Chapter 1

Sets and Classes

Let there be *classes*.

Given two classes A and B, let there be *functions* from A to B. We write $f : A \to B$ iff f is a function from A to B.

Given functions $f : A \to B$ and $g : B \to C$, let there be a function $g \circ f = gf : A \to C$, the *composite* of f and g.

Axiom 1.1 (Empty Class)**.** *There exists a class $\emptyset$ such that, for any class A, there exists a unique function $\emptyset \to A$.*

Axiom 1.2 (Terminal Class)**.** *There exists a class 1 such that, for any class A, there exists a unique function $A \to 1$.*

Axiom 1.3 (Disjoint Union)**.** *For any classes A and B, there exists a class $A + B$, the* disjoint union *of A and B, and functions $\kappa_1 : A \to A + B$, $\kappa_2 : B \to A + B$, the* injections*, such that, for any class X and functions $f : A \to X$, $g : B \to X$, there exists a unique function $[f, g] : A + B \to X$ such that*

$$[f, g]\kappa_1 = f, \qquad [f, g]\kappa_2 = g \ .$$

Axiom 1.4 (Pullback)**.** *For any classes A, B and C, and any functions $f : A \to C$ and $g : B \to C$, there exists a set $A \times_C B$, the* pullback *of f and g, and functions $\pi_1 : A \times_C B \to A$, $\pi_2 : A \times_C B \to B$, such that:*

- $f\pi_1 = g\pi_2$
- *for any set X and functions $x : X \to A$, $y : X \to B$ such that $fx = gy$, there exists a unique function $(x, y) : X \to A \times_C B$ such that $\pi_1 \circ (x, y) = x$ and $\pi_2 \circ (x, y) = y$.*