Radar 2020

7th Iranian Conference on Radar and Surveillance Systems

18 - 19 November 2020

Malek Ashtar University of Technology

2020

دانشگاه صنعتی مالک اشتر

Waveform Optimization Techniques for Radar Systems

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18 November 2020

SnT

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Lecturer

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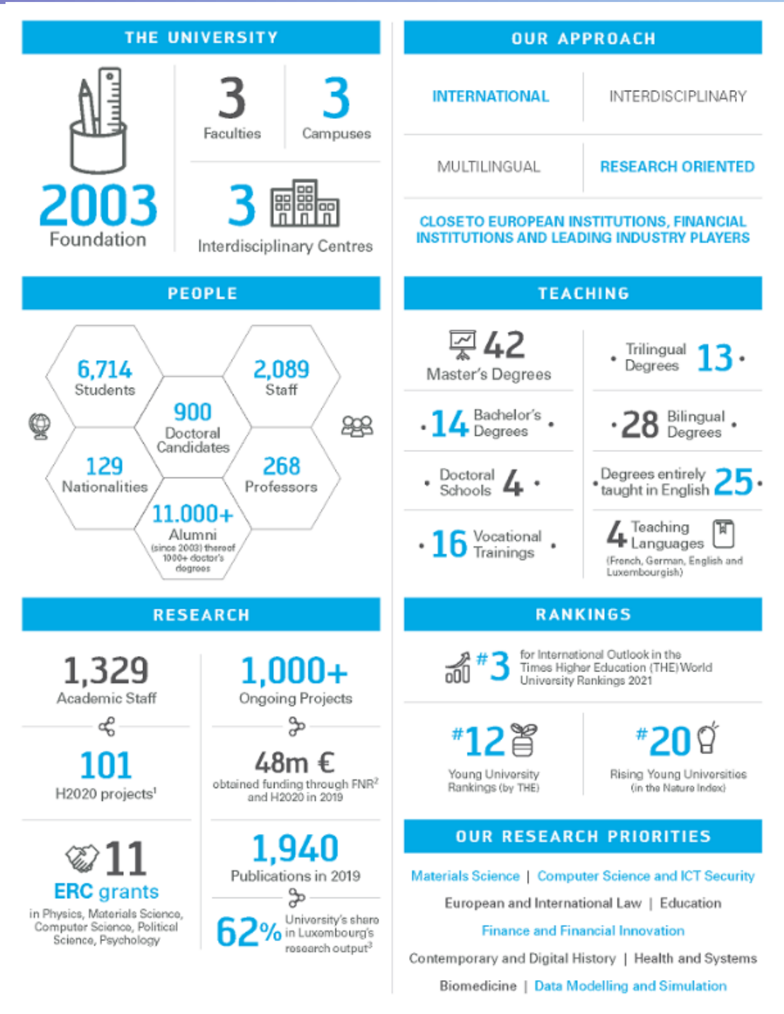
- In charge of radar laboratory at SnT-SIGCOM
- PhD, Isfahan University of Technology (https://www.iut.ac.ir/en/)
- More thank 10 years experience in radar systems

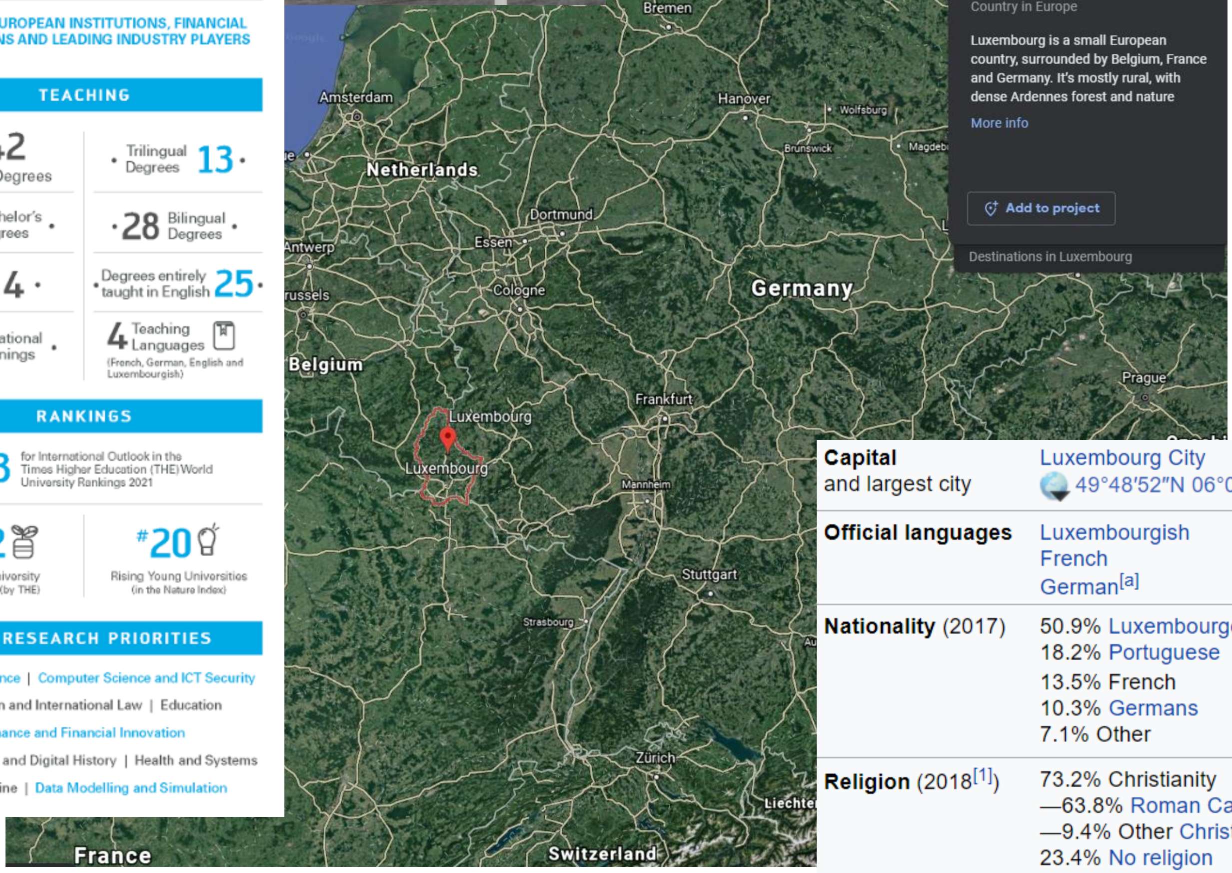
More information can be found in https://radarmimo.com/

I thank the **organizers** for the opportunity

University of Luxembourg

- ✓ It is regarded as one of the most international universities in Europe with students coming from about 130 countries from around the world.
- ✓ The University of Luxembourg ranked **89** in **Computer Science** among top 250 universities worldwide
- ✓ The University of Luxembourg ranks 12th in the *Times Higher Education Young University Ranking 2020*
- ✓ More than 1000 PhD since 2006





Capital and largest city	Luxembourg City 49°48′52″N 06°07′54″E
Official languages	Luxembourgish French German[a]
Nationality (2017)	50.9% Luxembourgers 18.2% Portuguese 13.5% French 10.3% Germans 7.1% Other
Religion (2018[1])	73.2% Christianity —63.8% Roman Catholic —9.4% Other Christian 23.4% No religion 2.6% Islam 0.6% Other

SnT and SIGCOM Group

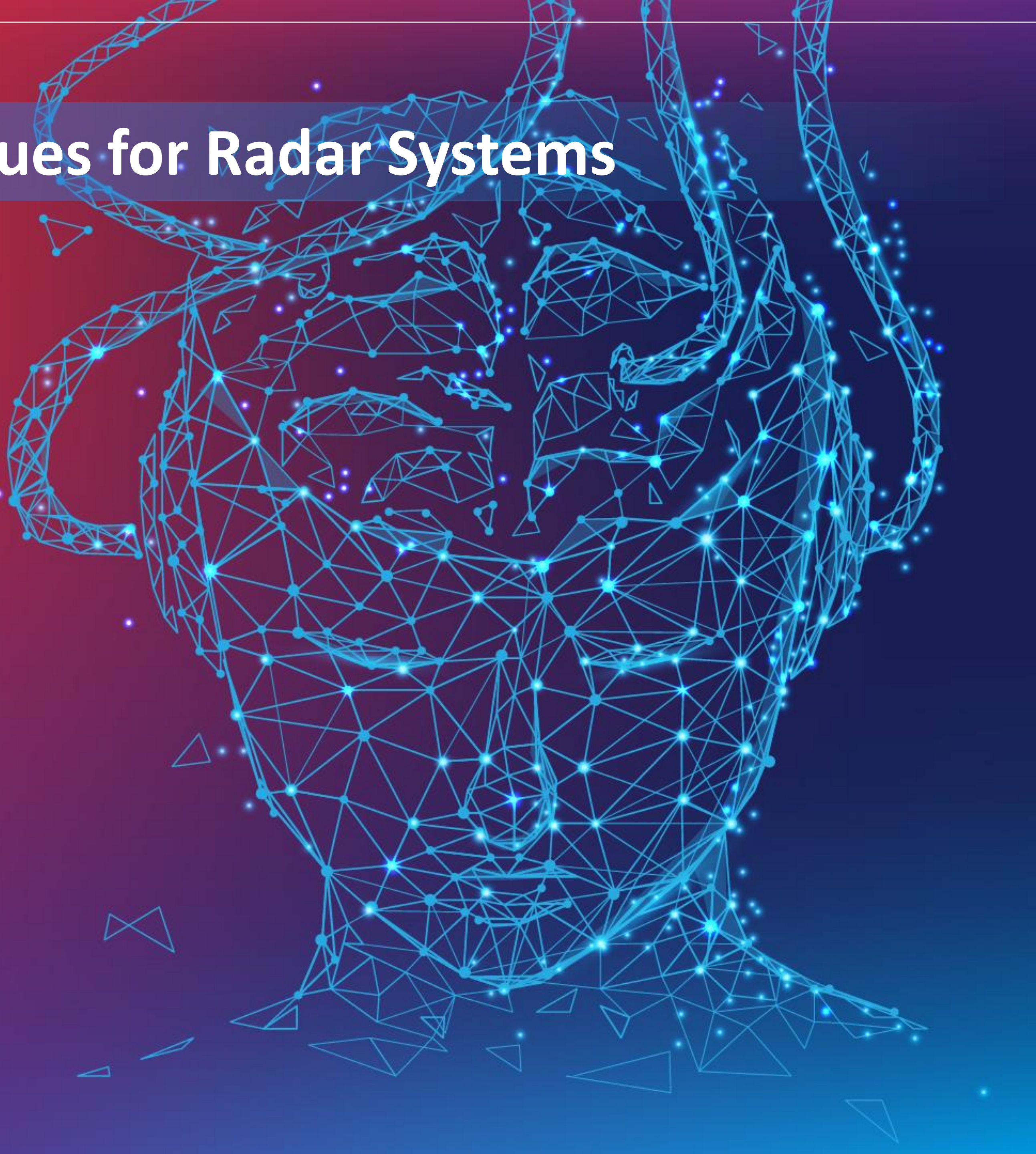
SnT - Interdisciplinary Centre for Security, Reliability and Trust

- **APSIA -** The Applied Security and Information Assurance Group (Prof. Peter Ryan)
- **Automation Research Group** (Prof Holger Voos)
- **CritiX** - Critical and Extreme Security and Dependability Research Group (Prof Paulo Esteves Veríssimo)
- **CryptoLux** (Prof. Alex Biryukov)
- **CVI2** - Computer Vision, Imaging and Machine Intelligence Research Group (Dr Djamila Aouada)
- **FINATRAX** - Digital Financial Services and Cross-Organisational Digital Transformations Research Group (Prof. Gilbert Fridgen)
- **IRiSC** - Sociotechnical Cybersecurity Interdisciplinary Research Group (Prof Dr Gabriele Lenzini)
- **PCOG** - Parallel Computing & Optimisation Research Group (Prof Pascal Bouvry)
- **RSA -** Remote Sensing Applications (Prof Dr Tonie Van Dam)
- **SEDAN** - Service and Data Management in Distributed Systems (Prof Dr Radu State)
- **SerVal** - Security Design and Validation Research Group (Prof Yves Le Traon)
- **SIGCOM** - Signal Processing and Communications (Prof Björn Ottersten)
- **Space R** - Space Robotics Research Group (Prof Dr Miguel Angel Olivares Mendez)
- **TRuX** (Prof Dr Jacques Klein)
- **V&V Lab -** Software Verification and Validation Research Group (Prof Lionel Briand)

- ❑**Personnel**: 75 (5 Permanent Scientists, 30 PostDocs, 35 PhDs, 5 Lab Technicians)
- ❑**Main Areas:** Wireless/Satellite Comms & Net, MIMO Radar
- ❑**>500 Publications : (6** Books / **>200** Journals / **>20** Book Chapters / **>300** Conferences / **<10** patents)

https://wwwen.uni.lu/snt/research/sigcom

Part I: Waveform design and a review of the optimization techniques



Timeline

Part II: CD optimization framework for waveform design

Part I: Waveform design and a review of the optimization techniques

Q&A

Waveform design and a review of the optimization techniques

Introduction

Waveform Design in radar systems

Optimization Techniques

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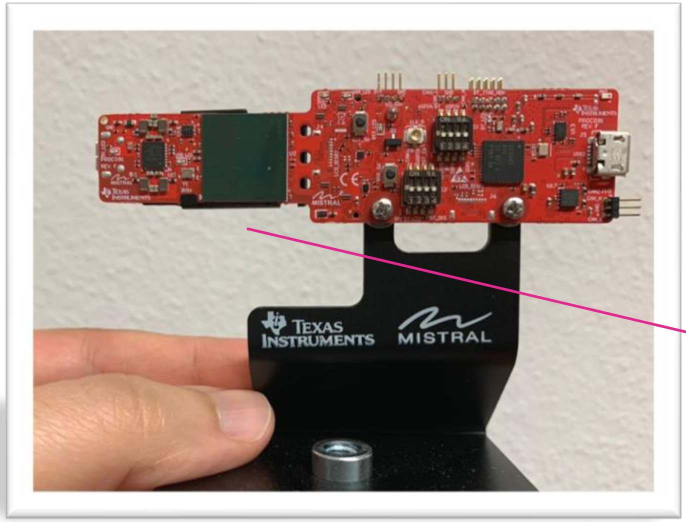
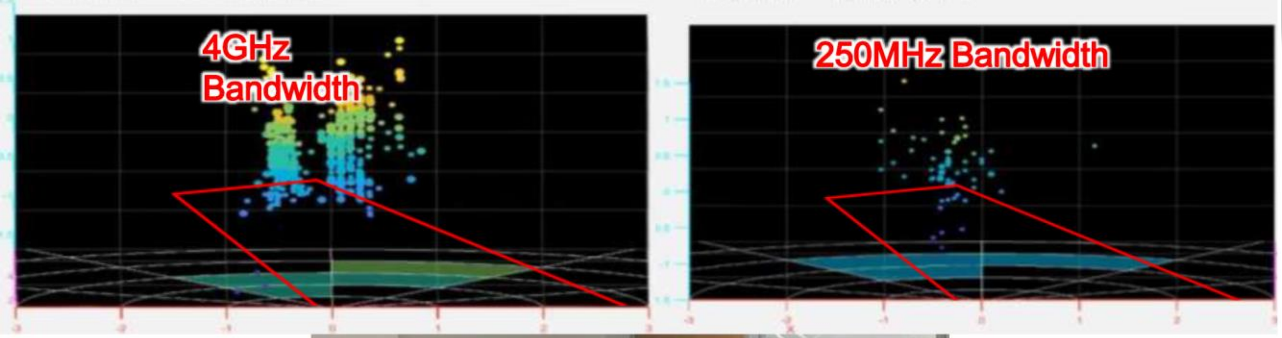
Radar Evolution

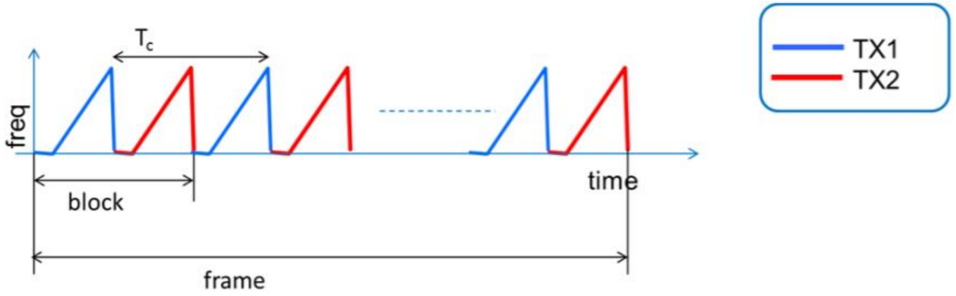
Automotive Radar (today)

- ✓ Operating in mmWave (60GHz, and 79 GHz)
- ✓ More than 5GHz bandwidth
- ✓ Radar on chip (ROC)
- ✓ MIMO capabilities



TDM-MIMO
ROC

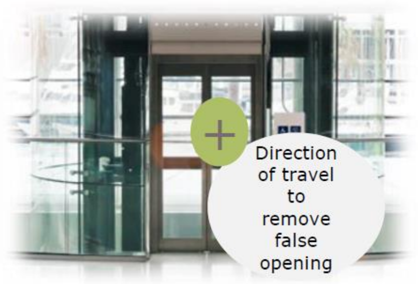




Radar on Chip applications

Smart appliances

Door opening



Security

Lighting

Traffic

Advanced motion detection & sensing

Streetlighting

Multicopter

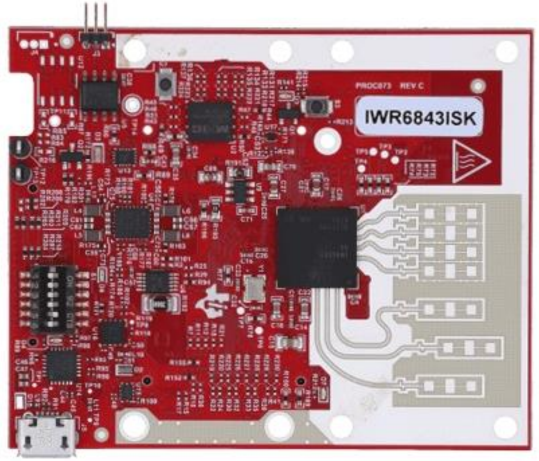
HVAC controls

Robotics

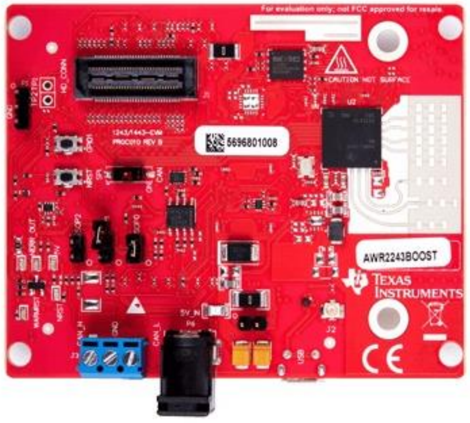
Vital sensing

More on available Automotive MIMO Radars

Operating Frequency	60GHz to 64GHz	76GHz to 81GHz
Chip	IWR 6443/6843	AWR2243
Evaluation Board	IWR6843ISK	AWR2243BOOST
Bandwidth	4 GHz	5GHz
Modulation	FMCW	FMCW
Antenna arrangement	3 Tx / 4 Rx	3 Tx / 4 Rx
USP	• High Resolution detection upto 3.75 cm • Small form factor	• High Resolution detection upto 3.75 cm • Small form factor
NDA	No	No
PPAP (Market availability)	Already available	Already available
Price	135 USD	349 USD
Link	https://www.ti.com/tool/IWR6843ISK#1	https://www.ti.com/tool/AWR2243BOOST#3

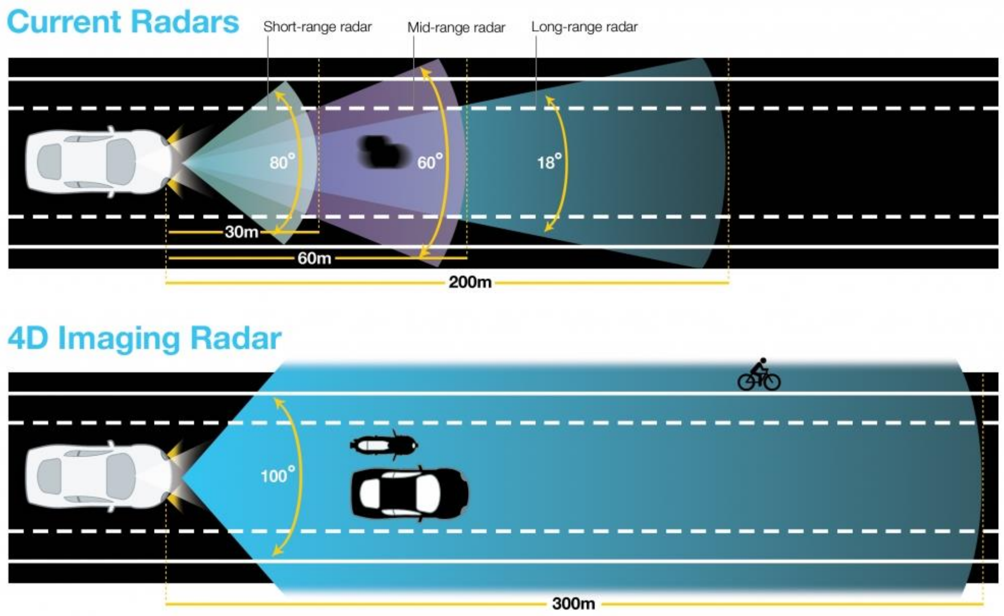


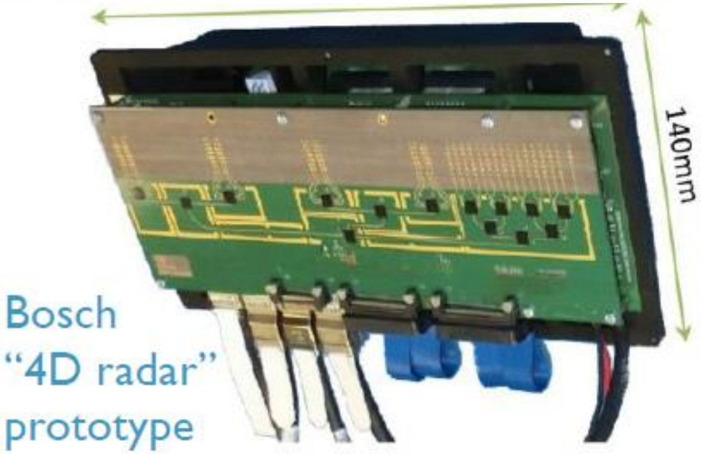
IWR6843ISK IWR6843 intelligent mmWave sensor standard antenna plug-in module top board image

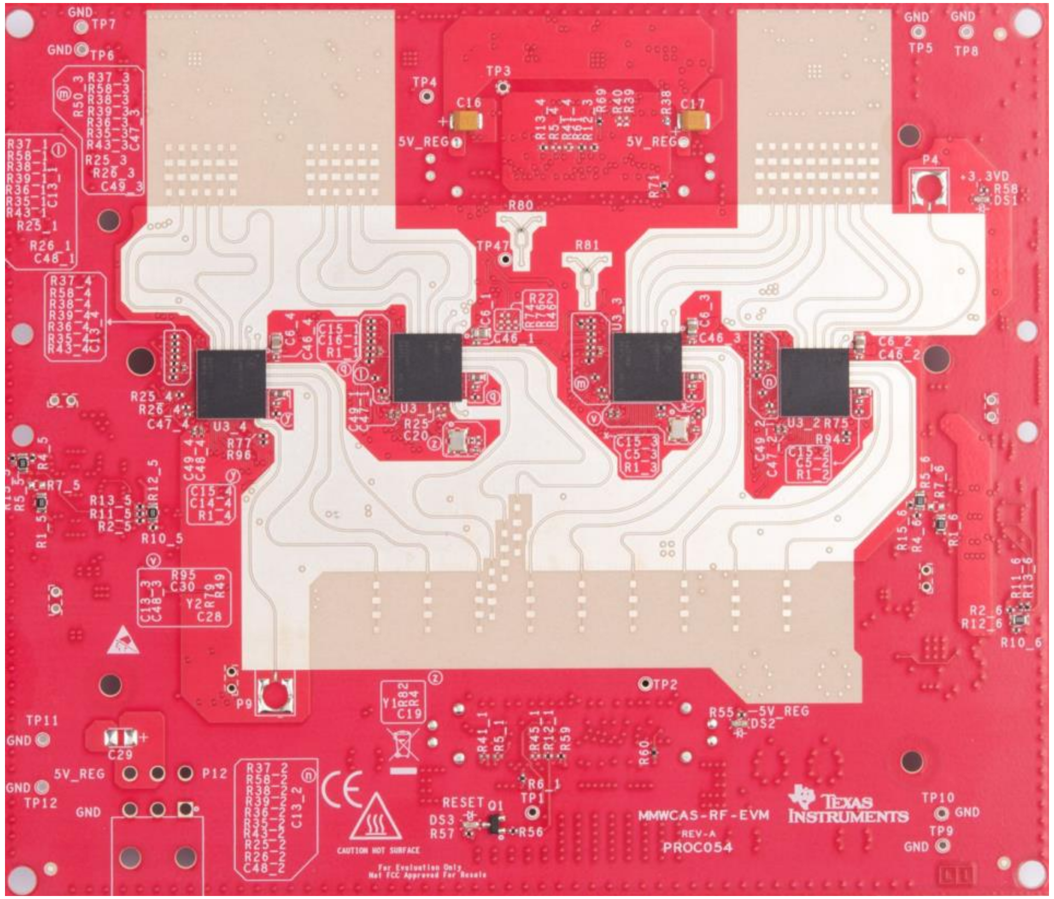


AWR2243BOOST AWR2243 second-generation 76-GHz to 81-GHz high-performance automotive MMIC evaluation module top board image

4D Imaging Automotive MIMO Radars (Tomorrow)







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Waveform design in radar systems

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Radar Pulse Compression, Good Waveforms, and Metrics

- Short pulses are required to have good range resolution.
- Short pulses = Decreased average power -> Limited receive SNR
- Limited receive SNR = Decreased detection capability.

Requirement

High average power + Good Range resolution

Radar Pulse Compression, Good Waveforms, and Metrics

- Higher average power is proportional to pulse width
- Better resolution is inversely proportional to pulse width

A long pulse can have the same bandwidth (resolution) as a short pulse if the long pulse be modulated with a "waveform"

energy of a long pulse + resolution of a short pulse

Radar Pulse Compression, Good Waveforms, and Metrics

What is a waveform?

a waveform is a structured modulation of the pulse, typically in frequency/phase (FM/PM), and sometimes also in amplitude (AM).

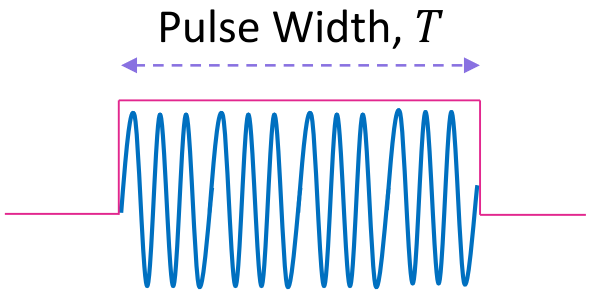
Waveform AM also necessitates linearity at the transmitter power amplifier (PA) to prevent waveform distortion

If a waveform has constant amplitude, the PA can be operated in saturation with much less distortion.

Radar Pulse Compression, Good Waveforms, and Metrics

Increasing the time-bandwidth product with waveforms

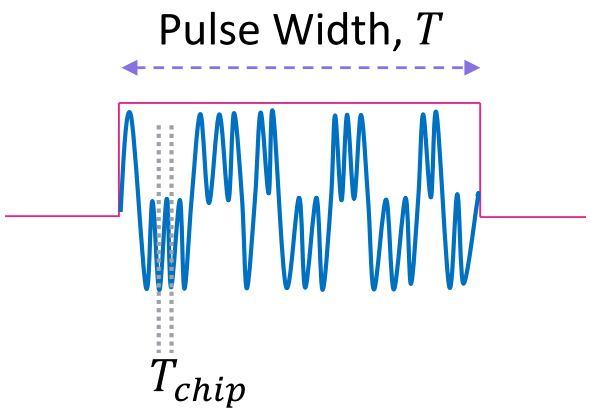
Square Pulse



Bandwidth = $\frac{1}{T}$

Time × Bandwidth = 1

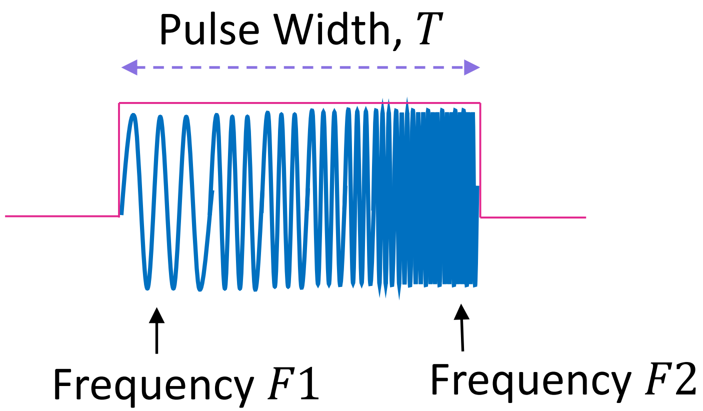
Phase Coded Waveform



Bandwidth = $\frac{1}{T_{chip}}$

Time × Bandwidth = $\frac{\mathrm{T}}{T_{chip}}$

Linear Frequency Modulated Waveform



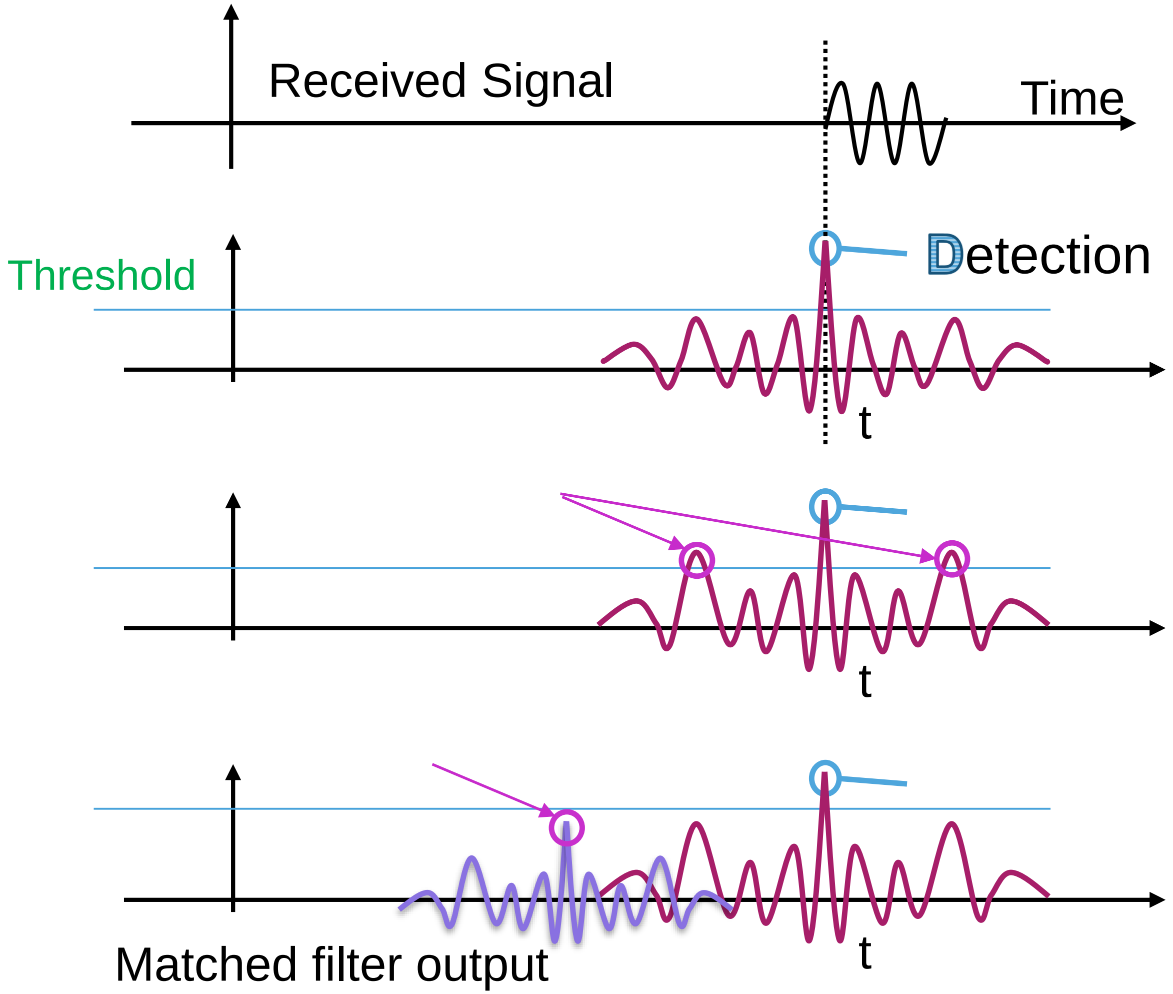
Bandwidth = $\Delta F = F2 - F1$

Time × Bandwidth = $T \times \Delta F$

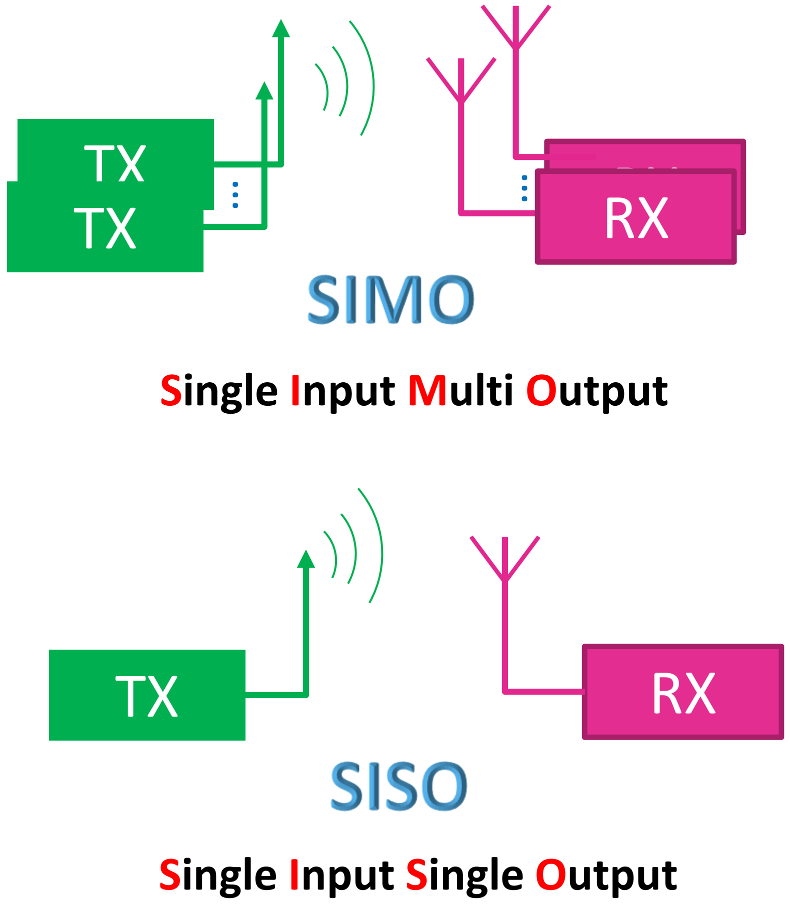
Radar Pulse Compression, Good Waveforms, and Metrics

Waveform type	Transmitted waveform in one transmit antenna	Detection principle	Tradeoff	MIMO	What Can be adapted
Continuous Wave (CW)	$e^{j2\pi f_c t}$	Conjugate mixing	- No range information - Tradeoff between duration and maximum unambiguous Doppler	Orthogonality can be achieved by using different frequencies in each transmit antennas.	- Duration - Frequency - Array Configuration
Pulsed Waveform (PW)	$\Pi(\tau)e^{j\,2\pi f_c t}$	Correlation	- Range-Doppler performance tradeoff - Coupling depends on nature of parameters	Orthogonality can be achieved by using either time division or frequency division in each transmit antennas.	- Duration - PRF - Frequency - Array Configuration
Phase-Coded Continuous Waveform (PCCW)	$\sum_{n=0}^{N-1}\Pi(t-n\tau)e^{-j\,[2\pi f_c(t-n\tau)+\phi_n]}$ $\phi_n \in \{0\,,2\pi\}$	Correlation	- Range-Doppler performance tradeoff - Doppler intolerance (Doppler introduces a phase slope across the pulse destroying it's coherence)	Orthogonality can be achieved by using code division in each transmit antennas.	- Duration - Frequency - Array Configuration - Code
Phase-Coded Step Frequency Waveform (PCSFW)	$\sum_{n=0}^{N-1}\Pi(t-n\tau)e^{-j\,[2\pi f_m(t-n\tau)+\phi_n]}\,,$ $f_m = f_c + (m-1)\Delta f$ $\phi_n \in \{0\,,2\pi\}$	Inverse Fourier transform and Correlation	- Δf decides maximum range - Doppler intolerance in each block	Orthogonality can achieved by using code and frequency division in each transmit antennas.	- Duration - Frequency - Array Configuration - Code
Frequency Modulation CW (FMCW)	$e^{j\,2\pi(f_c+0.5\,Kt)t}\,,$ $K = \frac{B}{T_0}$	Conjugate mixing	- Both Range and Doppler information cause to Range-Doppler coupling	Orthogonality can be achieved by using frequency division in each transmit antennas.	- Duration - Frequency - Array Configuration

Waveforms in SISO/SIMO (Phased-Array) Radar Systems



The matched filter output is the waveform's *autocorrelation* that possesses a *mainlobe* (the peak) surrounded by *sidelobes*



Radar Pulse Compression, Good Waveforms, and Metrics

Peak Sidelobe Level (PSL)

mitigate the deleterious effects of distributed clutter echoes which are close to the target of interest

PSL

Small

ISL

avoid masking of weak targets in range sidelobes of a strong return

Integrated Sidelobe Level (ISL)

Waveform Design and Optimization Problems

$$\boldsymbol{x} = [x_1, x_2, \dots., x_N]^T$$

Transmit waveform

Code length

$$r_k = \sum_{n=1}^{N-k} x_n^* x_{n+k}\,, \qquad k = 0. \dots. N-1$$

$$\mathrm{PSL} = \max_{k\neq 0} |r_k| \qquad \mathrm{ISL} = \sum_{k=1}^{N-1} r_k^2$$

Waveform Design and Optimization Problems

PSL Minimization Problem

$$\boldsymbol{x} = [x_1, x_2, \ldots, x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \underset{k \neq 0}{\max} |r_k| \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

Waveform Design and Optimization Problems

ISL Minimization Problem

$$\boldsymbol{x} = [x_1, x_2, \ldots., x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \sum_{k=1}^{N-1} r_k^2 \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

Waveform Design and Optimization Problems

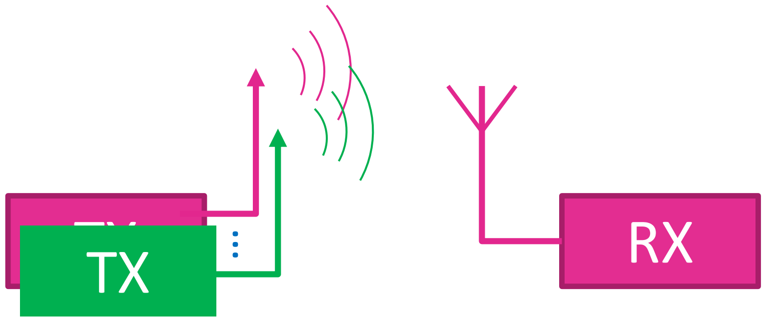
- Constraints
  - Energy
  - Peak-to-Average Power Ratio (PAPR, PAR)
  - Unimodularity (being Constant-Modulus)
  - Finite or Discrete-Alphabet (integer, binary, m-ary constellation)
  - …
- Challenges
  - How to handles signal constraints?
  - How to do it fast?

- Many of these problems are shown to be NP-hard
- Many others are deemed to be difficult

Waveform Design and Optimization Problems

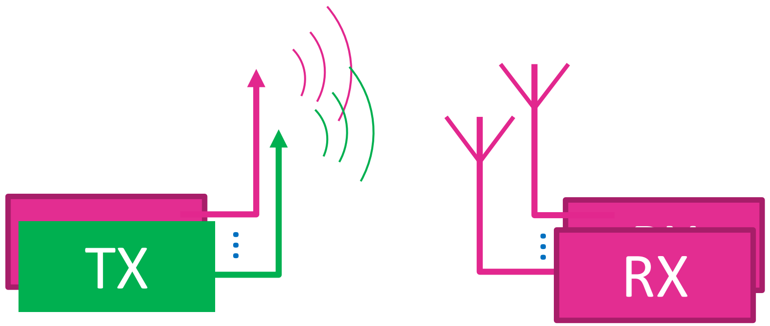
- Example: Waveform design with good correlation properties in MIMO radar systems
  - Transmitters should be observable at each receiver
  - Enabled by Orthogonal Waveforms

MISO



Multi Input Single Output

MIMO



Multi Input Multi Output

Waveform Design and Optimization Problems

- Orthogonal Waveforms in MIMO radar systems
  - Limit mutual interference
  - Enable cooperative operation
  - Provide visibility into paths between transmitter and receivers
  - Determines spatial distribution of energy

- Orthogonality achieved by division in time, frequency or code
  - FDM-, TDM-, DDM-, and CDM-MIMO

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Waveform Design and Optimization Problems

- Example: Waveform design with good correlation properties in MIMO radar systems

$$\boldsymbol{x}_m = [x_m(1), x_m(2), \ldots, x_m(N)]^T \in \mathbb{C}^N,$$

$$\boldsymbol{X} = [\boldsymbol{x}_1, \quad \boldsymbol{x}_2, \quad \ldots, \boldsymbol{x}_{N_T}] \in \mathbb{C}^{N \times N_T}$$

$$r_{ml}(k) = \sum_{n=1}^{N-k} x_m(n)\, x_l^*(n+k) = r_{lm}^*(-k)$$

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Waveform Design and Optimization Problems

- Example: Waveform design with good correlation properties in MIMO radar systems

$$\mathrm{PSL} = \max\left\{\max_{m}\max_{k\neq 0}|r_{mm}(k)|\ ,\max_{\substack{m,l\\ m\neq l}}\max_{k}|r_{ml}(k)|\right\}$$

$$\mathrm{ISL} = \sum_{m=1}^{N_T}\sum_{\substack{k=-N+1\\ k\neq 0}}^{N-1}|r_{mm}(k)|^2 + \sum_{\substack{m,l=1\\ m\neq l}}^{N_T}\sum_{k=-N+1}^{N-1}|r_{ml}(k)|^2$$

Waveform Design and Optimization Problems

PSL Minimization Problem in MIMO radar

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max\left\{\underset{m}{\max}\ \underset{k\neq 0}{\max}|r_{mm}(k)|\ ,\underset{\substack{m,l\\ m\neq l}}{\max}\ \underset{k}{\max}\ |r_{ml}(k)|\right\} \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

ISL Minimization Problem in MIMO radar

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \sum_{m=1}^{N_T} \sum_{\substack{k=-N+1\\ k\neq 0}}^{N-1} |r_{mm}(k)|^2 + \sum_{\substack{m,l=1\\ m\neq l}}^{N_T} \sum_{k=-N+1}^{N-1} |r_{ml}(k)|^2 \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

Waveform Design and Optimization Problems

- Waveform design related optimization problems
  - Beampattern shaping
  - Spectral shaping
  - Coexistence MIMO radar MIMO communications (MRMC)
  - Joint radar and communications (JRC)
  - …

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Waveform Design Techniques

- Gradient-Descent Based Methods (GD)
- Majorization-Minimization (MM)
- Coordinate Descent (CD)
- Alternating Direction Method of Multipliers (ADMM)
- Block Successive Upper-bound Minimization (BSUM)
- Several others ...

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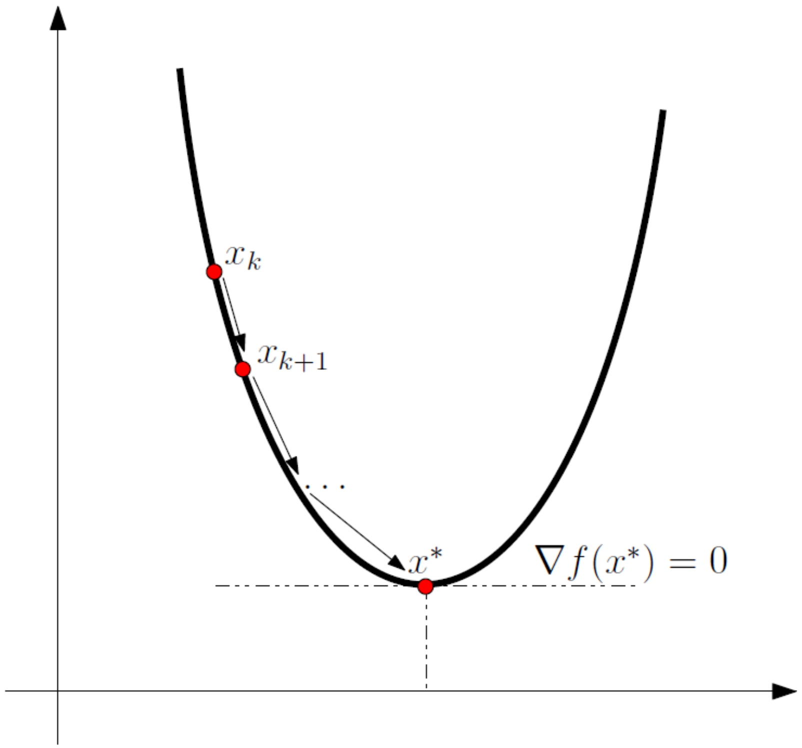
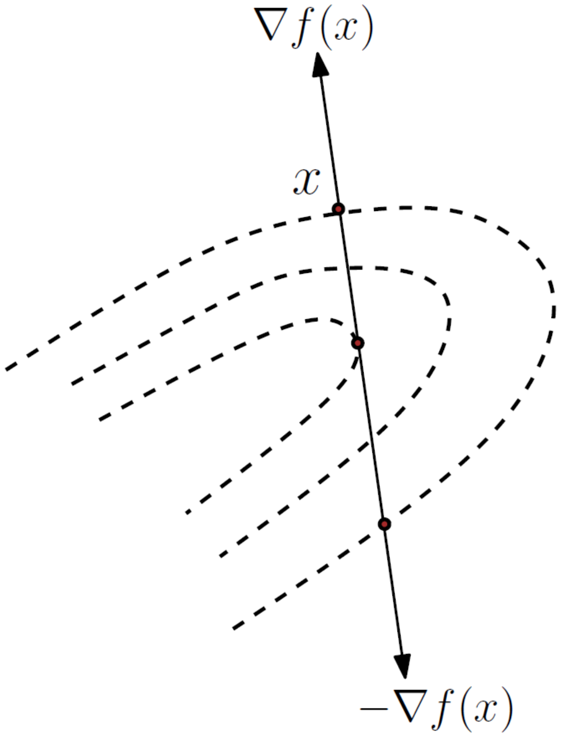
Gradient Descent

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Gradient-Based Methods





Gradient Descent

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Gradient-Based Methods

1. Start with some guess x^0;
2. For each $k = 0, 1, \ldots$
   - $x^{k+1} \leftarrow x^k + \alpha_k d^k$
   - Check when to stop (e.g., if $\nabla f(x^{k+1}) = 0$)

Gradient Descent

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Gradient-Based Methods

$$\boxed{x^{k+1} = x^k + \alpha_k d^k, \quad k = 0, 1, \ldots}$$

- **stepsize** $\alpha_k \geq 0$, usually ensures $f(x^{k+1}) < f(x^k)$
- **Descent direction** d^k satisfies

$$\langle \nabla f(x^k), d^k \rangle < 0$$

$$\boxed{\text{Numerous ways to select } \alpha_k \text{ and } d^k}$$

Gradient Descent – ISL and PSL minimization

For a real number $p \geq 1$, the **p-norm** or **L^p-norm** of x is defined by

$$\|x\|_p = \left(|x_1|^p + |x_2|^p + \cdots + |x_n|^p\right)^{1/p}.$$

The absolute value bars are unnecessary when *p* is a rational number and, in reduced form, has an even numerator.

The Euclidean norm from above falls into this class and is the 2-norm, and the 1-norm is the norm that corresponds to the rectilinear distance.

The **L^∞-norm** or maximum norm (or uniform norm) is the limit of the L^p-norms for $p \to \infty$. It turns out that this limit is equivalent to the following definition:

$$\|x\|_\infty = \max\left\{|x_1|, |x_2|, \ldots, |x_n|\right\}$$

ISL Minimization Problem using Gradient Descent

Sequences are designated as $\boldsymbol{a} = [a_1, a_2, a_3, \ldots, a_N]$.

$$(\boldsymbol{a} \star \boldsymbol{a})_m = \sum_{i=1}^{N} a_i^* a_{i+m-N} \quad (1 \le m \le 2N - 1)$$

$$ISL = \sum_{m=1}^{N-1} \left| \sum_{i=1}^{N} a_i^* a_{i+m-N} \right|^2$$

$$E = ||\boldsymbol{a} \star \boldsymbol{a}||_2^2$$

$$\nabla_{\angle a} E = 4\Im\left[\boldsymbol{a}^* \circ ((\boldsymbol{a} \star \boldsymbol{a}) \star \boldsymbol{a})_{k+N-1}\right]$$

ISL Minimization Problem using Gradient Descent

```
for m=1:maxIter
    fa      = fft(a,2*N-1);
    g       = fftshift(ifft(abs(fa).^2));
    gar     = fftshift(ifft(fft(g,2*N-1).*fa));
    grad    = -4*imag(a.*conj([gar(end) gar(1:N-1)]));
    grad    = grad/max(abs(grad));
    s       = -0.05 / m^0.5;
    a       = a .* exp(1i*s*grad);
end
```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PSL Minimization Problem using Gradient Descent

$$\begin{cases} \text{Minimize} \quad \mathit{maximum} \ |r_k| \quad k = 1, \dots, N-1 \\ \text{Subject to} \quad |x_i| = 1 \end{cases}$$

$$r_k = \sum_{i=1}^{N-k} x_i^* x_{i+k}, \qquad k = 0, \dots, N-1,$$

$$\begin{cases} \text{Minimize} \quad \|r_k\|_p \quad k = 1, \dots, N-1 \\ \text{Subject to} \quad |x_i| = 1 \end{cases}$$

PSL Minimization Problem using Gradient Descent

$$\begin{cases} \text{Minimize} & \|r_k\|_p \qquad k = 1, \dots, N-1 \\ \text{Subject to} & |x_i| = 1 \end{cases}$$

$$E = ||\boldsymbol{w} \circ (\boldsymbol{a} \star \boldsymbol{a})||_{2p}^{2p}$$

$$\nabla_{\angle \boldsymbol{a}} E = -4p\Im\left[\boldsymbol{a} \circ (\boldsymbol{g} \star \boldsymbol{a})^{*}_{k+N-1}\right]$$

where

$$\boldsymbol{g} = \boldsymbol{w}^{2p} \circ \boldsymbol{x} \circ |\boldsymbol{x}|^{2p-2}$$

J. M. Baden, B. O'Donnell and L. Schmieder, "Multiobjective Sequence Design via Gradient Descent Methods," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 54, no. 3, pp. 1237-1252, June 2018.

Majorization-Minimization (MM)

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Majorization-Minimization (MM)

An MM algorithm operates by creating a surrogate function that minorizes or majorizes the objective function. When the surrogate function is optimized, the objective function is driven uphill or downhill as needed.

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Majorization-Minimization (MM)

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Majorization-Minimization (MM); Example

Minimization of $\cos(x)$

Second order Taylor expansion

$$\cos(x) = \cos(x_n) - \sin(x_n)\,(x - x_n) - \frac{1}{2}\cos(z)\,(x - x_n)^2$$

Holds for some z between x and x_n

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Majorization-Minimization (MM)

Since $|\cos(z)| \leq 1$,

$$g(x|x_n) = \cos(x_n) - \sin(x_n)\,(x - x_n) + \frac{1}{2}(x - x_n)\text{^}2$$

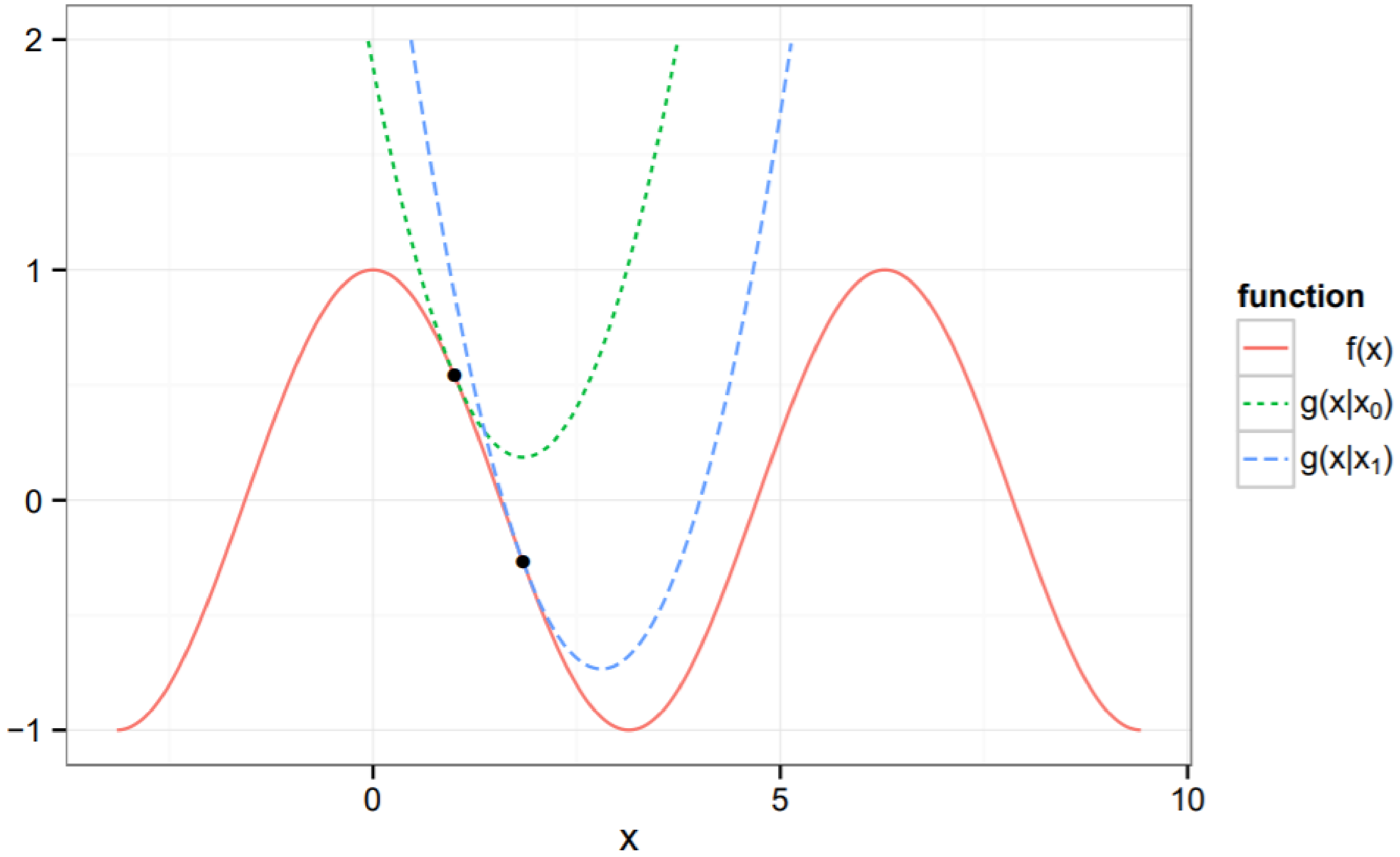
Can be selected as majorizer that majorizes $f(x)$

Solving $\frac{d}{dx} g(x|x_n) = 0$ gives the MM algorithm

$$x_{n+1} = x_n + \sin(x_n)$$

Majorization-Minimization (MM)

Minimum of cos(x)



Majorization-Minimization (MM)

Input: $\mathbf{x}_0 \in \mathcal{C}$.

1: **while** not converged **do**

2: Construct a surrogate function $g_k(\mathbf{x})$ of $f(\mathbf{x})$ at the current iterate $\mathbf{x}_k$.

3: Minimize the surrogate to get the next iterate: $\mathbf{x}_{k+1} = \operatorname{argmin}_{\mathbf{x}\in\mathcal{C}} g_k(\mathbf{x})$.

4: $k \leftarrow k + 1$.

5: **end while**

Output: The solution $\mathbf{x}_k$.

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ISL minimization using MM

$$\mathrm{WISL} = \sum_{k=1}^{N-1} w_k |r_k|^2,$$

$$\begin{aligned} \underset{x_n}{\text{minimize}} \quad & \mathrm{WISL} \\ \text{subject to} \quad & |x_n| = 1, n = 1, \ldots, N, \end{aligned}$$

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ISL minimization using MM

Let $\mathbf{L}$ be an $n \times n$ Hermitian matrix

$$\mathbf{x}^H \mathbf{L} \mathbf{x}$$

the quadratic function $\mathbf{x}^H \mathbf{L} \mathbf{x}$ is majorized

$$\mathbf{x}^H \mathbf{M} \mathbf{x} + 2\mathrm{Re}\left(\mathbf{x}^H (\mathbf{L} - \mathbf{M}) \mathbf{x}_0\right) + \mathbf{x}_0^H (\mathbf{M} - \mathbf{L}) \mathbf{x}_0$$

$$\mathbf{M} \succeq \mathbf{L}$$

ISL minimization using MM

Let us define $\mathbf{U}_k, k = 0, \ldots, , N-1$ to be $N \times N$ Toeplitz matrices with the kth diagonal elements being 1 and 0 elsewhere. Noticing that

$$r_k = \mathrm{Tr}(\mathbf{U}_k \mathbf{x}\mathbf{x}^H), k = 0, \ldots, N-1,$$

ISL optimization problem

$$\begin{aligned} \underset{\mathbf{X},\mathbf{x}}{\text{minimize}} \quad & \sum_{k=1}^{N-1} w_k |\mathrm{Tr}(\mathbf{U}_k \mathbf{X})|^2 \\ \text{subject to} \quad & \mathbf{X} = \mathbf{x}\mathbf{x}^H \\ & |x_n| = 1, n = 1, \ldots, N. \end{aligned}$$

ISL minimization using MM

$$\begin{aligned} \underset{\mathbf{x},\mathbf{X}}{\text{minimize}} \quad & \sum_{k=1-N}^{N-1} w_k \mathrm{vec}(\mathbf{X})^H \mathrm{vec}(\mathbf{U}_k) \mathrm{vec}(\mathbf{U}_k)^H \mathrm{vec}(\mathbf{X}) \\ \text{subject to} \quad & \mathbf{X} = \mathbf{x}\mathbf{x}^H \\ & |x_n| = 1, n = 1, \ldots, N. \end{aligned}$$

By defining

$$\mathbf{L} = \sum_{k=1-N}^{N-1} w_k \mathrm{vec}(\mathbf{U}_k) \mathrm{vec}(\mathbf{U}_k)^H$$

The objective $\mathbf{x}^H \mathbf{L} \mathbf{x}$

Majorization function $\mathbf{M} = \lambda_{\max}(\mathbf{L})\mathbf{I}$

ISL minimization using MM

```
for m = 1:maxIter
    f         = fft([x.', zeros(1,N)]).';
    r         = 1 / (2*N) * (ifft(abs(f).^2));
    c         = r .* [0,w,0,w(end:-1:1)].';
    mu        = fft(c);
    temp      = (ifft(mu .* f));
    lambda_mu = 1/2 * (max (mu(2:2:N)) + max(mu(1:2:N)));
    phi_n     = (p .* x - temp(1:N)) / (2*N * (lambda_mu - 
lambda_B));
    y_tilde = x + phi_n(1:N);
    x         = exp(1i * angle(y_tilde));
end
```

PSL minimization using MM

$$\begin{aligned} &\underset{x_n}{\text{minimize}} && \sum_{k=1}^{N-1} |r_k|^p \\ &\text{subject to} && |x_n| = 1, n = 1, \ldots, N. \end{aligned}$$

Let $f(x) = x^p$ with $p \geq 2$ and $x \in [0, t]$.

Then for any given $x_0 \in [0, t)$, $f(x)$ is majorized at x_0 over the interval $[0, t]$ by the following quadratic function

$$ax^2 + (px_0^{p-1} - 2ax_0)x + ax_0^2 - (p-1)x_0^p,$$

where

$$a = \frac{t^p - x_0^p - px_0^{p-1}(t - x_0)}{(t - x_0)^2}.$$

$$\begin{aligned} &\underset{x_n}{\text{minimize}} && \sum_{k=1}^{N-1} \left(a_k |r_k|^2 + b_k |r_k|\right) \\ &\text{subject to} && |x_n| = 1, n = 1, \ldots, N. \end{aligned}$$

$$\begin{aligned} &\underset{x_n}{\text{minimize}} && \mathbf{x}^H \left(\tilde{\mathbf{R}} - \lambda_{\max}(\mathbf{L})\mathbf{x}^{(l)}(\mathbf{x}^{(l)})^H\right) \mathbf{x} \\ &\text{subject to} && |x_n| = 1, n = 1, \ldots, N, \end{aligned}$$

where

$$\tilde{\mathbf{R}} = \sum_{k=1-N}^{N-1} \hat{w}_k r_{-k}^{(l)} \mathbf{U}_k.$$

J. Song, P. Babu and D. P. Palomar, "Sequence Design to Minimize the Weighted Integrated and Peak Sidelobe Levels," in *IEEE Transactions on Signal Processing*, vol. 64, no. 8, pp. 2051-2064, April15, 2016.

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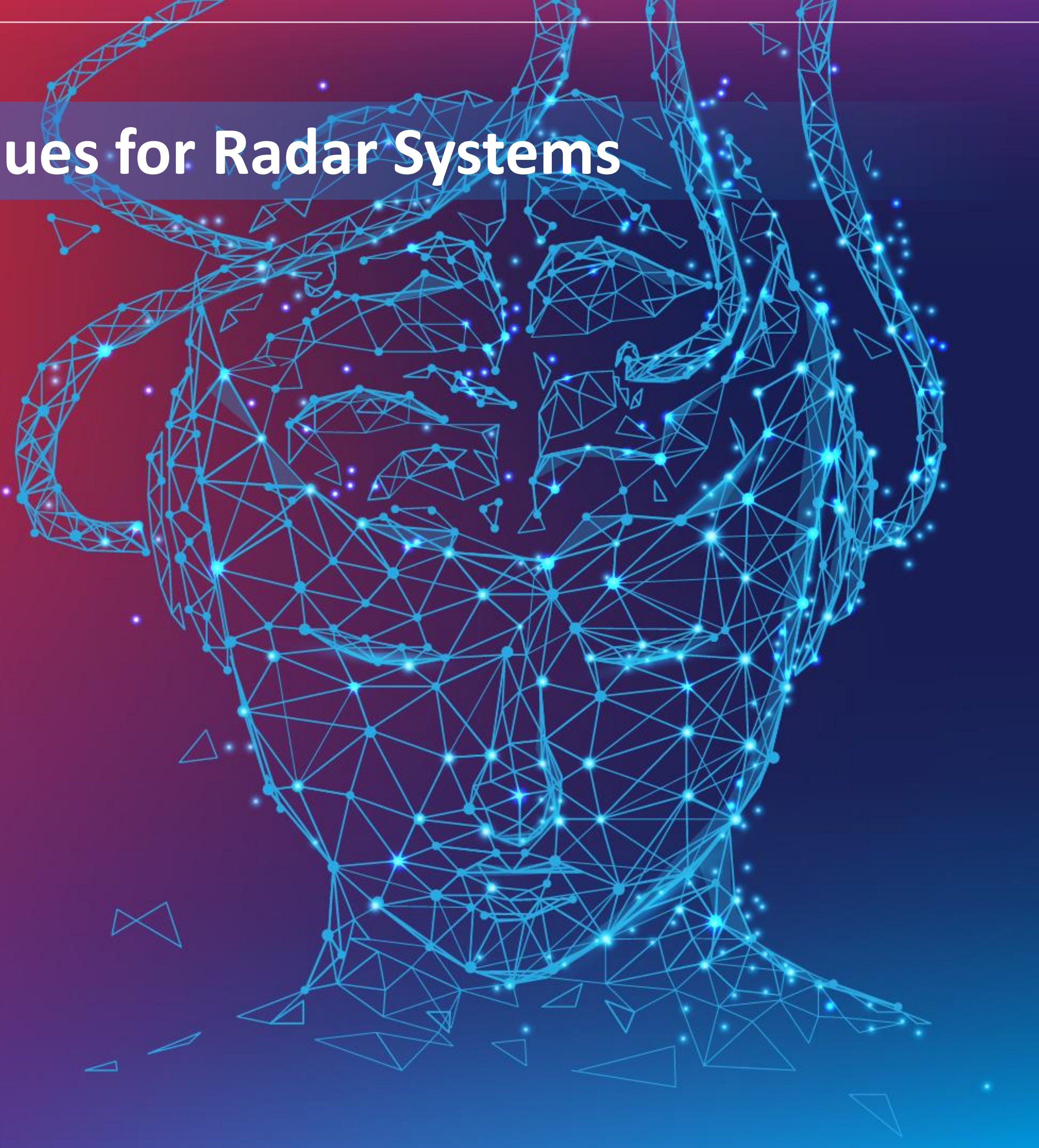
Waveform Design Techniques

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

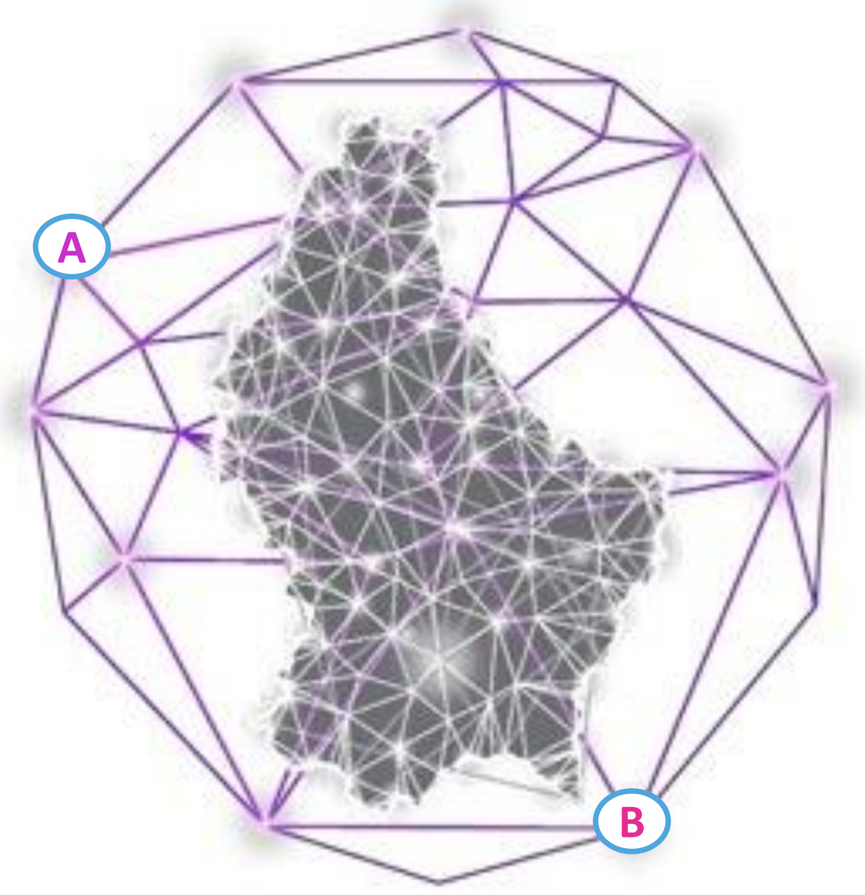
Coordinate Descent (CD)

Minimization of a multivariable function can be achieved by minimizing it along one direction at a time, i.e., solving univariate (or at least much simpler) optimization problems in a loop

Part II: Coordinate Descent (CD) Optimization Framework for Radar Waveform Design



Coordinate Descent (Ascent) Methods



Coordinate Descent

- Successively minimizes along coordinate directions
  - – Optimize each parameter separately, holding all the others fixed.
- Why is it used?
  - ✓ Very simple and easy to implement
  - ✓ Careful implementations can attain state-of-the-art
  - ✓ Scalable, don't need to keep data in memory, low memory requirements
  - ✓ Faster than gradient descent if iterations are N times cheaper

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Coordinate Descent

$$\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & f(\boldsymbol{x}) \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

idea: optimize over individual coordinates

Coordinate Descent – Steps

$$x_1^{(k)} \in \arg\min_{x_1} f(x_1, x_2^{(k-1)}, x_3^{(k-1)}, \ldots, x_N^{(k-1)})$$

$$x_2^{(k)} \in \arg\min_{x_2} f(x_1^{(k)}, x_2, x_3^{(k-1)}, \ldots, x_N^{(k-1)})$$

$$x_3^{(k)} \in \arg\min_{x_3} f(x_1^{(k)}, x_2^{(k)}, x_3, \ldots, x_N^{(k-1)})$$

$$\vdots$$

$$x_N^{(k)} \in \arg\min_{x_N} f(x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \ldots, x_N)$$

Note:

1- After we solve for $x_i^{(k)}$, we use its new value from then on

2- Can everywhere replace individual coordinates with blocks of coordinates (Block Coordinate Descent)

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Coordinate Descent – Algorithm

- Start from initial guess $\boldsymbol{x}^{(0)} = [x_1, x_2, \ldots, x_N]^T$

- For $k = 0, 1, \ldots$

  - Pick an index i from $\{1, \ldots, N\}$

  - Optimize the i-th coordinate

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f\Big(\underbrace{x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}}_{\text{done}}, \underset{\text{current}}{\underset{\uparrow}{\zeta}}, \underbrace{x_{i+1}^{(k)}, \ldots, x_N^{(k)}}_{\text{To do}}\Big)$$

- Decide when/how to stop; return $\boldsymbol{x}^{(k+1)}$

Gauss-Seidel and Jacobi

Gauss-Seidel style (One-at-a-time)

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$$

Jacobi style (all-at-once ; easy to parallelize)

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f(x_1^{(k)}, \dots, x_{i-1}^{(k)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$$

Which Coordinate? (One-at-a-time)

- **Greedy or Gauss-Southwell (Maximum Block Improvement)**
  - If f is differentiable, at iteration k, pick the index that minimizes $\nabla f(x_i^k)$
- **Derivative free rules**
  - **Cyclic** order $1, 2, \ldots, N, 1, \ldots$
  - **Double sweep**, $1, 2, \ldots, N$, then $N-1, \ldots, 1$, repeat
  - **Cyclic with permutation**, random order each cycle
  - **Random sampling**, pick random index at each iteration

Advantages

- Each iteration is usually cheap (single variable optimization)
- No extra storage vectors needed
- No stepsize tuning
- No other parameters that must be tuned
- In general, “derivative free”
- Simple to implement
- Works well for large-scale problems
- Currently quite popular; parallel version exist

Disadvantages

- Each sub-problem needs to be easily solvable. Tricky if single variable optimization is hard
- Can be "slow" if sub-problems cannot be solved efficiently
- Convergence theory can be complicated
  - "One-at-a-time" update scheme is critical, and "all-at-once" scheme does not necessarily converge
- Non-differentiable cases are more tricky

Convergence (One-at-a-time)

- The objective function values are non-decreasing, i. e.,

$$f\left(\boldsymbol{x}^{(0)}\right) \geq f\left(\boldsymbol{x}^{(1)}\right) \geq \ \ldots .$$

- If f is strictly convex and smooth, the algorithm converges to a global minimum (optimal solution).
- If f is strictly convex -> unique minimum -> local minimum = global minimum
  a. continuously differentiable over the feasible set,
  b. has separable constraints,
  c. has unique minimizer at each step,

  then CD method will converge to stationary points

[1] - Dimitri P, et al.. Nonlinear programming. Athena Scientific; 1999.

Other Alternating methods - Alternating Minimization

2 blocks is called alternative optimization

$$\boldsymbol{x} = [x_1, x_2]^T$$

$$x_1 \downarrow x \qquad x_2 \downarrow w$$

$$\mathcal{P}_{x,w} \begin{cases} \underset{x,w}{\text{minimize}} & f(x, w) \\ \text{subject to} & x \in \psi_1, w \in \psi_2 \end{cases}$$

Other Alternating methods - BSUM

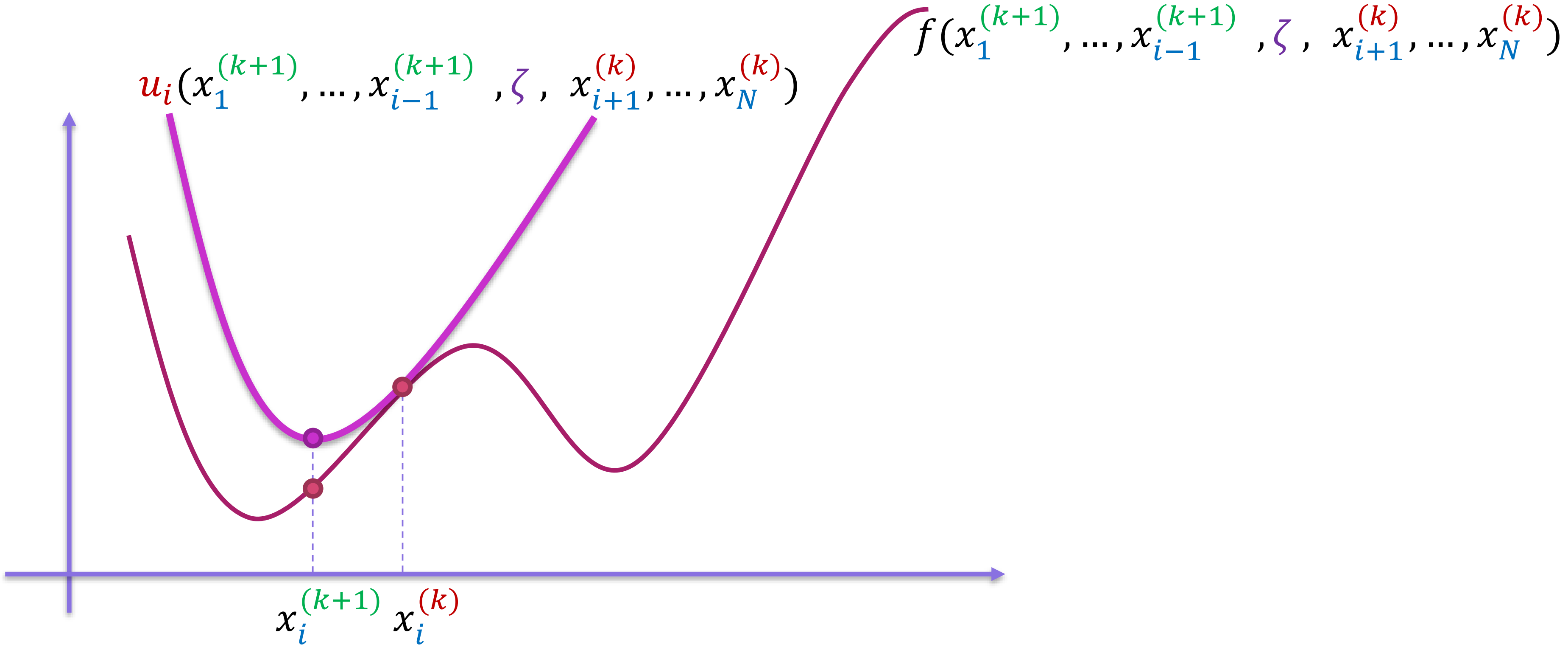
Block successive upper-bound minimization (BSUM)

$$\boldsymbol{x} = [x_1, x_2, \ldots., x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & f(\boldsymbol{x}) \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} \underbrace{u_i}_{\text{Local approximation of the objective function}}(x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \ldots, x_N^{(k)})$$

Other Alternating methods - BSUM



Upper-bound $\quad u_i\left(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)}\right) \geq f(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$

Block successive upper-bound minimization, block successive convex approximation, convex-concave procedure, majorization-minimization, dc-programming, BCGD,…

Other Alternating methods

- Alternating direction method of multipliers (ADMM)

$$\begin{cases} \underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} & f(\boldsymbol{x}) + g(\boldsymbol{z}) \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} = \boldsymbol{c} \end{cases}$$

$$L_\rho(\boldsymbol{x},\boldsymbol{z},\boldsymbol{y}) = f(\boldsymbol{x}) + g(\boldsymbol{z}) + y^T(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} - \boldsymbol{c}) + \left(\frac{\rho}{2}\right)\|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} - \boldsymbol{x}\|_2^2$$

$$\boldsymbol{x}^{(k+1)} \leftarrow \arg\min_{\boldsymbol{x}} L_\rho(\boldsymbol{x}, \boldsymbol{z}^{(k)}, \boldsymbol{y}^{(k)})$$

$$\boldsymbol{z}^{(k+1)} \leftarrow \arg\min_{\boldsymbol{z}} L_\rho(\boldsymbol{x}^{(k+1)}, \boldsymbol{z}, \boldsymbol{y}^{(k)})$$

$$\boldsymbol{y}^{(k+1)} \leftarrow \boldsymbol{y}^{(k)} + \rho(\boldsymbol{A}\boldsymbol{x}^{(k+1)} + \boldsymbol{B}\boldsymbol{z}^{(k+1)} - \boldsymbol{c})$$

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Examples

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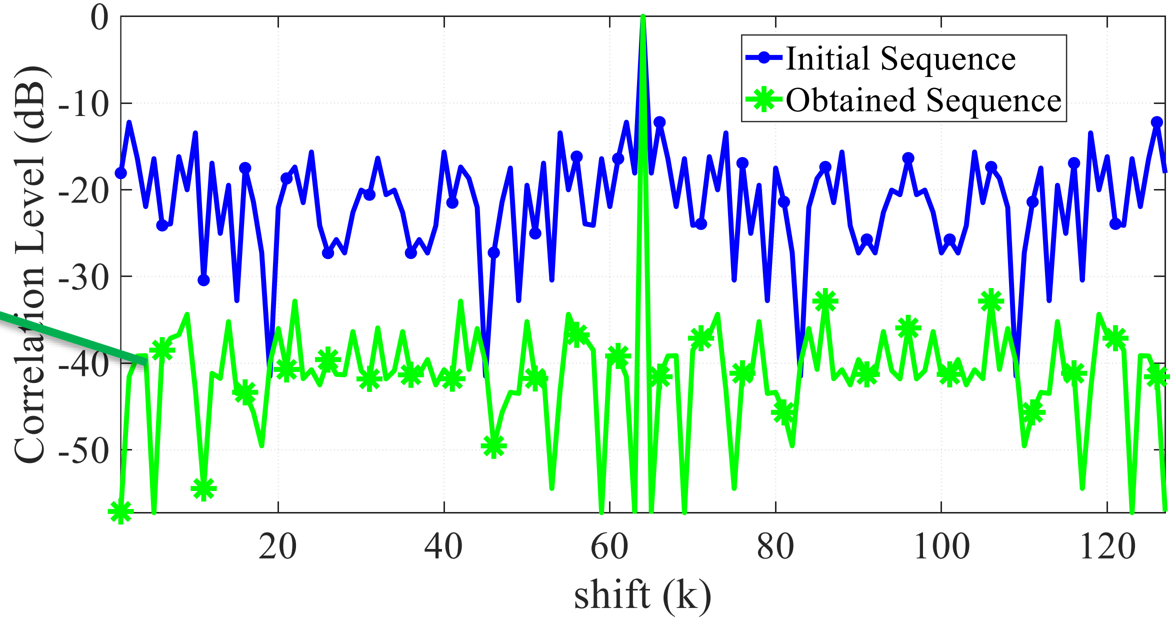
Example

Good Sequence



$$\boldsymbol{x} = [x_1. x_2 \dots . x_N]^T \in \mathbb{C}^N,$$

$$r_k = \sum_{n=1}^{N-k} x_n^* x_{n+k} . \quad k = 0. \dots . N-1$$

$$r_k^P = \sum_{n=1}^{N-k} x_n \, x_{n+k \bmod (N)}^* = r_{-k}^P$$

$$\text{PSL} = \max_{k \neq 0} |r_k| \qquad \text{ISL} = \sum_{k=1}^{N-1} r_k^2$$

How to design a sequence with small PSL / ISL ?

Example: PSL Minimization in SISO/SIMO Radar Systems

$$\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T \in \mathbb{C}^N,$$

$$\mathrm{f}(\mathbf{x}) = \max\{|r_k|\}_{k=1}^{N-1}$$

$$\mathcal{P}_{\boldsymbol{x}}^{M} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max\{|r_k|\}_{k=1}^{N-1} \\ \text{subject to} & x_{\mathrm{n}} \in \Omega_M \end{cases}$$

$$\mathcal{P}_{\boldsymbol{x}}^{\infty} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max\{|r_k|\}_{k=1}^{N-1} \\ \text{subject to} & |x_n| = 1 \end{cases}$$

$$\Omega_M = \left\{1, e^{\frac{j2\pi}{M}}, \dots, e^{\frac{j2\pi(M-1)}{M}}\right\}$$

Alphabet size

Non-Convex Multi-variable Constrained min-max optimization problems

Example: PSL Minimization in SISO/SIMO Radar Systems

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \max\{|r_k|\}_{k=1}^{N-1}$$

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad f(\boldsymbol{x})$$

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f\left(x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \ldots, x_N^{(k)}\right)$$

$$r_k\left(x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \ldots, x_N^{(k)}\right) = a_{ki}\,\zeta + b_{ki}\zeta^* + c_{ki}$$

$$\zeta = e^{j\phi}$$

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Example: PSL Minimization – Discrete Phase

$$\mathcal{P}_{\phi}^{(k+1)} \begin{cases} \underset{\phi}{\text{minimize}} & \max\left\{\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|\right\}_{k=1}^{N-1} \\ \text{subject to} & \phi \in \left\{0, \frac{2\pi}{M}, \dots, \frac{2\pi(M-1)}{M}\right\} \end{cases}$$

$$|r_k e^{-j\phi}| = |r_k|$$

$$r_k e^{-j\phi} = a_{ki} + b_{ki}e^{-j2\phi} + c_{ki}e^{-j\phi}$$

$$r_k e^{-j\phi} = a_{ki} + c_{ki}e^{-j\phi} + b_{ki}e^{-j2\phi}$$

Example: Matlab code example for discrete phase (potentially binary) code design

```
while (cnd)
    for d = 1 : N
        [ a_dk,b_dk,c_dk ]       = mabc( x,d );
        X1                   = [a_dk,c_dk,b_dk];
        Y                    = (fft(X1.',L));
        Y1                   = abs(Y).^2;
        Ytemp                = sum(Y1,2);
        Y2                   = repmat(Ytemp,1,N-1);
        Omega                = theta * Y1 + (1 - theta) * Y2;
        mYobj                = max(Omega,[],2);
        [~,iy]               = min(mYobj);
        phf1est              = 2*pi*(iy-1)/L;
        x(d)                 = exp(1i * phf1est);

    end
end
```

Matlab source codes can be downloaded from :

https://radarmimo.com/how-to-design-binary-codes-for-radar-systems/

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Example: PSL Minimization – Constant Modulus

$$\mathcal{P}_{\phi}^{(k+1)} \begin{cases} \underset{\phi}{\text{minimize}} & \max\left\{\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|^2\right\}_{k=1}^{N-1} \\ \text{subject to} & \phi \in [0, 2\pi) \end{cases}$$

$$\beta = \tan\frac{\phi}{2}$$

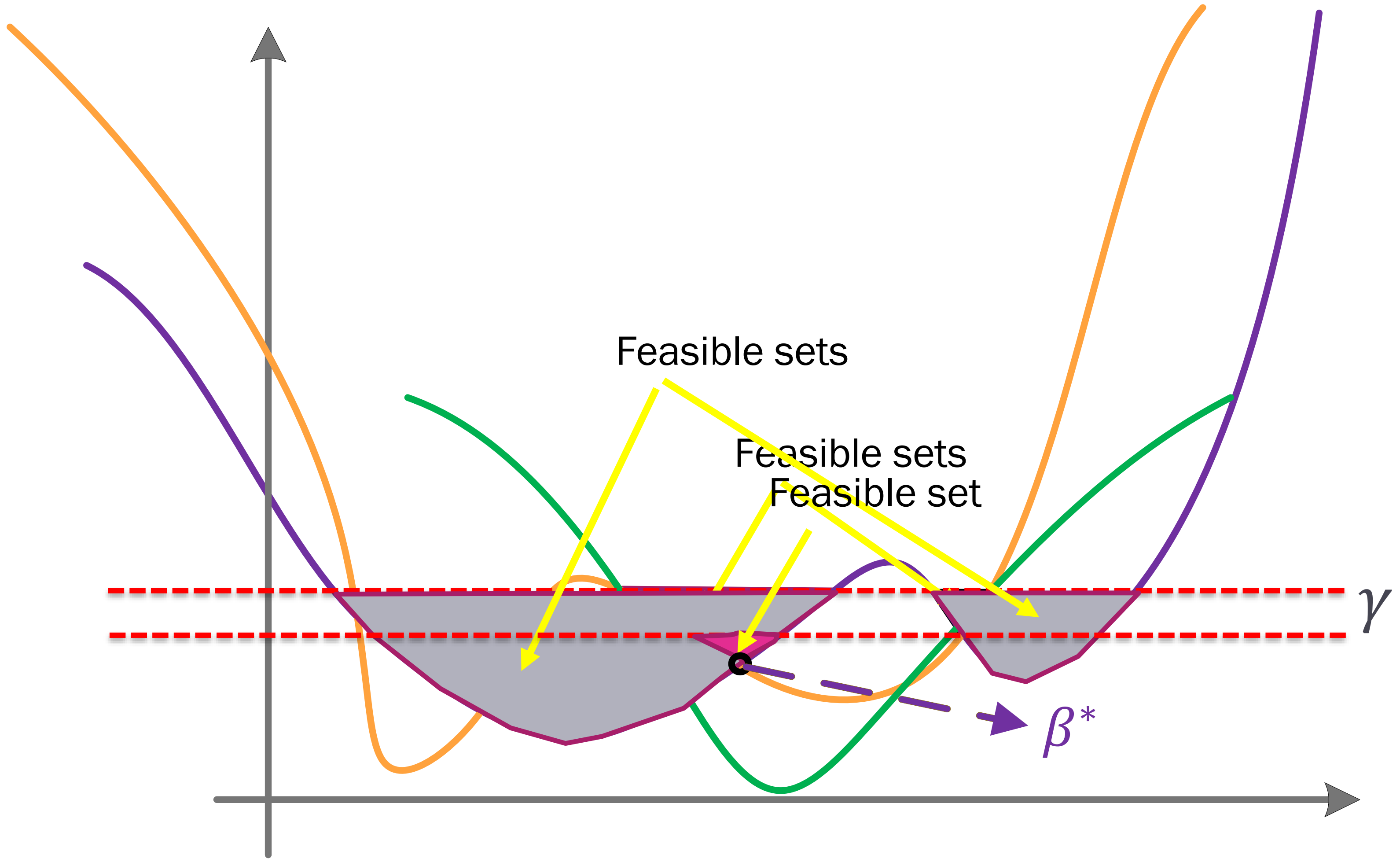
$$\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|^2 = \frac{\mu\beta^4 + \kappa\beta^3 + \delta\beta^2 + \eta\beta + \rho}{(1+\beta^2)^2}$$

Example: PSL Minimization – Constant Modulus

$$\underset{\beta}{\text{minimize}} \qquad \max\left\{\frac{\mu\beta^4 + \kappa\beta^3 + \delta\beta^2 + \eta\beta + \rho}{(1+\beta^2)^2}\right\}_{k=1}^{N-1}$$

$$\begin{cases} \text{find} & \beta \\ \text{subject to} & \dfrac{\mu\beta^4 + \kappa\beta^3 + \delta\beta^2 + \eta\beta + \rho}{(1+\beta^2)^2} \le \gamma \end{cases}$$

Example: PSL Minimization in SISO/SIMO Radar Systems



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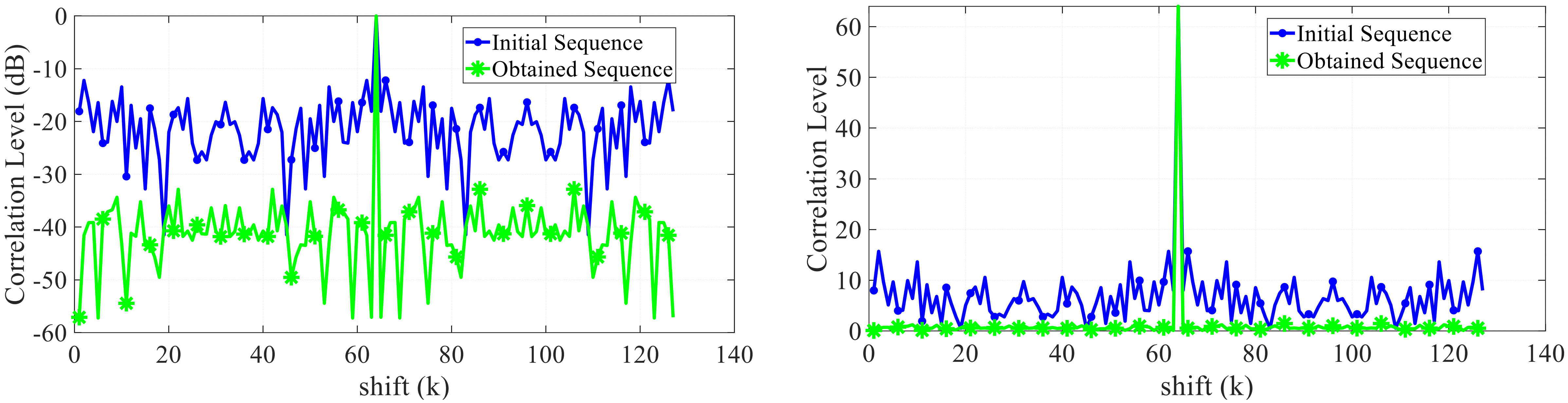
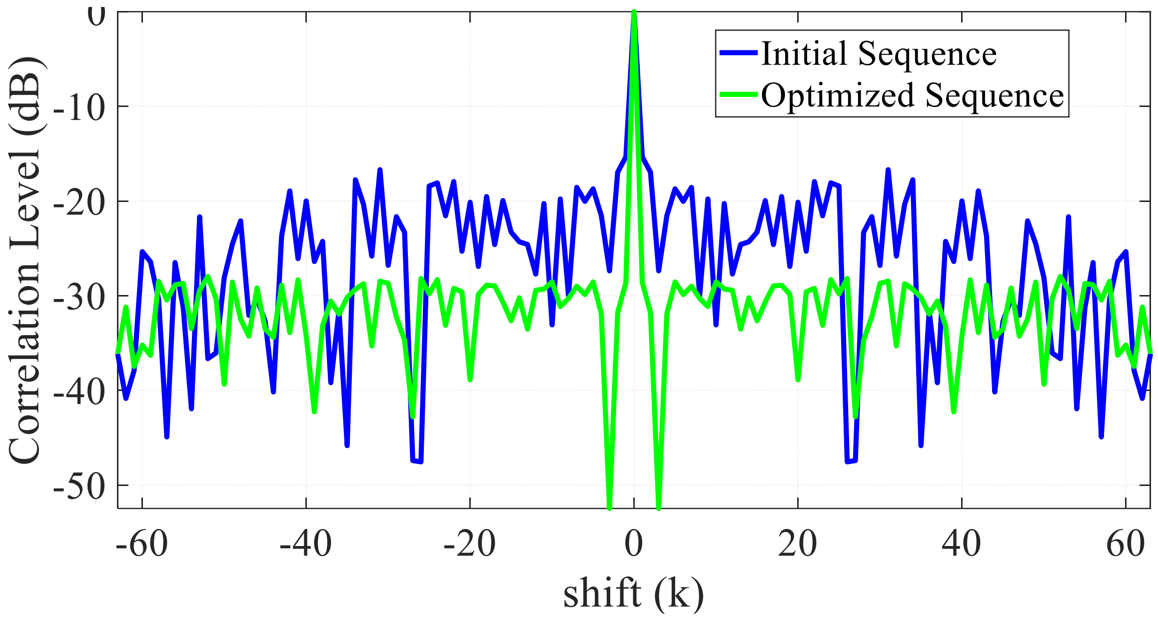
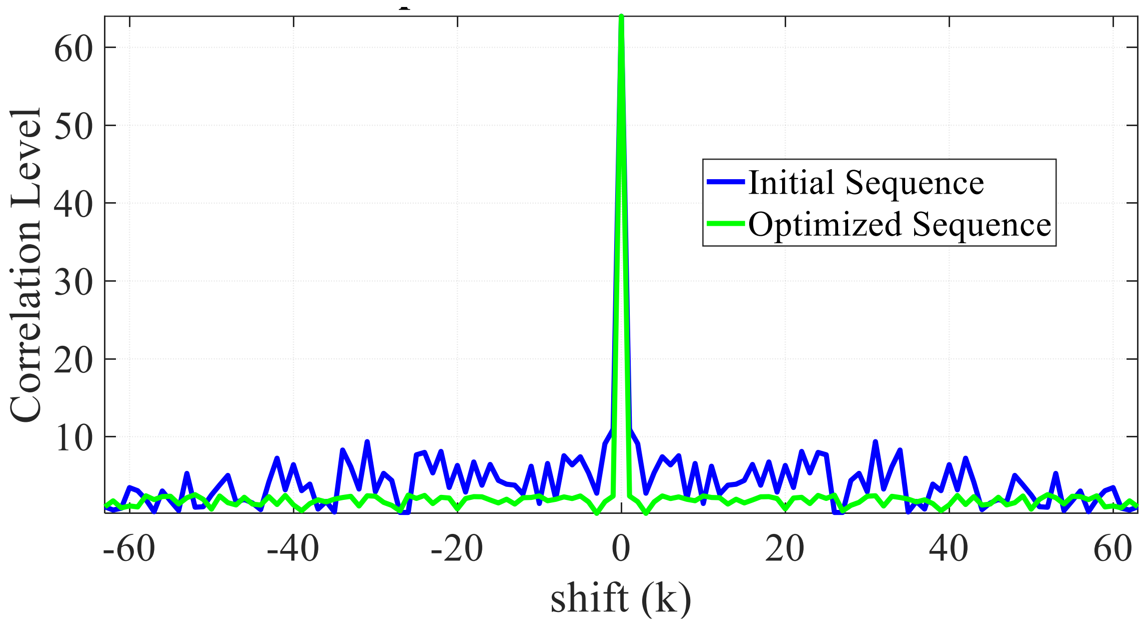
Example: Matlab code example for discrete phase (potentially binary) code design



Periodic ISL Minimization in SISO/SIMO Radar Systems

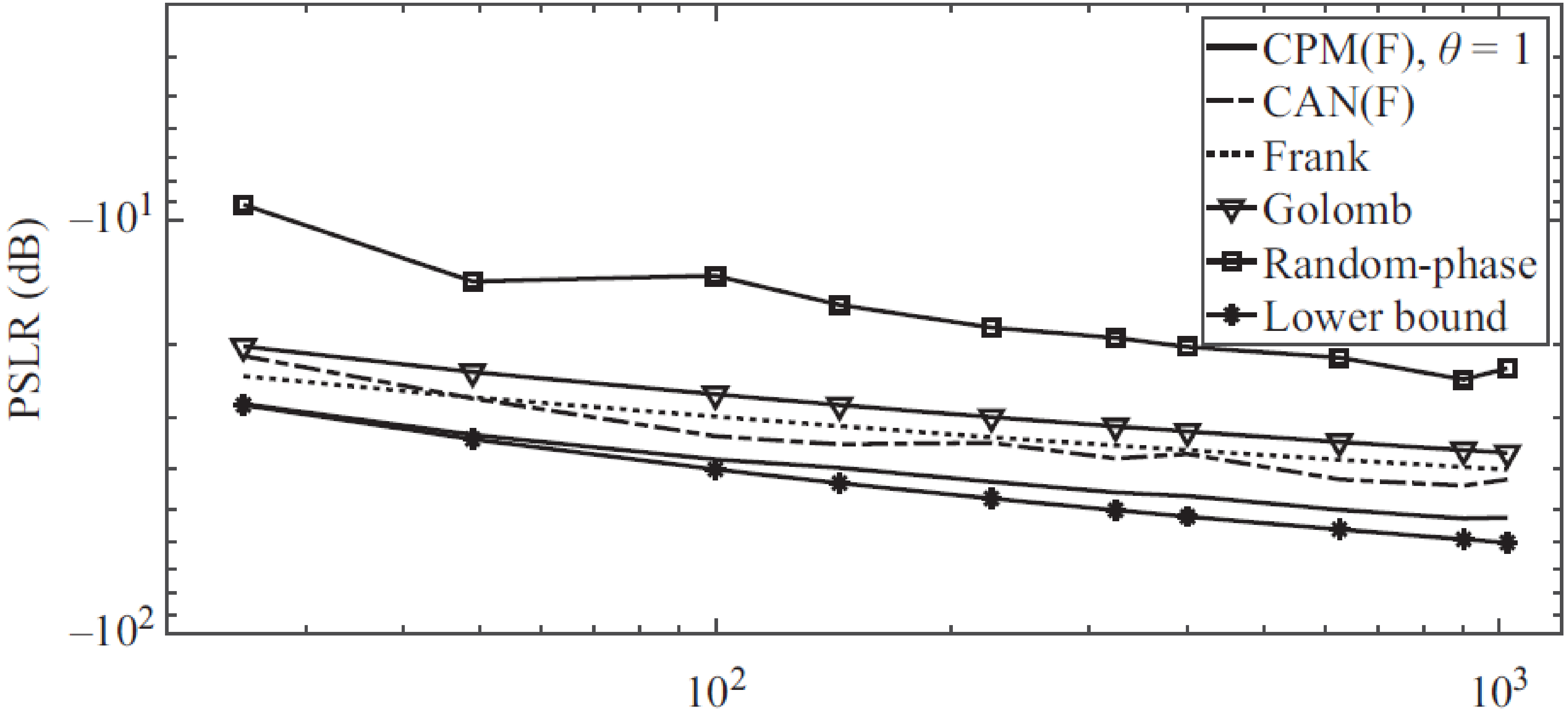
Example: Matlab code example for discrete phase (potentially binary) code design





Aperiodic PSL Minimization in SISO/SIMO Radar Systems

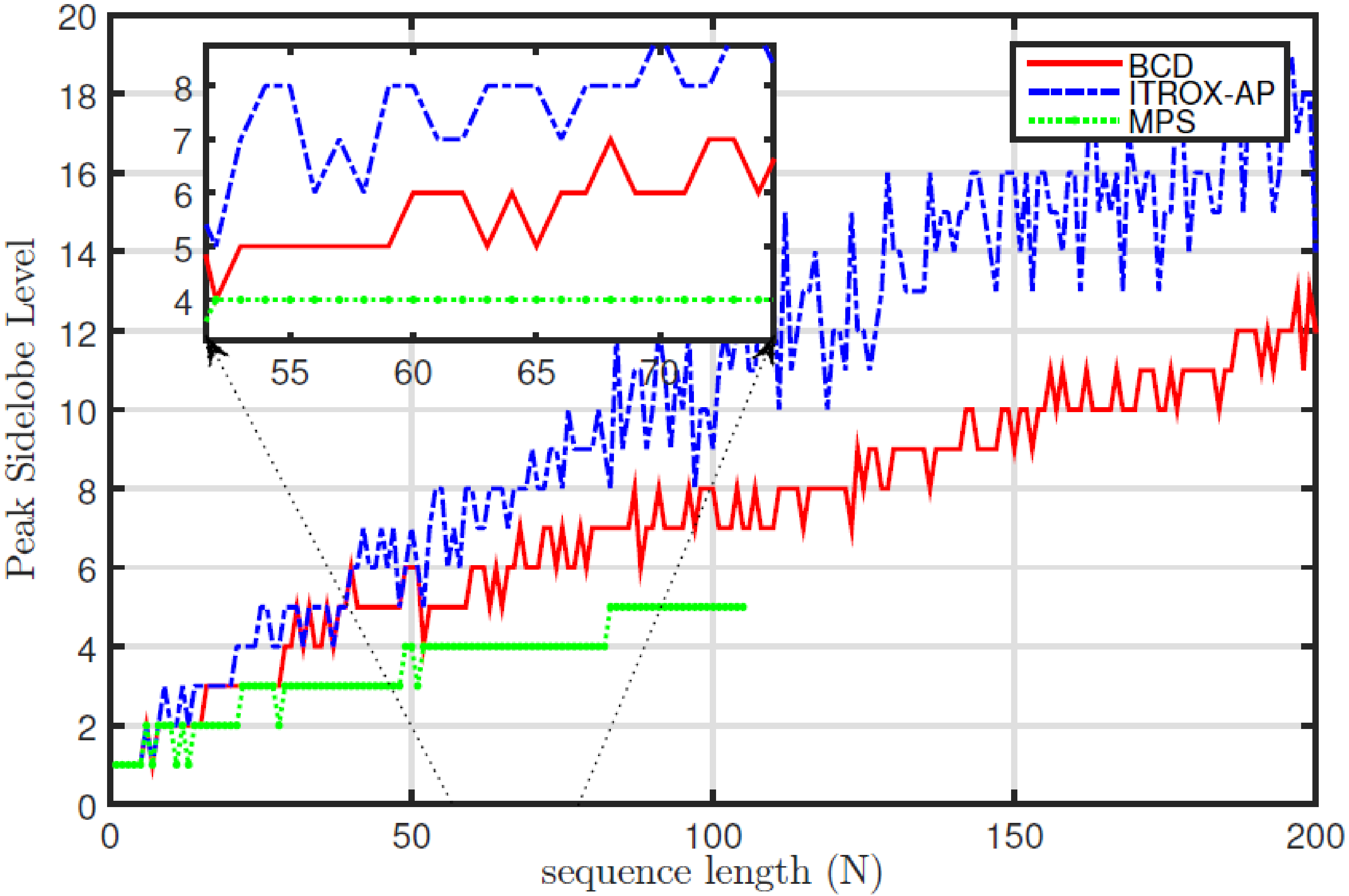
Example: PSL Minimization in SISO/SIMO Radar Systems



Matlab source codes can be downloaded from :

https://radarmimo.com/how-to-design-binary-codes-for-radar-systems/

Example: Binary codes obtained by PSL Minimization



Example: PSL Minimization in SISO/SIMO Radar Systems

N	**Sample Binary Codes with Good PSL (in HEX)**
320	3398D83F635CC5A0D5727CB53A97D39896CFD7C6F1EF86C9AEDE20400F546DF8AB49D7D0879C21BB
360	6D4A71524C40837C9DA7F101F7580E457FFE23696BFA3B7DB9A957CA7923E185985396572CCB9AAD7347A38682
400	B6686A4E6FEA1CF29CBFE6ECA477E2A5D7A8F448A108A5F3F593E63ABC7917D84CA736F15C447BD2072CABA99F127CA5185C
440	73B8B3397676BB952A97A519AEB64C7C544D00242B2A8180BFCB610F4AE6D1C0740F1D8904DE217F4F79248D054B2C7FB490C3CE10BC67
480	64A83F1A672F6E4A4CF5824F5FDD9FBE73FC48322A4D930E17702F859E67911CFD2E12415ACBB55159C229E8ACFF70C25227A379A92CAA17A712B91D

Waveforms in (Colocated/Widely Separated) MISO/MIMO Radar Systems

- ✓ Transmitters should be observable at each receiver
- ✓ Enabled by Orthogonal Waveforms
  - Limit mutual interference
  - Enable cooperative operation
  - Provide visibility into paths between transmitter and receivers
  - Determines spatial distribution of energy
  - Orthogonality achieved by division in time, frequency or code
- ✓ FDM-, TDM-, DDM-, and CDM-MIMO

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$$r_{ml}(k) = \sum_{n=1}^{N-k} x_m(n)\, x_l^*(n+k) = r_{lm}^*(-k)$$

CDM-MIMO Waveform Design Problem

$$\boldsymbol{x}_m = [x_m(1), x_m(2), \dots, x_m(N)]^T \in \mathbb{C}^N,$$

$$\boldsymbol{X} = \left[\boldsymbol{x}_1, \quad \boldsymbol{x}_2, \quad \dots, \boldsymbol{x}_{N_T}\right] \in \mathbb{C}^{N \times N_T}$$

$$r_{ml}(k) = \sum_{n=1}^{N-k} x_m(n)\, x_l^*(n+k) = r_{lm}^*(-k)$$

CDM-MIMO Waveform Design Problem

$$\mathrm{PSL} = \max\left\{\max_{m}\max_{k\neq 0}|r_{mm}(k)|\ ,\max_{\substack{m,l\\ m\neq l}}\max_{k}|r_{ml}(k)|\right\}$$

$$\mathrm{ISL} = \sum_{m=1}^{N_T}\sum_{\substack{k=-N+1\\ k\neq 0}}^{N-1}|r_{mm}(k)|^2 + \sum_{\substack{m,l=1\\ m\neq l}}^{N_T}\sum_{k=-N+1}^{N-1}|r_{ml}(k)|^2$$

How to design set of sequences with small PSL / ISL ?

[2] - M. Alaee-Kerahroodi, M. Modarres-Hashemi and M. M. Naghsh, "Designing Sets of Binary Sequences for MIMO Radar Systems," in *IEEE Transactions on Signal Processing*, vol. 67, no. 13, pp. 3347-3360, 1 July1, 2019.

CDM-MIMO Waveform Design Problem

Waveform Design with ISL Minimization in MIMO Radar

$$\mathrm{ISL} = \sum_{m=1}^{N_T} \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} |r_{mm}(k)|^2 + \sum_{\substack{m,l=1 \\ m\neq l}}^{N_T} \sum_{k=-N+1}^{N-1} |r_{ml}(k)|^2$$

$$P_{\mathbf{X}} = \begin{cases} \min\limits_{\mathbf{X}} & f(\mathbf{X}) \\ \text{s.t.} & |x_m(n)| = 1 \end{cases}$$

CDM-MIMO Waveform Design Problem using ISL Minimization

$$\mathrm{ISL} = \sum_{m=1}^{N_T} \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} |r_{mm}(k)|^2 + \sum_{\substack{m,l=1 \\ m\neq l}}^{N_T} \sum_{k=-N+1}^{N-1} |r_{ml}(k)|^2$$

$$f(x_t(d)) = \boxed{\sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} |a_{dkt} x_t(d) + b_{dkt} x_t^*(d) + c_{dkt}|^2 + \sum_{l=1}^{N_T} \sum_{k=-N+1}^{N-1} |a_{dkl} x_t(d) + c_{dkl}|^2}$$
$$+ \sum_{\substack{m=1 \\ m\neq t}}^{N_T} \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} |r_{mm}(k)|^2 + \sum_{\substack{m,l=1 \\ m\neq \{t,l\}}}^{N_T} \sum_{k=-N+1}^{N-1} |r_{ml}(k)|^2,$$

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CDM-MIMO Waveform Design Problem using ISL Minimization

$$\tilde{f}(x_t(d)) = \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} \left|a_{dkt}x_t(d) + b_{dkt}x_t^*(d) + c_{dkt}\right|^2 + \sum_{l=1}^{N_T}\sum_{k=-N+1}^{N-1} \left|a_{dkl}x_t(d) + c_{dkl}\right|^2$$

$$\widetilde{P}_{x_t(d)} = \begin{cases} \min\limits_{x_t(d)} & \tilde{f}(x_t(d)) \\ \text{s.t.} & |x_t(d)| = 1 \end{cases}$$

Still non-convex!!!

CDM-MIMO Waveform Design Problem using ISL Minimization

$$\widetilde{P}_{\phi_t(d)} = \begin{cases} \min\limits_{\phi_t(d)} & \sum\limits_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} \left|a_{dkt}e^{j\phi_t(d)} + b_{dkt}e^{-j\phi_t(d)} + c_{dkt}\right|^2 + \sum\limits_{l=1}^{N_T} \sum\limits_{k=-N+1}^{N-1} \left|a_{dkl}e^{j\phi_t(d)} + c_{dkl}\right|^2 \\ \text{s.t.} & \phi_t(d) \in [0, 2\pi) \end{cases}$$

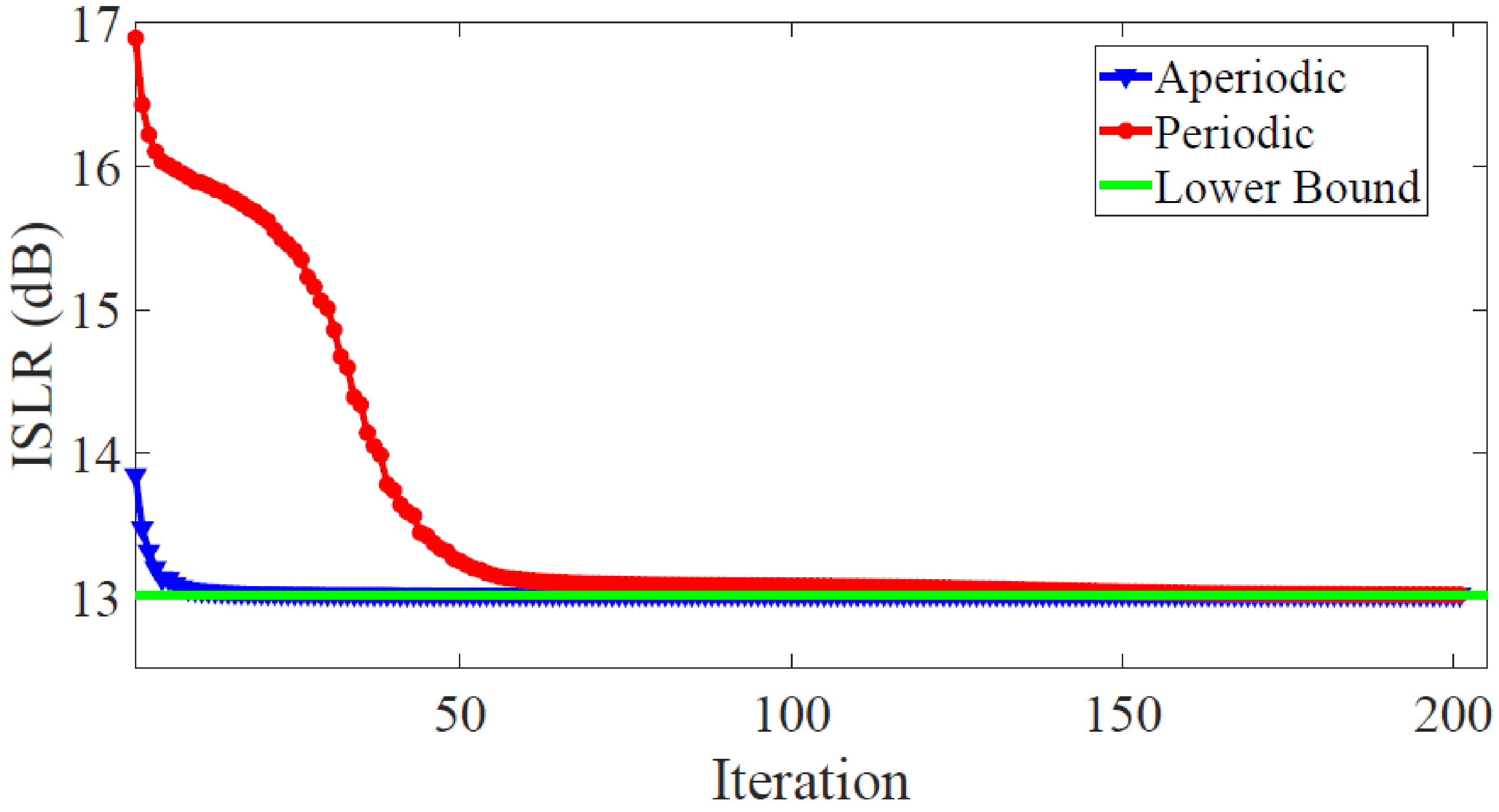
$$\beta_d = \tan\frac{\phi_t(d)}{2}$$

$$\boxed{\widetilde{P}_{\beta_d} = \begin{cases} \min\limits_{\beta_d} & \dfrac{\mu_{dk}\beta_d^4 + \kappa_{dk}\beta_d^3 + \xi_{dk}\beta_d^2 + \eta_{dk}\beta_d + \rho_{dk}}{(1+\beta_d^2)^2} \\ \text{s.t.} & \beta_d \in \mathbb{R} \end{cases}}$$

CDM-MIMO Waveform Design Problem using ISL Minimization

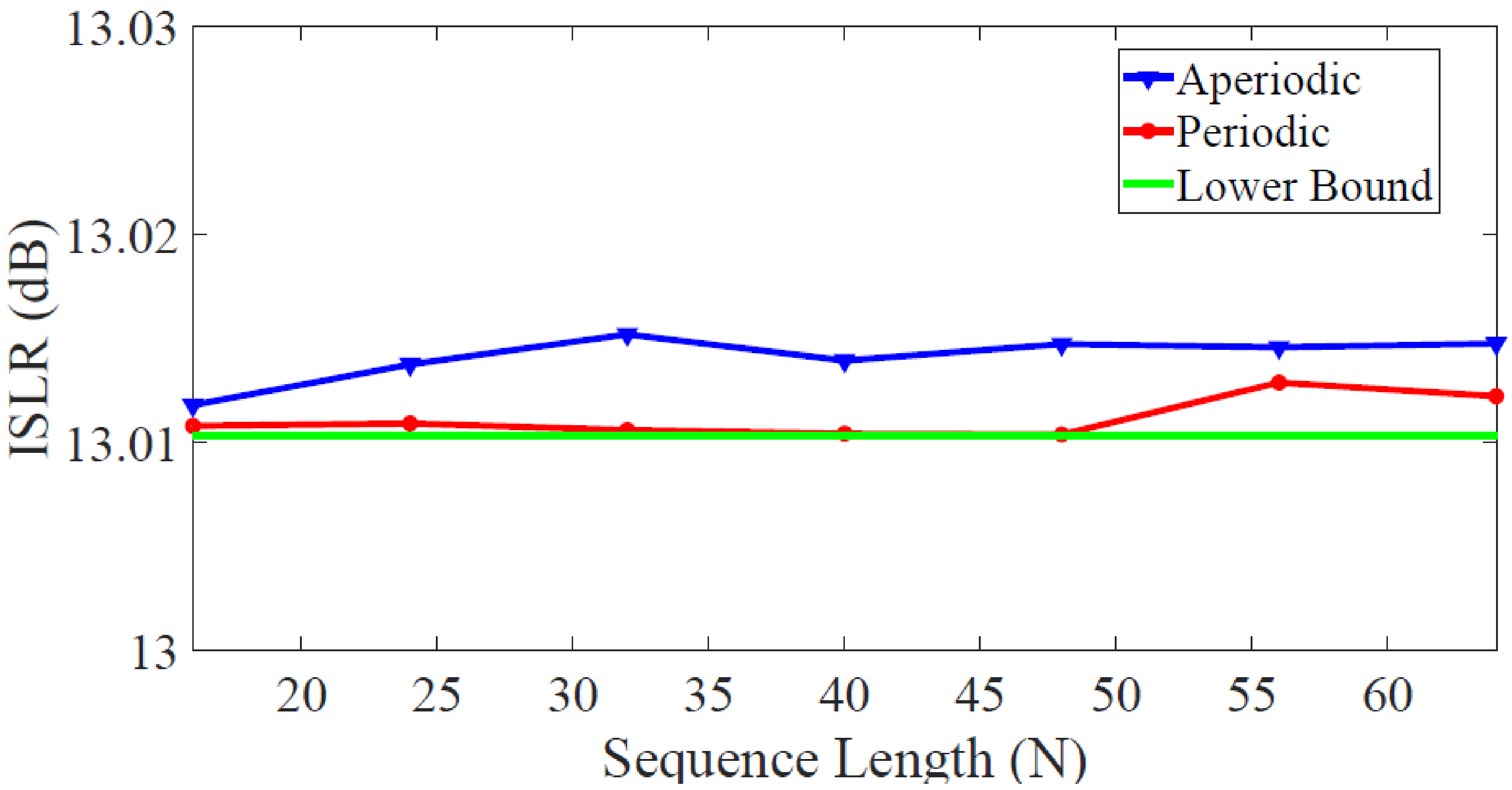
- **do**
  - Find the real roots of the first order derivative of the objective function, and evaluate the objective function in these points, and obtain optimal $\beta_d^\star$ where the objective function is minimum;
  - Set $\phi_t^\star(d) = 2\,\mathrm{atan}(\beta_d^\star)$ and $x_t^\star(d) = e^{j\phi_t^\star(d)}$;
  - Set $d = d + 1$;
- **until** $d > N$;
- $t = t + 1$ and set code entry index $d = 1$;

CDM-MIMO Waveform Design Problem using ISL Minimization



Convergence behavior of the proposed algorithm ($N = 64, N_T = 5$)

CDM-MIMO Waveform Design Problem using ISL Minimization



ISLR values of the obtained set of $N_T = 5$ sequences through the proposed algorithm averaged over 10 independent trails, comparing with the lower bound.

CDM-MIMO Waveform Design Problem using ISL Minimization

ISLR(dB) values obtained via the proposed algorithm initialized by random-phase sequence of length $N = 64$ averaged over 10 independent trails, in comparison with the lower bound.

Set Size (N_T)	2	3	4	5	6	7	8	9	10
Aperiodic	3.117	7.807	10.803	13.016	14.774	16.234	17.483	18.574	19.543
Periodic	3.022	7.797	10.798	13.013	14.775	16.236	17.482	18.574	19.546
Lower Bound	3.010	7.781	10.791	13.010	14.771	16.232	17.481	18.573	19.542

Conclusion

- Radar waveform design is long standing problem, but still there are challenges that needs to a research to be addressed.
- GD, MM, CD, BSUM, and ADMM, are iterative approaches that was found to be successful in solving many non-convex optimization problems, particularly radar waveform design
- Beampattern shaping, joint radar and communications, …

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Thank you

and

Question?

Get in touch for more info

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