th Iranian National Conference on Radar and Surveillance Systems

رادار و سامانه های مراقبتی

17 - 18 November
Islamic Azad University, Yadegar Imam Khomeini (RAH) branch
Shahr-e-ray

Mathematical Techniques for Signal Design in Modern Radar Systems

Mohammad Alaee
University of Luxembourg, SnT, SPARC

17 November 2021

SnT

UNIVERSITY OF LUXEMBOURG

About me!

Mohammad Alaee-Kerahroodi

- Research Scientist, SnT-SIGCOM/ SPARC, **University of Luxembourg - LUXEMBOURG**
- **SPARC** (**S**ignal **P**rocessing **A**pplications for **R**adar and **C**ommunications): from the January 2022
- In charge of radar lab activities at SnT-SIGCOM/SPARC (see https://radarmimo.com/)
- PhD, Isfahan University of Technology (https://www.iut.ac.ir/en/)
- More thank 12 years experience in radar systems

My condolences on the passing of Dr. Majid Okhovat

I thank the **organizers** for the opportunity!

Hope to have physical meetings in early future!

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UNIVERSITY OF LUXEMBOURG

Luxembourg

Capital and largest city	Luxembourg City[1] 49°48′52″N 06°07′54″E
Official languages	Luxembourgish French German[a]
Nationality (2017)	50.9% Luxembourgers 18.2% Portuguese 13.5% French 10.3% Germans 7.1% Other

Luxembourg

Country in Europe

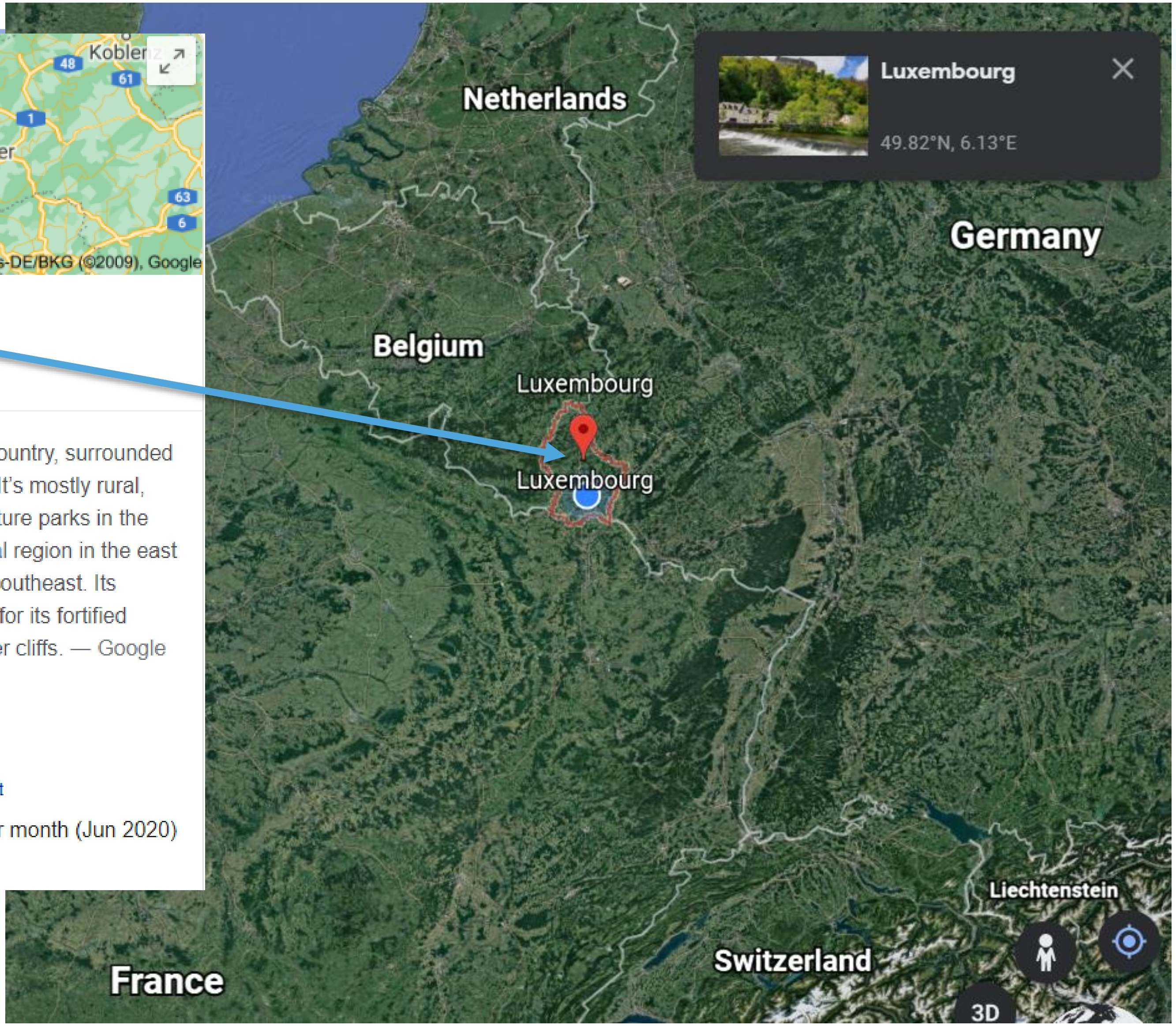
Luxembourg is a small European country, surrounded by Belgium, France and Germany. It's mostly rural, with dense Ardennes forest and nature parks in the north, rocky gorges of the Mullerthal region in the east and the Moselle river valley in the southeast. Its capital, Luxembourg City, is famed for its fortified medieval old town perched on sheer cliffs. — Google

Capital: Luxembourg

Area: 2,586 km²

Population: 613,894 (2019) Eurostat

Minimum wage: 2,141.99 EUR per month (Jun 2020) Eurostat



Luxembourg is the wealthiest country in the European Union, per capita, and its citizens enjoy a high standard of living.

#3 worldwide for its international outlook in the Times Higher Education (THE) World University Rankings 2022.

World University Ranking by subject 2022
The **University of Luxembourg** ranked:

- ✓ 92 in Law (not listed previously)
- ✓ 93 in Computer Science
- ✓ 126-150 in Engineering & Technology
- ✓ 101-125 in Life Sciences
- ✓ 201-250 in Physical Sciences
- ✓ 126-150 in Psychology
- ✓ 176-200 in Social Sciences
- ✓ 251-300 in Business and Economics
- ✓ 176-200 in Education
- ✓ 201-250 in Arts and Humanities

SnT and SIGCOM Group

SnT - Interdisciplinary Centre for Security, Reliability and Trust

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Director of SnT
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- **APSIA -** The Applied Security and Information Assurance Group (Prof. Peter Ryan)
- **Automation Research Group** (Prof Holger Voos)
- **CritiX** - Critical and Extreme Security and Dependability Research Group (Prof Paulo Esteves Veríssimo)
- **CryptoLux** (Prof. Alex Biryukov)
- **CVI2** - Computer Vision, Imaging and Machine Intelligence Research Group (Dr Djamila Aouada)
- **FINATRAX** - Digital Financial Services and Cross-Organisational Digital Transformations Research Group (Prof. Gilbert Fridgen)
- **IRiSC** - Sociotechnical Cybersecurity Interdisciplinary Research Group (Prof Dr Gabriele Lenzini)
- **PCOG** - Parallel Computing & Optimisation Research Group (Prof Pascal Bouvry)
- **RSA -** Remote Sensing Applications (Prof Dr Tonie Van Dam)
- **SEDAN** - Service and Data Management in Distributed Systems (Prof Dr Radu State)
- **SerVal** - Security Design and Validation Research Group (Prof Yves Le Traon)
- **SIGCOM** - Signal Processing and Communications (Prof. Symeon CHATZINOTAS)
- **SPARC** – Signal Processing Applications for Radar and Communications (Dr. Bhavani Shankar)
- **Space R** - Space Robotics Research Group (Prof Dr Miguel Angel Olivares Mendez)
- **TRuX** (Prof Dr Jacques Klein)
- **V&V Lab** - Software Verification and Validation Research Group (Prof Lionel Briand)

- **Projects**
  - **2** EC (ERC & ERC PoC)
  - **2** National projects – Fundamental Research (1 bilateral with Germany)
  - **2** National projects – Collaboration with Industry
  - **1** MC COST Action
- **17 Personnel (end of current hiring)**
  - 7 PhD, 6 Post Docs, 2 Research Scientists, 1 Research Developer, 1 Honorary member

SPARC – Research Topics

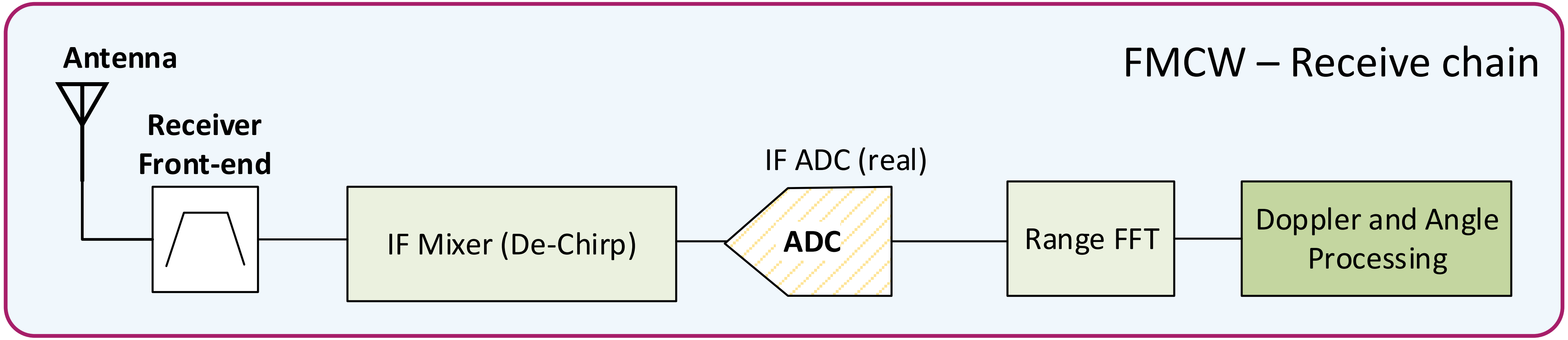
- Waveform Design
  - Cognitive MIMO/ Phased Array radars / **4D Imaging automotive radars**
- Signal Processing in Distributed Radar Systems
  - Sensor placement/ localization/ **distributed Imaging**/ fusion and tracking
- Joint/Integrated/Dual Radar and Communications
  - Co-existence and **co-design**
- Radar Signal Processing
  - **Vital Signs Monitoring**/ **Interference Analysis** / Sparse Sensing / **Unlimited Sampling**
- Scene Generation
  - **Raw data simulation** for indoor sensing scenarios
- Prototyping
  - mmWave COTS/ **USRPs**/ Custom Built Radars

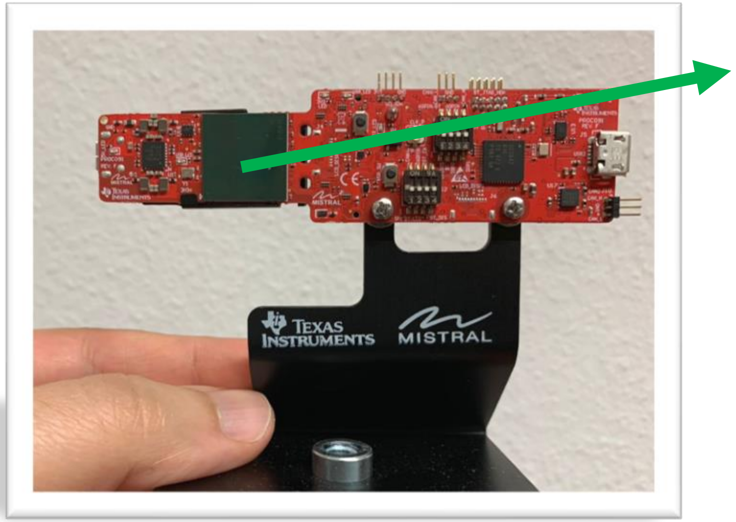
Mathematical Techniques for Signal Design in Modern Radar Systems

Background

	Title	Presented in	Slide/ Video
T4	Waveform Optimization Techniques for Radar Systems	7th Iranian Radar Conference (2020)	download here!
T3	Non-Convex Optimization for Practical Signal Design in Radar Systems with Emerging Applications	2020 IEEE Radar Conference	--
T2	Joint Automotive MIMO-Radar-MIMO-Communications Signal Processing	2020 IEEE INTERNATIONAL RADAR CONFERENCE	--
T1	Waveform optimization techniques for radar systems		

Radar On Chip (Automotive Applications)





Radar On Chip (ROC)

- ✓ Operating in mmWave (60GHz, and 79 GHz)
- ✓ More than 5GHz bandwidth
- ✓ MIMO capabilities

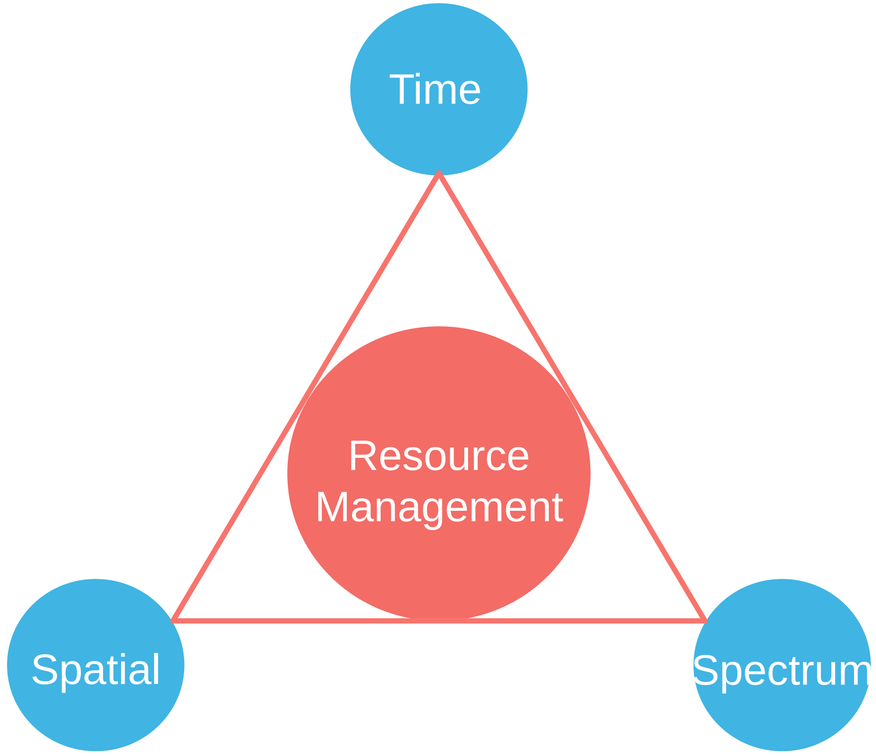
In Automotive Applications

- FMCW radars can do the sampling in few MHz rates
- PMCW radars need the minimum sampling rate of BW in every I & Q receive channels

- Building FMCW radars is typically easier than PMCW radars
- The performance of the both radars can be similar for many cases
- Novel applications encourage using PMCW waveforms

PMCW Provides more degree of freedom to manage the resources

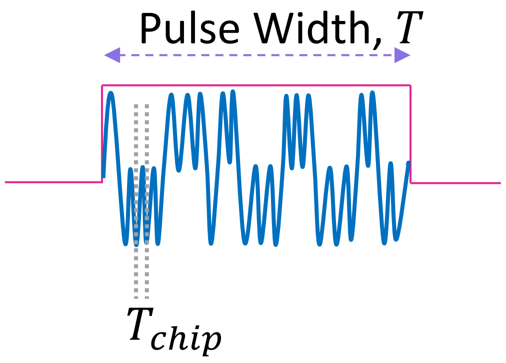
One important aspect in Cognitive radar



Waveform Diversity and MIMO Radars

Waveform Diversity

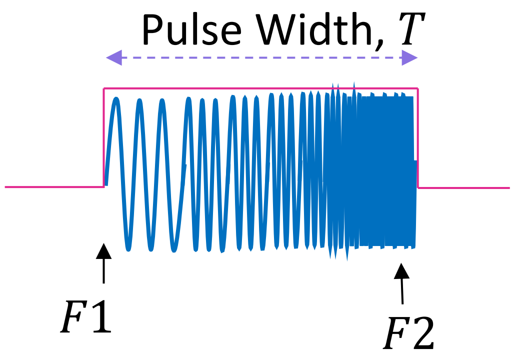
Phase Coded Waveform



Bandwidth = $\frac{1}{T_{chip}}$

Time × Bandwidth = $\frac{T}{T_{chip}}$

Linear Frequency Modulated Waveform

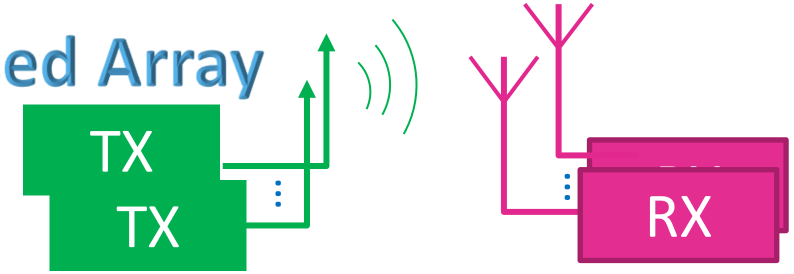


Bandwidth = $\Delta F = F2 - F1$

Time × Bandwidth = $T \times \Delta F$

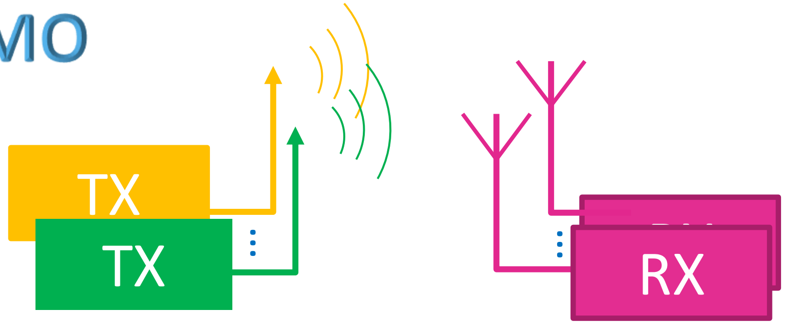
Multiple Antennas

Phased Array



Single **I**nput **M**ulti **O**utput

MIMO



Multi **I**nput **M**ulti **O**utput

Classical Radar Problems?

Pulse Compression

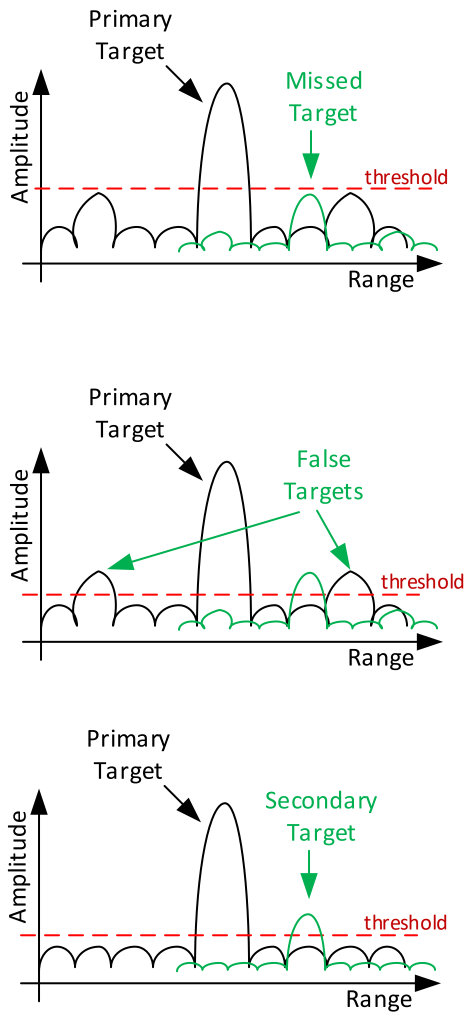
$$\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T$$

Transmit waveform

Code length

$$r_k = \sum_{n=1}^{N-k} x_n^* x_{n+k}\,, \qquad k = 0, \dots, N-1$$

$$\begin{cases} \min\limits_{\boldsymbol{x}} & \max\limits_{k \neq 0} |r_k| \\ \text{s.t.} & x_n \in \psi_n \end{cases} \qquad \begin{cases} \min\limits_{\boldsymbol{x}} & \sum_k r_k^2 \\ \text{s.t.} & x_n \in \psi_n \end{cases}$$



Classical Radar Problems?

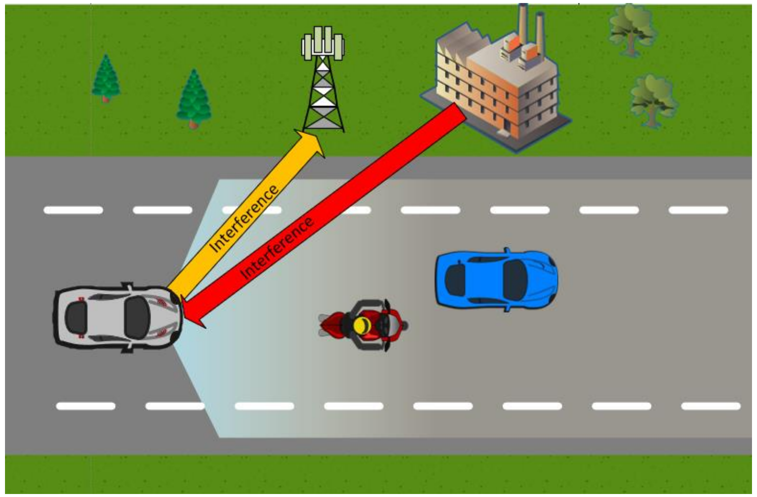
Spectral Shaping

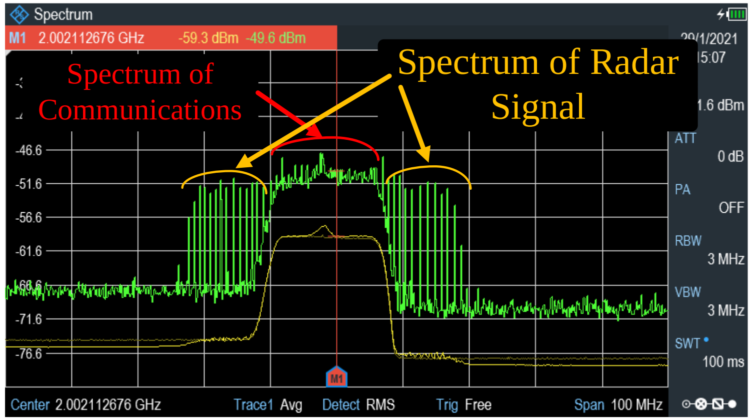
$$\boldsymbol{x}_m = \left[x_{1,\mathrm{m}}, x_{2,\mathrm{m}}, \dots, x_{N,m}\right]^T \in \mathbb{C}^N$$

DFT matrix $\quad \boldsymbol{F} \triangleq [\boldsymbol{f}_0, \dots, \boldsymbol{f}_{N-1}] \in \mathbb{C}^{N\times N}$

$$\boldsymbol{f}_k \triangleq \left[1, e^{-j\frac{2\pi k}{N}}, \dots, e^{-j\frac{2\pi k(N-1)}{N}}\right]^T \in \mathbb{C}^N$$

$$\begin{cases} \min\limits_{\boldsymbol{x}} & \dfrac{\sum_m \left\|\boldsymbol{f}_k^H \boldsymbol{x}_m\right\|^2 \mid k \in U}{\sum_m \left\|\boldsymbol{f}_k^H \boldsymbol{x}_m\right\|^2 \mid k \in V} \\ \text{s.t.} & x_n \in \psi_n \end{cases}$$





Classical Radar Problems?

Transmit Beampattern Shaping

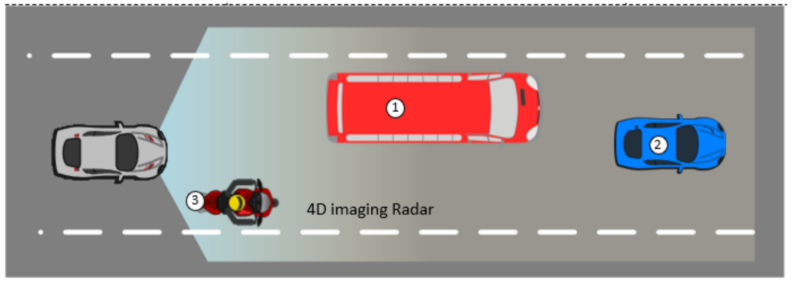
$$\boldsymbol{x}_n = \left[x_{1,\mathrm{n}}, x_{2,\mathrm{n}}, \dots, x_{M,n}\right]^T \in \mathbb{C}^M$$

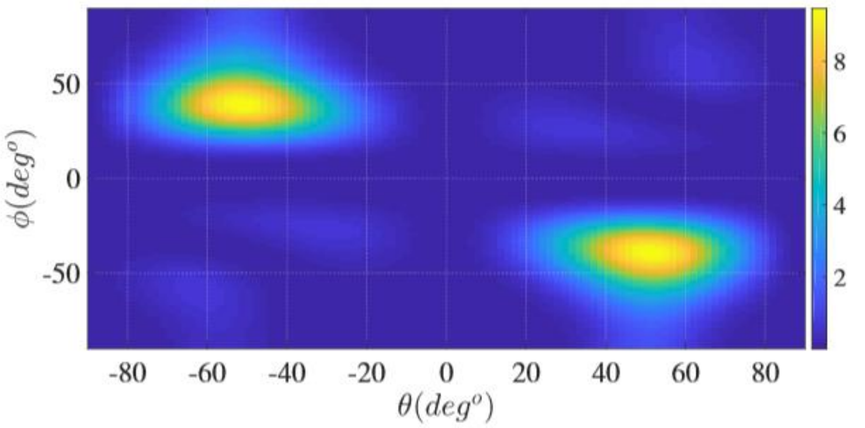
Steering Vector $\boldsymbol{a}(\theta,\phi) \triangleq [a_1(\theta,\phi), \dots, a_M(\theta,\phi)] \in \mathbb{C}^M$

Transmit Beampattern $\Sigma_n |\boldsymbol{a}^H(\theta,\phi)\boldsymbol{x}_n|^2 = \Sigma_n\, \boldsymbol{x}_n\, \boldsymbol{A}(\theta,\phi)\, \boldsymbol{x}_n$

$$\boldsymbol{A}(\theta,\phi) = \boldsymbol{a}(\theta,\phi)\, \boldsymbol{a}^H(\theta,\phi)$$

$$\begin{cases} \min\limits_{\boldsymbol{x}} & \dfrac{\Sigma_n\, \boldsymbol{x}_n\, \boldsymbol{A}_U(\theta_u,\phi_u)\, \boldsymbol{x}_n}{\Sigma_n\, \boldsymbol{x}_n\, \boldsymbol{A}_D(\theta_d,\phi_d)\, \boldsymbol{x}_n} \\ \mathrm{s.t.} & x_n \in \psi_n \end{cases}$$





Waveform Design and Optimization Problems

- Constraints
  - Energy
  - Peak-to-Average Power Ratio (PAPR, PAR)
  - Unimodularity (being Constant-Modulus)
  - Finite or Discrete-Alphabet (integer, binary, m-ary constellation)
  - …
- Challenges
  - How to handles signal constraints?
  - How to do it fast?

- Many of these problems are shown to be **NP-hard**
- Many others are deemed to be difficult

Optimization Techniques for Waveform Design

- Gradient-Descent Based Methods (GD)
- Majorization-Minimization (MM)
- Coordinate Descent (CD)
- Alternating Direction Method of Multipliers (ADMM)
- Block Successive Upper-bound Minimization (BSUM)
- Several others ...

Recall: Coordinate Descent

$$x_1^{(k)} \in \arg\min_{x_1} f\left(x_1, x_2^{(k-1)}, x_3^{(k-1)}, \ldots., x_N^{(k-1)}\right)$$

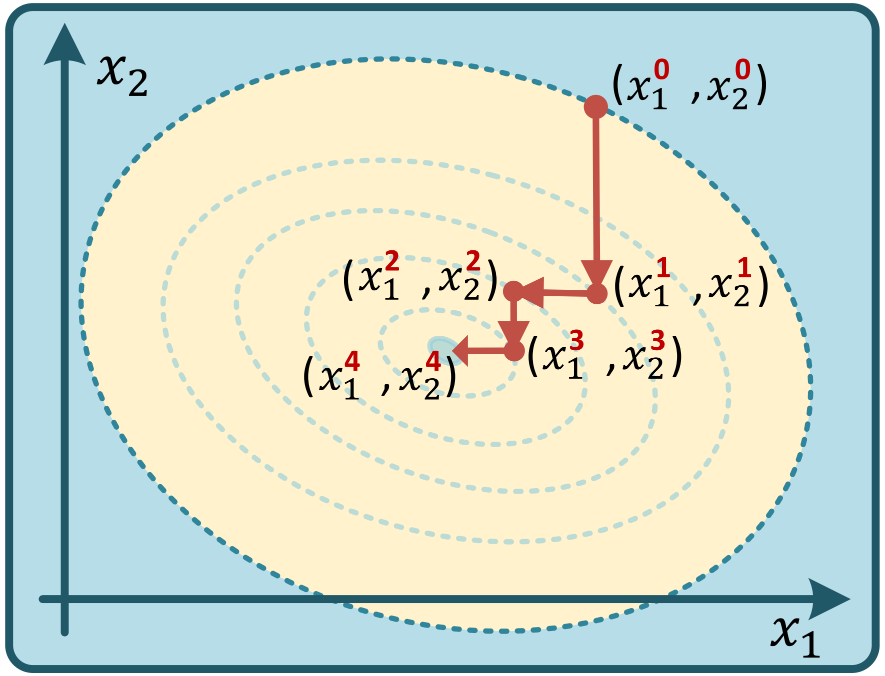
$$x_2^{(k)} \in \arg\min_{x_2} f\left(x_1^{(k)}, x_2, x_3^{(k-1)}, \ldots., x_N^{(k-1)}\right)$$

$$x_3^{(k)} \in \arg\min_{x_3} f\left(x_1^{(k)}, x_2^{(k)}, x_3, \ldots., x_N^{(k-1)}\right)$$

$$\vdots$$

$$x_N^{(k)} \in \arg\min_{x_N} f\left(x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \ldots., x_N\right)$$

Successively minimizes along coordinate directions



$$y = x_1^2 + 2\,x_2^2 - 9$$

No stepsize tuning!

Recall: Majorization-Minimization

Minimization of $\cos(x)$

Second order Taylor expansion

$$\cos(x) = \cos(x_n) - \sin(x_n)\,(x - x_n) - \frac{1}{2}\cos(z)\,(x - x_n)^2$$

Since $|\cos(z)| \leq 1$,

$$g(x|x_n) = \cos(x_n) - \sin(x_n)\,(x - x_n) + \frac{1}{2}(x - x_n)\hat{}2$$

Solving $\frac{d}{dx} g(x|x_n) = 0$ gives the MM algorithm

$$x_{n+1} = x_n + \sin(x_n)$$

An MM algorithm operates by creating a surrogate function that minorizes or majorizes the objective function. When the surrogate function is optimized, the objective function is driven uphill or downhill as needed.

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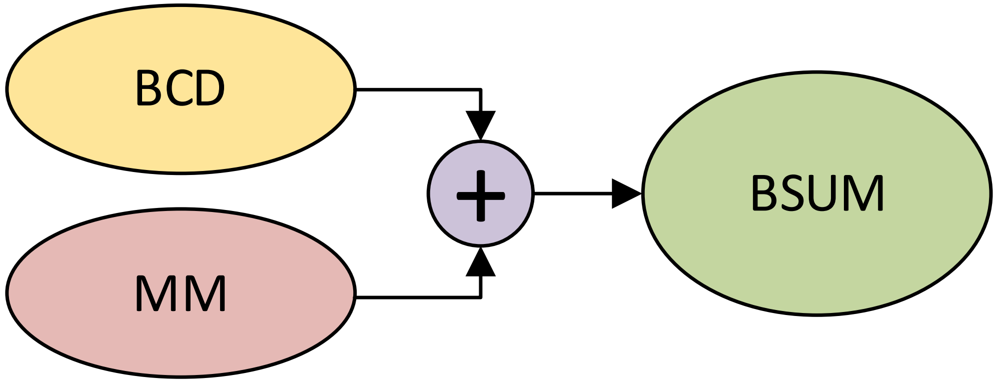
Block successive upper-bound minimization (BSUM)

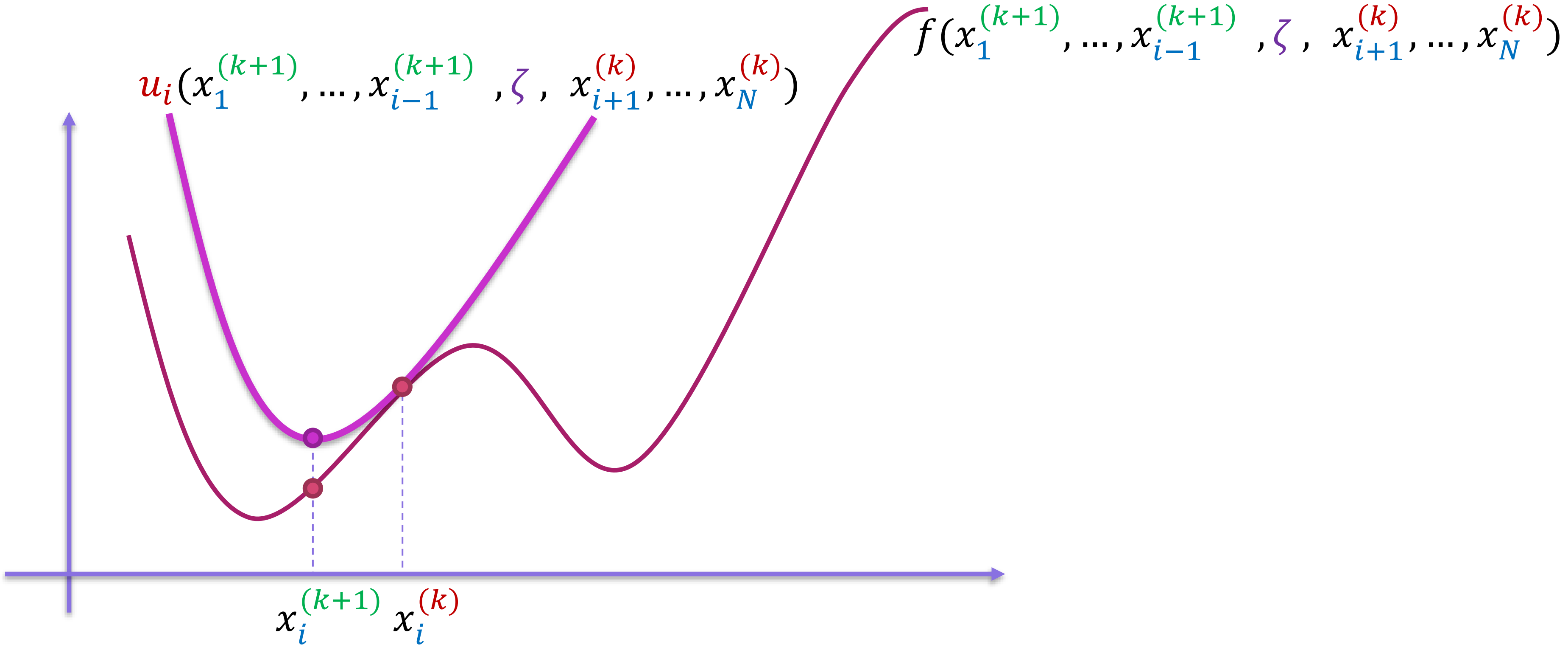


$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & f(\boldsymbol{x}) \\ \text{subject to} & x_n \in \psi_n \end{cases} \qquad \boldsymbol{x} = [x_1, x_2, \dots, x_N]^T$$

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} u_i\left(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)}\right)$$

Local approximation of the objective function

Block successive upper-bound minimization (BSUM)



Upper-bound $u_i\left(x_1^{(k+1)},\dots,x_{i-1}^{(k+1)},\zeta,\ x_{i+1}^{(k)},\dots,x_N^{(k)}\right) \geq f(x_1^{(k+1)},\dots,x_{i-1}^{(k+1)},\zeta,\ x_{i+1}^{(k)},\dots,x_N^{(k)})$

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BSUM – Algorithm

- Start from initial guess $\boldsymbol{x}^{(0)} = [x_1, x_2, \dots, x_N]^T$
- For $k = 0, 1, \dots$

  - Pick an index i from $\{1, \dots, N\}$

  - Optimize the i-th coordinate

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} u_i(\underbrace{x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}}_{\text{done}}, \underset{\uparrow \text{ current}}{\zeta}, \underbrace{x_{i+1}^{(k)}, \dots, x_N^{(k)}}_{\text{To do}})$$

- Decide when/how to stop; return $\boldsymbol{x}^{(k+1)}$

Which Coordinate in BSUM?

Gauss-Seidel style (One-at-a-time)

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} u_i(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$$

Jacobi style (all-at-once ; easy to parallelize)

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} u_i(x_1^{(k)}, \dots, x_{i-1}^{(k)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$$

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Example- Lp-Norm Minimization

$$\mathbf{x} = [x_1, x_2, \ldots, x_N]^T \in \mathbb{C}^N$$

$$r_k = \sum_{n=1}^{N-k} x_n x_{n+k}^* \qquad 0 \leq k \leq N-1$$

$$\mathcal{P}_h \begin{cases} \min\limits_{\mathbf{x}} & \sum\limits_{k=1}^{N-1} |r_k|^p \\ s.t. & \mathbf{x} \in \Omega_h \end{cases}$$

Example- Lp-Norm Minimization

Let $f(x) = x^p$ *with* $p \geq 2$ *and* $x \in [0, t]$.

Then for any given $x_0 \in [0, t)$, $f(x)$ *is majorized at* x_0 *over the interval* $[0, t]$ *by*

$$\boxed{u(x) = ax^2 + \left(px_0^{p-1} - 2ax_0\right) x + ax_0^2 - (p-1)\, x_0^p}$$

with $a = \dfrac{t^p - x_0^p - px_0^{p-1}(t - x_0)}{(t - x_0)^2}$

Using the above lemma, following will be a majorizer for $|r_k|^p$

$$\boxed{\sum_{k=1}^{N-1} a_k |r_k|^2 + \sum_{k=1}^{N-1} b_k |r_k|}$$

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Example- Lp-Norm Minimization

$$\widetilde{\mathcal{P}}_h \begin{cases} \min_{\mathbf{x}} & \sum_{k=1}^{N-1} a_k \left|r_k\right|^2 + \sum_{k=1}^{N-1} b_k \mathrm{Re}\left\{ r_k^* \frac{r_k^{(\ell)}}{\left|r_k^{(\ell)}\right|} \right\} \\ s.t. & \mathbf{x} \in \Omega_h \end{cases}$$

$$r_k(x_d) = a_{1k} x_d + a_{2k} x_d^* + a_{3k},$$

The only variable in the current iteration

$$\mathbf{x}_{-d} = [x_1^{(i+1)}, \ldots, x_{d-1}^{(i+1)}, 0, x_{d+1}^{(i)}, \ldots x_N^{(i+1)}]^T \in \mathbb{C}^N,$$

Example- Lp-Norm Minimization

$$\boxed{x_d = e^{j\phi_d}}$$

$$\boxed{\widetilde{r}_k(\phi_d) = a_{1k}e^{j\phi_d} + a_{2k}e^{-j\phi_d} + a_{3k},}$$

$$\boxed{\widetilde{\mathcal{H}}_h^{(i+1)} \begin{cases} \min\limits_{\phi_d} & \sum\limits_{k=1}^{N-1} a_k \left|\widetilde{r}_k(\phi_d)\right|^2 + \sum\limits_{k=1}^{N-1} b_k \mathrm{Re}\left\{\widetilde{r}_k(\phi_d)^* \dfrac{r_k^{(\ell)}}{|r_k^{(\ell)}|}\right\} \\ s.t. & \phi_d \in \Phi_h \end{cases}}$$

$$\Phi_\infty = [-\pi, \pi) \qquad \Phi_M = \{0, \frac{2\pi}{M}, \frac{4\pi}{M}, \dots, \frac{2\pi(M-1)}{M}\}$$

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Lp-Norm Minimization – Continuous Phase Code Design

$$\beta_d \triangleq \tan\left(\frac{\phi_d}{2}\right) \qquad |\widetilde{r}_k(\phi_d)|^2 = \frac{\widetilde{p}_k(\beta_d)}{q(\beta_d)}, \qquad \mathrm{Re}\left\{\widetilde{r}_k^*(\beta_d)\frac{r_k^{(i)}}{\left|r_k^{(i)}\right|}\right\} = \frac{\bar{p}_k(\beta_d)}{q(\beta_d)},$$

$$\widetilde{p}_k(\beta_d) = \mu_{1k}\beta_d^4 + \mu_{2k}\beta_d^3 + \mu_{3k}\beta_d^2 + \mu_{4k}\beta_d + \mu_{5k},$$

$$\bar{p}_k(\beta_d) = \kappa_{1k}\beta_d^4 + \kappa_{2k}\beta_d^3 + \kappa_{3k}\beta_d^2 + \kappa_{4k}\beta_d + \kappa_{5k},$$

$$q(\beta_d) = (1 + \beta_d^2)^2,$$

$$\boxed{\begin{cases} \min\limits_{\beta_d} & \dfrac{1}{q(\beta_d)} \displaystyle\sum_{k=1}^{N-1} a_k\widetilde{p}_k(\beta_d) + b_k\bar{p}_k(\beta_d) \\ s.t. & \beta_d \in \mathbb{R} \end{cases}}$$

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Lp-Norm Minimization – Discrete Phase Code Design

$$\zeta_{dk} = \begin{cases} \left| \mathrm{FFT}\ [a_{1k}, a_{3k}, a_{2k}, \mathbf{0}_{1\times(M-3)}]^T \right| \in \mathbb{R}^M & M \geq 3, \\ \left| \mathrm{FFT}\ [a_{1k} + a_{2k}, a_{3k}]^T \right| \in \mathbb{R}^2 & M = 2, \end{cases}$$

$$\boldsymbol{\alpha}_{dk} = \sum_{k=1}^{N-1} \left(a_k \zeta_{dk}^2 + b_k \zeta_{dk} \right) \in \mathbb{R}^M,$$

$$\widetilde{m}^\star = \arg \min_{m=1,\dots,M} \boldsymbol{\alpha}_{dk},$$

$$\phi_d^\star = \frac{2\pi(\widetilde{m}^\star - 1)}{M}, \; x_d^\star = e^{j\phi_d^\star}$$

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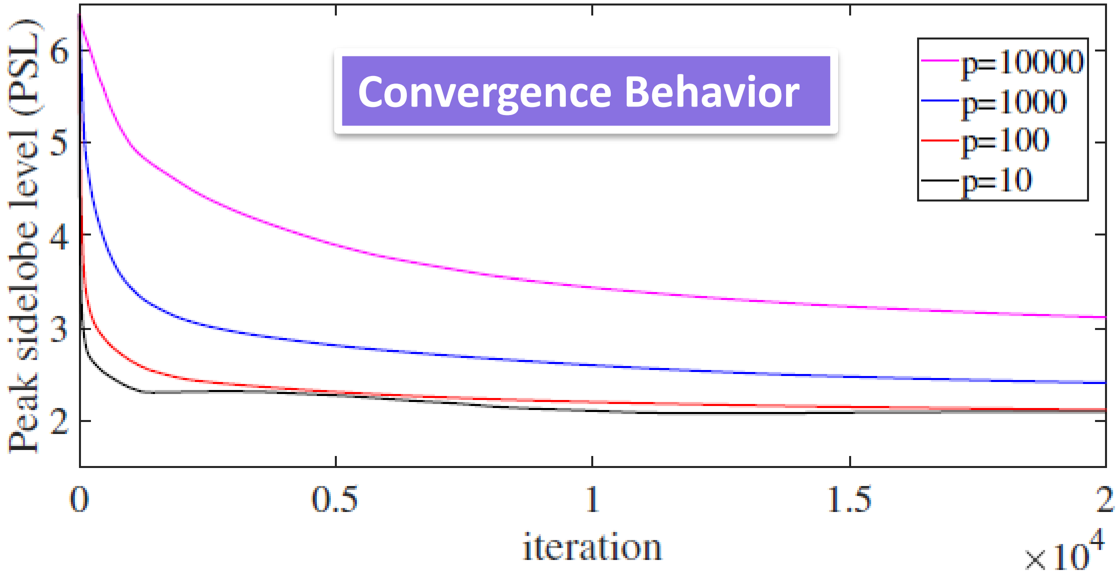
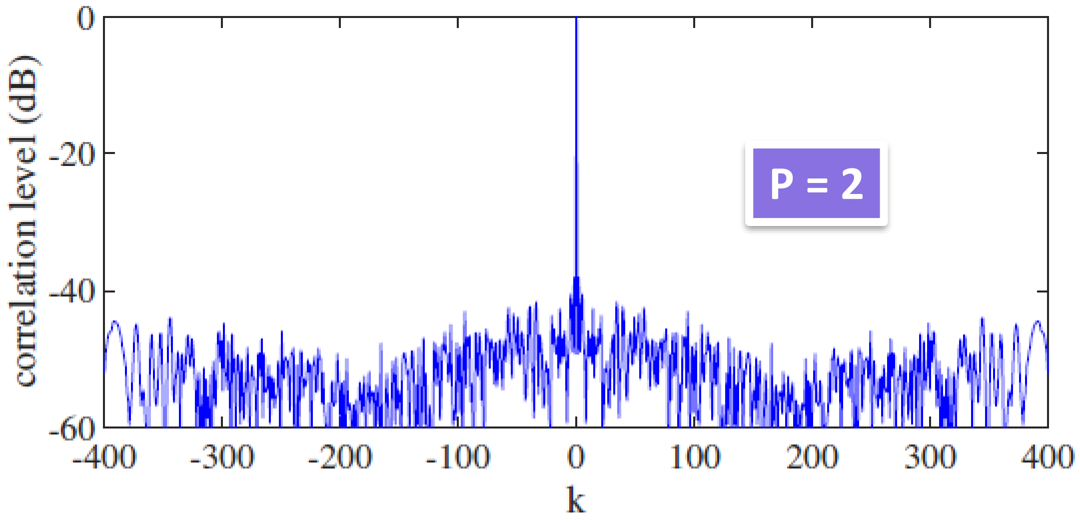
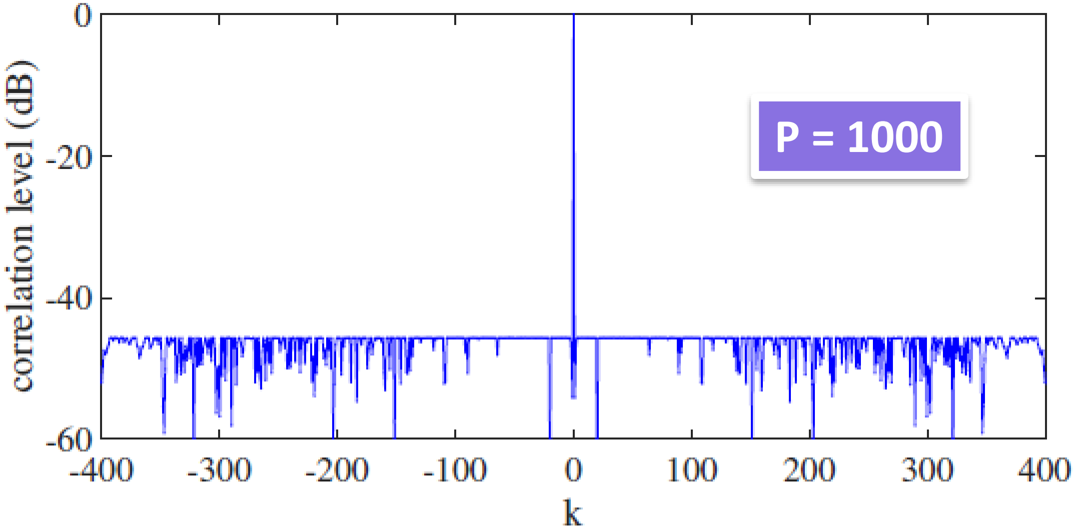
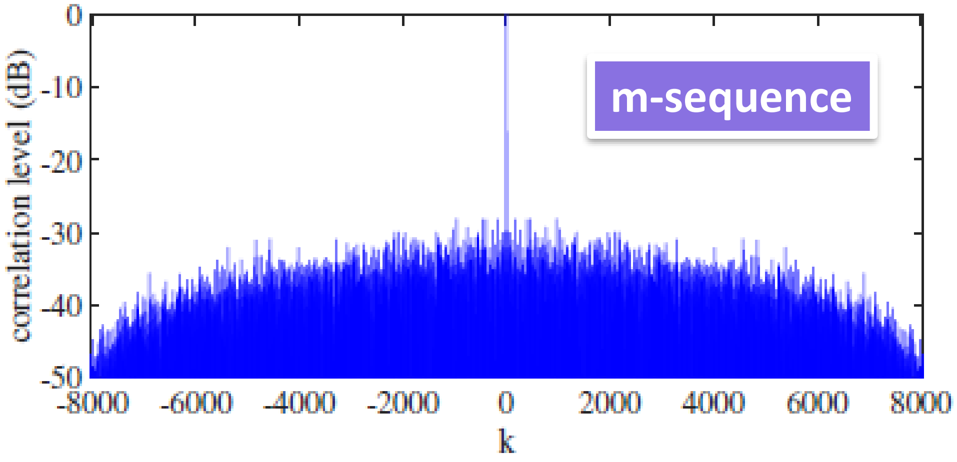
Lp-Norm Minimization – Results

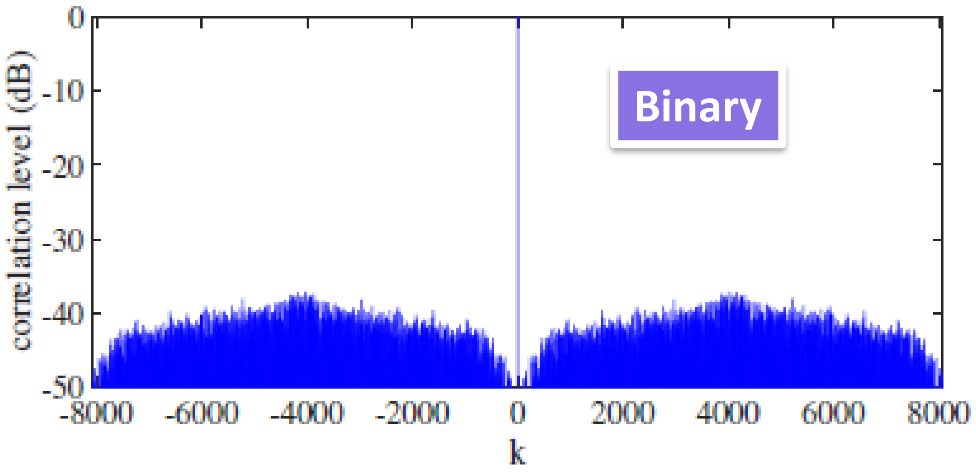


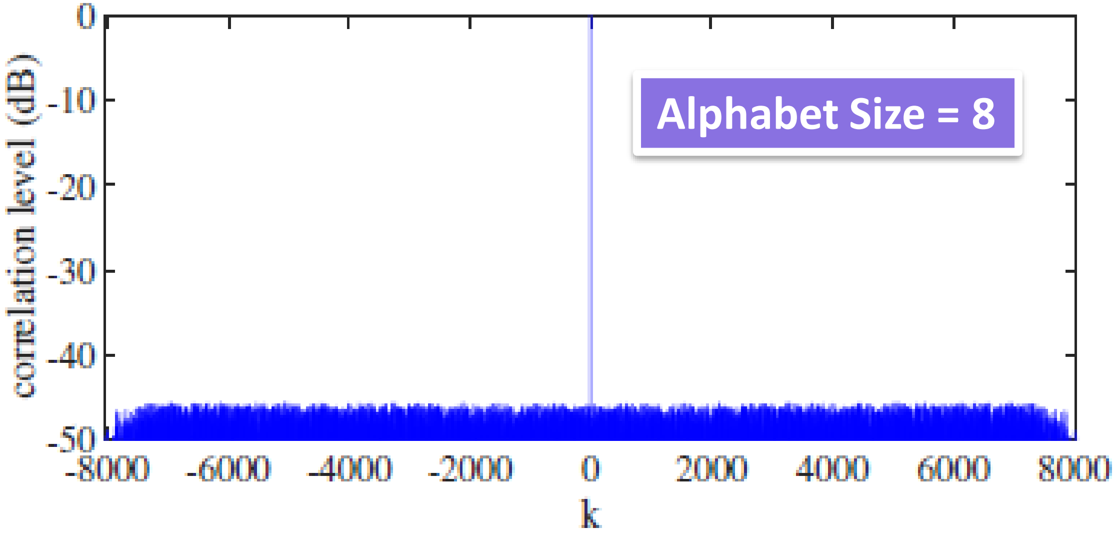




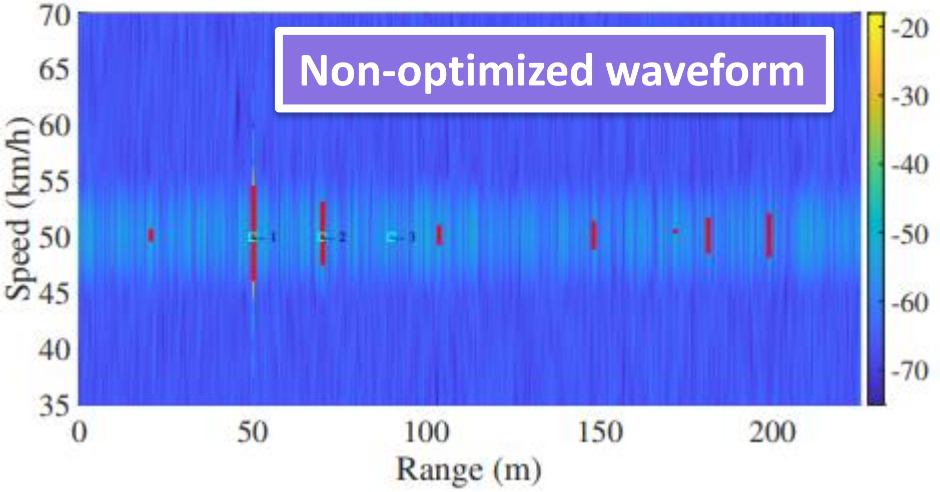
Lp-Norm Minimization – Results

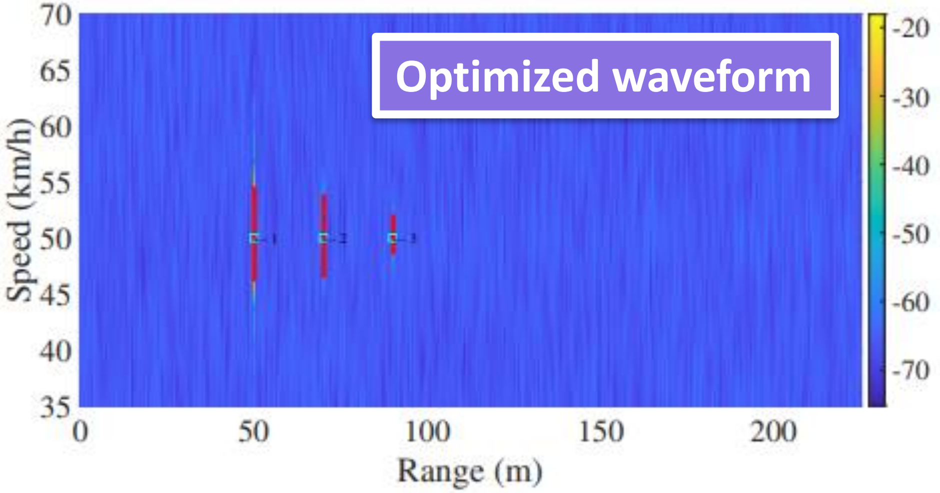






Performance in Practice





System parameters	Value
Operating frequency	79 (GHz)
Transmitting power	12 (dBm)
Antenna gain	10 (dB)
Maximum detection range	225 (m)
Bandwidth	300 (MHz)
Range resolution	0.5 (m)
Receiver noise figure	15 (dB)
Transmission time	27.3 (μs)
Inter-pulse duration	10.7 (μs)
PMCW code length	8191
Number of pulses	256
Doppler FFT size	512
Max umambiguous relative velocity	89 (km/h)
Total active frame time	7.68 (ms)

Conclusion

- Radar **waveform design** is long standing problem, but still there are challenges that needs to a research to be addressed.
- GD, MM, CD, **BSUM**, and ADMM, are iterative approaches that was found to be successful in solving many related problems
- Many research have been done, but the area is still **alive** and the research is ongoing
- Some new problems can be considered in the context of **integrated sensing and communications**

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Thank you

and

Question?

Get in touch for more info

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