C1 Set Theory

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1 Primitive Notions

Let there be *sets*.

Let there be a binary relation called *membership*, $\in$. When $x \in y$ holds, we say x is a *member* or *element* of y. We write $x \notin y$ iff x is not a member of y.

2 The Axioms

Axiom 1 (Extensionality)**.** *If two sets have exactly the same members, then they are equal.*

Definition 2 (Empty Set)**.** The *empty set*, $\emptyset$, is the set with no elements.

Definition 3. Given objects $x_1, \ldots, x_n$, we write $\{x_1, \ldots, x_n\}$ for the set whose elements are exactly $x_1, \ldots, x_n$.

Definition 4 (Union)**.** The *union* of sets A and B, $A \cup B$, is the set whose elements are exactly the things that are members of A or members of B.

Definition 5 (Intersection)**.** The *intersection* of sets A and B, $A \cap B$, is the set whose elements are exactly the things that are members of both A and B.

Definition 6 (Disjoint)**.** Two sets A and B are *disjoint* iff they have no common members.

Definition 7 (Subset)**.** A set A is a *subset* of a set B or is *included* in B, $A \subseteq B$, iff every member of A is a member of B.

Proposition 8. *For any set A we have $A \subseteq A$.*

Proof: Every member of A is a member of A. $\square$

Proposition 9. *For any set A we have $\emptyset \subseteq A$.*

Proof: Vacuously, every member of $\emptyset$ is a member of A. $\square$

Definition 10 (Power Set)**.** The *power set* of A, $\mathcal{P}A$, is the set whose elements are the subsets of A.

Definition 11 (Abstraction)**.** Given a property $P(x)$, we write $\{x \mid P(x)\}$ for the set whose elements are all the objects x such that $P(x)$ is true.