Summary of Halmos' Naive Set Theory

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August 20, 2023

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Chapter 1

The Axiom of Extension

Let there be *sets*. We assume that everything is a set.

Let there be a binary relation of *membership*, $\in$. If $x \in A$ we say that x *belongs* to A, x is an *element* of A, or x is *contained* in A.

Axiom 1.1 (Axiom of Extension)**.** *Two sets are equal if and only if they have the same elements.*

Definition 1.2 (Subset)**.** Let A and B be sets. We say that A is a *subset* of B, or B *includes* A, and write $A \subset B$ or $B \supset A$, iff every element of A is an element of B.

Theorem 1.3. *For any set A, we have $A \subset A$.*

PROOF: Every element of A is an element of A. □

Theorem 1.4. *For any sets A, B and C, if $A \subset B$ and $B \subset C$ then $A \subset C$.*

PROOF: If every element of A is an element of B, and every element of B is an element of C, then every element of A is an element of C. □

Theorem 1.5. *For any sets A and B, if $A \subset B$ and $B \subset A$ then $A = B$.*

PROOF: If every element of A is an element of B, and every element of B is an element of A, then A and B have the same elements, and therefore are equal by the Axiom of Extension. □

Definition 1.6 (Proper Subset)**.** Let A and B be sets. We say that A is a *proper* subset of B, or B *properly* includes A, iff $A \subset B$ and $A \neq B$.