0.027922 -0.093407  0.304344  0.244770  0.340319  1.000000  0.827577
USO   -0.042101 -0.161657  0.262248  0.078460  0.191200  0.827577  1.000000

[24 rows x 24 columns]
```



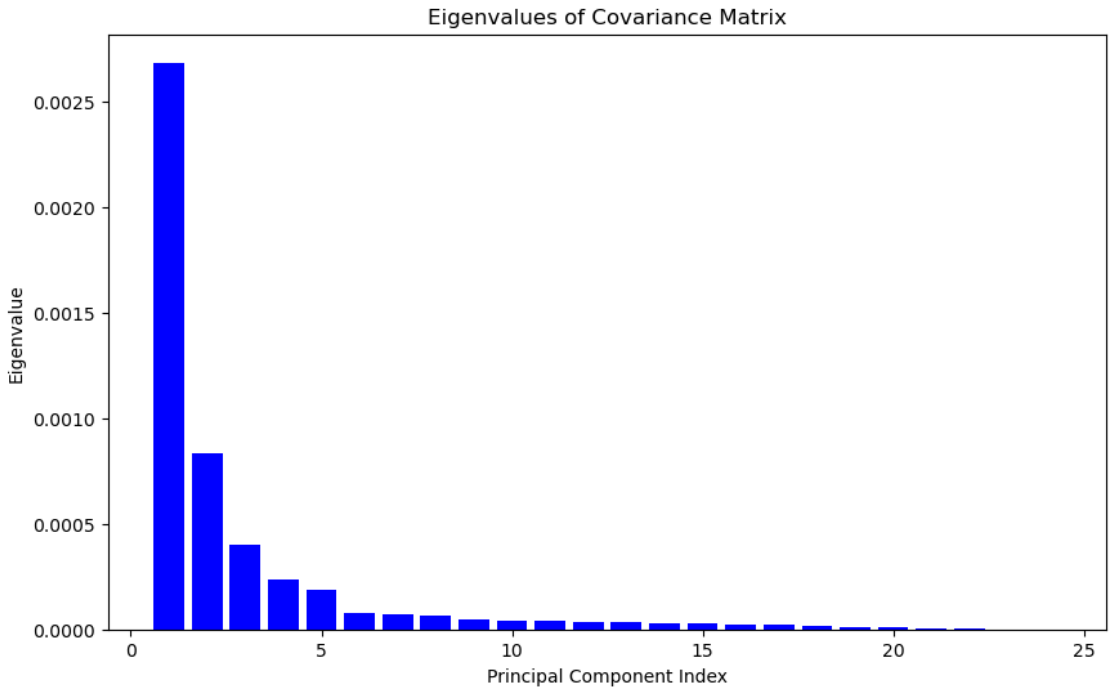
2 Zadatak 2 - Analiza glavnih komponenti

2.1. Za analizu glavnih komponenti potrebno je izračunati svojstvenu dekompoziciju, koju možete pronaći u sklopu biblioteke NumPy https://numpy.org/doc/stable/reference/generated/numpy.linalg.eig.html.

Izračunajte svojstvene vektore i pripadajuće svojstvene vrijednosti matrice kovarijance povrata Σ. Poredajte komponente padajući po svojstvenim vrijednostima i prikažite svojstvene vrijednosti grafički.

[4]:
```
# calculate eigenvalues and eigenvectors
eigenvalues, eigenvectors = np.linalg.eig(covariance_matrix)

# sort eigenvalues and corresponding eigenvectors in descending order
sorted_indices = np.argsort(eigenvalues)[::-1]
eigenvalues = eigenvalues[sorted_indices]
eigenvectors = eigenvectors[:, sorted_indices]

# print the eigenvalues
print("Eigenvalues:")
print(eigenvalues)

# plot the eigenvalues
plt.figure(figsize=(10, 6))
plt.bar(range(1, len(eigenvalues) + 1), eigenvalues, color='blue')
plt.title('Eigenvalues of Covariance Matrix')
plt.xlabel('Principal Component Index')
plt.ylabel('Eigenvalue')
plt.show()
```

```
Eigenvalues:
[2.68665277e-03 8.31976027e-04 4.01307581e-04 2.38512955e-04
 1.87916883e-04 7.93705677e-05 7.34527881e-05 6.48461289e-05
 4.98178157e-05 4.32038374e-05 4.07701847e-05 3.70788113e-05
 3.50096587e-05 2.97918414e-05 2.84744055e-05 2.43922231e-05
 2.15450339e-05 1.73556808e-05 1.02231492e-05 8.74032562e-06
 4.69766257e-06 2.04639608e-06 4.48673254e-07 2.82416669e-08]
```



2.2. **Izračunajte koliki udio varijance objašnjavaju prve tri komponente?**

[5]:
```
# sum of eigenvalues
total_eigenvalues_sum = sum(eigenvalues)

# proportion of variance explained by first 3 components
variance_explained = sum(eigenvalues[:3]) / total_eigenvalues_sum

print(f'Proportion of variance explained by first 3 components:␣
  ↪{variance_explained:.2%}')
```

```
Proportion of variance explained by first 3 components: 79.71%
```

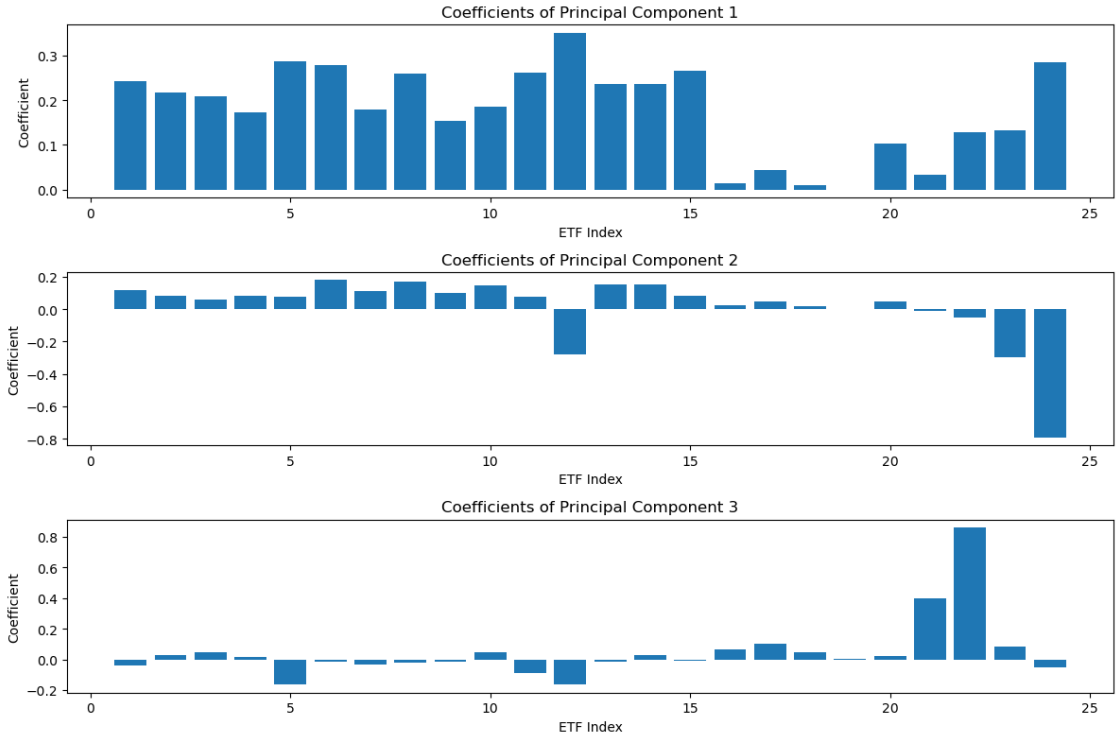
2.3. Komponente PCA će u financijama često opisivati neke zajedničke faktore u podatcima, što je moguće analizirati promatranjem pojedinih elemenata svojstvenih vektora. Ako je neki element određenog svojstvenog vektora velik po magnitudi (pozitivan ili negativan), to znači da ta komponenta opisuje odgovarajuću vrijednosnicu i objašnjava njenu varijancu, za razliku od slučaja kad je element blizu 0, što znači da razmatrana vrijednosnica ne ovisi previše o toj komponenti.

Prikažite grafički (npr. stupčastim dijagramom za svaku komponentu posebno) koeficijente prve 3 glavne komponente (elemente prva tri svojstvena vektora).

S obzirom na to koje vrijednosnice opisuju prve tri komponente, razmislite možete li zaključiti kakve zajedničke faktore u tržištu opisuju razmatrane komponente?

NAPOMENA: pripazite na to što vraća funkcija koju koristite i u kojoj se dimenziji (stupac ili red) nalaze svojstveni vektori.

[6]:
```
# plot coefficients of the first 3 principal components
plt.figure(figsize=(12, 8))

for i in range(3):
    plt.subplot(3, 1, i + 1)
    plt.bar(range(1, len(eigenvectors) + 1), eigenvectors[:, i])
    plt.title(f'Coefficients of Principal Component {i + 1}')
    plt.xlabel('ETF Index')
    plt.ylabel('Coefficient')

plt.tight_layout()
plt.show()
```



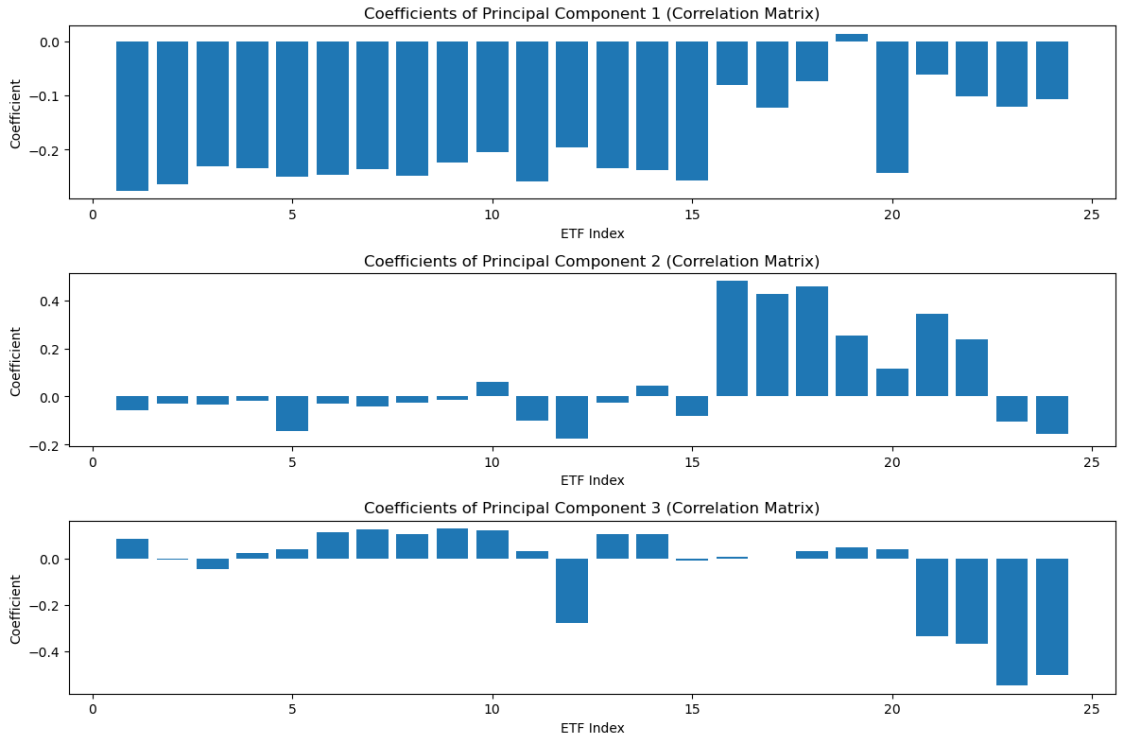
2.4. **Ponovite prethodnu analizu za matricu korelacije povrata C (prikažite svojstvene vrijednosti, udio varijance i koeficijente pojedinih komponenti za prve tri komponente).**

Usporedite rezultate - mijenjaju li se interpretacije komponenti?

[7]:
```
# for the correlation matrix
eigenvalues_corr, eigenvectors_corr = np.linalg.eig(correlation_matrix)

# sort in descending order
sorted_indices_corr = np.argsort(eigenvalues_corr)[::-1]
eigenvalues_corr = eigenvalues_corr[sorted_indices_corr]
```

```
eigenvectors_corr = eigenvectors_corr[:, sorted_indices_corr]

# eigenvalues for correlation matrix
print("Eigenvalues (Correlation Matrix):")
print(eigenvalues_corr)

# calculate the proportion of variance explained by the first 3 components
variance_explained_corr = sum(eigenvalues_corr[:3]) / sum(eigenvalues_corr)
print(f'Proportion of variance explained by the first three components␣
 ↪(Correlation Matrix): {variance_explained_corr:.2%}')

# plot coefficients of the first 3 principal components for correlation matrix
plt.figure(figsize=(12, 8))

for i in range(3):
    plt.subplot(3, 1, i + 1)
    plt.bar(range(1, len(eigenvectors_corr) + 1), eigenvectors_corr[:, i])
    plt.title(f'Coefficients of Principal Component {i + 1} (Correlation␣
 ↪Matrix)')
    plt.xlabel('ETF Index')
    plt.ylabel('Coefficient')

plt.tight_layout()
plt.show()
```

```
Eigenvalues (Correlation Matrix):
[1.22699323e+01 2.96295467e+00 2.01360322e+00 1.18735196e+00
 1.00143306e+00 8.65576176e-01 6.27675797e-01 3.97530473e-01
 3.45783916e-01 3.24897841e-01 2.76070196e-01 2.35837944e-01
 2.22869191e-01 1.90703448e-01 1.76334517e-01 1.58453188e-01
 1.50328950e-01 1.36322331e-01 1.18049724e-01 1.06644230e-01
 9.41515496e-02 7.32676508e-02 6.18218163e-02 2.40580290e-03]
Proportion of variance explained by the first three components (Correlation
Matrix): 71.86%
```



3 Zadatak 3 - Svojstveni portfelji

U primjeni PCA i svojstvenoj dekompoziciji kovarijance u financijama, svojstveni vektori se često zovu i tzv. svojstveni portfelji.

Općenito, portfelj je vektor $w = [w_1, ..., w_N]$ u kojem svaki element predstavlja težinu ili udio kapitala u određenoj vrijednosnici. Same težine svojstvenih portfelja mogu biti rotirane i skalirane u odnosu na elemente svojstvenih vektora.

U ovoj analizi ćemo pomnožiti njihove težine s predznakom njihove sume - na taj način zapravo samo "okrećemo" predznak svojstvenog vektora tako da mu je suma pozitivna (konačni PCA rastav je i dalje isti ako svojstveni vektor pomnožimo s -1). Također, dobro je i skalirati svojstvene portfelje sa sumom njihovih apsolutnih vrijednosti:

$\tilde{w}_i = \frac{w_i}{\sum_j^N |w_j|}$.

Na taj način se osigurava da visoke magnitude pojedinih elemenata ne uzrokuju velike razlike u volatilnostima svojstvenih portfelja.

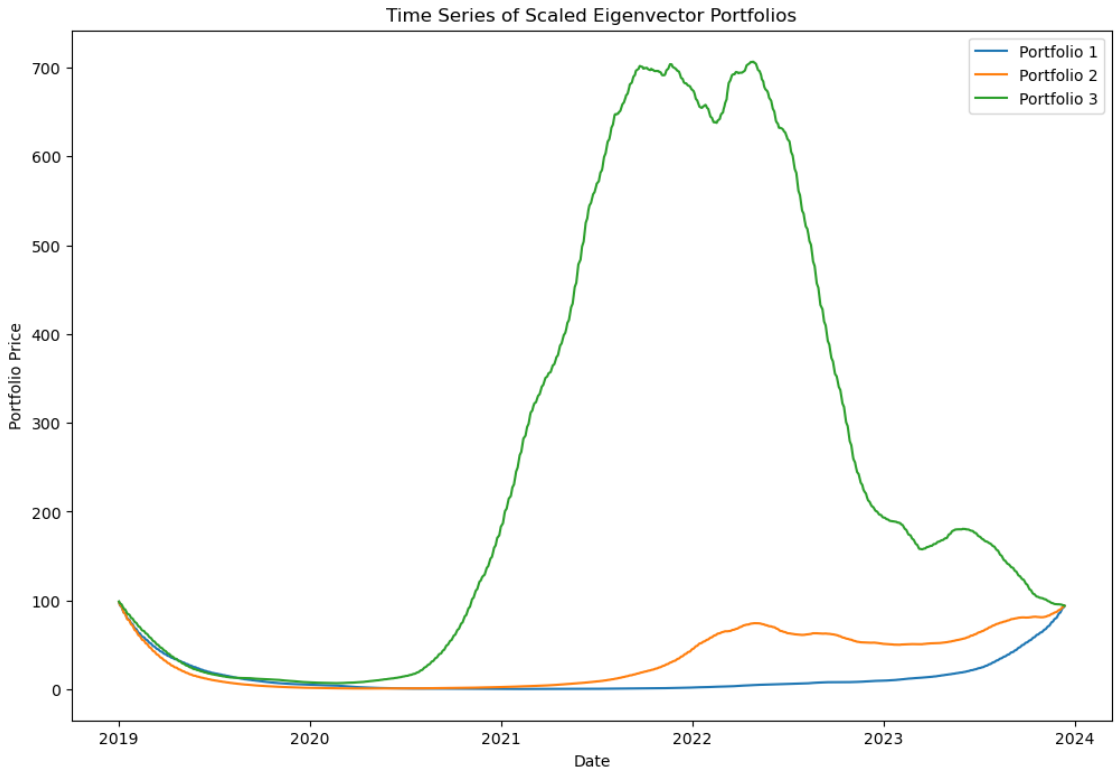
Ukoliko znamo povrate $R \in \mathbb{R}^{T \times N}$ (gdje je $R_i \in \mathbb{R}^T$ vektor povrata za vrijednosnicu i) za N vrijednosnica u nekom vremenskom periodu od T dana, povrate portfelja w u tom istom periodu možemo izračunati kao:

$R_p = \sum R_i w_i = R \cdot w.$

Izračunajte skalirane svojstvene portfelje $\tilde{w}$ koji proizlaze iz prve tri glavne kompo-

nente dobivene iz matrice kovarijance Σ. Za ta tri svojstvena portfelja izračunajte povijesne povrate kroz razmatrani period. Grafički prikažite vremensko kretanje njihovih vrijednosti (njihove povrate "vratite" natrag u cijene, s tim da početna cijena bude jednak za oba portfelja, npr. 100).

[8]:
```
# Select the first three eigenvectors
selected_eigenvectors = eigenvectors[:, :3]

# Calculate scaled eigenvectors
scaled_eigenvectors = selected_eigenvectors / np.sum(np.
 ↪abs(selected_eigenvectors), axis=0)

# calculate historical returns of the scaled eigenvectors
historical_returns = np.dot(prices.values, scaled_eigenvectors)

# normalize returns before calculating cumulative products
normalized_returns = (historical_returns - np.mean(historical_returns, axis=0))␣
 ↪/ np.std(historical_returns, axis=0)

# convert returns to prices, assuming an initial price of 100 for both␣
 ↪portfolios
portfolio_prices = 100 * np.cumprod(1 + normalized_returns / 100, axis=0)

# plot the time series of portfolio prices
plt.figure(figsize=(12, 8))
plt.plot(prices.index, portfolio_prices)
plt.title('Time Series of Scaled Eigenvector Portfolios')
plt.xlabel('Date')
plt.ylabel('Portfolio Price')
plt.legend([f'Portfolio {i + 1}' for i in range(3)])
plt.show()
```



Ako usporedite dobivene rezultate s kretanjem cijena originalnih vrijednosnica, vidjet ćete sličnosti između vrijednosnica koje pripadaju određenim klasama imovina i pojedinih svojstvenih portfelja. Svojstveni portfelji dakle predstavljaju niže-dimenzionalan prostor tzv. sintetičkih vrijednosnica (u našem slučaju 3 umjesto originalnih 24) koje najbolje opisuju cijeli razmatrani skup podataka. Dobra procjena tih komponenti je ključna u razumijevanju zajedničkog kretanja većih skupova dionica i upravljanju financijskim rizikom.

Razmislite što to znači za tržište koje smo analizirali - koji su glavni izvori rizika prevladavali u razmatranom periodu?