Waveform Optimization Techniques for Radar Systems

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Timeline

Part II: CD optimization framework for transceiver design (50 mins)

Part III: MM optimization framework for waveform design (40 mins)

Part V: Summary and open challenges, and Q&A (20 mins)

Q&A

Q&A

Q&A

Part I: A brief review of optimization principles, active sensing scenarios and problem formulation (50 mins)

Part IV: Waveform optimization in mm-Wave sensing and communications (40 mins)

Slot 1

Mohammad Alaee

Slot 2

Bhavani Shankar M. R.

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Slot 1

Part I: A brief review of optimization principles, active sensing scenarios and problem formulation

Radar Pulse Compression, Good Waveforms, and Metrics

Waveform Design and Optimization Problems

Waveform Design Techniques

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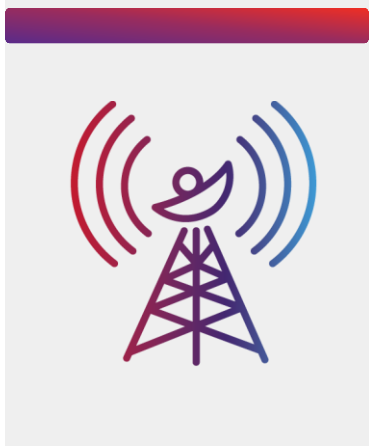
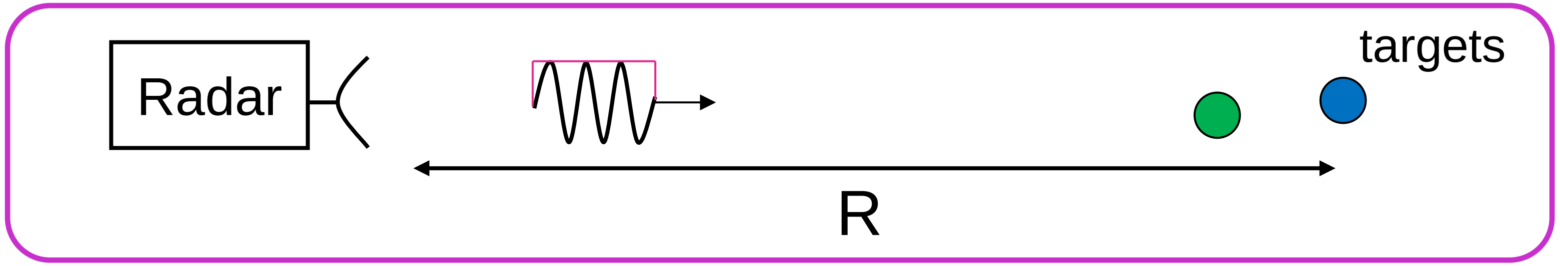
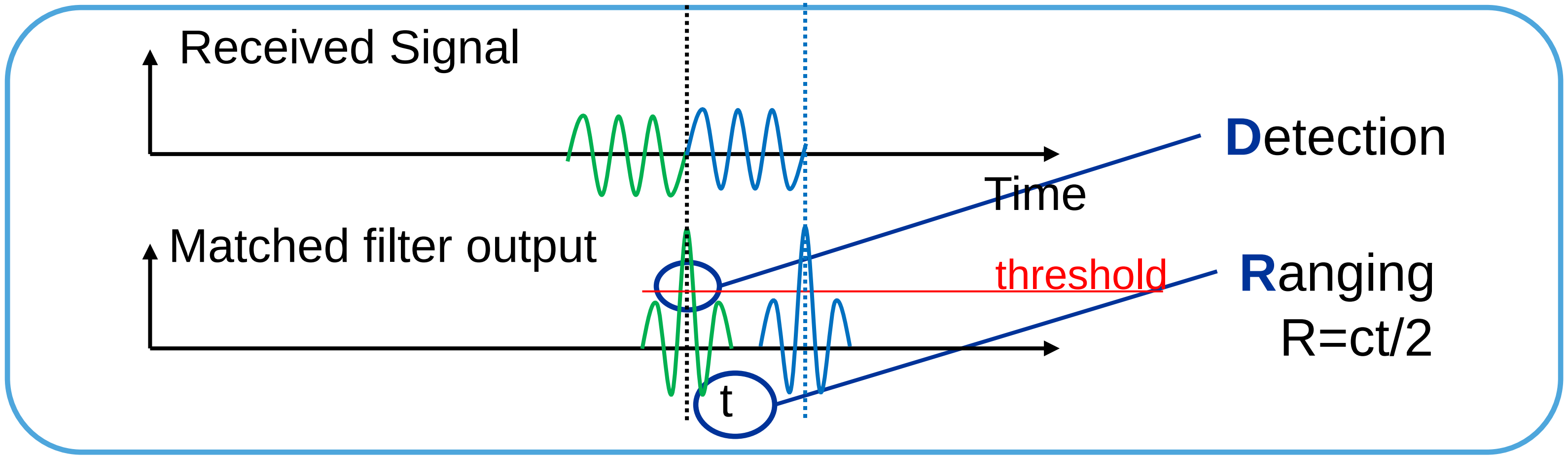
Radar Pulse Compression, Good Waveforms, and Metrics

Radar was an acronym for **Ra**dio **D**etection **a**nd **R**anging.







Matched Filter maximizes the peak-signal to mean noise ratio

Radar Pulse Compression, Good Waveforms, and Metrics

- Short pulses are required to have good range resolution.
- Short pulses = Decreased average power -> Limited receive SNR
- Limited receive SNR = Decreased detection capability.

Requirement

High average power + Good Range resolution

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Radar Pulse Compression, Good Waveforms, and Metrics

- Higher average power is proportional to pulse width
- Better resolution is inversely proportional to pulse width

A long pulse can have the same bandwidth (resolution) as a short pulse if the long pulse be modulated with a "waveform"

energy of a long pulse + resolution of a short pulse

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Radar Pulse Compression, Good Waveforms, and Metrics

What is a waveform?

a waveform is a structured modulation of the pulse, typically in frequency/phase (FM/PM), and sometimes also in amplitude (AM).

Waveform AM also necessitates linearity at the transmitter power amplifier (PA) to prevent waveform distortion

If a waveform has constant amplitude, the PA can be operated in saturation with much less distortion.

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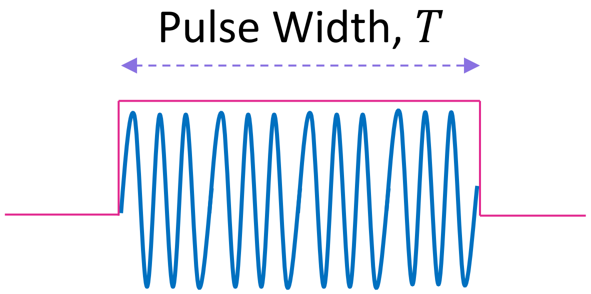
Radar Pulse Compression, Good Waveforms, and Metrics

Increasing the time-bandwidth product

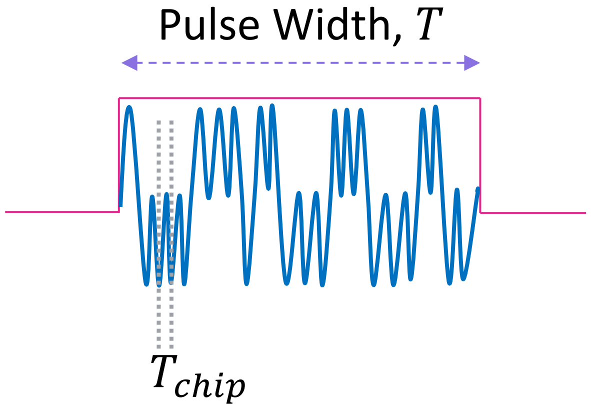
Square Pulse



Bandwidth = $\frac{1}{T}$

Time × Bandwidth = 1

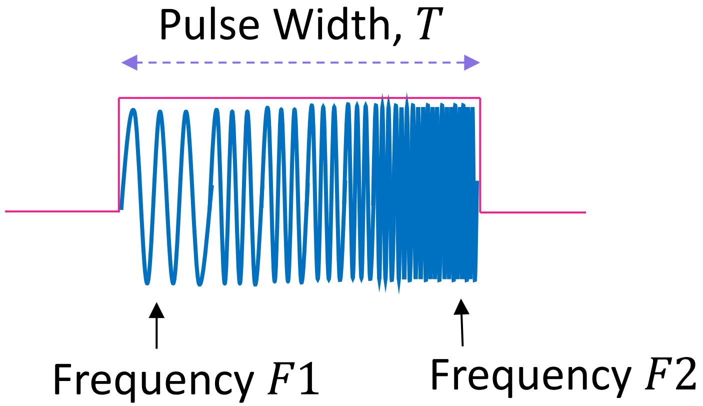
Phase Coded Waveform



Bandwidth = $\frac{1}{T_{chip}}$

Time × Bandwidth = $\frac{T}{T_{chip}}$

Linear Frequency Modulated Waveform

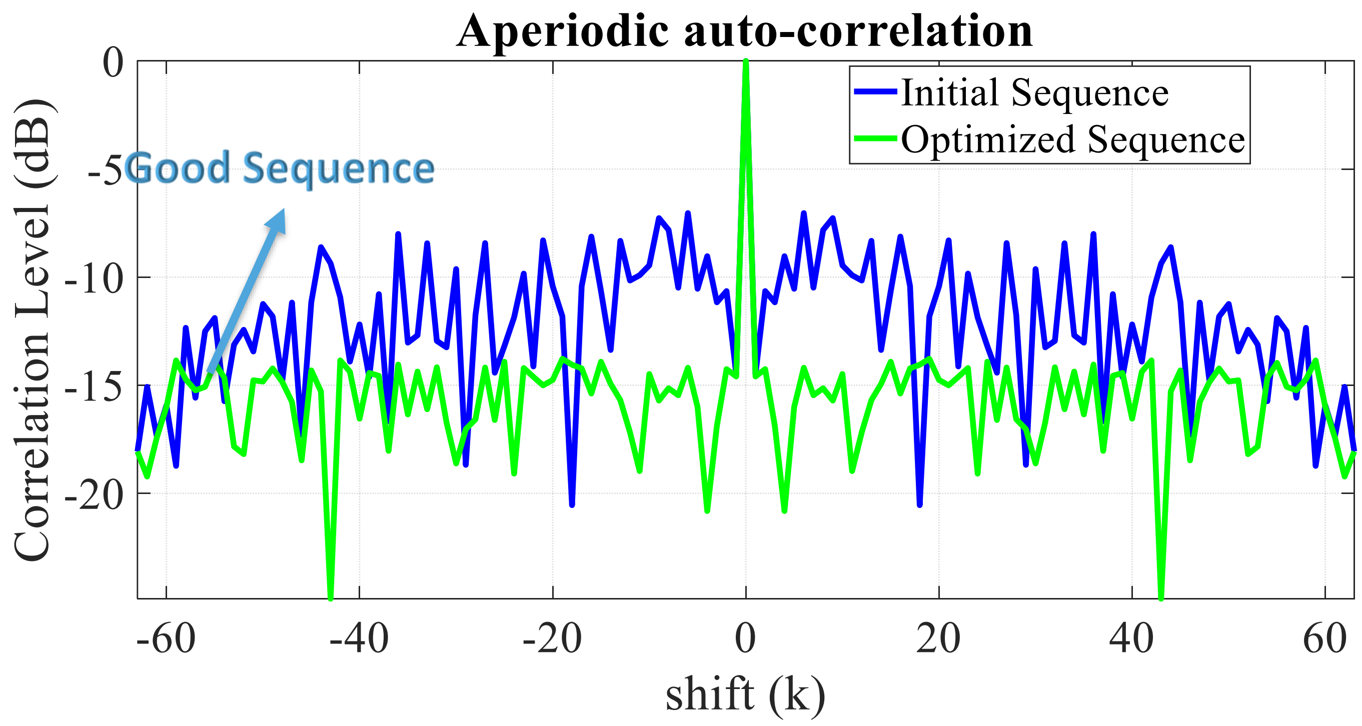


Bandwidth = $\Delta F = F2 - F1$

Time × Bandwidth = $T \times \Delta F$

Radar Pulse Compression, Good Waveforms, and Metrics

The matched filter output is the waveform’s *autocorrelation* that possesses a *mainlobe* (the peak) surrounded by *sidelobes*



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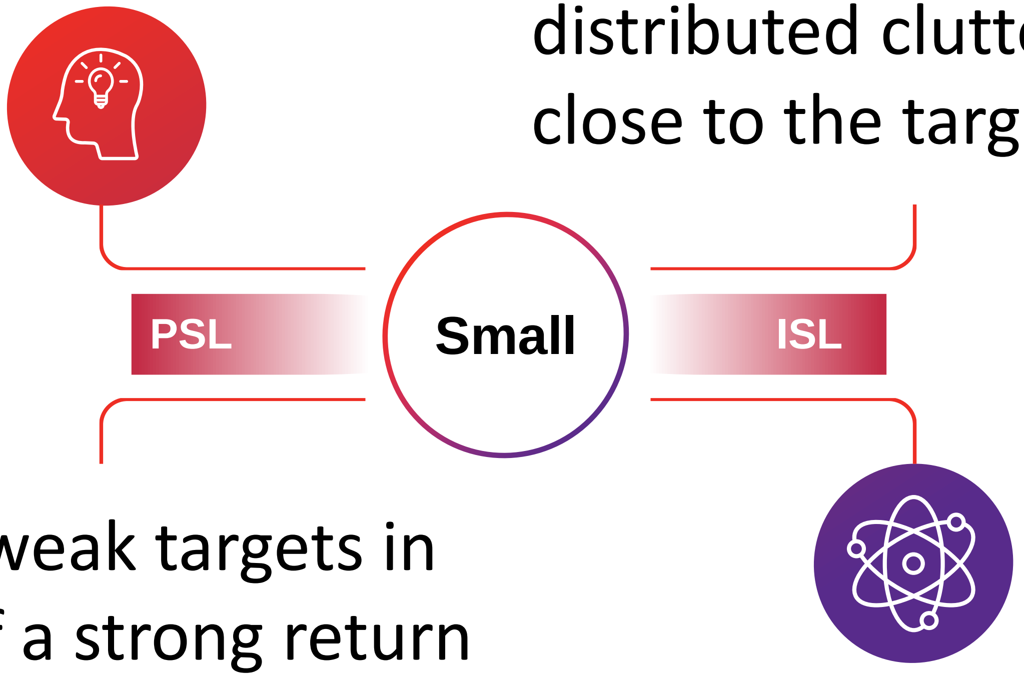
Radar Pulse Compression, Good Waveforms, and Metrics

Peak Sidelobe Level (PSL)

mitigate the deleterious effects of distributed clutter echoes which are close to the target of interest



avoid masking of weak targets in range sidelobes of a strong return

Integrated Sidelobe Level (ISL)

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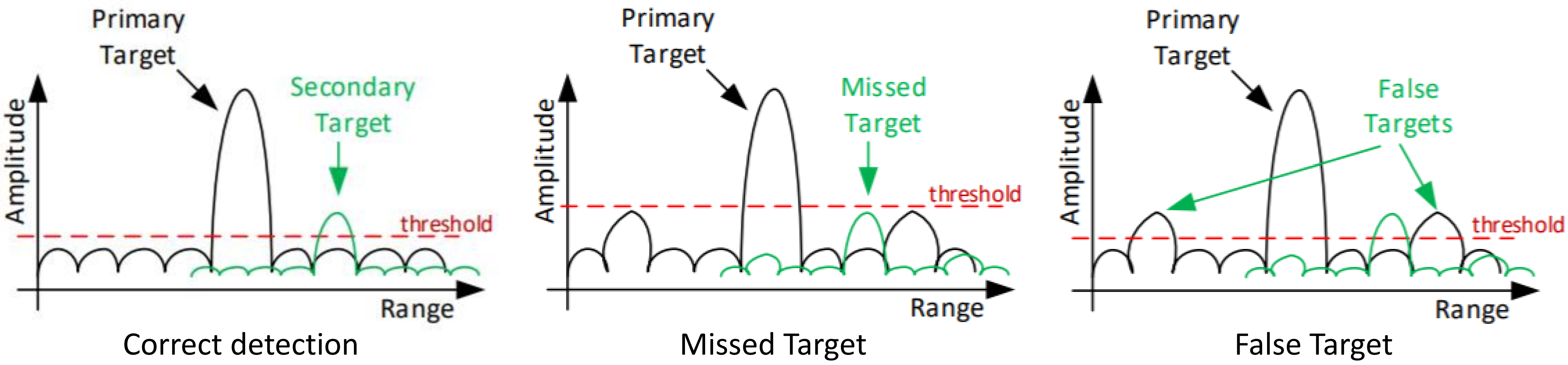
Radar Pulse Compression, Good Waveforms, and Metrics



Sketch of auto-correlation function, displaying the effects of chossing waveforms for detecting weak signals

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Waveform Design and Optimization Problems

$$\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T \qquad r_k = \sum_{n=1}^{N-k} x_n^* x_{n+k} \,. \quad k = 0. \dots. N-1$$

Waveform Design and Optimization Problems

$$\mathrm{PSL} = \max_{k \neq 0} |r_k|$$

$$\mathrm{ISL} = \sum_{k=1}^{N-1} r_k^2$$

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Waveform Design and Optimization Problems

PSL Minimization Problem

$$\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max_{k \neq 0} |r_k| \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

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Waveform Design and Optimization Problems

ISL Minimization Problem

$$\boldsymbol{x} = [x_1, x_2, \dots., x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \sum_{k=1}^{N-1} r_k^2 \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

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Waveform Design and Optimization Problems

- Constraints
  - Energy
  - Peak-to-Average Power Ratio (PAPR, PAR)
  - Unimodularity (being Constant-Modulus)
  - Finite or Discrete-Alphabet (integer, binary, m-ary constellation)
  - …
- Challenges
  - How to handles signal constraints?
  - How to do it fast?

- Many of these problems are shown to be NP-hard
- Many others are deemed to be difficult

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Waveform Design and Optimization Problems

- Example: Waveform design with good correlation properties in MIMO radar systems
  - Transmitters should be observable at each receiver
  - Enabled by Orthogonal Waveforms

Multi Input Single Output

Multi Input Multi Output

Waveform Design and Optimization Problems

- Orthogonal Waveforms in MIMO radar systems
  - Limit mutual interference
  - Enable cooperative operation
  - Provide visibility into paths between transmitter and receivers
  - Determines spatial distribution of energy

- Orthogonality achieved by division in time, frequency or code
  - FDM-, TDM-, DDM-, and CDM-MIMO

Waveform Design and Optimization Problems

- Example: Waveform design with good correlation properties in MIMO radar systems

$$\boldsymbol{x}_m = [x_m(1), x_m(2), \ldots, x_m(N)]^T \in \mathbb{C}^N,$$

$$\boldsymbol{X} = [\boldsymbol{x}_1, \quad \boldsymbol{x}_2, \quad \ldots, \boldsymbol{x}_{N_T}] \in \mathbb{C}^{N \times N_T}$$

$$r_{ml}(k) = \sum_{n=1}^{N-k} x_m(n)\, x_l^*(n+k) = r_{lm}^*(-k)$$

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Waveform Design and Optimization Problems

- Example: Waveform design with good correlation properties in MIMO radar systems

$$\mathrm{PSL} = \max\left\{\max_{m}\max_{k\neq 0}|r_{mm}(k)|\ ,\max_{\substack{m,l\\ m\neq l}}\max_{k}|r_{ml}(k)|\right\}$$

$$\mathrm{ISL} = \sum_{m=1}^{N_T}\sum_{\substack{k=-N+1\\ k\neq 0}}^{N-1}|r_{mm}(k)|^2 + \sum_{\substack{m,l=1\\ m\neq l}}^{N_T}\sum_{k=-N+1}^{N-1}|r_{ml}(k)|^2$$

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Waveform Design and Optimization Problems

PSL Minimization Problem in MIMO radar

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max\left\{\max_{m}\max_{k\neq 0}|r_{mm}(k)|\;,\max_{\substack{m,l\\ m\neq l}}\max_{k}|r_{ml}(k)|\right\} \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

ISL Minimization Problem in MIMO radar

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \sum_{m=1}^{N_T}\sum_{\substack{k=-N+1\\ k\neq 0}}^{N-1}|r_{mm}(k)|^2 + \sum_{\substack{m,l=1\\ m\neq l}}^{N_T}\sum_{k=-N+1}^{N-1}|r_{ml}(k)|^2 \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

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Waveform Design and Optimization Problems

- Waveform design related optimization problems
  - Beampattern shaping
  - Spectral shaping
  - Coexistence MIMO radar MIMO communications (MRMC)
  - Joint radar and communications (JRC)
  - …

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Question?

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Waveform Design Techniques

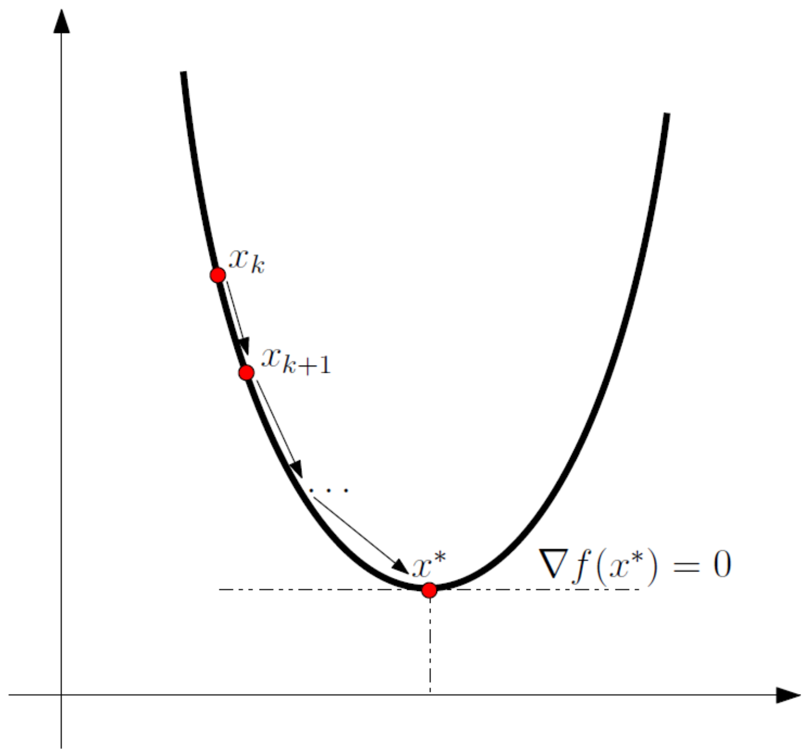
- Gradient-Descent Based Methods (GD)
- Majorization-Minimization (MM)
- Coordinate Descent (CD)
- Alternating Direction Method of Multipliers (ADMM)
- Block Successive Upper-bound Minimization (BSUM)
- Several others ...

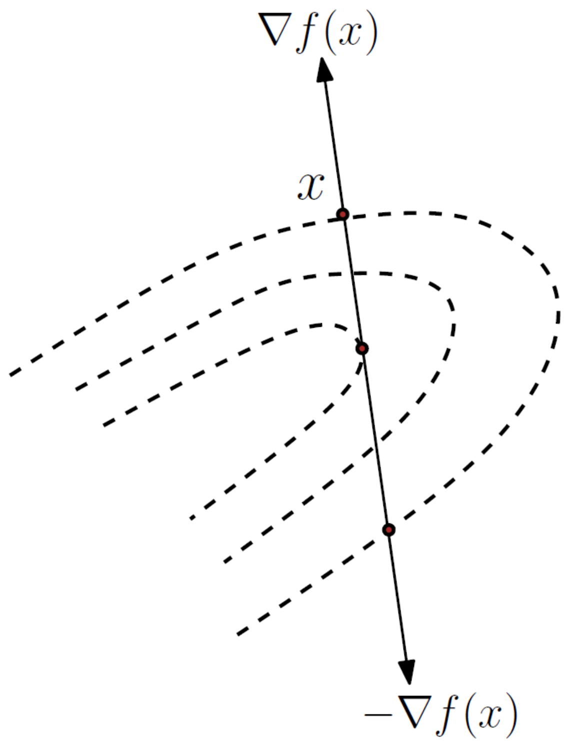
Waveform Design Techniques

Waveform Design Techniques

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Gradient-Based Methods





Waveform Design Techniques

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Gradient-Based Methods

1. Start with some guess x^0;
2. For each $k = 0, 1, \ldots$
   - $x^{k+1} \leftarrow x^k + \alpha_k d^k$
   - Check when to stop (e.g., if $\nabla f(x^{k+1}) = 0$)

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Waveform Design Techniques

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Gradient-Based Methods

$$\boxed{x^{k+1} = x^k + \alpha_k d^k, \quad k = 0, 1, \ldots}$$

- **stepsize** $\alpha_k \geq 0$, usually ensures $f(x^{k+1}) < f(x^k)$
- **Descent direction** d^k satisfies

$$\langle \nabla f(x^k), d^k \rangle < 0$$

$$\boxed{\text{Numerous ways to select } \alpha_k \text{ and } d^k}$$

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Waveform Design Techniques

$$\text{minimize} \left|\left|\boldsymbol{Ax} - \boldsymbol{b}\right|\right|^2$$

Gradient-Based Methods; Example

1) Least Squares Solution

$$\boldsymbol{x} = \boldsymbol{A}^H\left(\boldsymbol{A}\boldsymbol{A}^H\right)^{-\mathbf{1}}\boldsymbol{b}$$

2) CVX

```
cvx_begin
    variable x_cvx(n)
    minimize( norm( A * x_cvx - b, 2 ) )
cvx_end
```

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Waveform Design Techniques

$$\text{minimize} \; \left|\left|\boldsymbol{Ax} - \boldsymbol{b}\right|\right|^2$$

Gradient-Based Methods; Example

3) Gradient Descent

$$\boxed{x^{k+1} = x^k + \alpha_k d^k, \quad k = 0, 1, \ldots}$$

```
for k = 1 : maxIter
    dk = A' * (A * x_grad - b);
    alpha_k = -0.05 / k^0.5;
    x_grad = x_grad + alpha_k * dk;
end
```


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Waveform Design Techniques

$$\text{minimize} \left|\left|\boldsymbol{Ax} - \boldsymbol{b}\right|\right|^2$$

Gradient-Based Methods; Example

3) Gradient Descent

$$\boxed{x^{k+1} = x^k + \alpha_k d^k, \quad k = 0, 1, \ldots}$$

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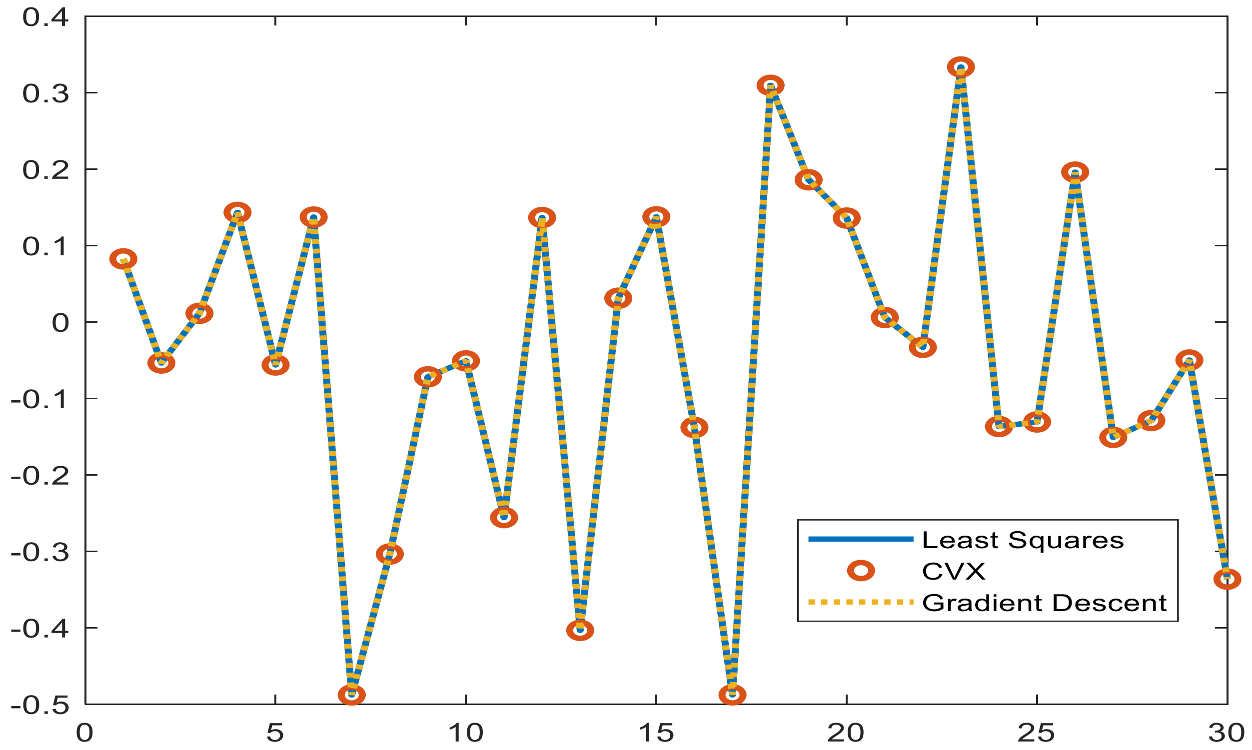
end
```

Waveform Design Techniques

$$\text{minimize } ||\boldsymbol{Ax} - \boldsymbol{b}||^2$$

Gradient-Based Methods; Example



Waveform Design Techniques

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Majorization-Minimization (MM)

An MM algorithm operates by creating a surrogate function that minorizes or majorizes the objective function. When the surrogate function is optimized, the objective function is driven uphill or downhill as needed.

Will be discussed more in **Slot 2**

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Waveform Design Techniques

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Majorization-Minimization (MM); Example

Minimization of $\cos(x)$

Second order Taylor expansion

$$\cos(x) = \cos(x_n) - \sin(x_n)\,(x - x_n) - \frac{1}{2}\cos(z)\,(x - x_n)^2$$

Holds for some z between x and x_n

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Waveform Design Techniques

Since $|\cos(z)| \leq 1$,

$$g(x|x_n) = \cos(x_n) - \sin(x_n)\,(x - x_n) + \frac{1}{2}(x - x_n)\text{^}2$$

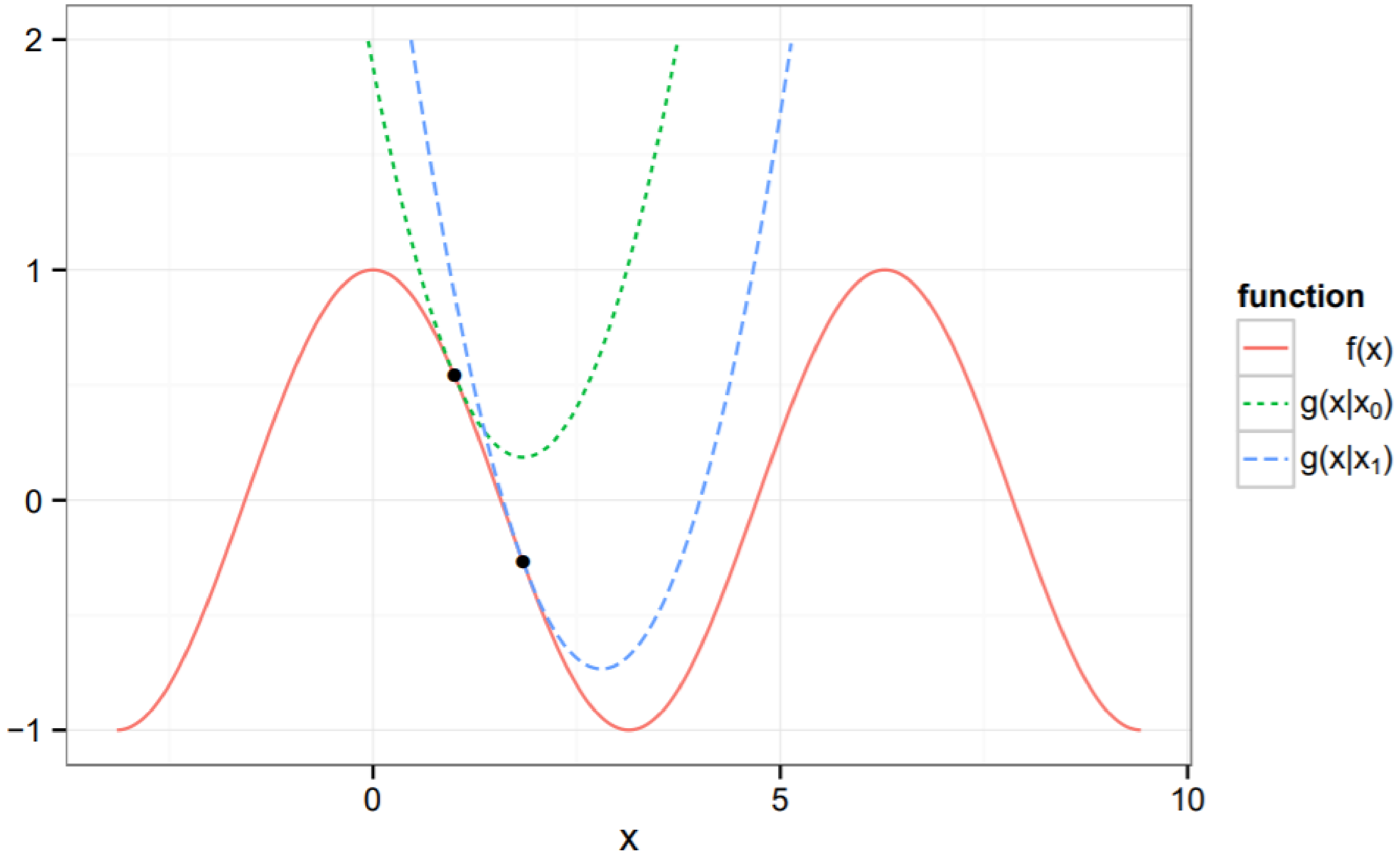
Can be selected as majorizer that majorizes $f(x)$

Solving $\frac{d}{dx} g(x|x_n) = 0$ gives the MM algorithm

$$x_{n+1} = x_n + \sin(x_n)$$

Waveform Design Techniques

Minimum of cos(x)



Waveform Design Techniques

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x})$$

Coordinate Descent (CD)

Minimization of a multivariable function can be achieved by minimizing it along one direction at a time, i.e., solving univariate (or at least much simpler) optimization problems in a loop

Question?

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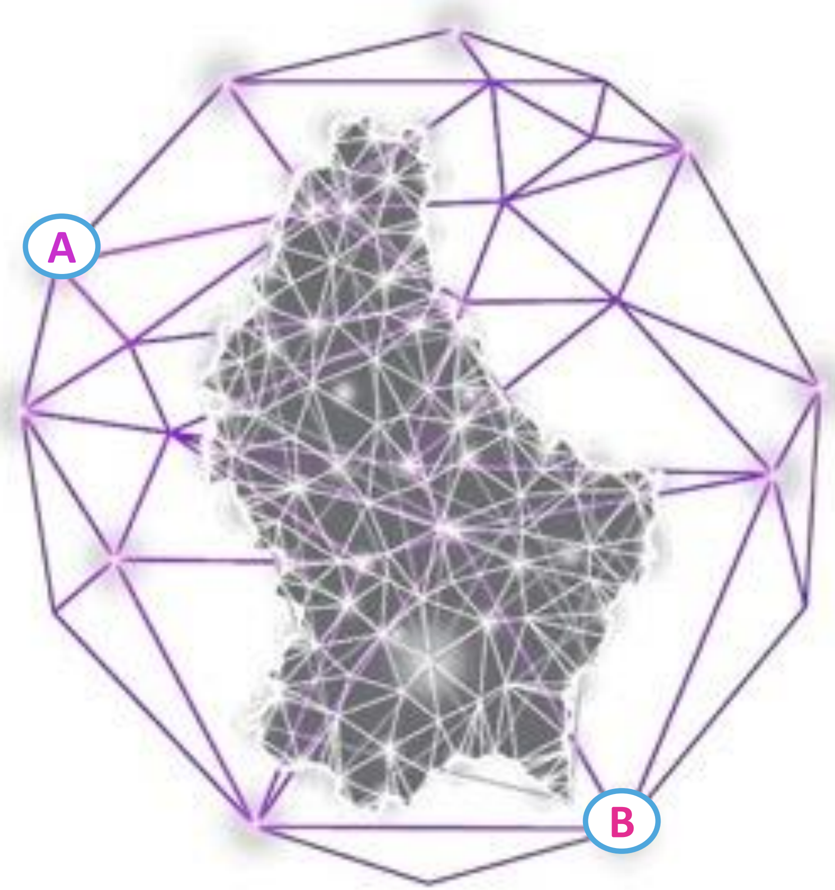
Slot 1

Part II: Coordinate Descent (CD) Optimization Framework for Transceiver Design

Coordinate Descent (Ascent) Methods



Coordinate Descent

- Successively minimizes along coordinate directions
  - Optimize each parameter separately, holding all the others fixed.
- Why is it used?
  - ✓ Very simple and easy to implement
  - ✓ Careful implementations can attain state-of-the-art
  - ✓ Scalable, don't need to keep data in memory, low memory requirements
  - ✓ Faster than gradient descent if iterations are N times cheaper

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Coordinate Descent

$$\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & f(\boldsymbol{x}) \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

idea: optimize over individual coordinates

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Coordinate Descent – Steps

$$x_1^{(k)} \in \arg\min_{x_1} f\left(x_1, x_2^{(k-1)}, x_3^{(k-1)}, \dots, x_N^{(k-1)}\right)$$

$$x_2^{(k)} \in \arg\min_{x_2} f\left(x_1^{(k)}, x_2, x_3^{(k-1)}, \dots, x_N^{(k-1)}\right)$$

$$x_3^{(k)} \in \arg\min_{x_3} f\left(x_1^{(k)}, x_2^{(k)}, x_3, \dots, x_N^{(k-1)}\right)$$

$$\vdots$$

$$x_N^{(k)} \in \arg\min_{x_N} f\left(x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \dots, x_N\right)$$

Note:

1- After we solve for $x_i^{(k)}$, we use its new value from then on

2- Can everywhere replace individual coordinates with blocks of coordinates (Block Coordinate Descent)

Coordinate Descent – Algorithm

- ❑ Start from initial guess $\boldsymbol{x}^{(0)} = [x_1, x_2, \dots, x_N]^T$

- ❑ For $k = 0, 1, \dots$

  - Pick an index i from $\{1, \dots, N\}$

  - Optimize the i-th coordinate

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f(\underbrace{x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}}_{\text{done}}, \underbrace{\zeta}_{\text{current}}, \underbrace{x_{i+1}^{(k)}, \dots, x_N^{(k)}}_{\text{To do}})$$

- ❑ Decide when/how to stop; return $\boldsymbol{x}^{(k+1)}$

Gauss-Seidel and Jacobi

Gauss-Seidel style (One-at-a-time)

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$$

Jacobi style (all-at-once ; easy to parallelize)

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f(x_1^{(k)}, \dots, x_{i-1}^{(k)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$$

Which Coordinate? (One-at-a-time)

- **Greedy or Gauss-Southwell (Maximum Block Improvement)**
  - If f is differentiable, at iteration k, pick the index that minimizes $\nabla f(x_i^k)$
- **Derivative free rules**
  - **Cyclic** order $1, 2, \ldots, N, 1, \ldots$
  - **Double sweep**, $1, 2, \ldots, N$, then $N - 1, \ldots, 1$, repeat
  - **Cyclic with permutation**, random order each cycle
  - **Random sampling**, pick random index at each iteration

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Advantages

- Each iteration is usually cheap (single variable optimization)
- No extra storage vectors needed
- No stepsize tuning
- No other parameters that must be tuned
- In general, “derivative free”
- Simple to implement
- Works well for large-scale problems
- Currently quite popular; parallel version exist

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Disadvantages

- Each sub-problem needs to be easily solvable. Tricky if single variable optimization is hard
- Can be “slow” if sub-problems cannot be solved efficiently
- Convergence theory can be complicated
  - “One-at-a-time” update scheme is critical, and “all-at-once” scheme does not necessarily converge
- Non-differentiable cases are more tricky

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Convergence (One-at-a-time)

- The objective function values are non-decreasing, i. e.,

$$f\left(\boldsymbol{x}^{(0)}\right) \geq f\left(\boldsymbol{x}^{(1)}\right) \geq \ \dots.$$

- If f is strictly convex and smooth, the algorithm converges to a global minimum (optimal solution).
- If f is strictly convex -> unique minimum -> local minimum = global minimum
  a. continuously differentiable over the feasible set,
  b. has separable constraints,
  c. has unique minimizer at each step,

  then CD method will converge to stationary points

[1] - Dimitri P, et al.. Nonlinear programming. Athena Scientific; 1999.

Other Alternating methods - Alternating Minimization

2 blocks is called alternative optimization

$$\boldsymbol{x} = [x_1, x_2\,]^T$$

$$x \quad w$$

$$\mathcal{P}_{x,w} \begin{cases} \underset{x,w}{\text{minimize}} & f(x, w) \\ \text{subject to} & x \in \psi_1\,, w \in \psi_2 \end{cases}$$

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Other Alternating methods - BSUM

Block successive upper-bound minimization (BSUM)

$$\boldsymbol{x} = [x_1, x_2, \ldots, x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & f(\boldsymbol{x}) \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} u_i\left(x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \ldots, x_N^{(k)}\right)$$

Local approximation of the objective function

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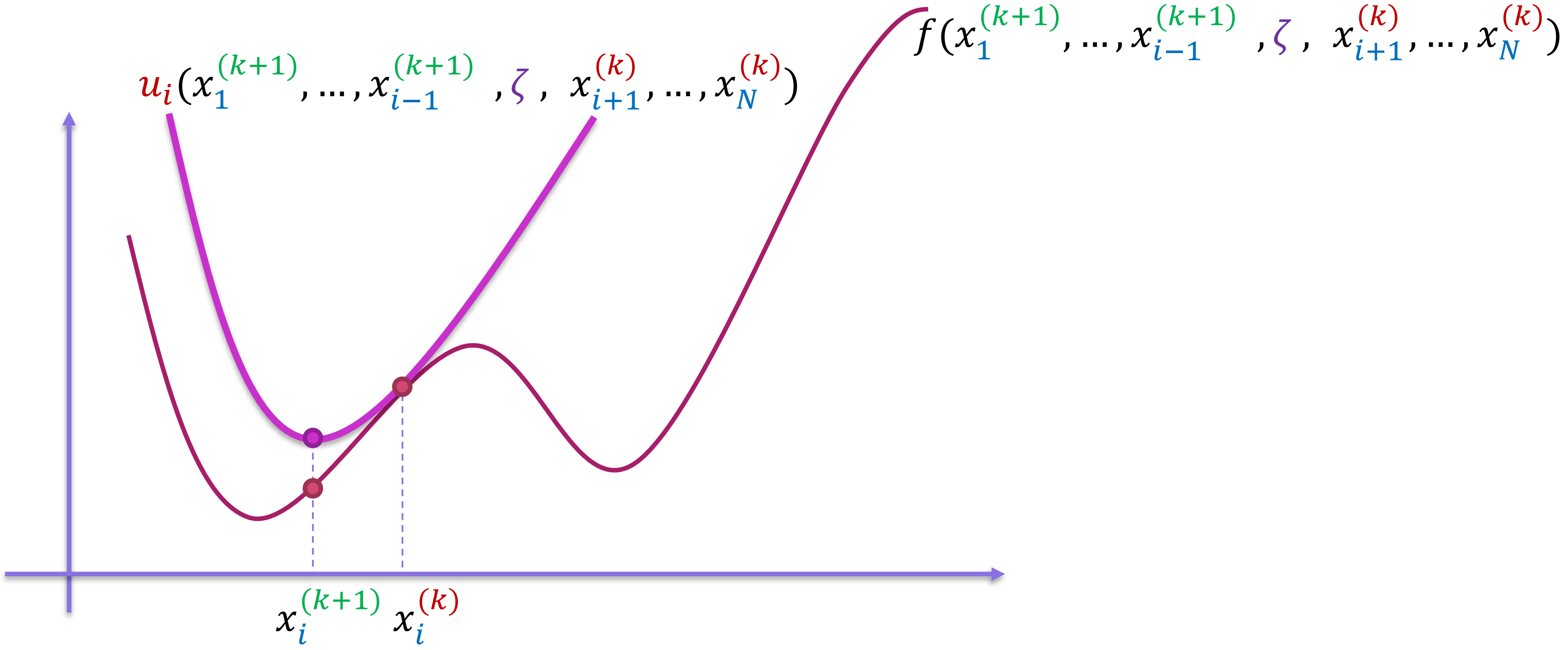
Other Alternating methods - BSUM



Upper-bound $u_i\left(x_1^{(k+1)},\dots,x_{i-1}^{(k+1)},\zeta,\ x_{i+1}^{(k)},\dots,x_N^{(k)}\right) \geq f(x_1^{(k+1)},\dots,x_{i-1}^{(k+1)},\zeta,\ x_{i+1}^{(k)},\dots,x_N^{(k)})$

Block successive upper-bound minimization, block successive convex approximation, convex-concave procedure, majorization-minimization, dc-programming, BCGD,…

Other Alternating methods

- Alternating direction method of multipliers (ADMM)

$$\begin{cases} \underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} & f(\boldsymbol{x}) + g(\boldsymbol{z}) \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} = \boldsymbol{c} \end{cases}$$

$$L_\rho(\boldsymbol{x},\boldsymbol{z},\boldsymbol{y}) = f(\boldsymbol{x}) + g(\boldsymbol{z}) + y^T(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} - \boldsymbol{c}) + \left(\frac{\rho}{2}\right)\|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} - \boldsymbol{x}\|_2^2$$

$$\boldsymbol{x}^{(k+1)} \leftarrow \arg\min_{\boldsymbol{x}} L_\rho(\boldsymbol{x}, \boldsymbol{z}^{(k)}, \boldsymbol{y}^{(k)})$$

$$\boldsymbol{z}^{(k+1)} \leftarrow \arg\min_{\boldsymbol{z}} L_\rho(\boldsymbol{x}^{(k+1)}, \boldsymbol{z}, \boldsymbol{y}^{(k)})$$

$$\boldsymbol{y}^{(k+1)} \leftarrow \boldsymbol{y}^{(k)} + \rho(\boldsymbol{A}\boldsymbol{x}^{(k+1)} + \boldsymbol{B}\boldsymbol{z}^{(k+1)} - \boldsymbol{c})$$

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Question?

Waveform Design with Good Correlation Properties in Radar Systems

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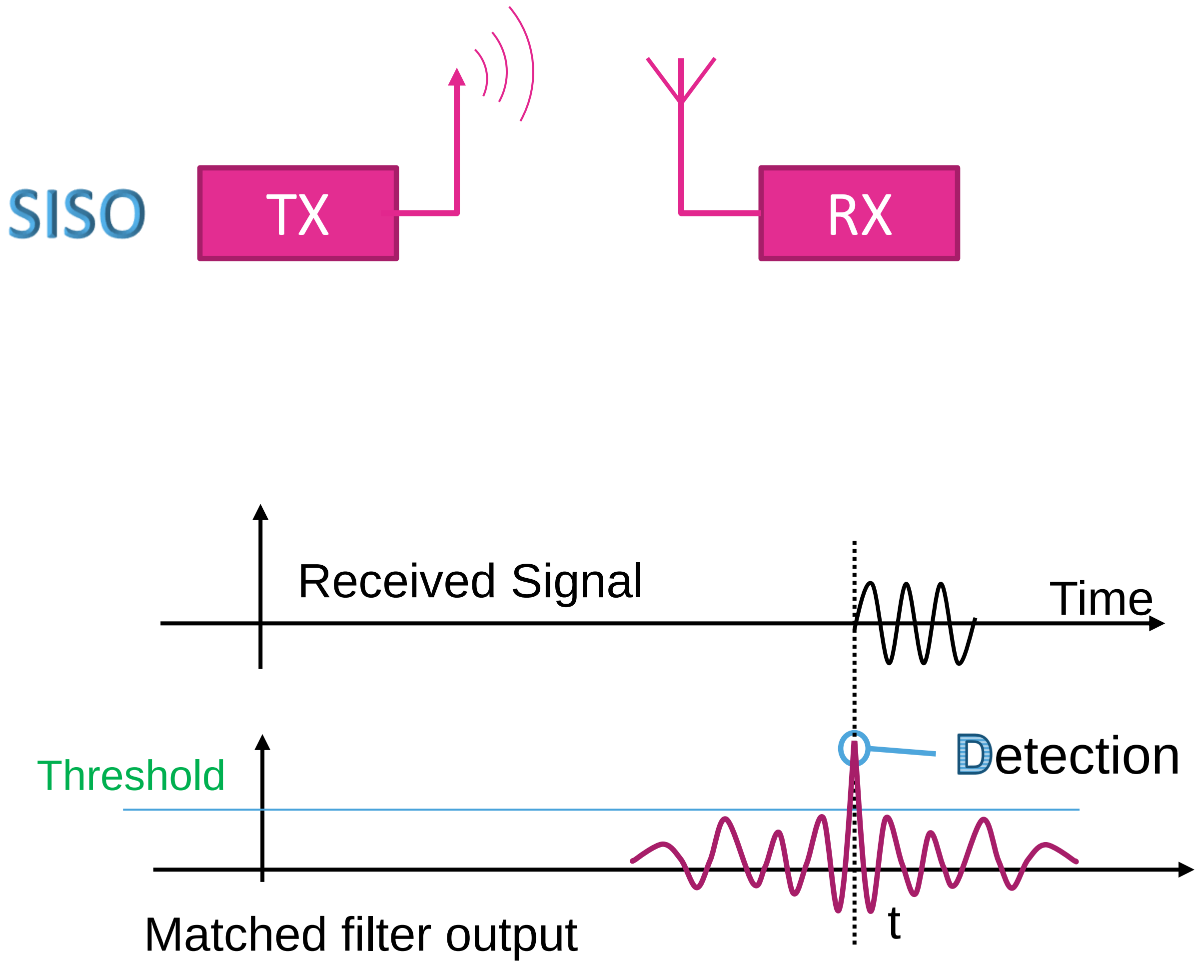
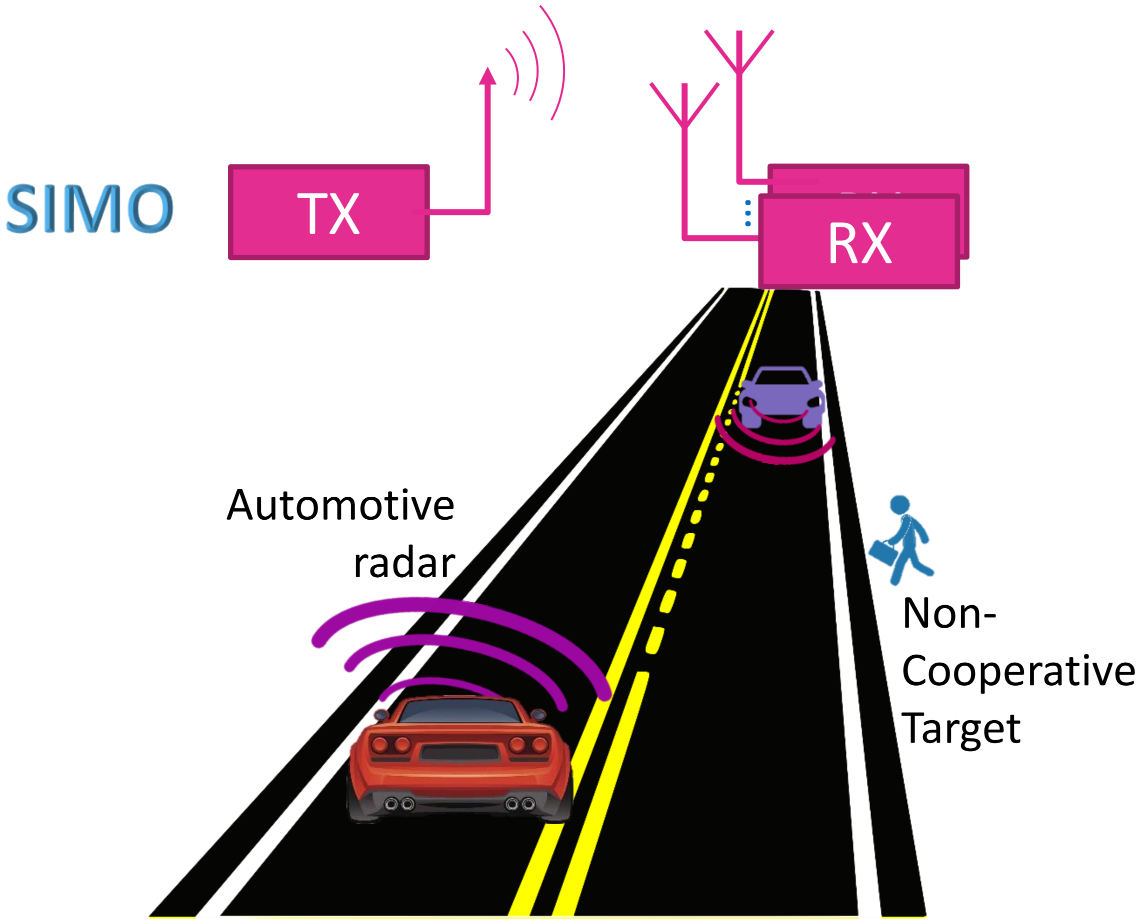
Waveforms in SISO/SIMO Radar Systems

Single Input Single Output

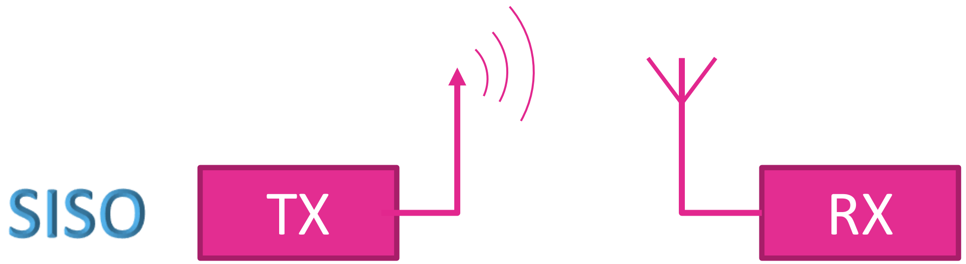


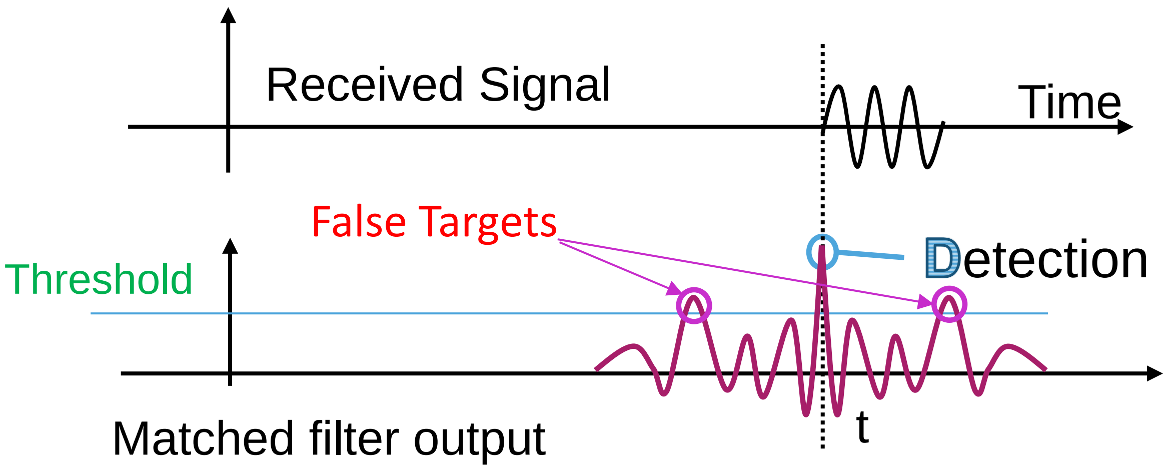
Single Input Multi Output



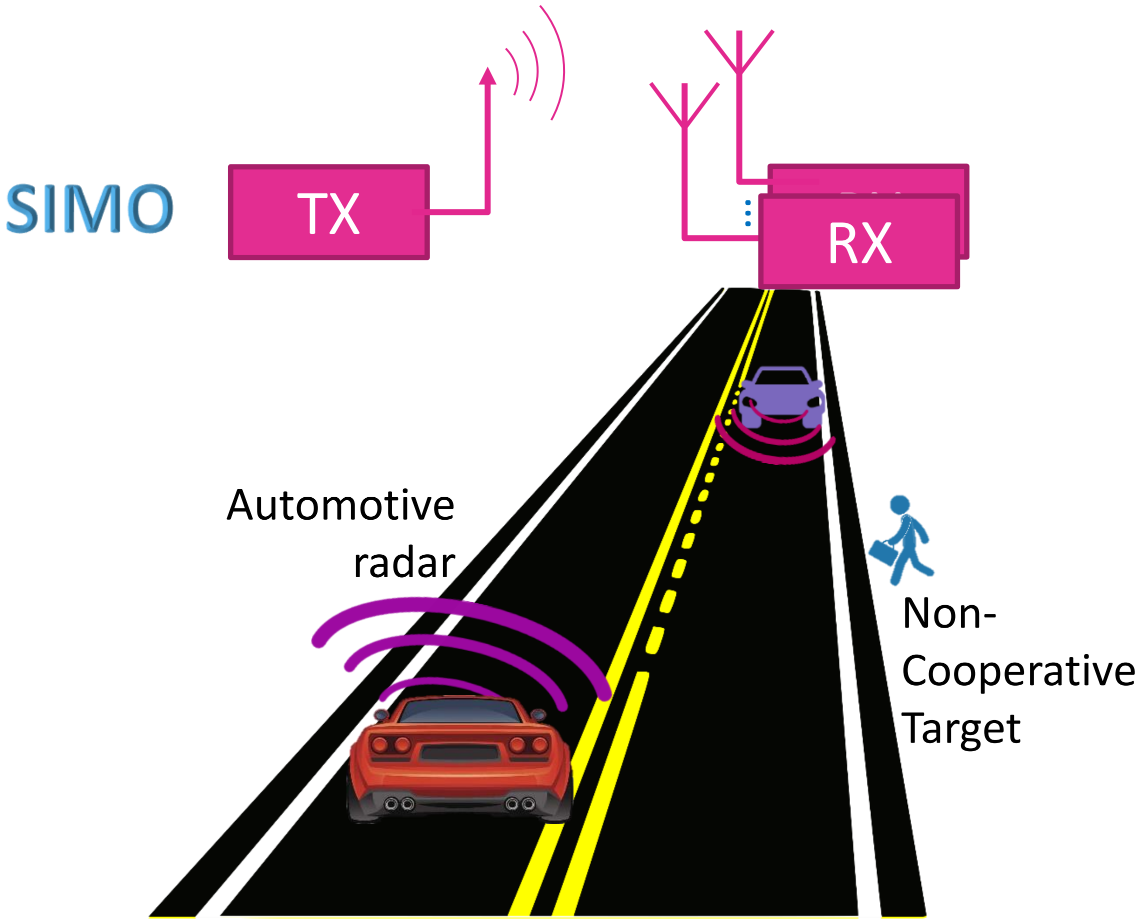
Waveforms in SISO/SIMO Radar Systems

Single Input Single Output

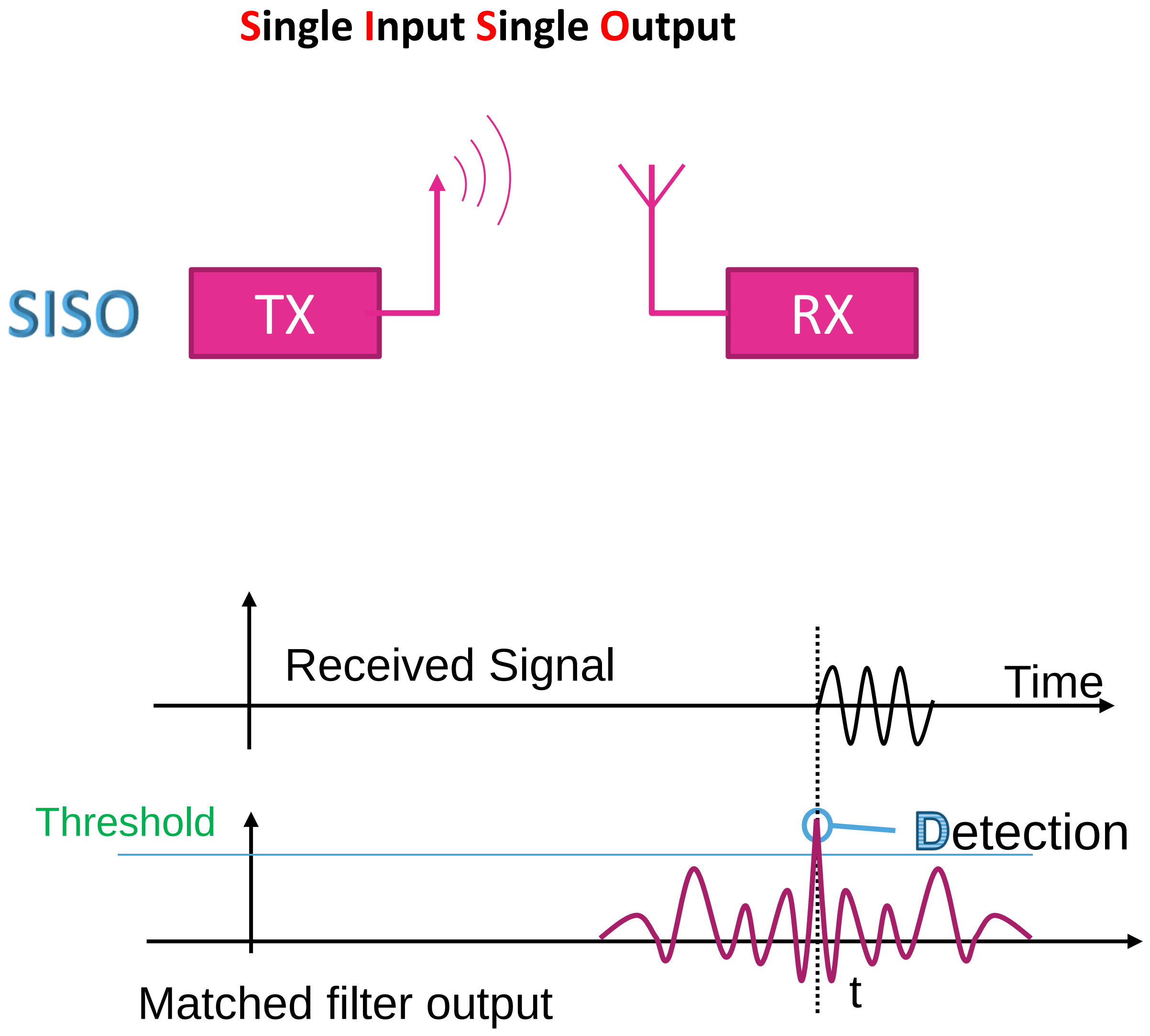


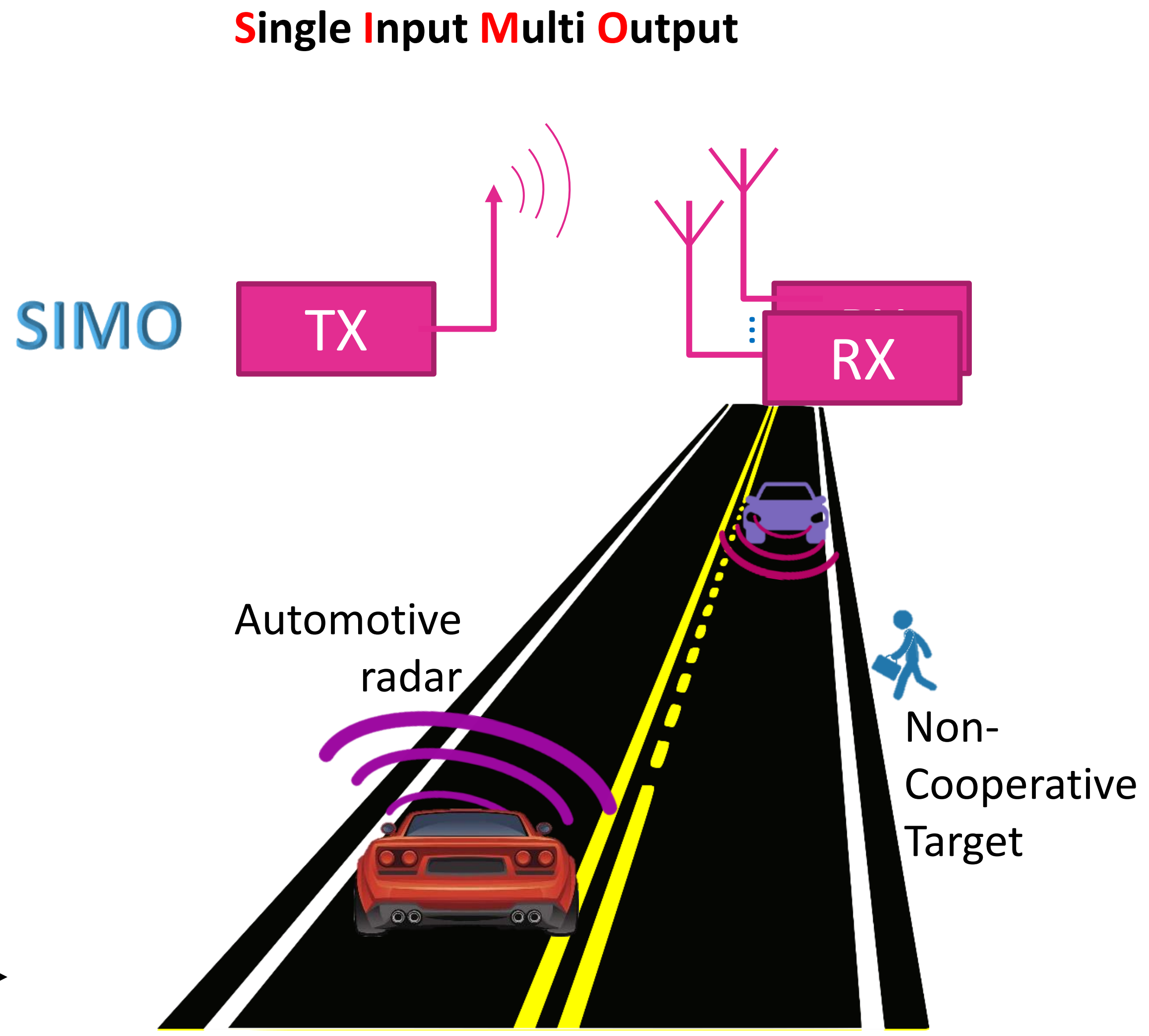


Single Input Multi Output



Waveforms in SISO/SIMO Radar Systems





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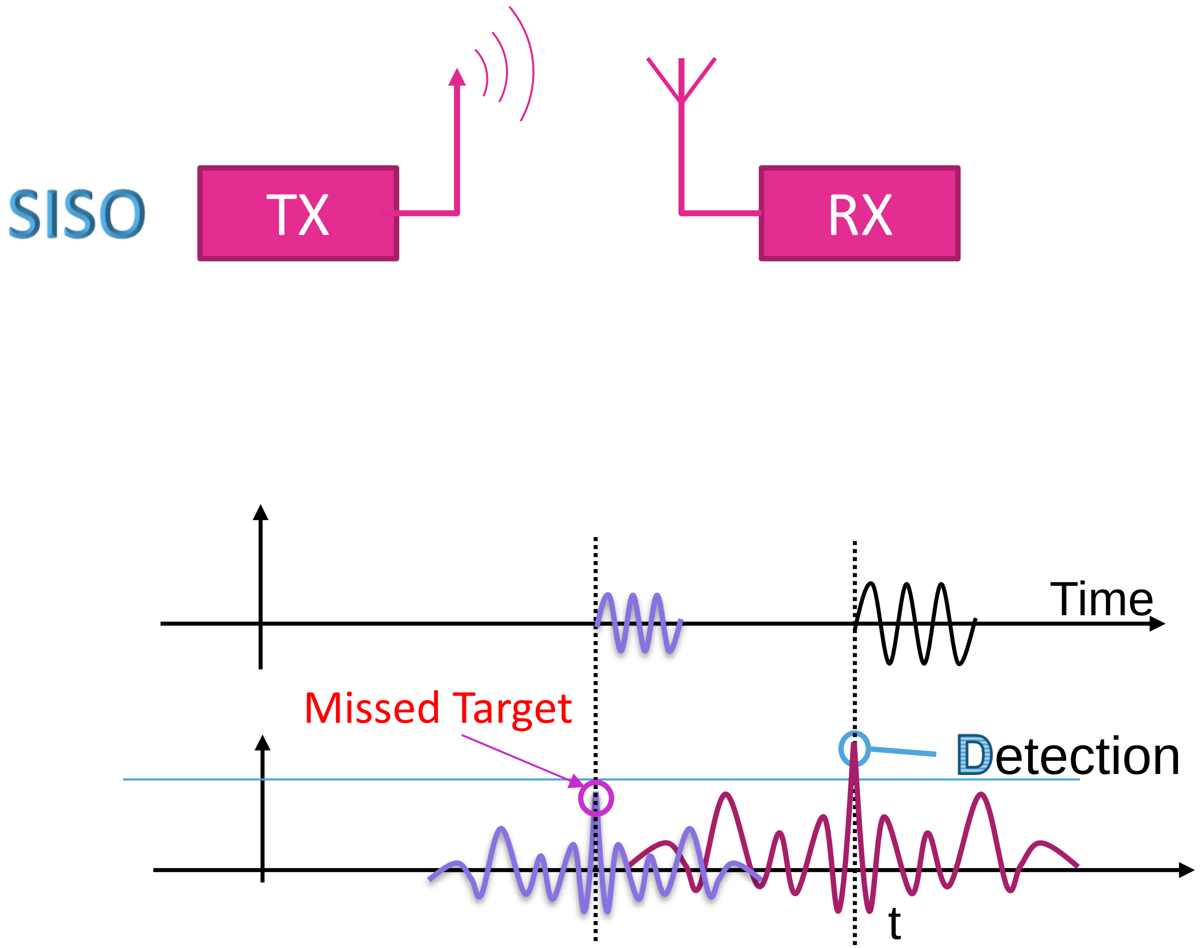
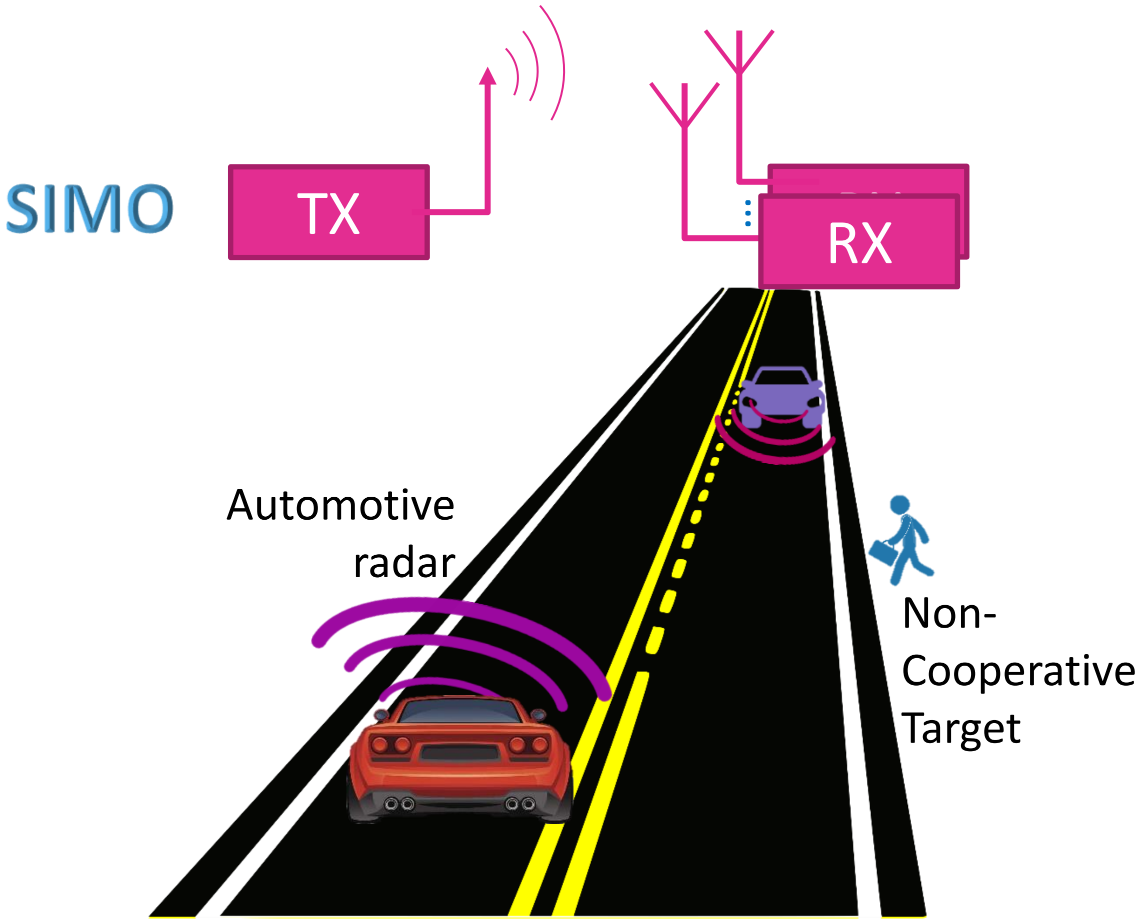
Waveforms in SISO/SIMO Radar Systems

Single Input Single Output

Single Input Multi Output





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Metrics for Good Waveforms

- Small
  - Peak Sidelobe Level (PSL)
  - Integrated Sidelobe Level (ISL)

Small PSL	• avoid masking of weak targets in range sidelobes of a strong return
Low ISL	• mitigate the deleterious effects of distributed clutter echoes which are close to the target of interest

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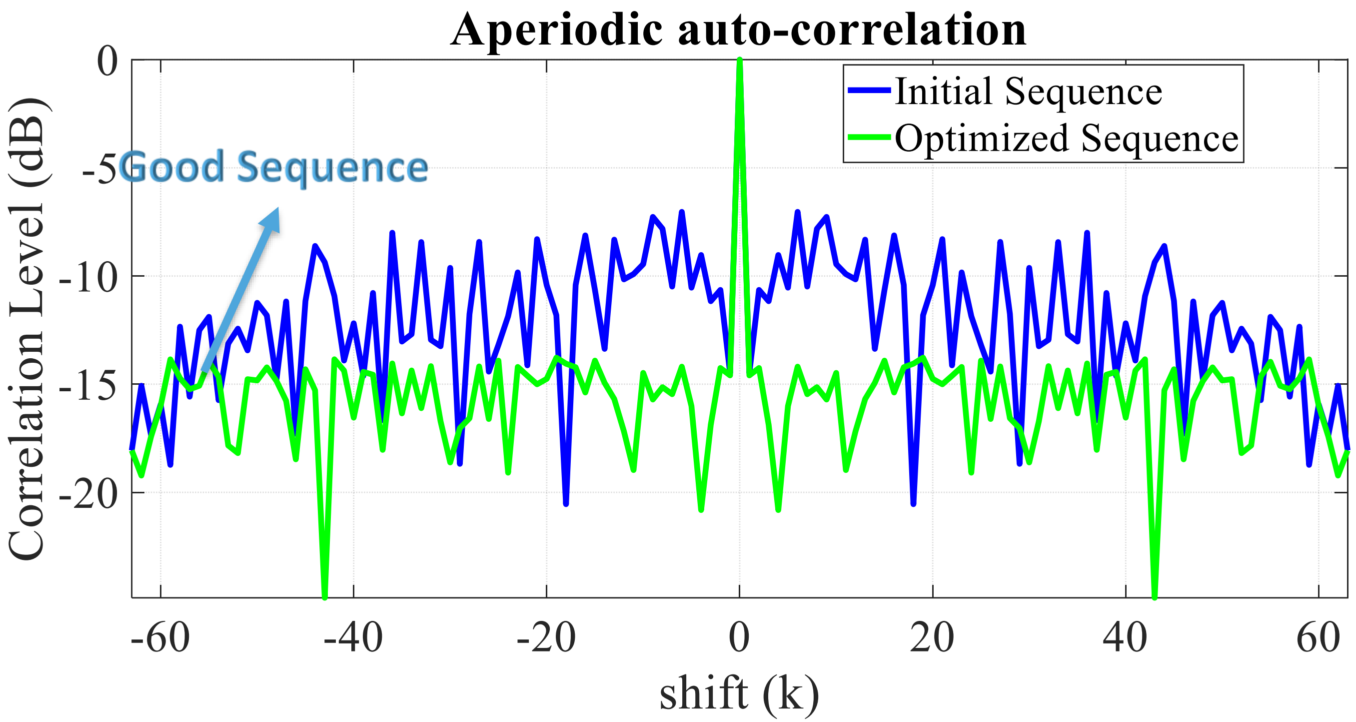
Example



$$\boldsymbol{x} = [x_1. x_2 \dots . x_N]^T \in \mathbb{C}^N,$$

$$r_k = \sum_{n=1}^{N-k} x_n^* x_{n+k} . \quad k = 0. \dots . N-1$$

$$r_k^P = \sum_{n=1}^{N-k} x_n \, x_{n+k \bmod (N)}^* = r_{-k}^P$$

$$\mathrm{PSL} = \max_{k \neq 0} |r_k| \qquad \mathrm{ISL} = \sum_{k=1}^{N-1} r_k^2$$

How to design a sequence with small PSL / ISL ?

Example: PSL Minimization in SISO/SIMO Radar Systems

$$\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T \in \mathbb{C}^N,$$

$$\mathrm{f}(\mathbf{x}) = \max\{|r_k|\}_{k=1}^{N-1}$$

$$\mathcal{P}_{\boldsymbol{x}}^{M} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max\{|r_k|\}_{k=1}^{N-1} \\ \text{subject to} & x_{\mathrm{n}} \in \Omega_M \end{cases}$$

$$\mathcal{P}_{\boldsymbol{x}}^{\infty} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max\{|r_k|\}_{k=1}^{N-1} \\ \text{subject to} & |x_n| = 1 \end{cases}$$

$$\Omega_M = \left\{1, e^{\frac{j2\pi}{M}}, \dots, e^{\frac{j2\pi(M-1)}{M}}\right\}$$

Alphabet size

Non-Convex Multi-variable Constrained min-max optimization problems

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Example: PSL Minimization in SISO/SIMO Radar Systems

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \max\{|r_k|\}_{k=1}^{N-1}$$

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad f(\boldsymbol{x})$$

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$$

$$r_k\left(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)}\right) = a_{ki}\,\zeta + b_{ki}\zeta^* + c_{ki}$$

$$\zeta = e^{j\phi}$$

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Example: PSL Minimization – Discrete Phase

$$
\mathcal{P}_{\phi}^{(k+1)}
\begin{cases}
\underset{\phi}{\text{minimize}} & \max\left\{\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|\right\}_{k=1}^{N-1} \\
\text{subject to} & \phi \in \left\{0, \dfrac{2\pi}{M}, \dots, \dfrac{2\pi(M-1)}{M}\right\}
\end{cases}
$$

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Example: PSL Minimization – Constant Modulus

$$\mathcal{P}_{\phi}^{(k+1)} \begin{cases} \underset{\phi}{\text{minimize}} & \max\left\{\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|^2\right\}_{k=1}^{N-1} \\ \text{subject to} & \phi \in [0, 2\pi) \end{cases}$$

$$\beta = \tan\frac{\phi}{2}$$

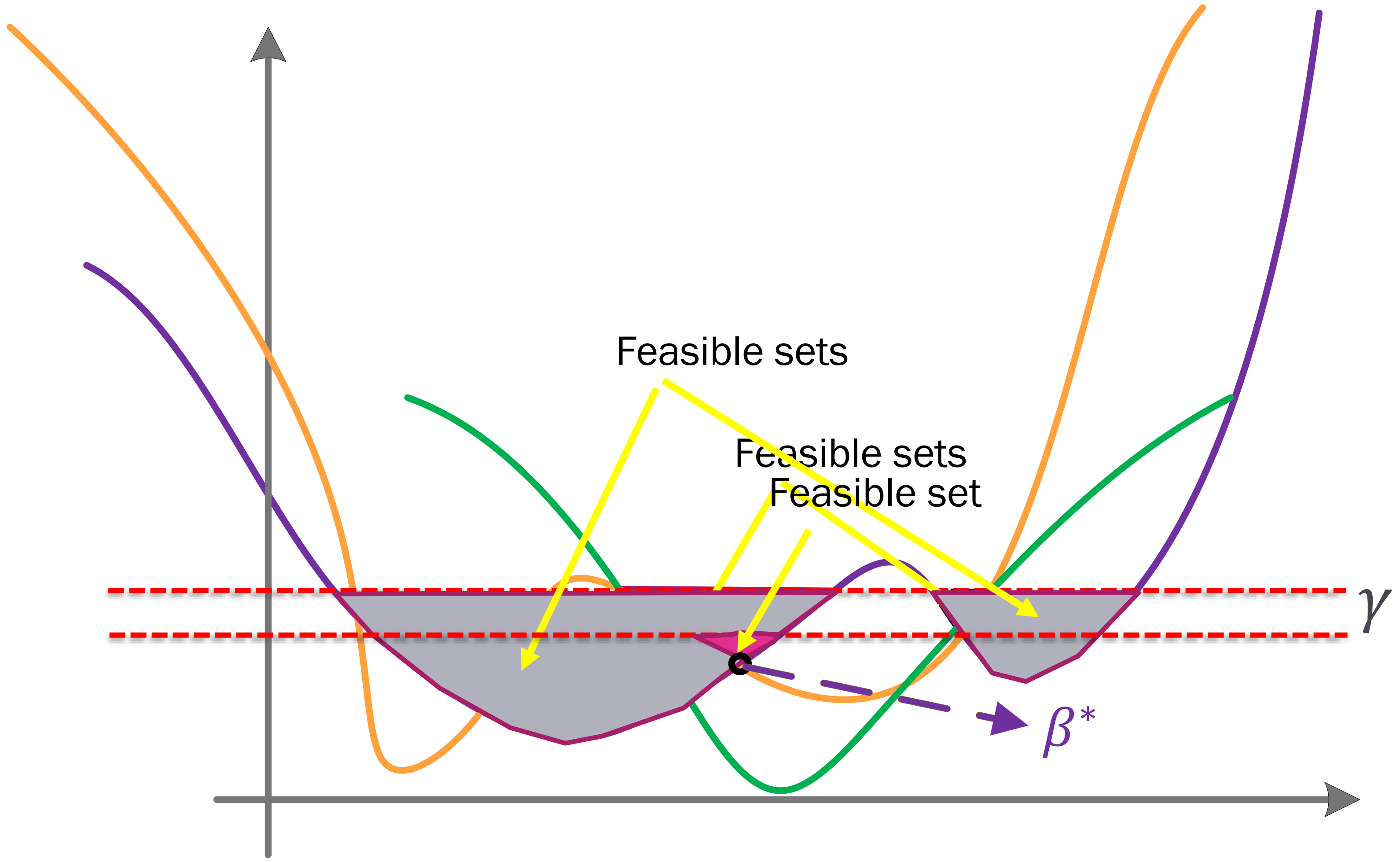
$$\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|^2 = \frac{\mu\beta^4 + \kappa\beta^3 + \delta\beta^2 + \eta\beta + \rho}{(1+\beta^2)^2}$$

Example: PSL Minimization – Constant Modulus

$$\underset{\beta}{\text{minimize}} \qquad \max\left\{\frac{\mu\beta^4+\kappa\beta^3+\delta\beta^2+\eta\beta+\rho}{(1+\beta^2)^2}\right\}_{k=1}^{N-1}$$

$$\begin{cases} \text{find} & \beta \\ \text{subject to} & \dfrac{\mu\beta^4+\kappa\beta^3+\delta\beta^2+\eta\beta+\rho}{(1+\beta^2)^2} \leq \gamma \end{cases}$$

Example: PSL Minimization in SISO/SIMO Radar Systems



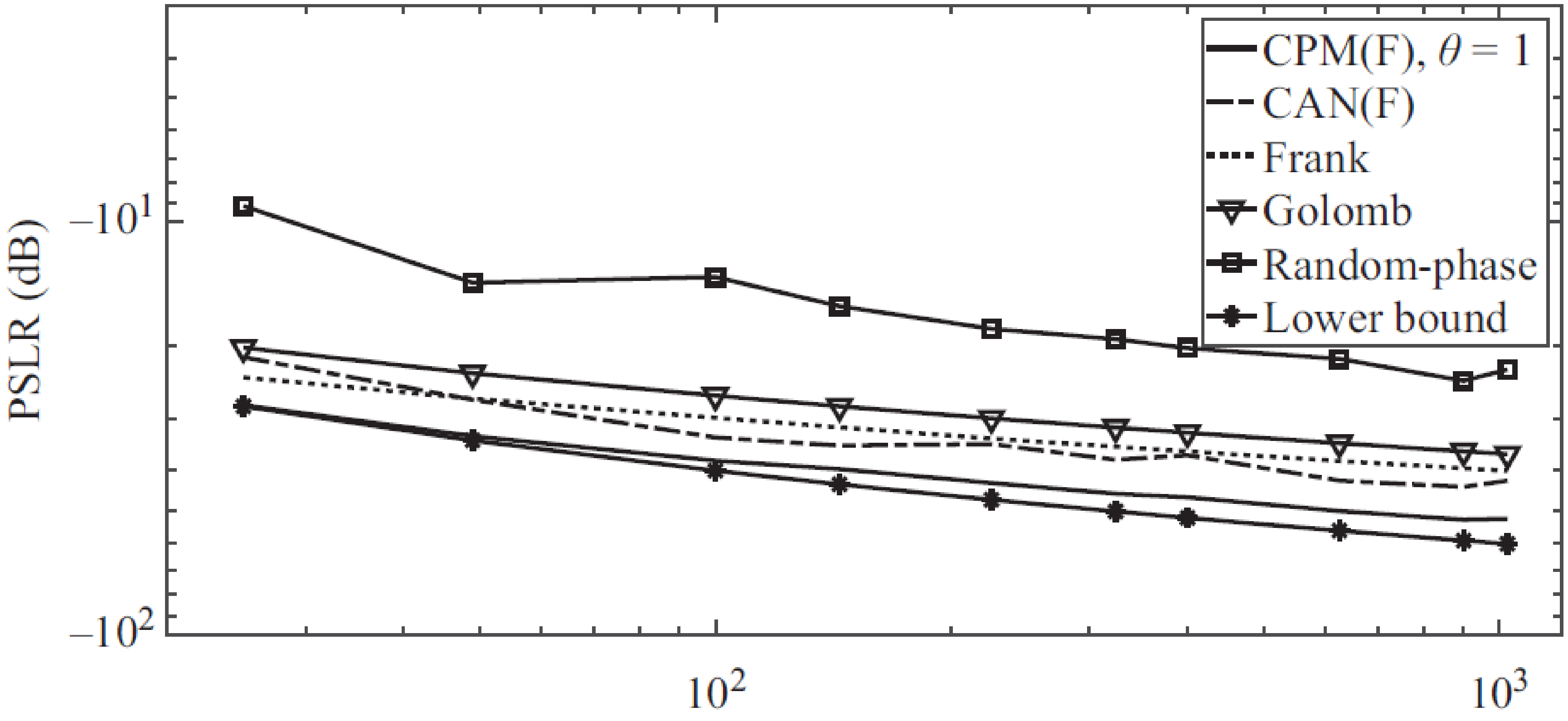
Example: PSL Minimization in SISO/SIMO Radar Systems

N	Sample Binary Codes with Good PSL (in HEX)
320	3398D83F635CC5A0D5727CB53A97D39896CFD7C6F1EF86C9AEDE20400F546DF8AB49D7D0879C21BB
360	6D4A71524C40837C9DA7F101F7580E457FFE23696BFA3B7DB9A957CA7923E185985396572CCB9AAD7347A38682
400	B6686A4E6FEA1CF29CBFE6ECA477E2A5D7A8F448A108A5F3F593E63ABC7917D84CA736F15C447BD2072CABA99F127CA5185C
440	73B8B3397676BB952A97A519AEB64C7C544D00242B2A8180BFCB610F4AE6D1C0740F1D8904DE217F4F79248D054B2C7FB490C3CE10BC67
480	64A83F1A672F6E4A4CF5824F5FDD9FBE73FC48322A4D930E17702F859E67911CFD2E12415ACBB55159C229E8ACFF70C25227A379A92CAA17A712B91D

Matlab source codes can be downloaded from :

https://radarmimo.com/how-to-design-binary-codes-for-radar-systems/

Example: PSL Minimization in SISO/SIMO Radar Systems

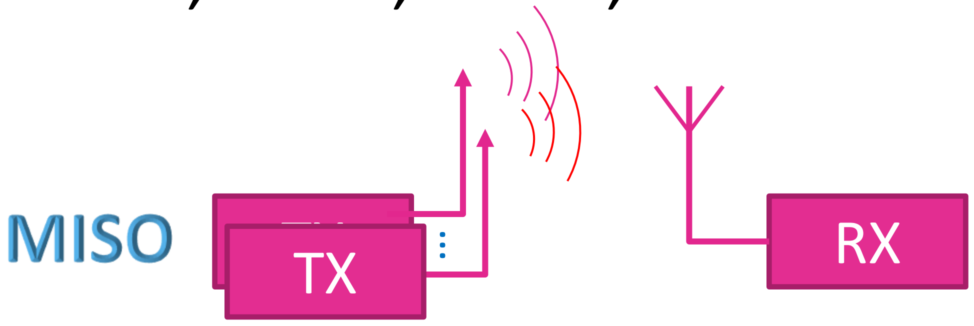


Matlab source codes can be downloaded from :

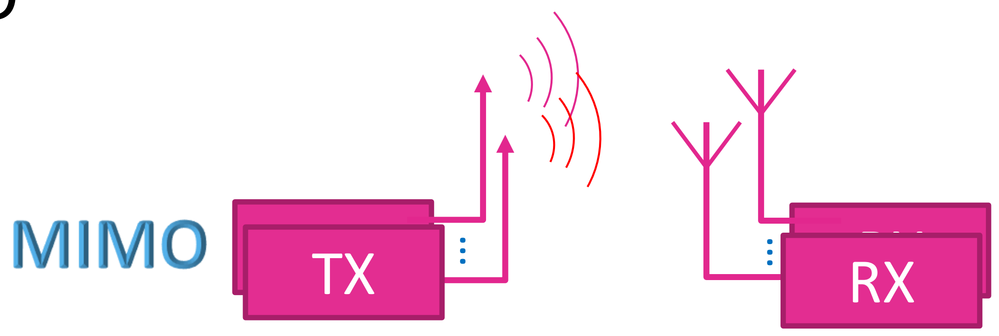
https://radarmimo.com/how-to-design-binary-codes-for-radar-systems/

Waveforms in (Colocated/Widely Separated) MISO/MIMO Radar Systems

- ✓ Transmitters should be observable at each receiver
- ✓ Enabled by Orthogonal Waveforms
  - Limit mutual interference
  - Enable cooperative operation
  - Provide visibility into paths between transmitter and receivers
  - Determines spatial distribution of energy
  - Orthogonality achieved by division in time, frequency or code
- ✓ FDM-, TDM-, DDM-, and CDM-MIMO



Multi Input Single Output



Multi Input Multi Output

$$r_{ml}(k) = \sum_{n=1}^{N-k} x_m(n)\, x_l^*(n+k) = r_{lm}^*(-k)$$

CDM-MIMO Waveform Design Problem

$$\boldsymbol{x}_m = [x_m(1), x_m(2), \ldots, x_m(N)]^T \in \mathbb{C}^N,$$

$$\boldsymbol{X} = \left[\boldsymbol{x}_1, \quad \boldsymbol{x}_2, \quad \ldots, \boldsymbol{x}_{N_T}\right] \in \mathbb{C}^{N \times N_T}$$

$$r_{ml}(k) = \sum_{n=1}^{N-k} x_m(n)\, x_l^*(n+k) = r_{lm}^*(-k)$$

CDM-MIMO Waveform Design Problem

$$\text{PSL} = \max\left\{\max_{m}\max_{k\neq 0}|r_{mm}(k)|\ ,\max_{\substack{m,l\\ m\neq l}}\max_{k}|r_{ml}(k)|\right\}$$

$$\text{ISL} = \sum_{m=1}^{N_T}\sum_{\substack{k=-N+1\\ k\neq 0}}^{N-1}|r_{mm}(k)|^2 + \sum_{\substack{m,l=1\\ m\neq l}}^{N_T}\sum_{k=-N+1}^{N-1}|r_{ml}(k)|^2$$

How to design set of sequences with small PSL / ISL ?

[2] - M. Alaee-Kerahroodi, M. Modarres-Hashemi and M. M. Naghsh, "Designing Sets of Binary Sequences for MIMO Radar Systems," in *IEEE Transactions on Signal Processing*, vol. 67, no. 13, pp. 3347-3360, 1 July1, 2019.

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CDM-MIMO Waveform Design Problem

Waveform Design with ISL Minimization in MIMO Radar

$$\mathrm{ISL} = \sum_{m=1}^{N_T} \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} |r_{mm}(k)|^2 + \sum_{\substack{m,l=1 \\ m\neq l}}^{N_T} \sum_{k=-N+1}^{N-1} |r_{ml}(k)|^2$$

$$P_{\mathbf{X}} = \begin{cases} \min\limits_{\mathbf{X}} & f(\mathbf{X}) \\ \text{s.t.} & |x_m(n)| = 1 \end{cases}$$

CDM-MIMO Waveform Design Problem

$$f(x_t(d)) = \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} \left| a_{dkt} x_t(d) + b_{dkt} x_t^*(d) + c_{dkt} \right|^2 + \sum_{l=1}^{N_T} \sum_{k=-N+1}^{N-1} \left| a_{dkl} x_t(d) + c_{dkl} \right|^2$$

$$+ \sum_{\substack{m=1 \\ m\neq t}}^{N_T} \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} \left| r_{mm}(k) \right|^2 + \sum_{\substack{m,l=1 \\ m\neq \{t,l\}}}^{N_T} \sum_{k=-N+1}^{N-1} \left| r_{ml}(k) \right|^2,$$

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CDM-MIMO Waveform Design Problem

$$\tilde{f}(x_t(d)) = \sum_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} \left| a_{dkt} x_t(d) + b_{dkt} x_t^*(d) + c_{dkt} \right|^2 + \sum_{l=1}^{N_T} \sum_{k=-N+1}^{N-1} \left| a_{dkl} x_t(d) + c_{dkl} \right|^2$$

$$\widetilde{P}_{x_t(d)} = \begin{cases} \min\limits_{x_t(d)} & \tilde{f}(x_t(d)) \\ \text{s.t.} & |x_t(d)| = 1 \end{cases}$$

Still non-convex!!!

CDM-MIMO Waveform Design Problem

$$\widetilde{P}_{\phi_t(d)} = \begin{cases} \min\limits_{\phi_t(d)} & \sum\limits_{\substack{k=-N+1 \\ k\neq 0}}^{N-1} \left| a_{dkt} e^{j\phi_t(d)} + b_{dkt} e^{-j\phi_t(d)} + c_{dkt} \right|^2 + \sum\limits_{l=1}^{N_T} \sum\limits_{k=-N+1}^{N-1} \left| a_{dkl} e^{j\phi_t(d)} + c_{dkl} \right|^2 \\ \text{s.t.} & \phi_t(d) \in [0, 2\pi) \end{cases}$$

$$\beta_d = \tan \frac{\phi_t(d)}{2}$$

$$\widetilde{P}_{\beta_d} = \begin{cases} \min\limits_{\beta_d} & \dfrac{\mu_{dk}\beta_d^4 + \kappa_{dk}\beta_d^3 + \xi_{dk}\beta_d^2 + \eta_{dk}\beta_d + \rho_{dk}}{(1+\beta_d^2)^2} \\ \text{s.t.} & \beta_d \in \mathbb{R} \end{cases}$$

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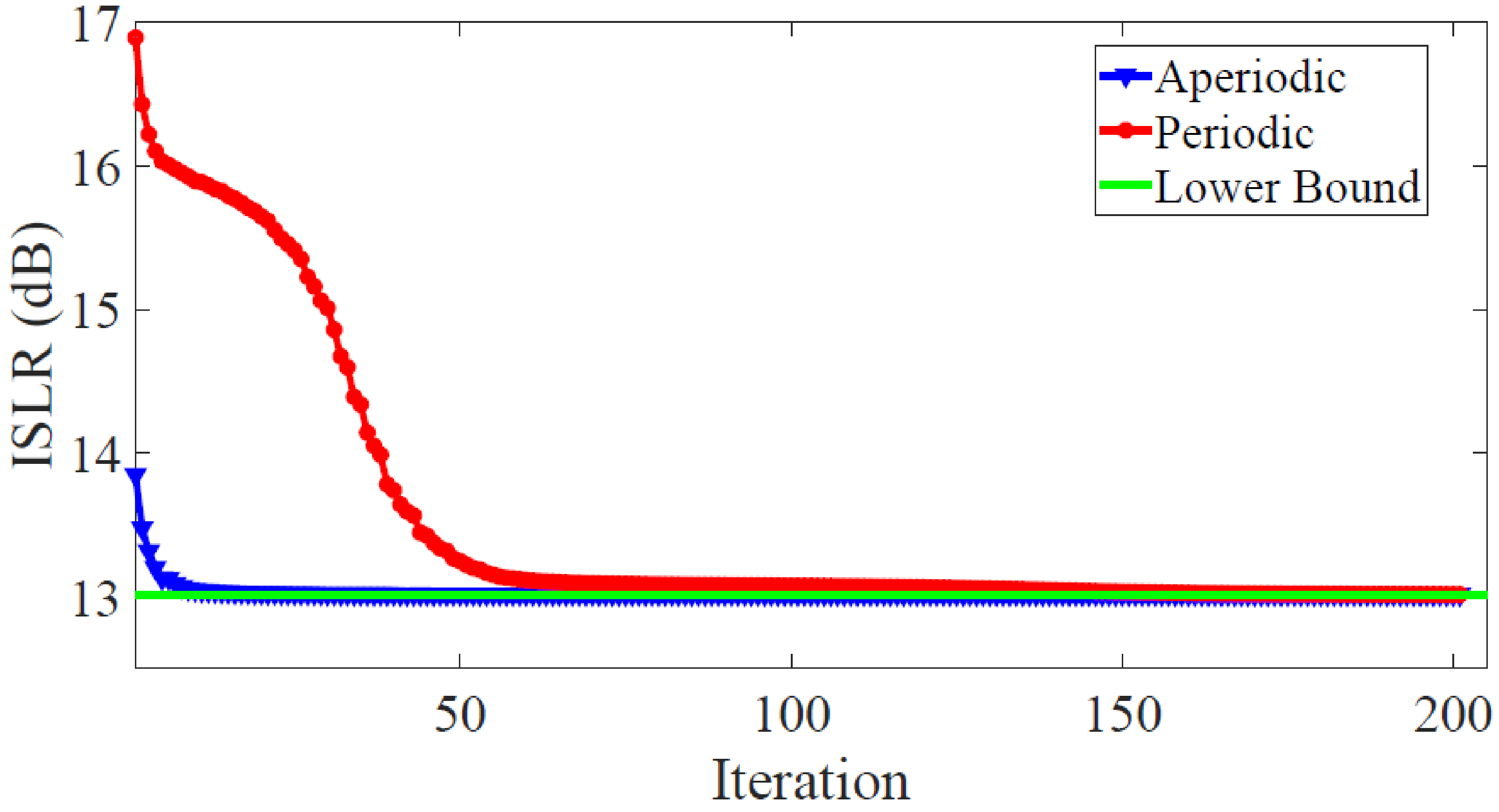
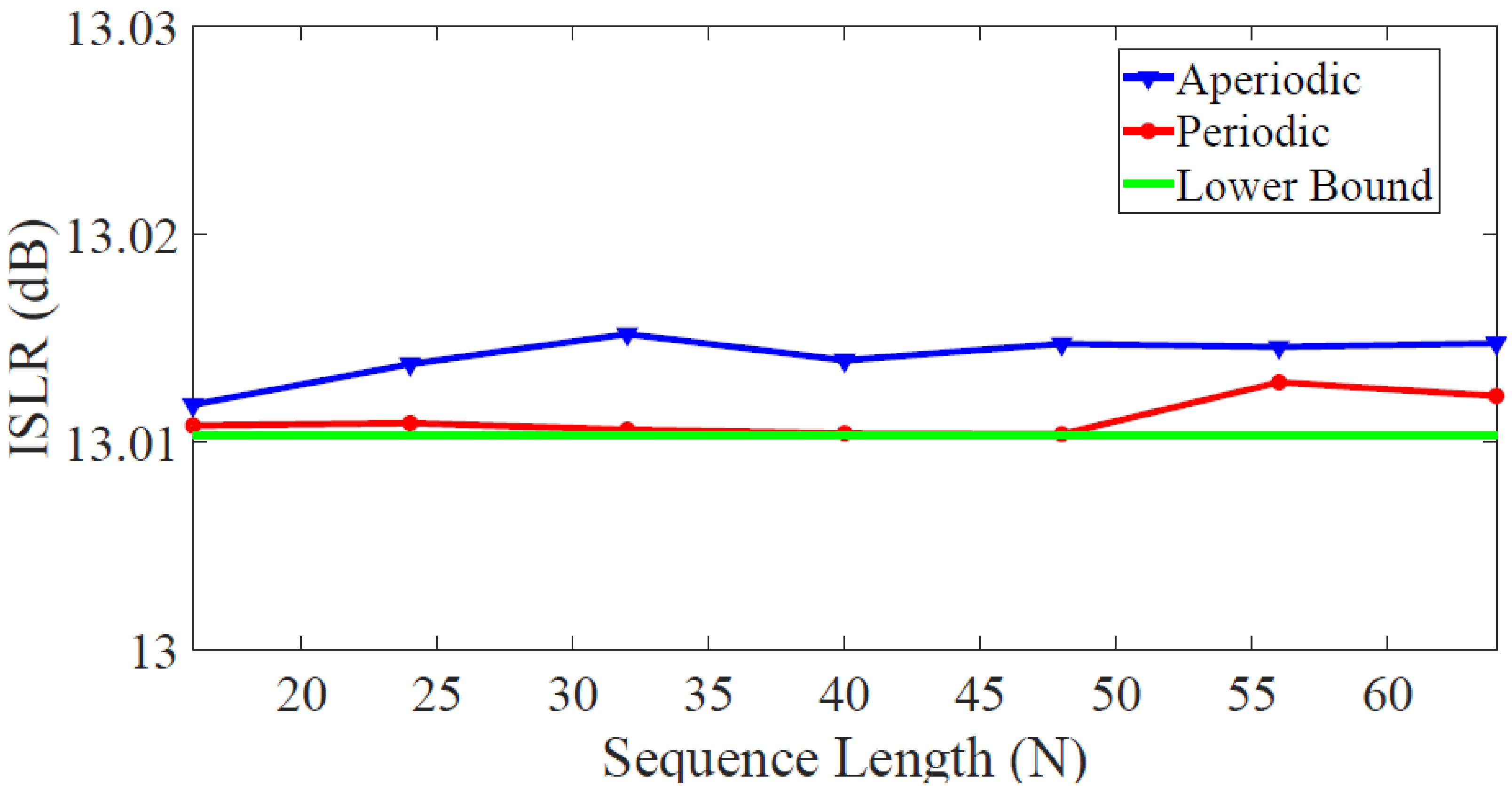
CDM-MIMO Waveform Design Problem



Convergence behavior of the proposed algorithm $(N = 64, N_T = 5)$

CDM-MIMO Waveform Design Problem



ISLR values of the obtained set of $N_T = 5$ sequences through the proposed algorithm averaged over 10 independent trails, comparing with the lower bound.

CDM-MIMO Waveform Design Problem

ISLR(dB) values obtained via the proposed algorithm initialized by random-phase sequence of length $N = 64$ averaged over 10 independent trails, in comparison with the lower bound.

Set Size (N_T)	2	3	4	5	6	7	8	9	10
Aperiodic	3.117	7.807	10.803	13.016	14.774	16.234	17.483	18.574	19.543
Periodic	3.022	7.797	10.798	13.013	14.775	16.236	17.482	18.574	19.546
Lower Bound	3.010	7.781	10.791	13.010	14.771	16.232	17.481	18.573	19.542

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Question?

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