

Radar Signal Processing Mastery

Theory and Hands-On Applications with mmWave MIMO Radar Sensors

Date: 7-11 October 2024

Time: 9:00AM-11:00AM ET (New York Time)



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Outline



Time: 9:00AM-11:00AM ET (New York Time)

Lecture	Duration	Date
Lecture 1: Radar Systems Fundamental	2 Hours	October 7 th , 2024
Lecture 2: Advanced Radar Systems	2 Hours	October 8 th , 2024
Lecture 3: Practical Radar Signal Processing - Motion Detection	2 Hours	October 9 th , 2024
Lecture 4: Practical Radar Signal Processing - Breathing and Heart Rate Estimation	2 Hours	October 10 th , 2024
Lecture 5: Practical Radar Signal Processing – Angle estimation with MIMO radar	2 Hours	October 11 th , 2024



Lecture 1

Radar Fundamentals



Lecture 1: Radar Systems Fundamental

What we learn in Lecture 1

- Pulsed Radar
- Pulse Doppler Radar
- Pulse Compression Radar
- Continuous Wave Radar
- Frequency Modulated Continuous Wave Radar



Scan the QR code for access to the codes

- Pulse Repletion Interval (PRI), Pulse Repletion Frequency (PRF)
- Range resolution and Unambiguous Range
- Doppler resolution and Unambiguous Doppler
- Simple pulse, LFM, Binary Codes, Poly phase codes
- Fast-time, Slow-time, and ambiguity function
- Matched Filter, Range-Doppler Processing
- De-chirp/Stretch Processing

Introduction to Radar Systems



Overview of radar applications

RADAR: RAdio Detection And Ranging

the acronym created in 1940 by the United States Navy

Radar is a system that uses radio waves to determine the **distance** (ranging), **direction** (azimuth and elevation angles), and **radial velocity** of objects relative to the site.

Applications

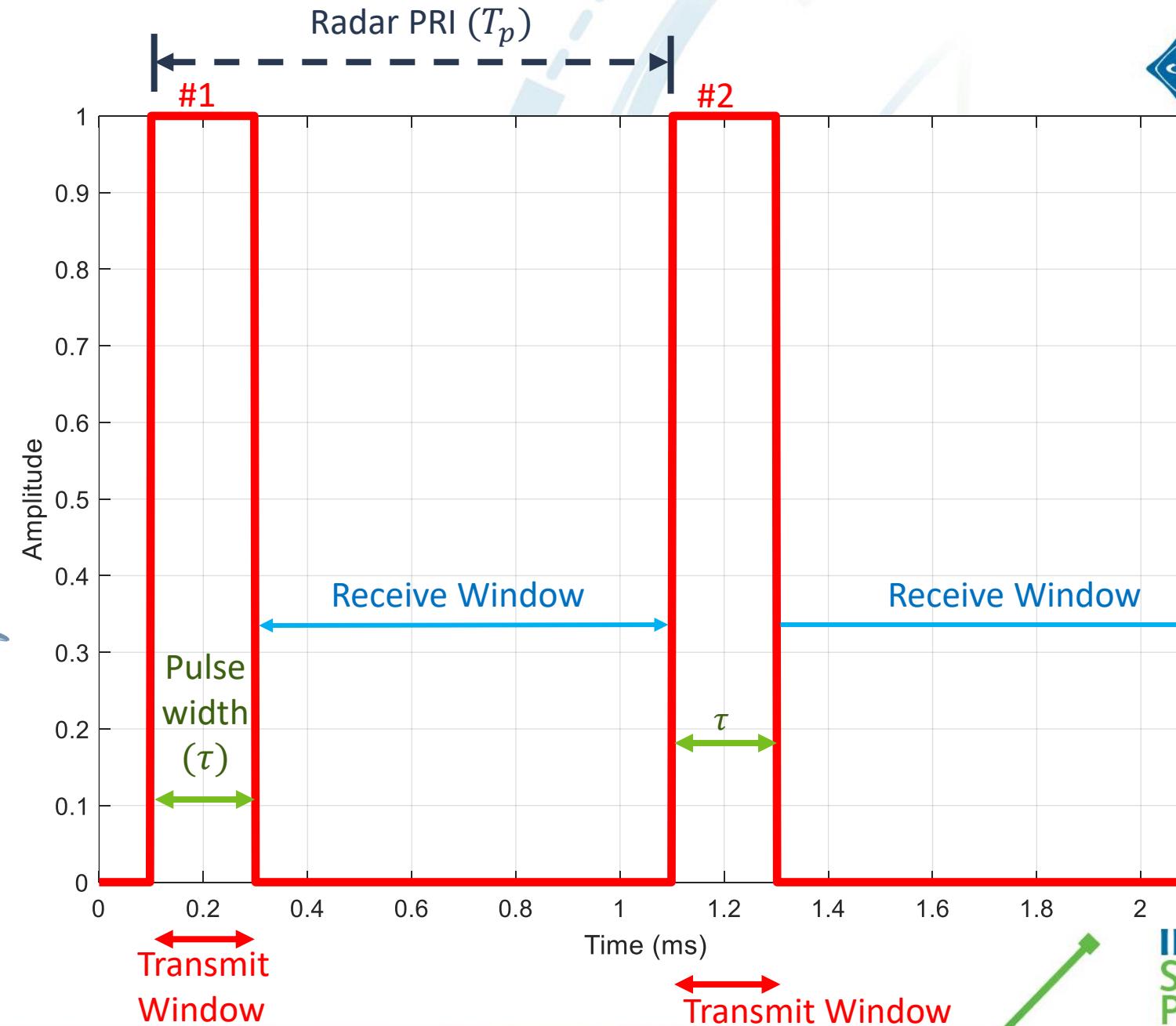
- Aerospace and Aviation
- Military and Defense
- Maritime
- Meteorology
- Space Exploration
- Geological and Environmental
- Healthcare and Biomedical
- Law Enforcement and Security
- Automotive
- Industrial



Pulsed Radar

Transmit signal

- transmits energy in short bursts, or pulses
- measures distances and detect the presence of objects



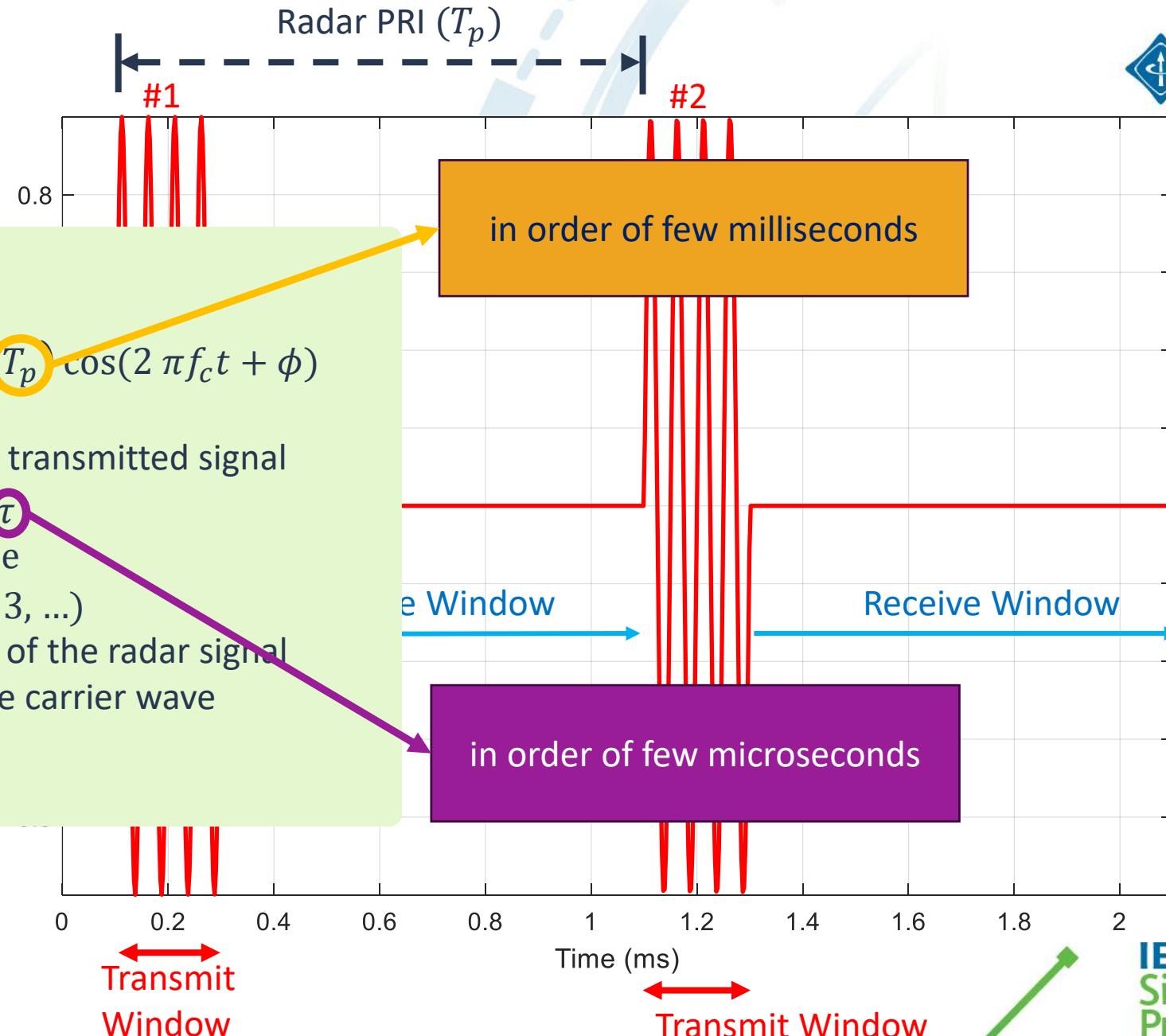
Pulsed Radar

Transmit signal

$$s_{tx}(t) = A_t \operatorname{rect}(t - (i - 1)T_p) \cos(2\pi f_c t + \phi)$$

- A_t is the amplitude of the transmitted signal
- $\operatorname{rect}(t) = \begin{cases} 1 & 0 < t < \tau \\ 0 & \text{otherwise} \end{cases}$
- i is the pulse index ($1, 2, 3, \dots$)
- f_c is the carrier frequency of the radar signal
- ϕ is the initial phase of the carrier wave

Lect1_example1.m

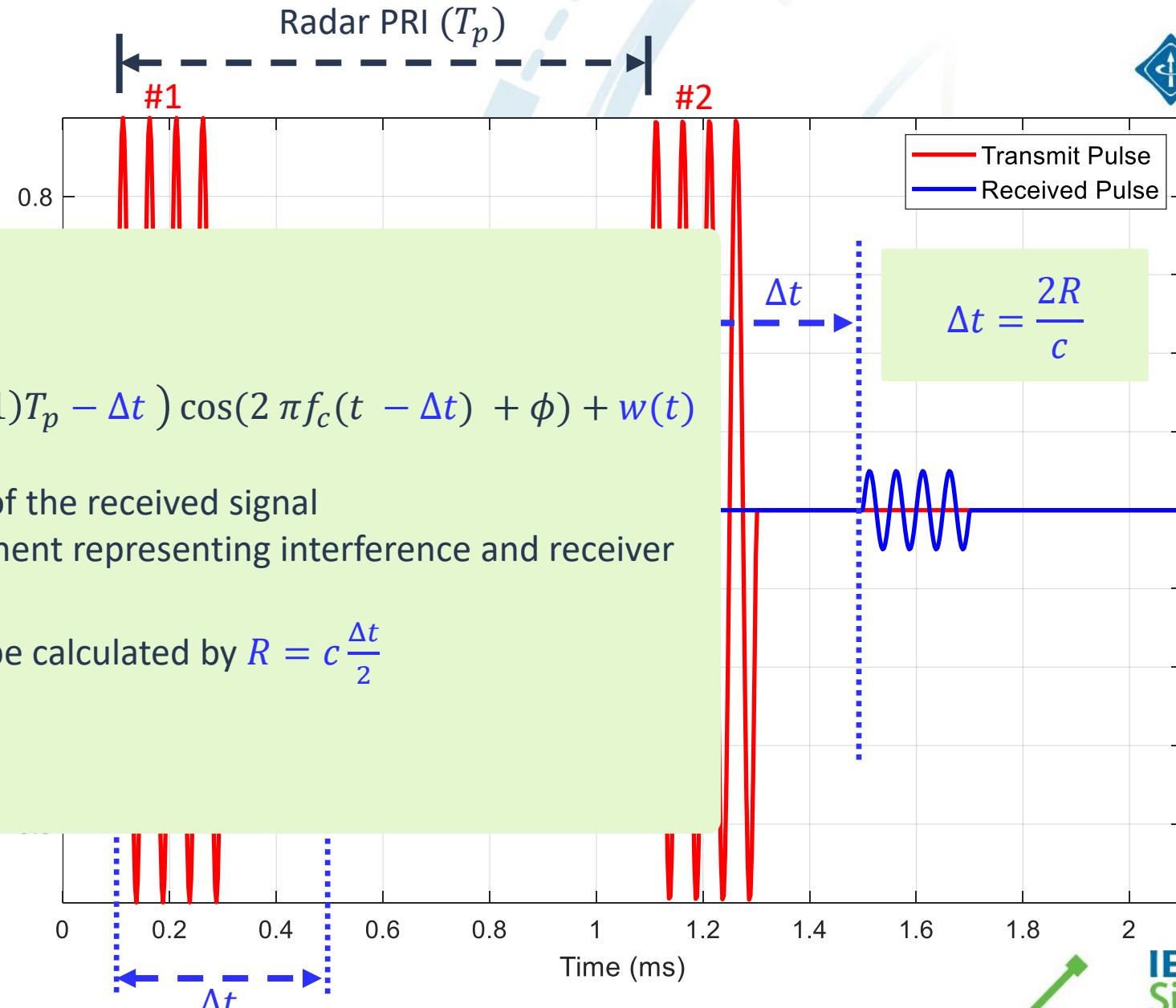


Pulsed Radar

Received signal

$$s_{rx}(t) = A_r \operatorname{rect}\left(t - (i-1)T_p - \Delta t\right) \cos(2\pi f_c(t - \Delta t) + \phi) + w(t)$$

- A_r is the amplitude of the received signal
- $w(t)$ is noise component representing interference and receiver noise
- Range of target can be calculated by $R = c \frac{\Delta t}{2}$



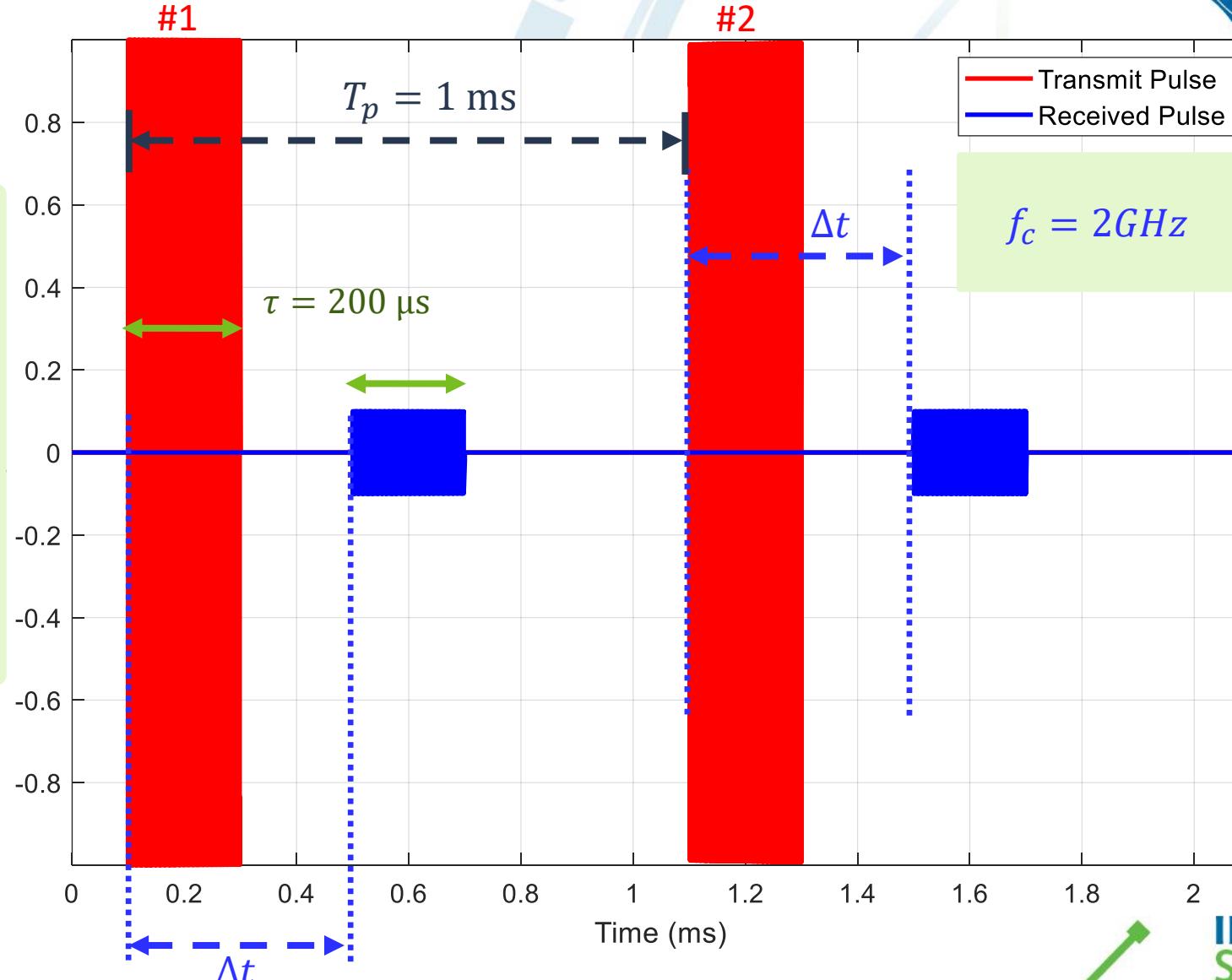
Pulsed Radar

Received signal

$$T_p = 1 \text{ ms} \Rightarrow \frac{cT_p}{2} = 150 \text{ Km}$$

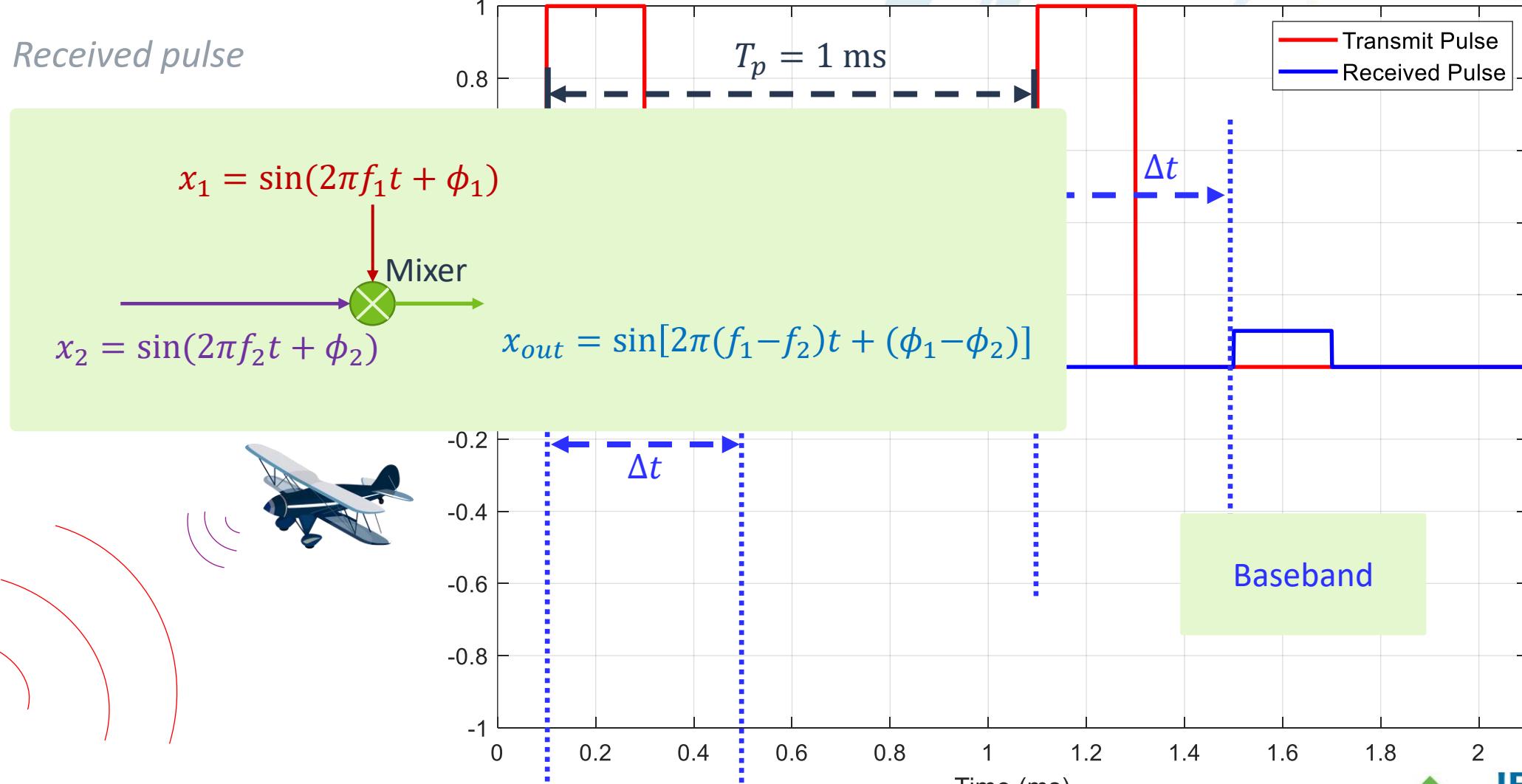
$$\tau = 200 \mu\text{s} \Rightarrow \frac{c\tau}{2} = 30 \text{ Km}$$

$$\Delta t = 400 \mu\text{s} \Rightarrow \frac{c\Delta t}{2} = 60 \text{ Km}$$



Pulsed Radar

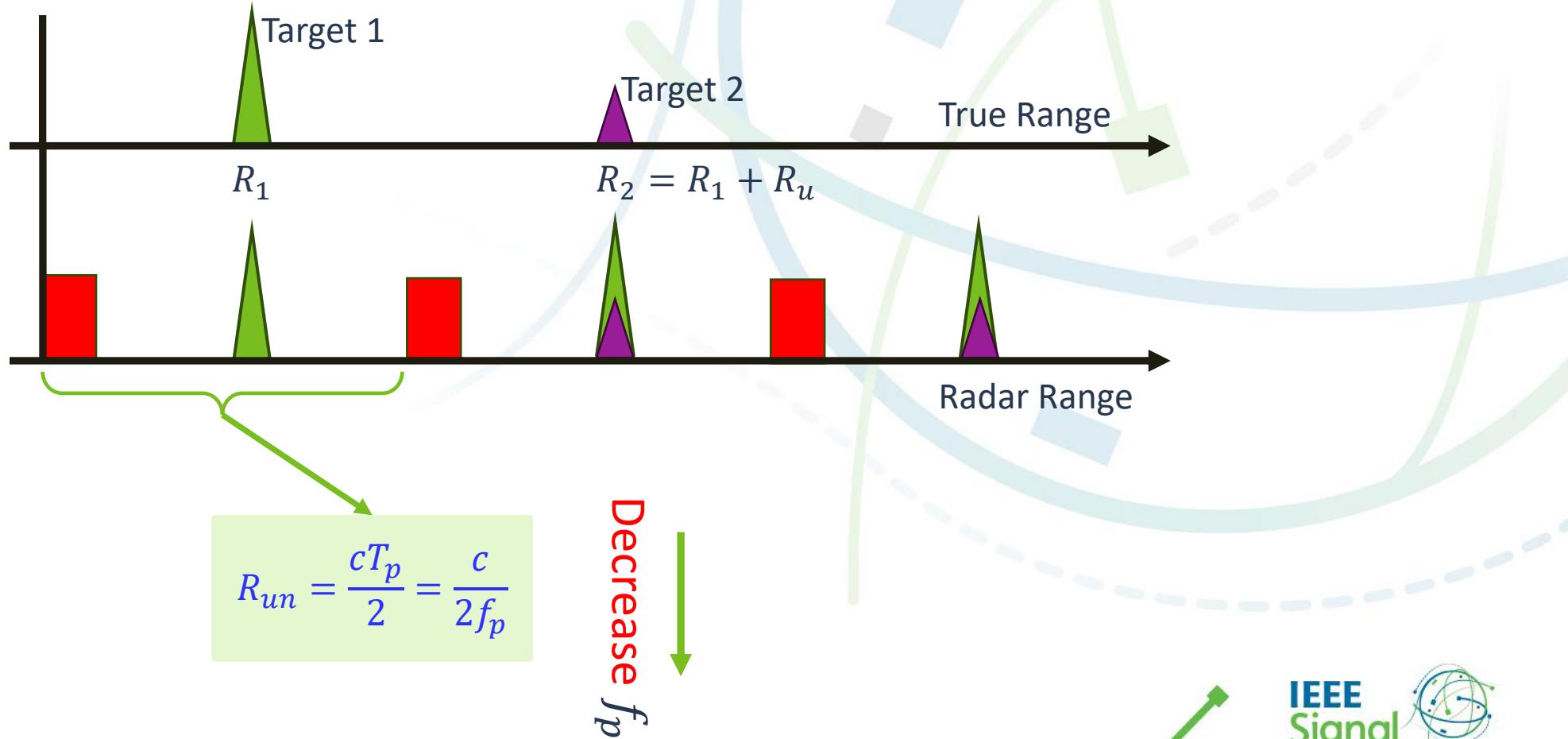
Received pulse



Pulsed Radar

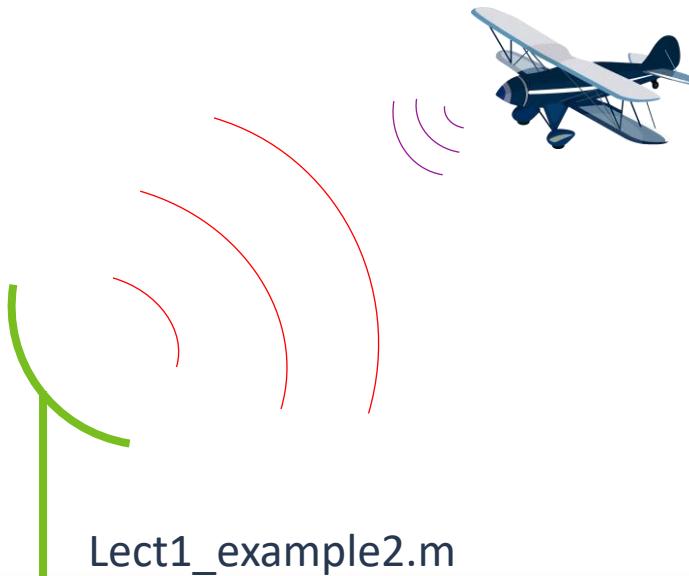
Unambiguous Range

Range ambiguities occur when echoes from one pulse are not all received before the next pulse

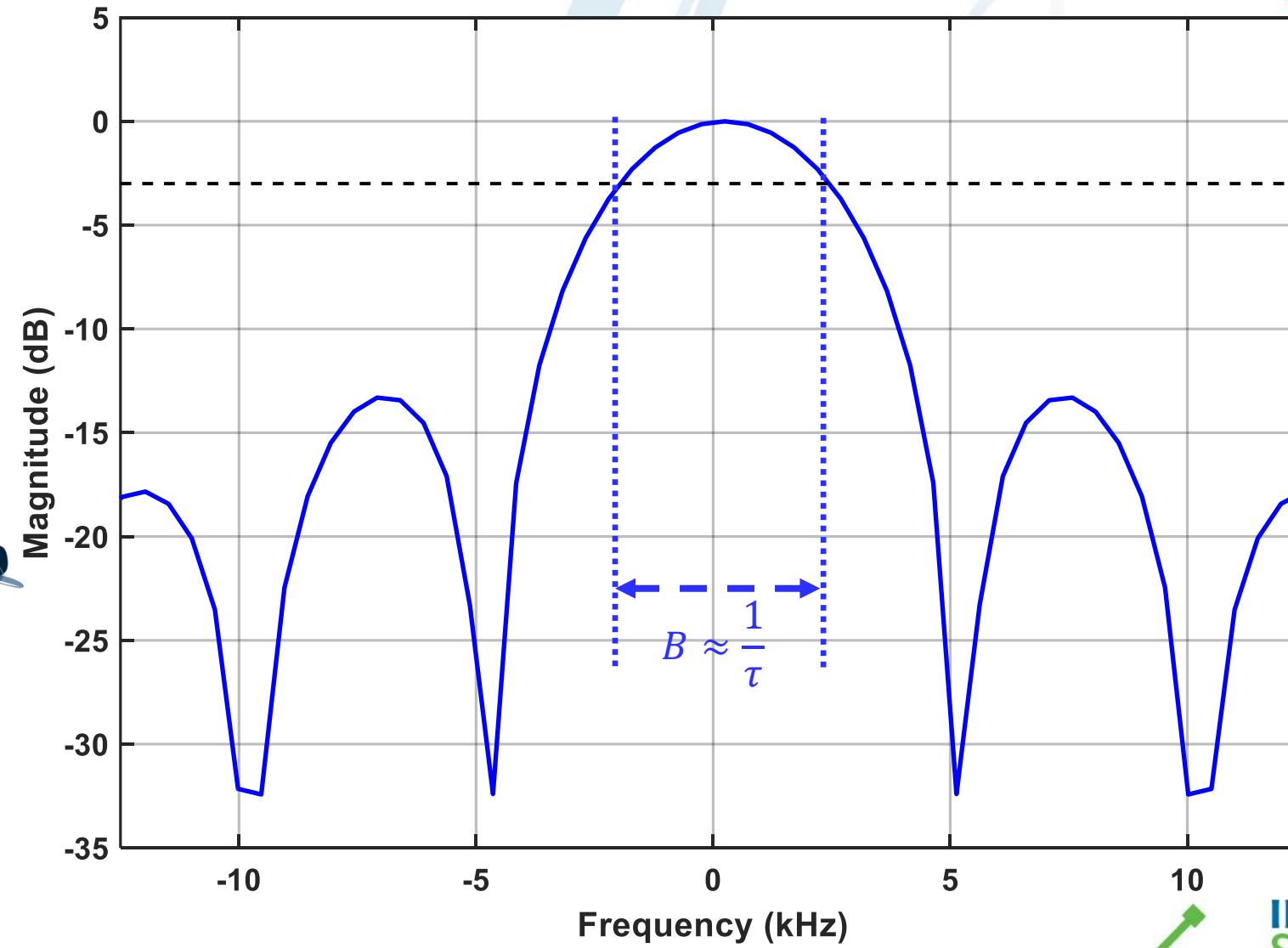


Pulsed Radar

Received pulse



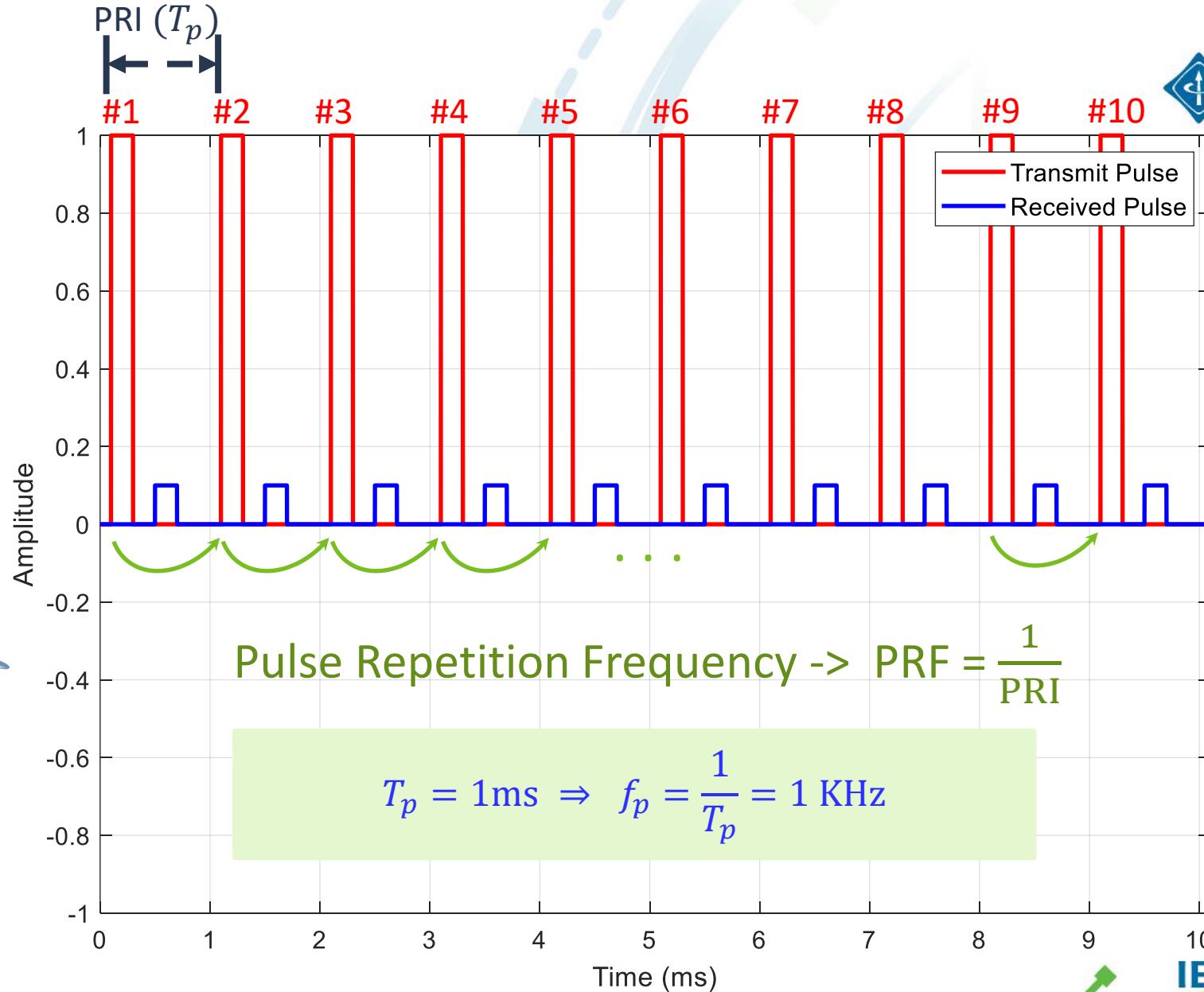
12



Lect1_example2.m

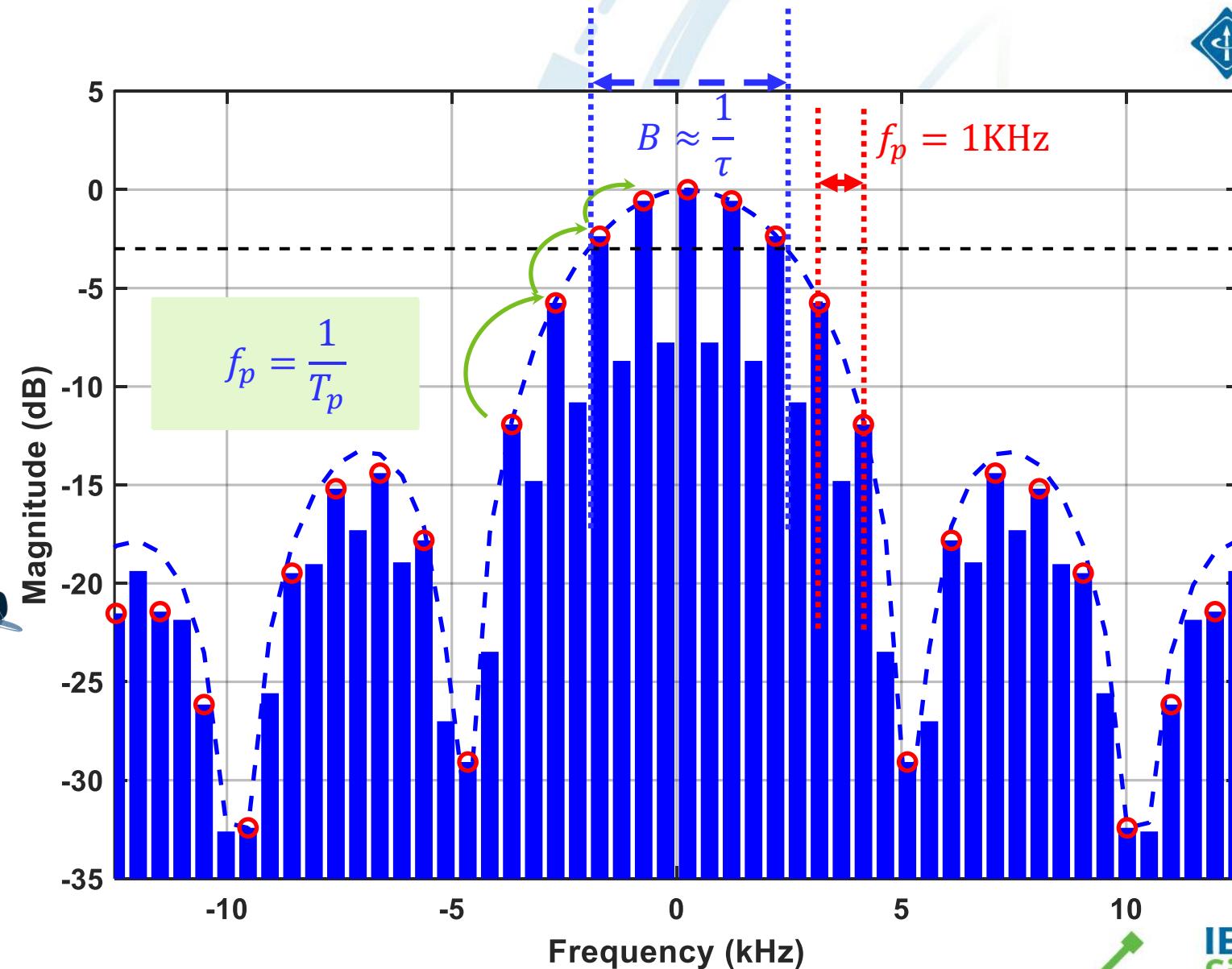
Pulsed Radar

Received pulses



Pulsed Radar

Received pulses



Doppler effect

Received pulses

$$f_d = \frac{2v_r}{\lambda}$$



Christian Andreas Doppler

(/ˈdɒplər/; 29 November 1803 – 17 March 1853)^[1] was an Austrian mathematician and physicist. He formulated the principle – now known as the Doppler effect – that the observed frequency of a wave depends on the relative speed of the source and the observer.

https://en.wikipedia.org/wiki/Christian_Doppler

Christian Doppler



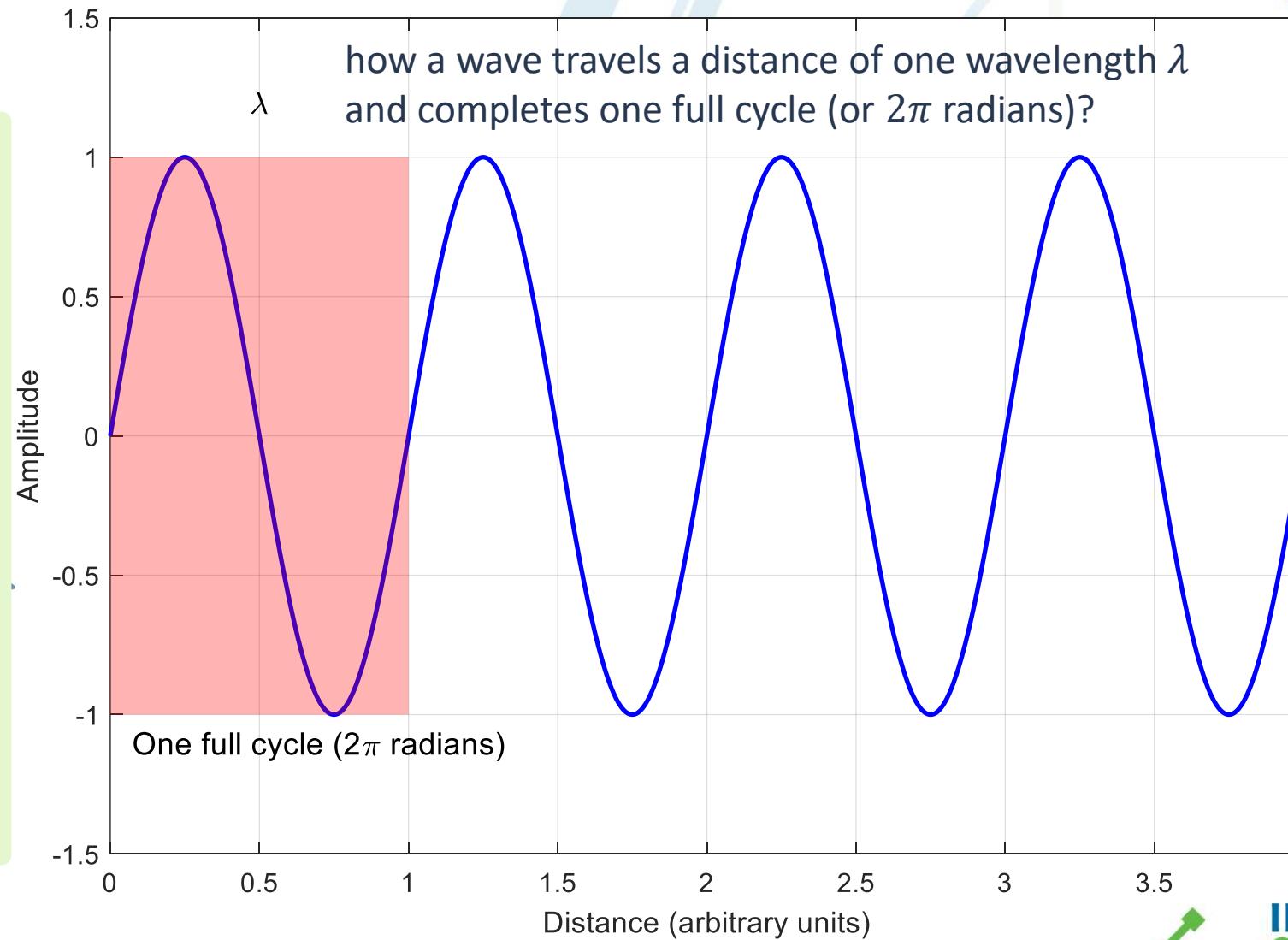
Doppler effect

phase difference between the transmitted signal and received signal

$$\phi = 2R \times \frac{2\pi}{\lambda}$$

$$\frac{d\phi}{dt} = 2 \frac{dR}{dt} \frac{2\pi}{\lambda}$$

$$f_d = \frac{1}{2\pi} \frac{d\phi}{dt} = 2 \frac{v_r}{\lambda}$$



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Pulsed Doppler Radar

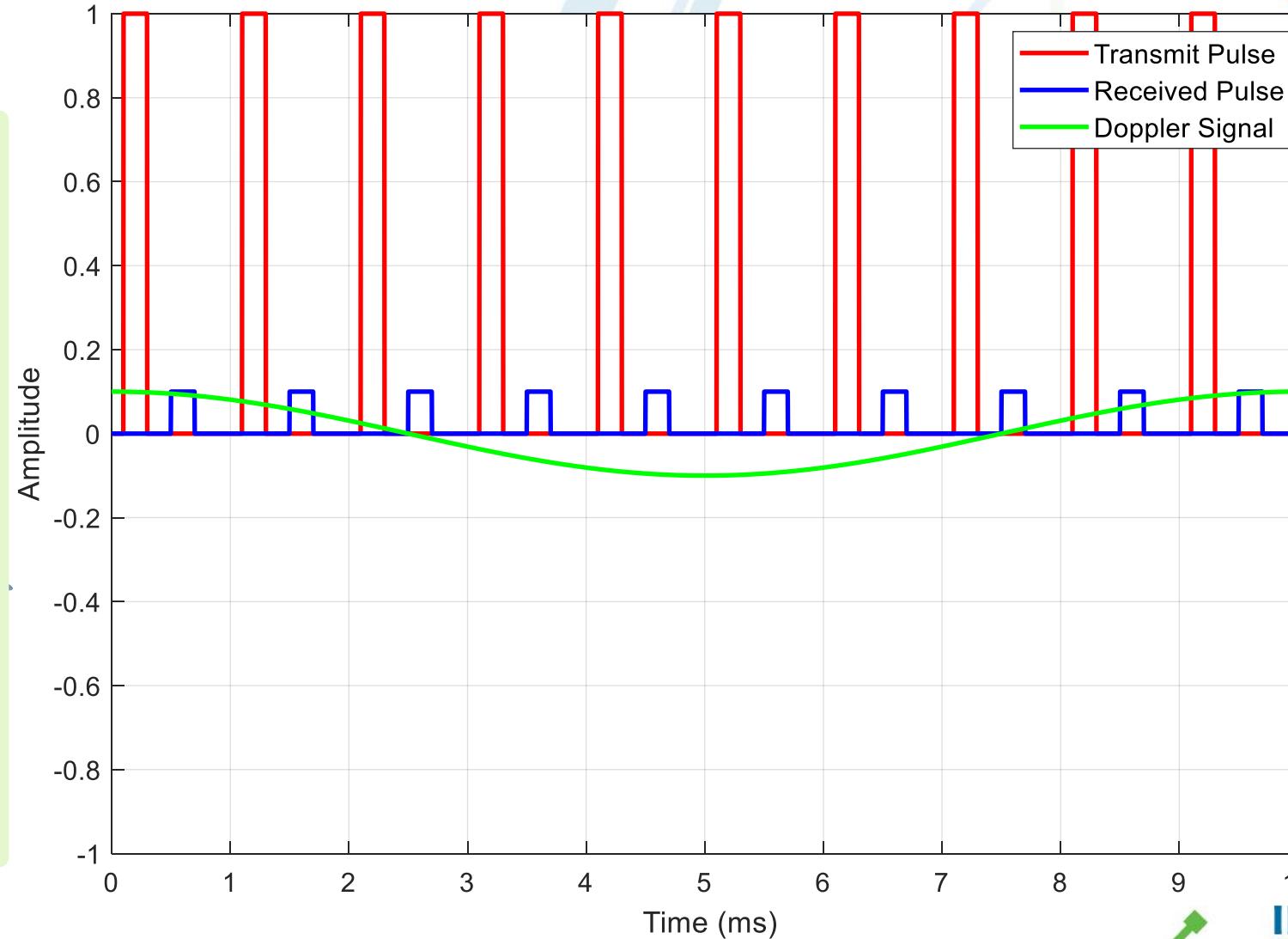
Received signal

$$f_c = 150 \text{ MHz}$$

$$\lambda = \frac{c}{f_c} = 2 \text{ m}$$

$$v_r = 100 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 100 \text{ Hz}$$



Pulsed Doppler Radar

Received signal

$$f_c = 150 \text{ MHz}$$

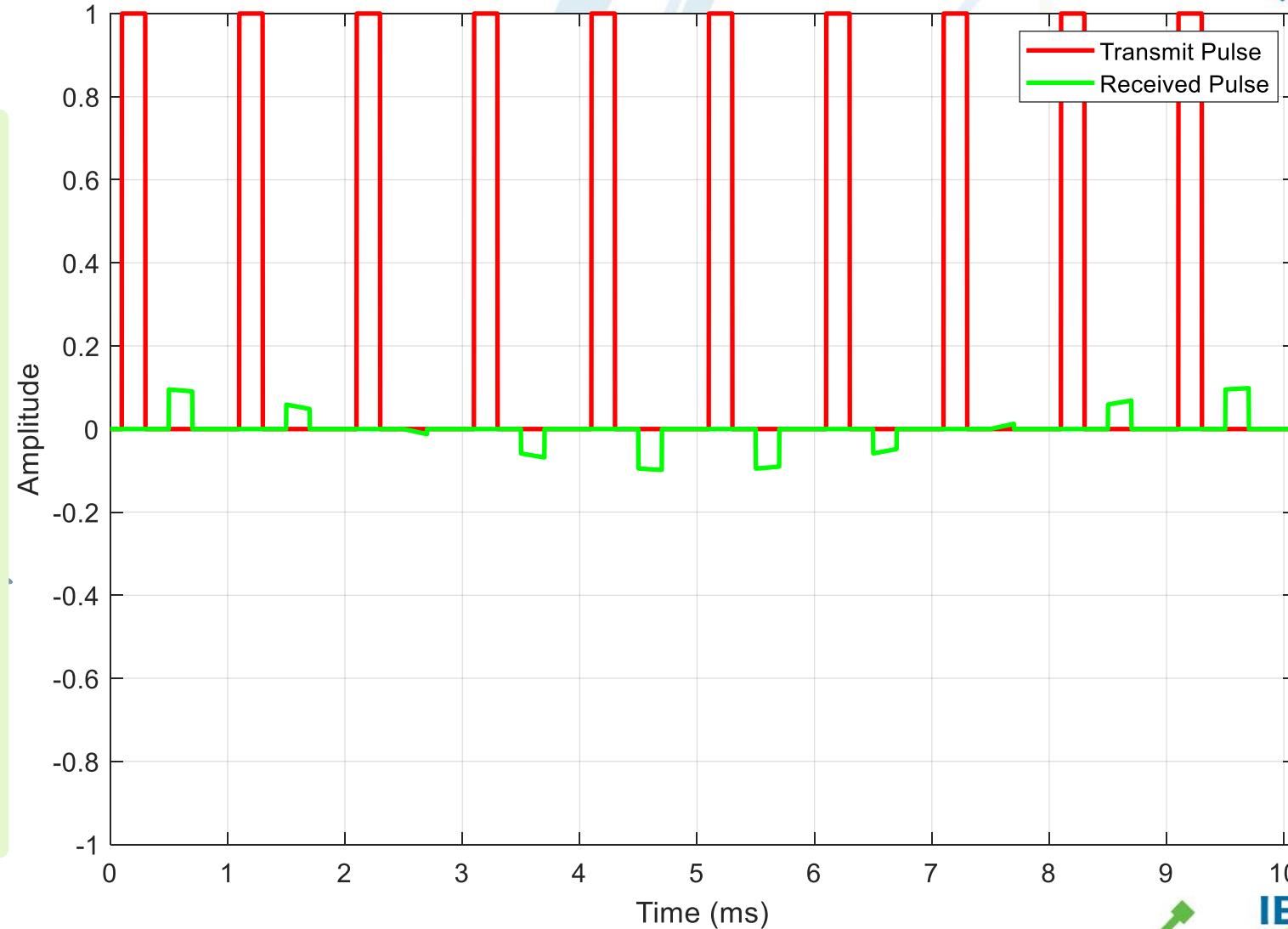
$$\lambda = \frac{c}{f_c} = 2 \text{ m}$$

$$v_r = 100 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 100 \text{ Hz}$$



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Pulsed Doppler Radar

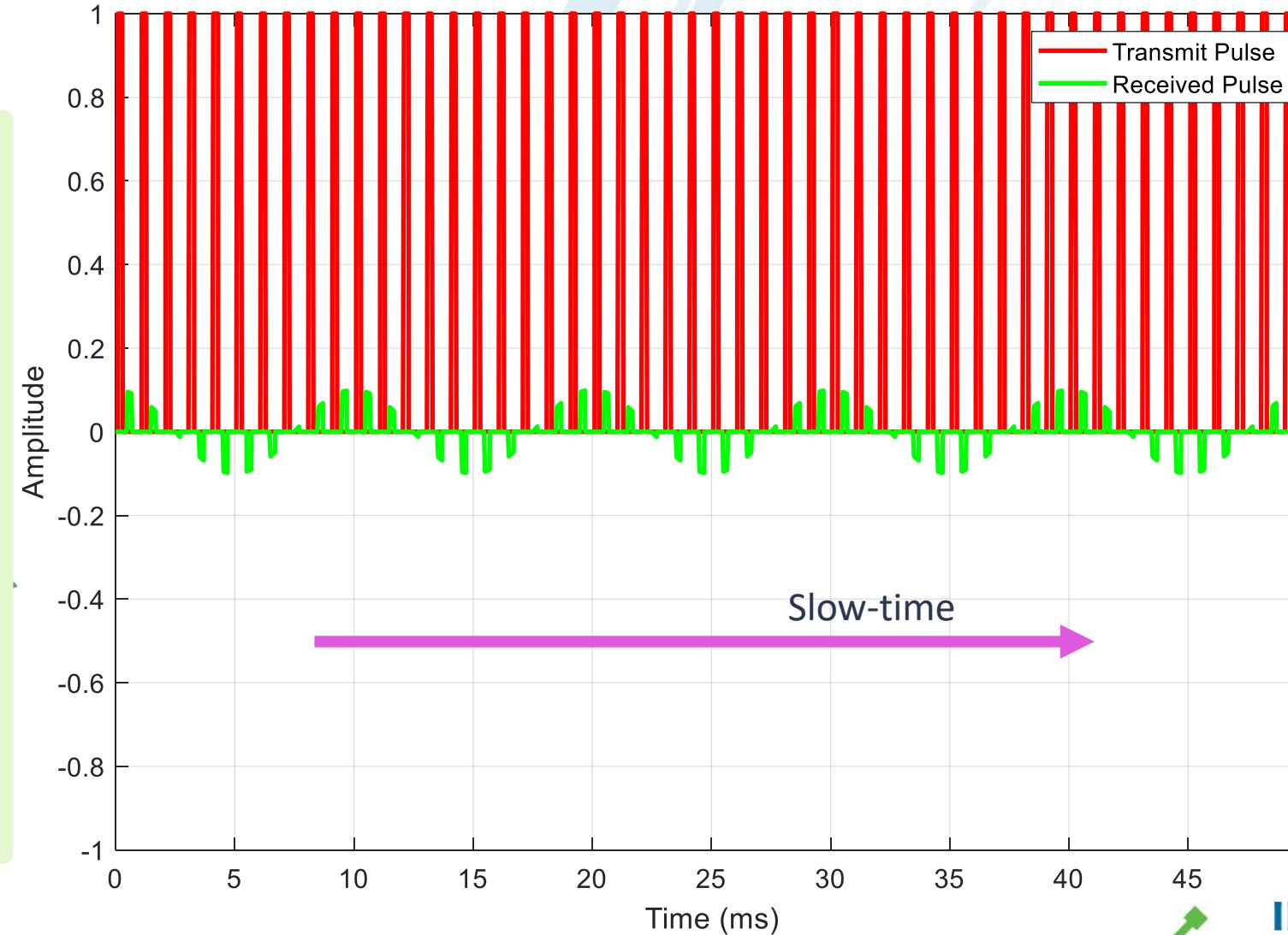
Received signal

$$f_c = 150 \text{ MHz}$$

$$\lambda = \frac{c}{f_c} = 2 \text{ m}$$

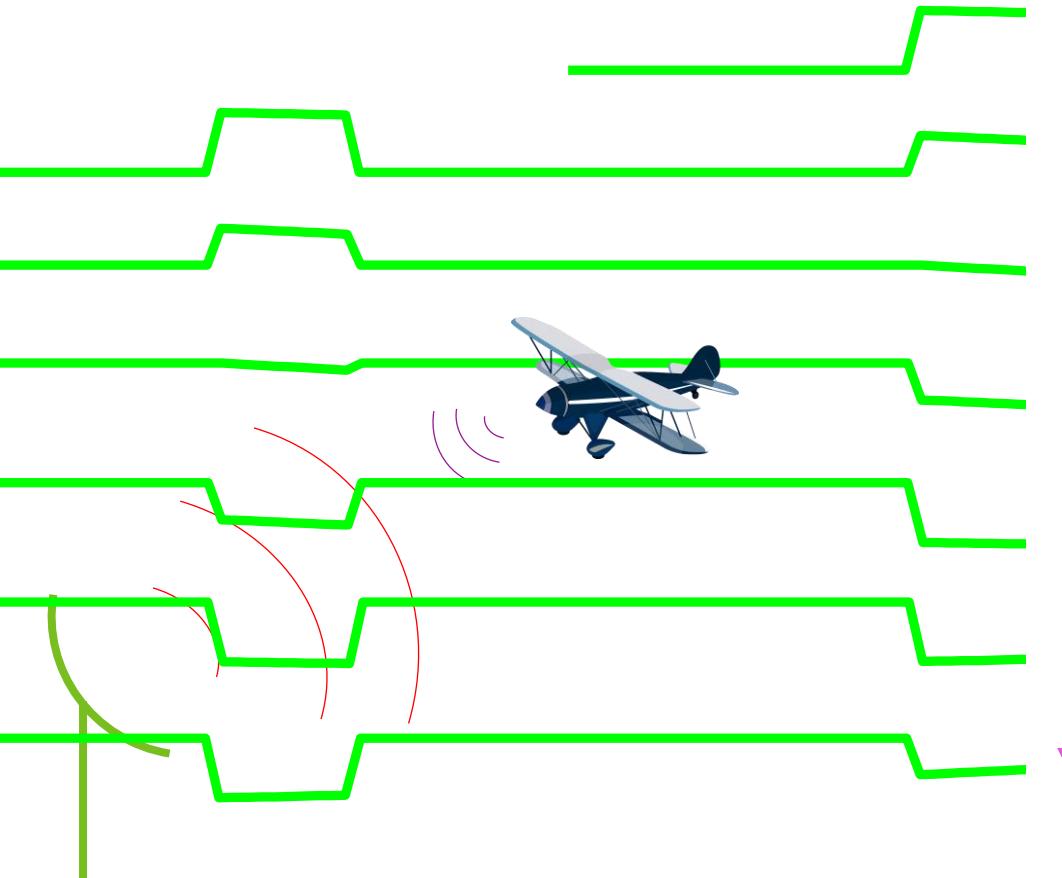
$$v_r = 100 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 100 \text{ Hz}$$

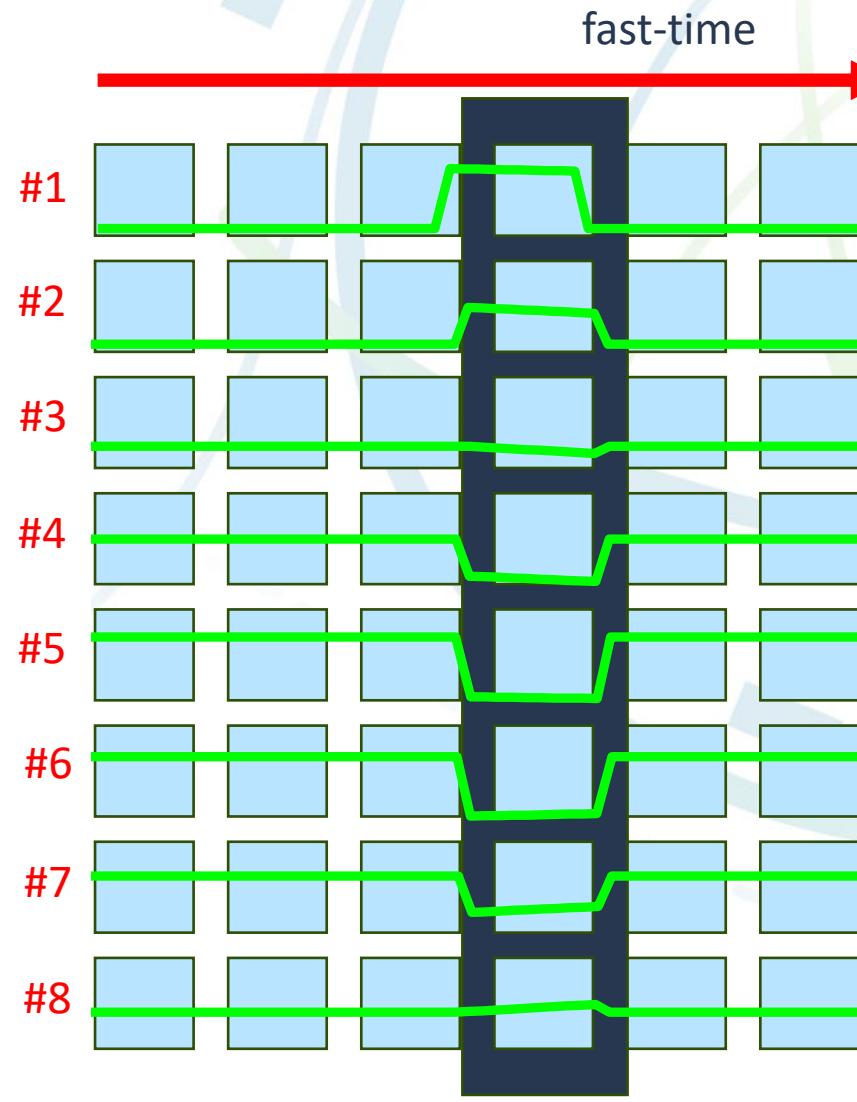


Pulsed Doppler Radar

Received signal



Slow-time



Pulsed Doppler Radar

Received signal

$$f_c = 150 \text{ MHz}$$

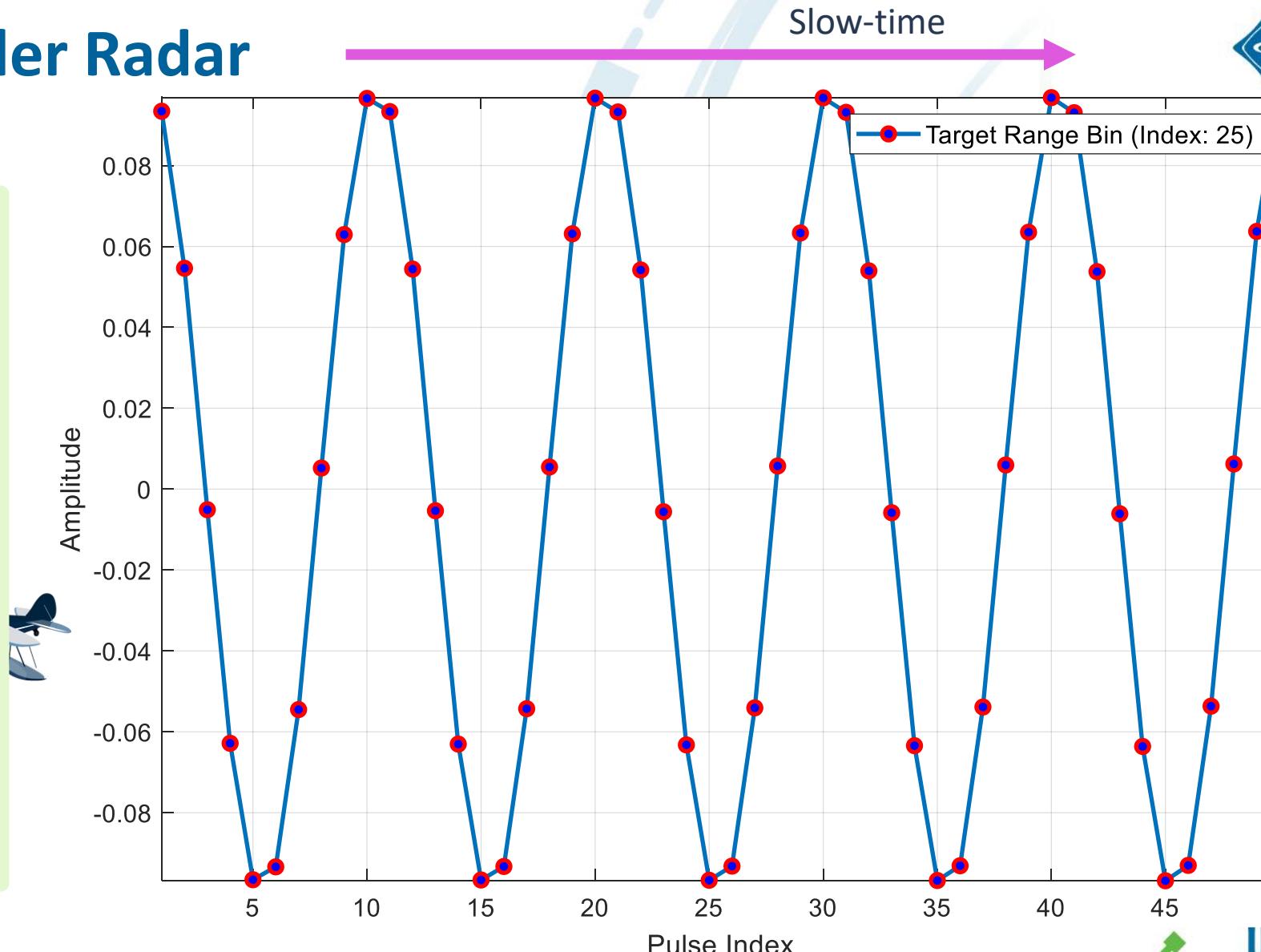
$$\lambda = \frac{c}{f_c} = 2 \text{ m}$$

$$v_r = 100 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 100 \text{ Hz}$$



Lect1_example4.m



Pulsed Doppler Radar

Received signal

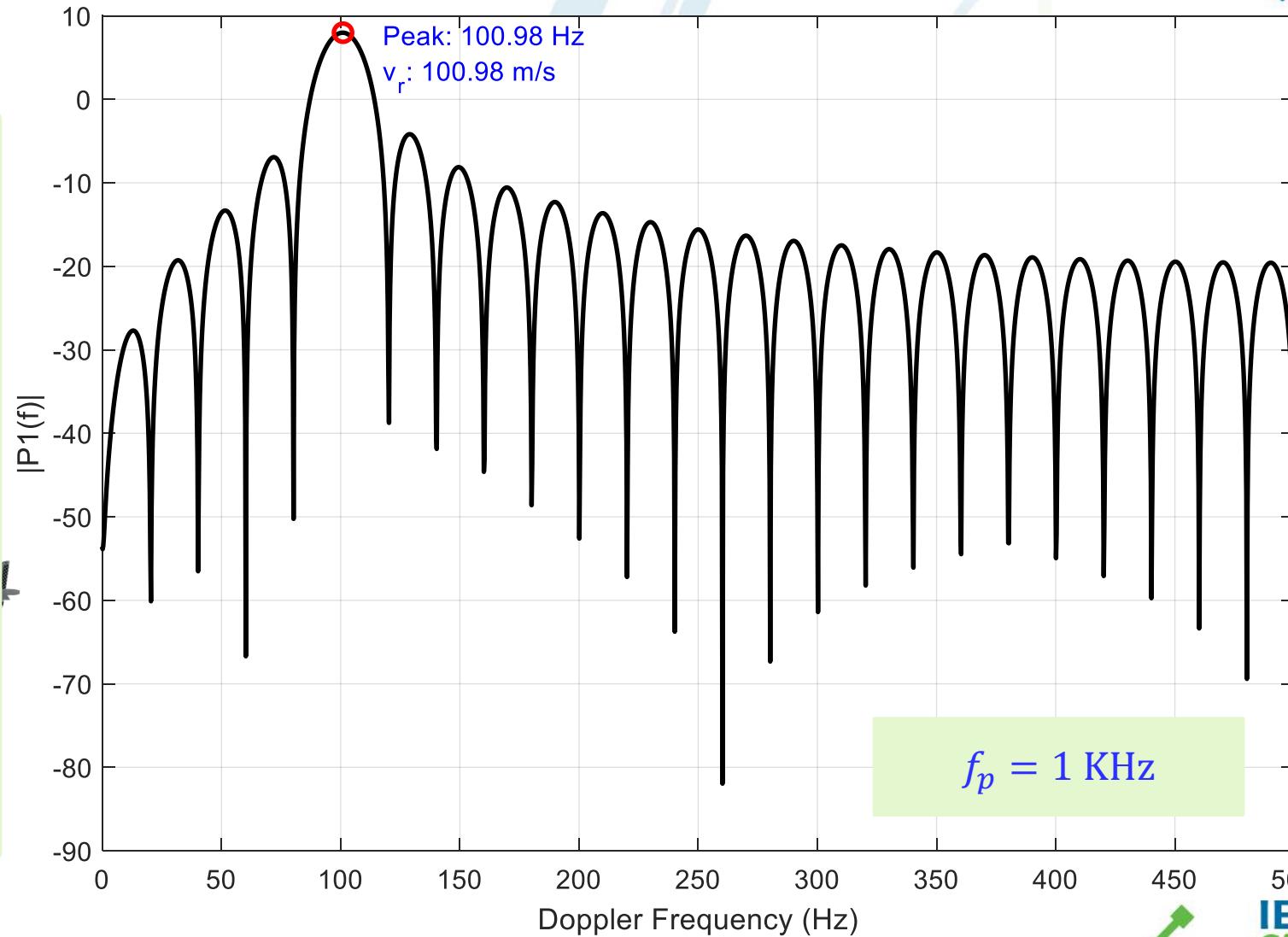
$$f_c = 150 \text{ MHz}$$

$$\lambda = \frac{c}{f_c} = 2 \text{ m}$$

$$v_r = 100 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 100 \text{ Hz}$$

Lect1_example4.m



Pulsed Doppler Radar

Received signal

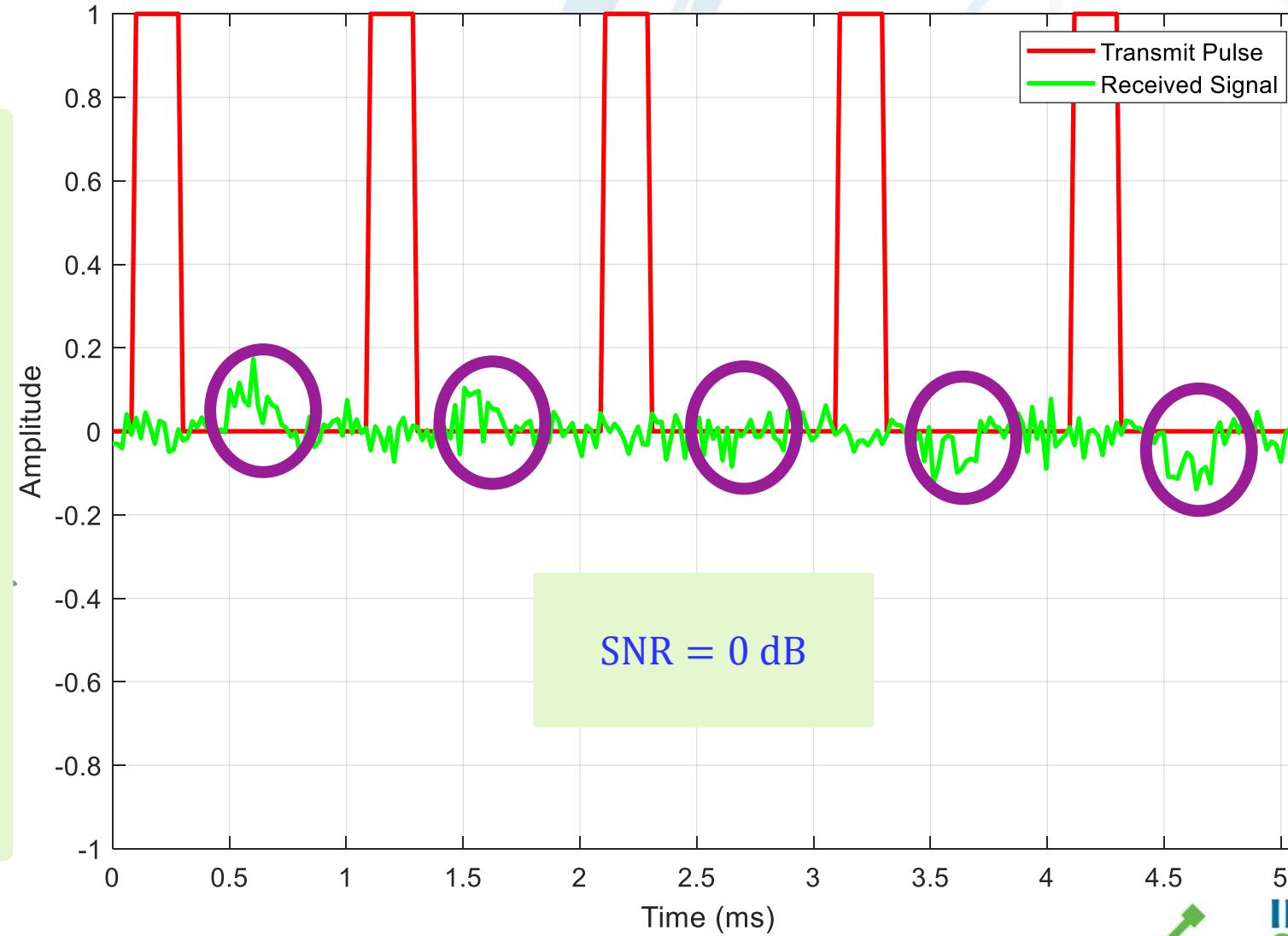
$$f_c = 150 \text{ MHz}$$

$$\lambda = \frac{c}{f_c} = 2 \text{ m}$$

$$v_r = 100 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 100 \text{ Hz}$$

Lect1_example4.m



Pulsed Doppler Radar

Received signal

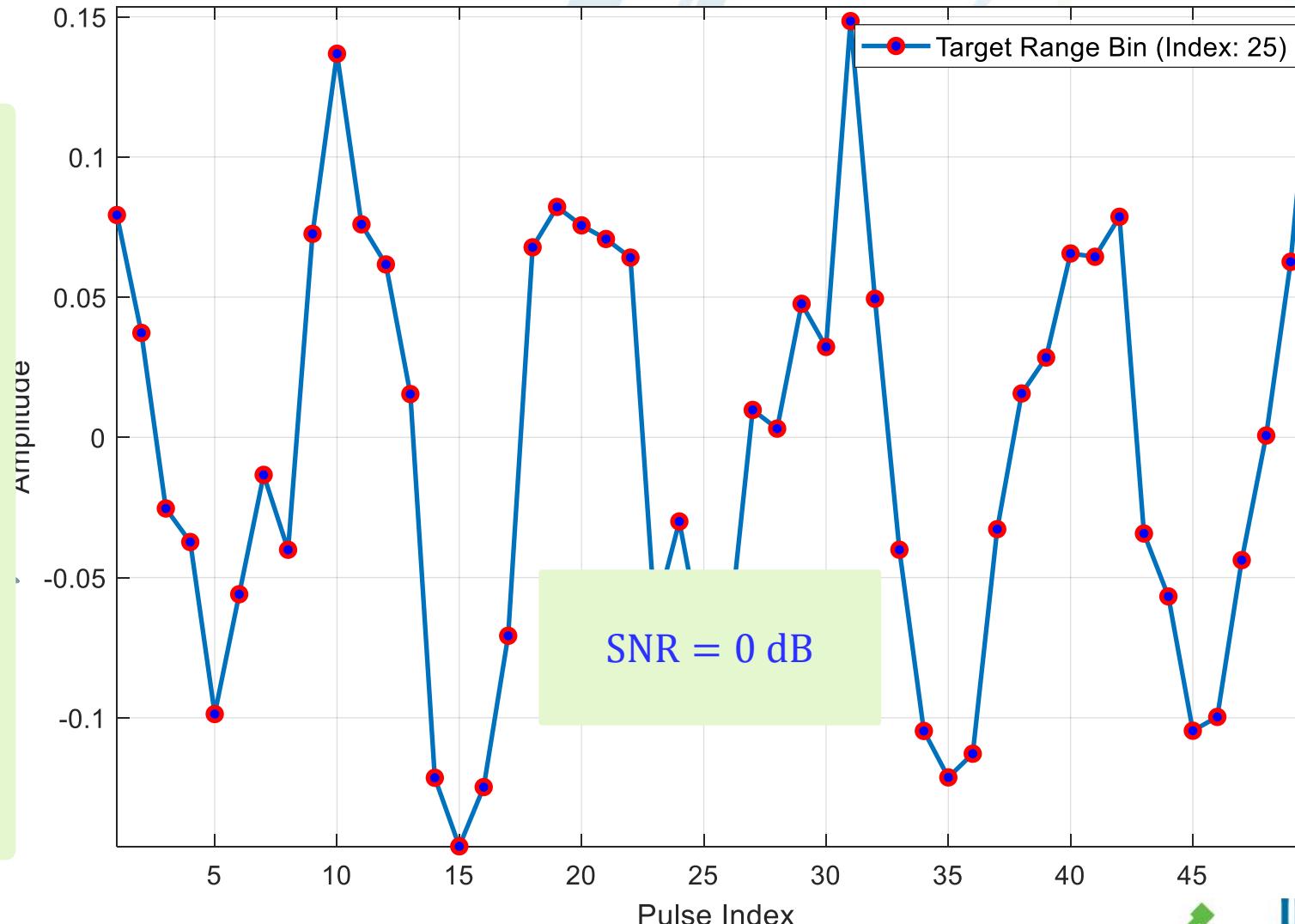
$$f_c = 150 \text{ MHz}$$

$$\lambda = \frac{c}{f_c} = 2 \text{ m}$$

$$v_r = 100 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 100 \text{ Hz}$$

Lect1_example4.m



Pulsed Doppler Radar

Received signal

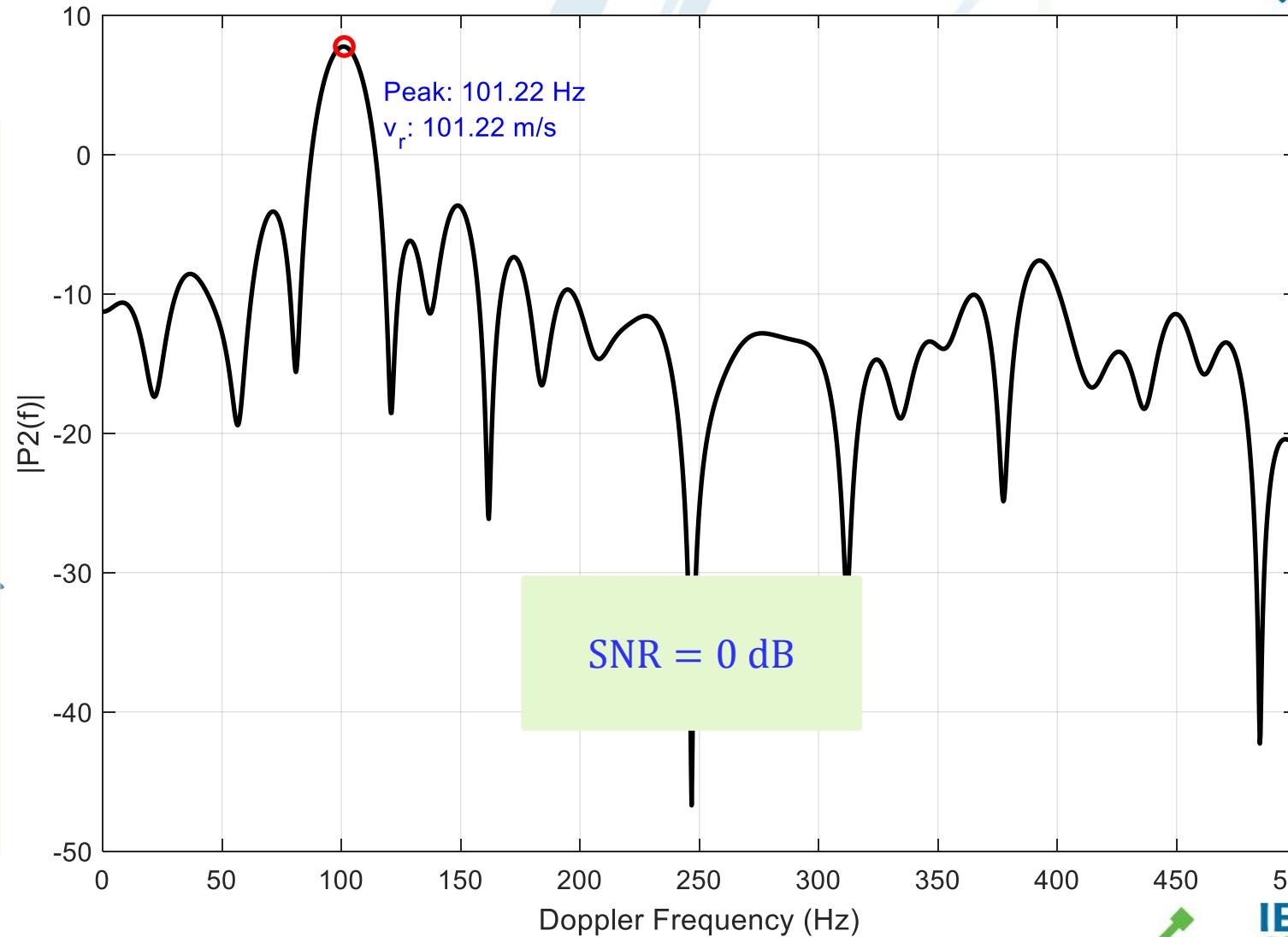
$$f_c = 150 \text{ MHz}$$

$$\lambda = \frac{c}{f_c} = 2 \text{ m}$$

$$v_r = 100 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 100 \text{ Hz}$$

Lect1_example4.m



Pulsed Doppler Radar

Doppler Sampling frequency

Doppler Sampling frequency = PRF

$$f_p = \frac{1}{T_p}$$

$$f_{d_{max}} \leq \frac{f_p}{2}$$

$$f_d = 2 \frac{v_r}{\lambda}$$

$$v_{r_{max}} = \lambda \frac{f_{d_{max}}}{2} \leq \frac{\lambda f_p}{4}$$

Increase f_p

Slow-time

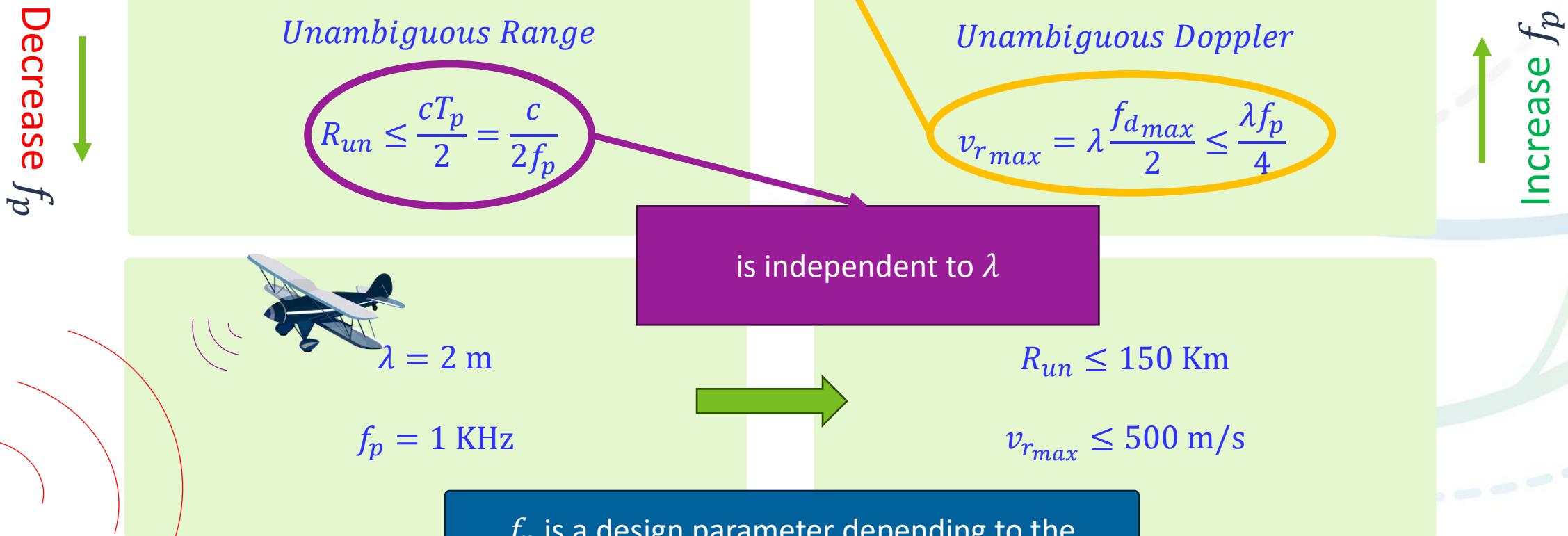
fast-time

#1
#2
#3
#4
#5
#6
#7
#8

$$PRI = T_p$$

Pulsed Doppler Radar

Unambiguous Range and Doppler

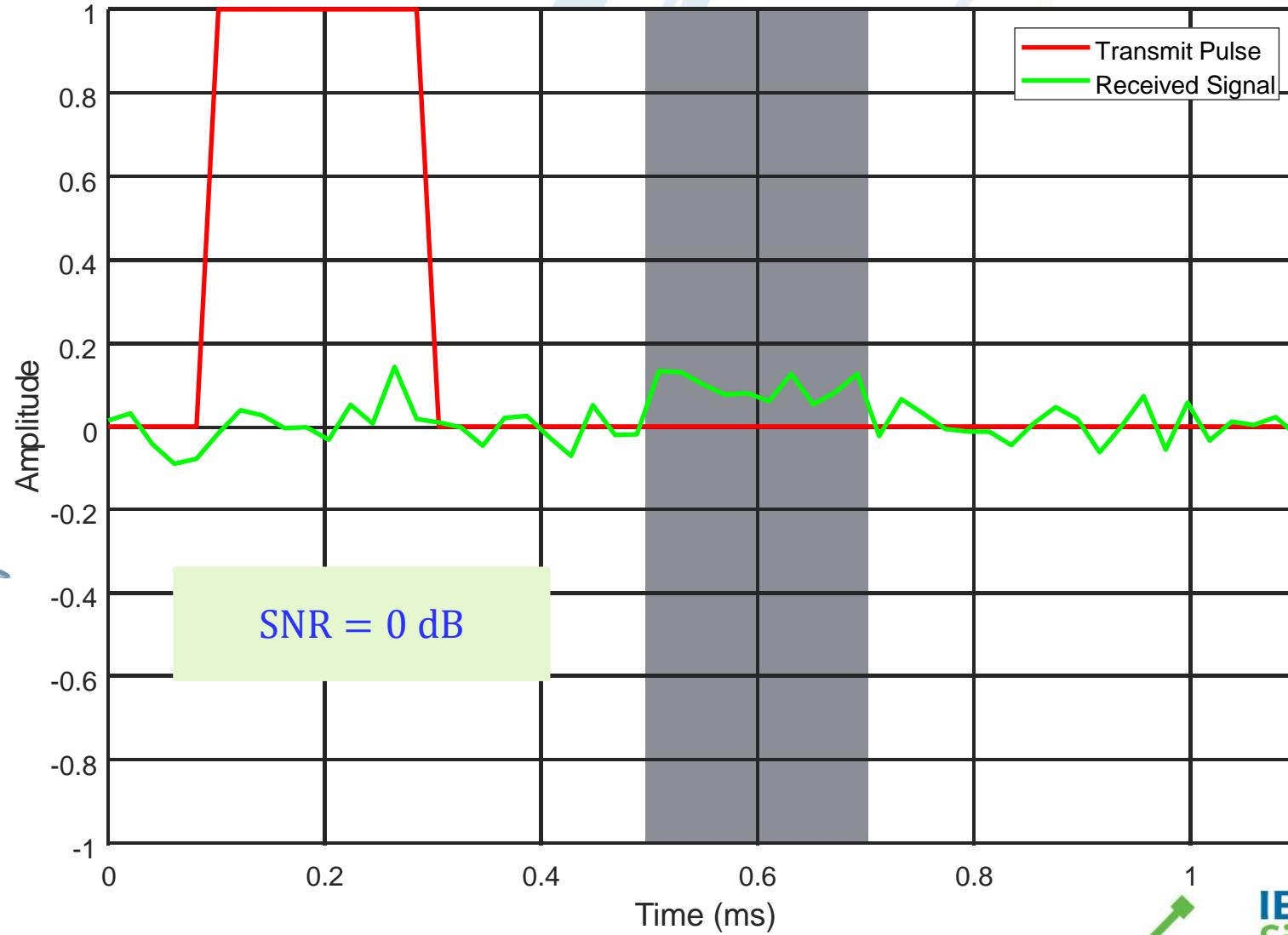


Pulsed Doppler Radar

Range Resolution

$$\tau = 200 \mu\text{s}$$

$$T_p = 1 \text{ ms}$$



Pulsed Doppler Radar

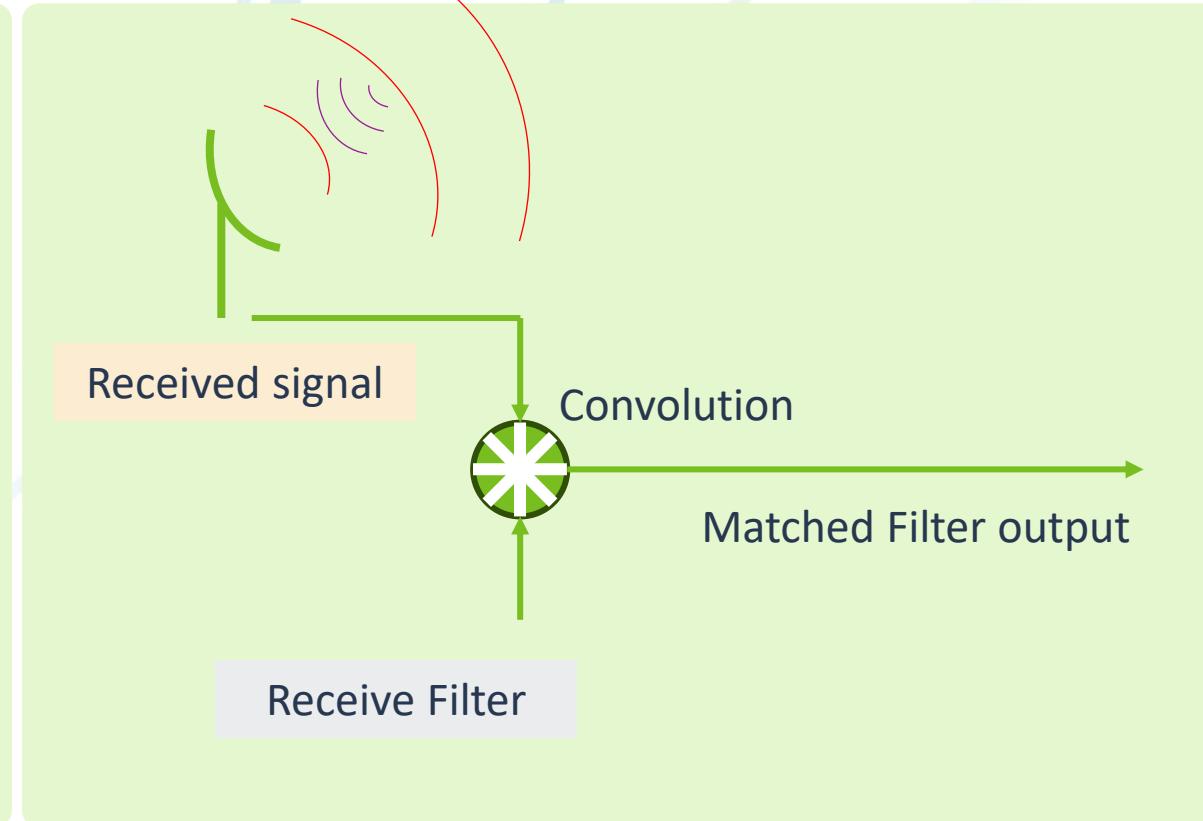
Matched Filter

$$s_{tx}[n] = A_t \text{rect} [n - (i - 1)T_p]$$

$$s_{rx}[n] = A_r \text{ rect} [n - (i - 1)T_p - \Delta n] + w(n)$$

$$h[n] = s_{tx}^*[N - 1 - n]$$

- $h[n]$ is the impulse response of the matched filter
- $s_{tx}^*[n]$ is the complex conjugate of the transmitted signal $s_{tx}[n]$
- N is the total number of samples of the transmitted signal



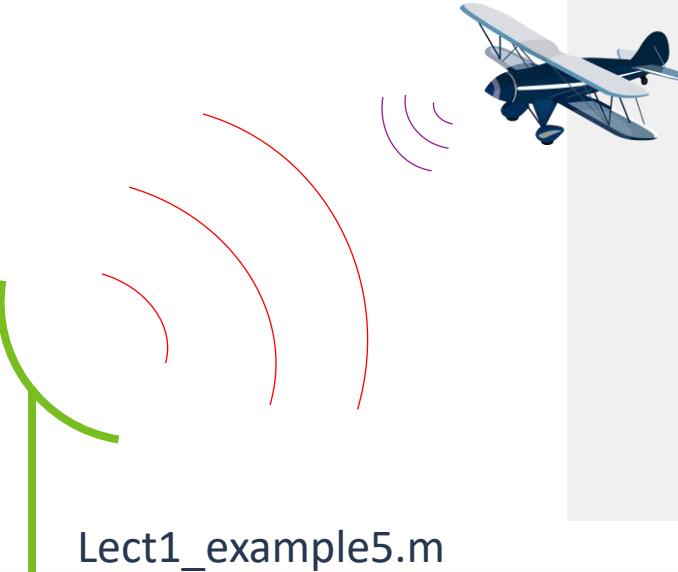
Lect1_example4.m

$$y[n] = \sum_{k=0}^{N-1} s_{rx}[k] h[n - k] = \sum_{k=0}^{N-1} s_{rx}[k] s_{tx}^*[N - 1 - (n - k)]$$

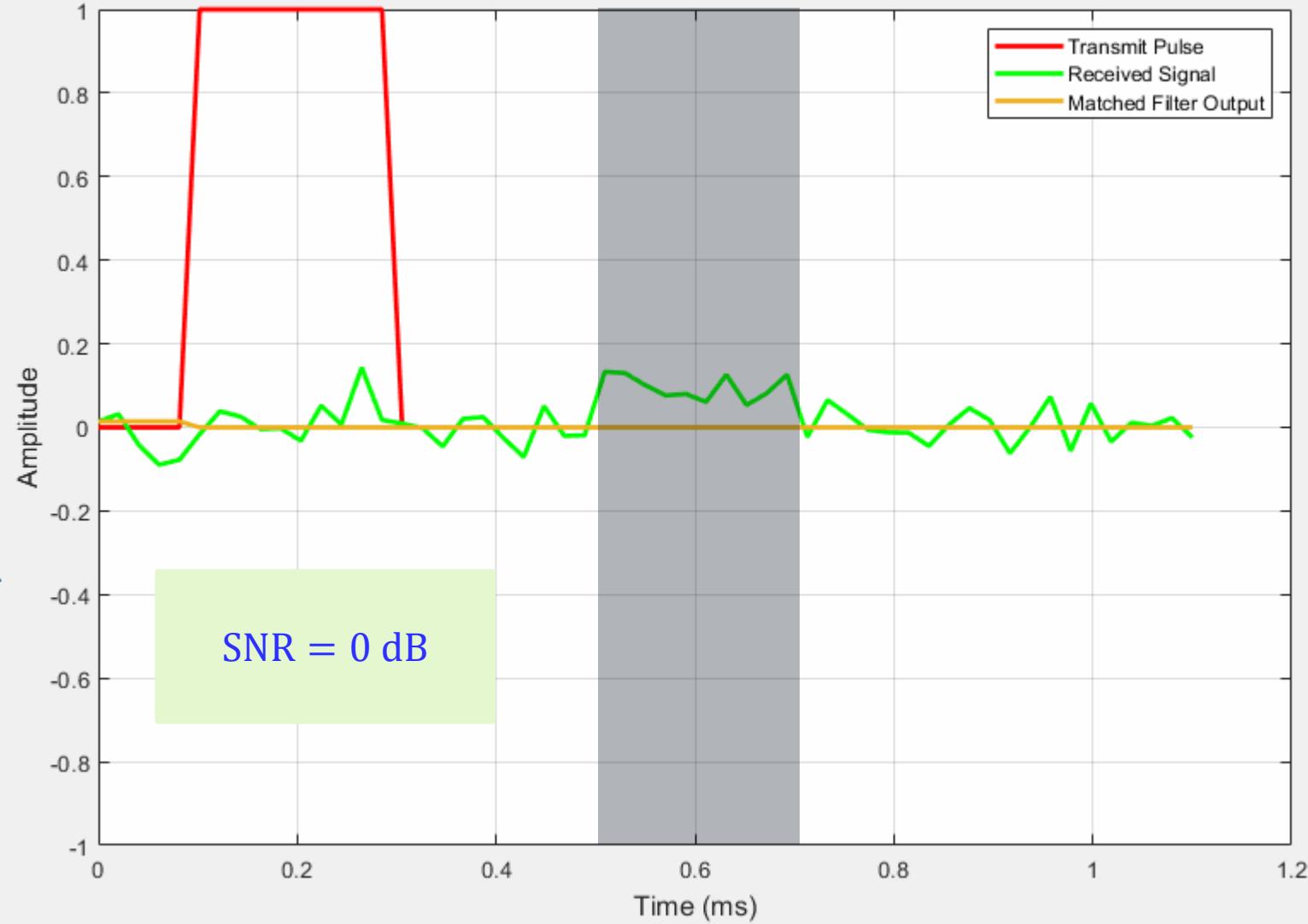
Pulsed Doppler Radar

Matched Filter

How close the second target could be?



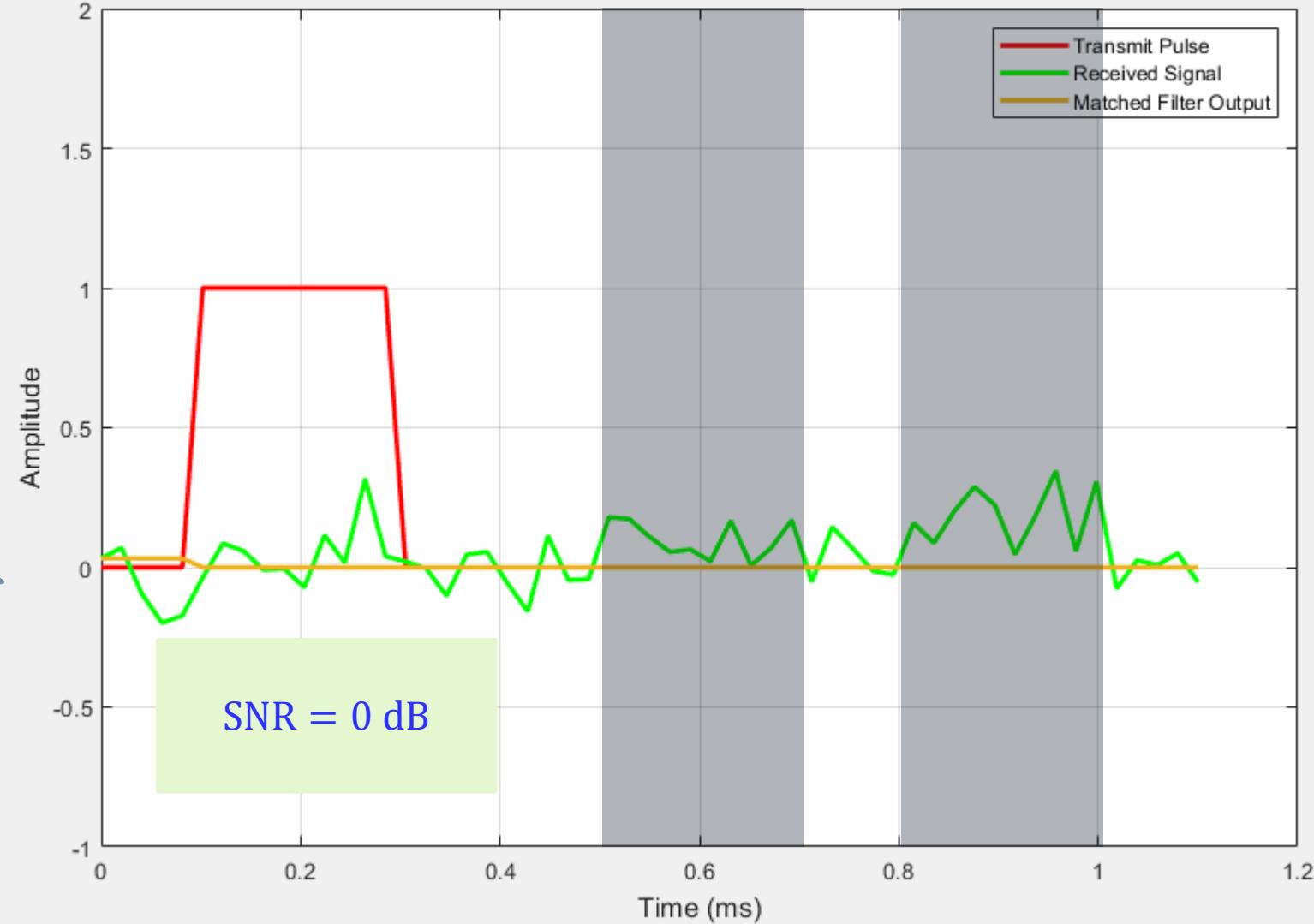
Lect1_example5.m



Pulsed Doppler Radar

Matched Filter

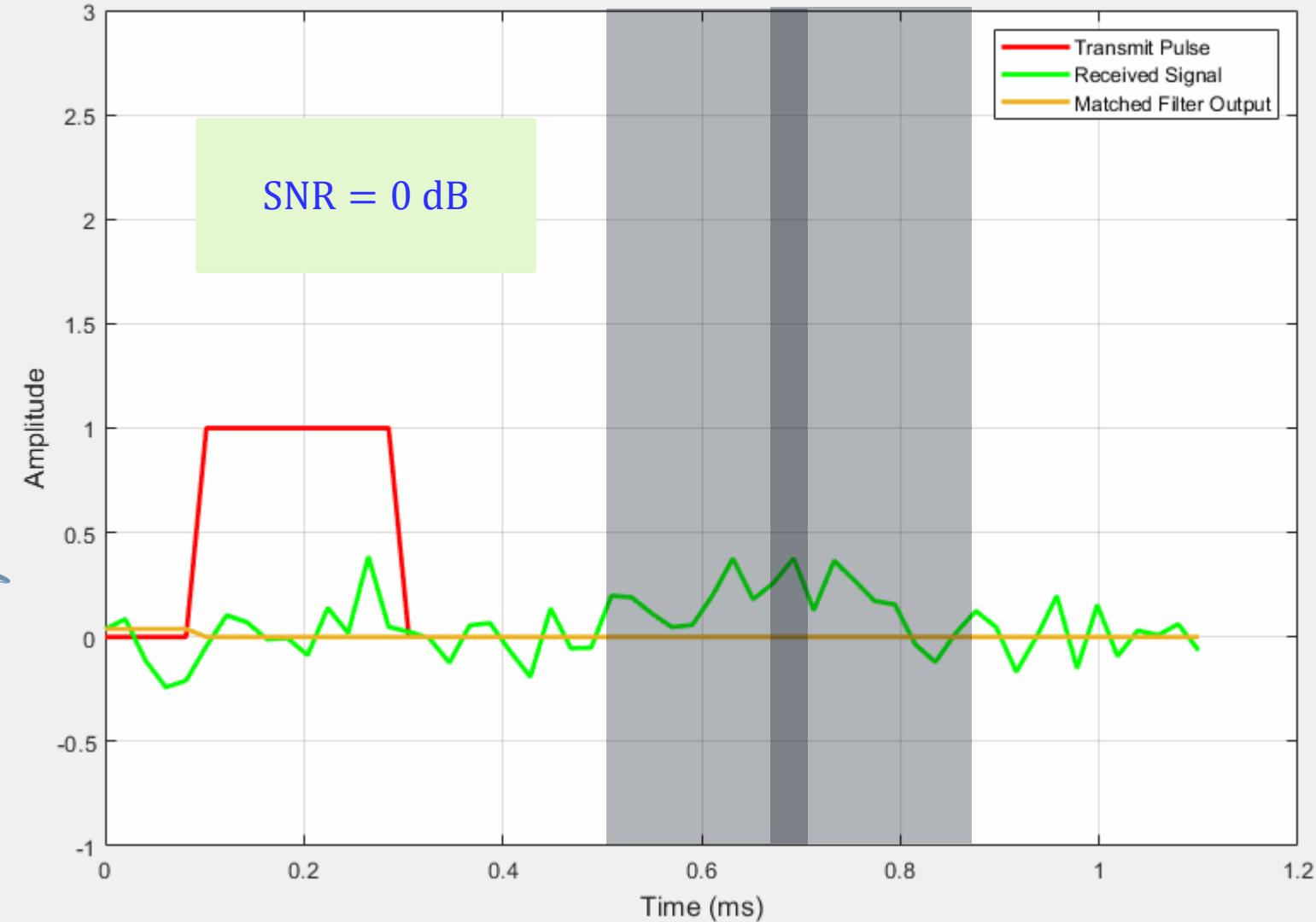
How close the second target could be?



Pulsed Doppler Radar

Matched Filter

How close the second target could be?



Pulsed Doppler Radar

Matched Filter

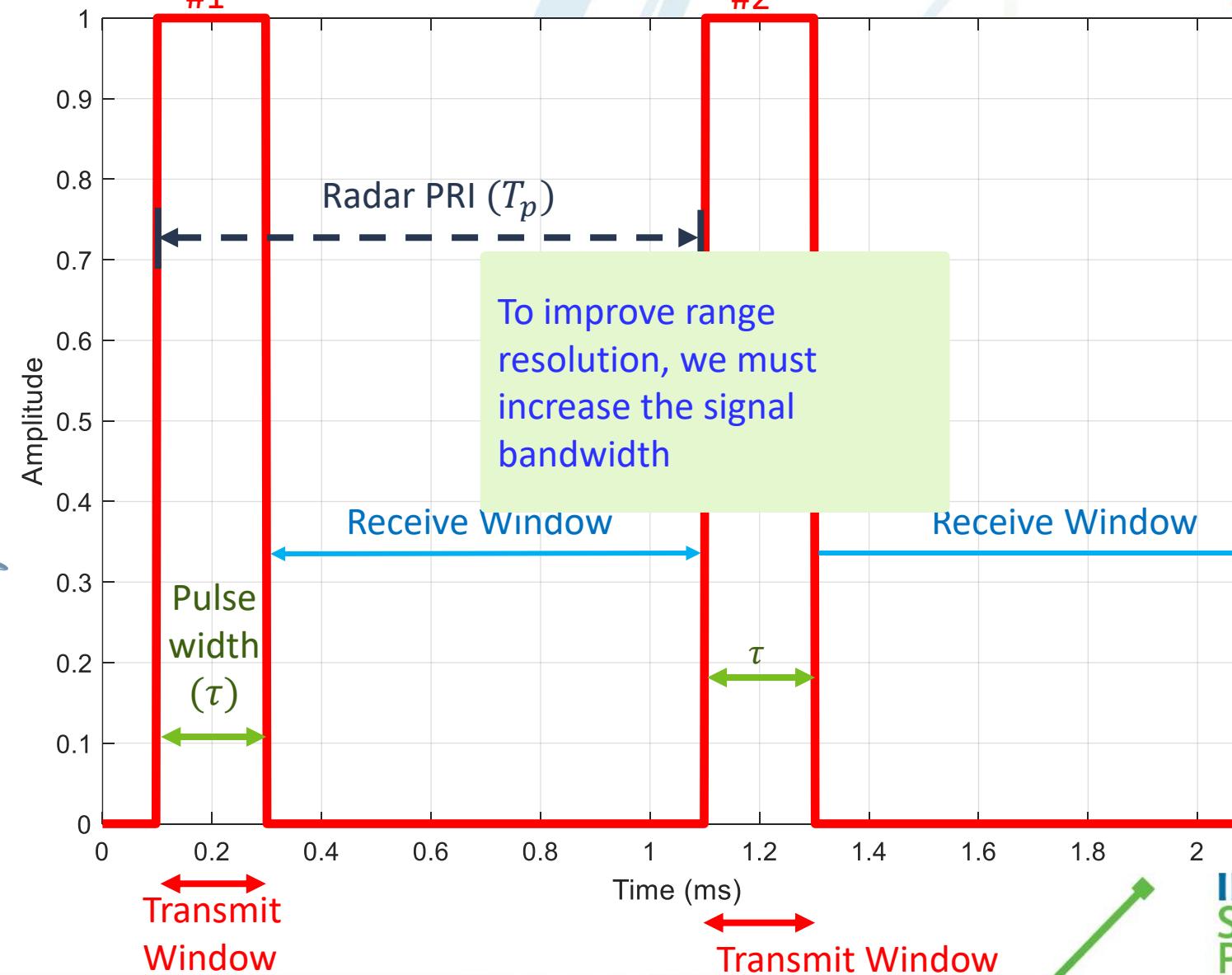
$$\text{Range resolution } \Delta R = \frac{c\tau}{2}$$

$$B \approx \frac{1}{\tau}$$

$$\Delta R = \frac{c}{2B}$$



Lect1_example5.m



Pulse Compression Radar

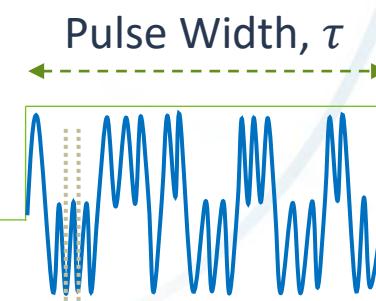
intra-pulse modulation

Square Pulse



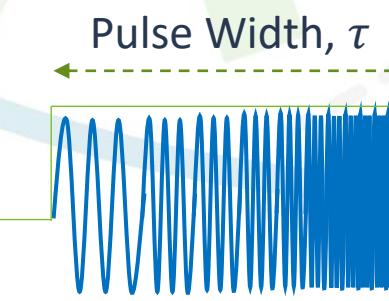
$$\text{Bandwidth} \approx \frac{1}{\tau}$$
$$\text{Time} \times \text{Bandwidth} = 1$$

Phase Coded Waveform



$$\text{Bandwidth} \approx \frac{1}{T_{chip}}$$
$$\text{Time} \times \text{Bandwidth} = \frac{T}{T_{chip}}$$

Linear Frequency Modulated (LFM) Waveform



$$\text{Bandwidth} = \Delta F = F_2 - F_1$$

$$\text{Time} \times \text{Bandwidth} = \tau \times \Delta F$$

Pulse Compression Radar

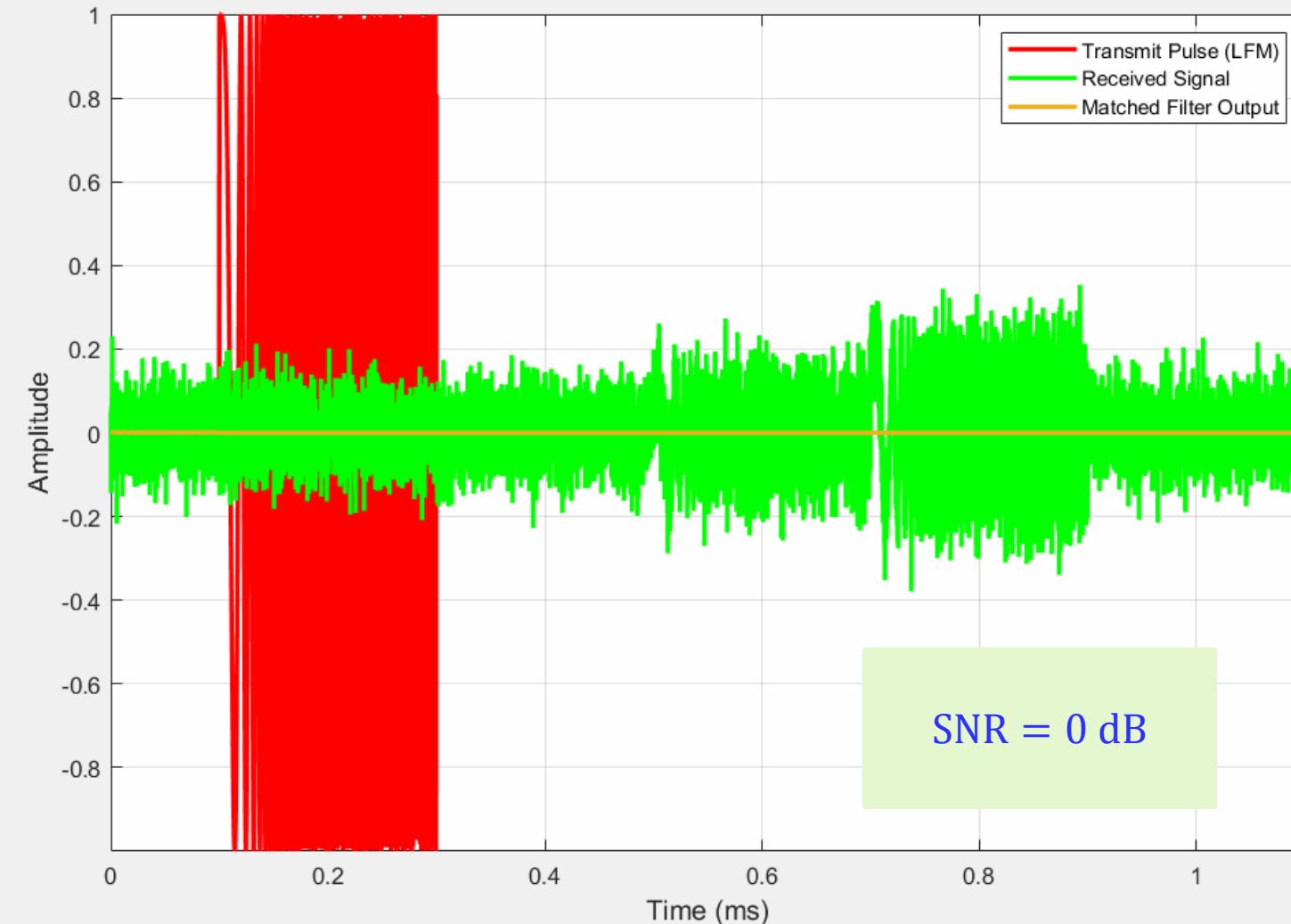
LFM Waveform

$$B = 1 \text{ MHz}$$

$$\Delta R = \frac{c}{2B} = 150 \text{ m}$$



Lect1_example6.m

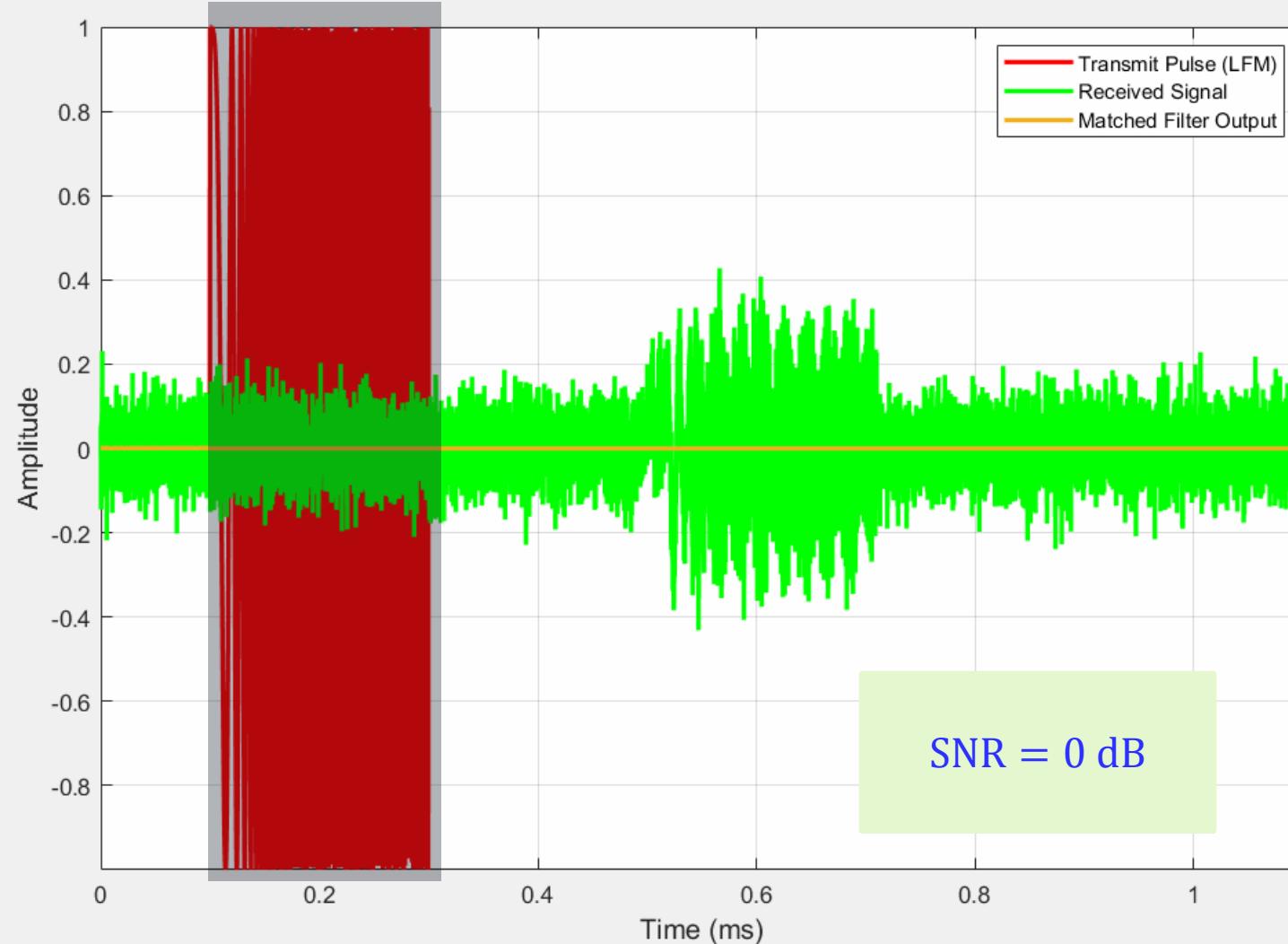


Pulse Compression Radar

LFM Waveform

$$B = 1 \text{ MHz}$$

$$\Delta R = \frac{c}{2B} = 150 \text{ m}$$



LFM Waveform

Slope

$$B = 1 \text{ MHz}$$

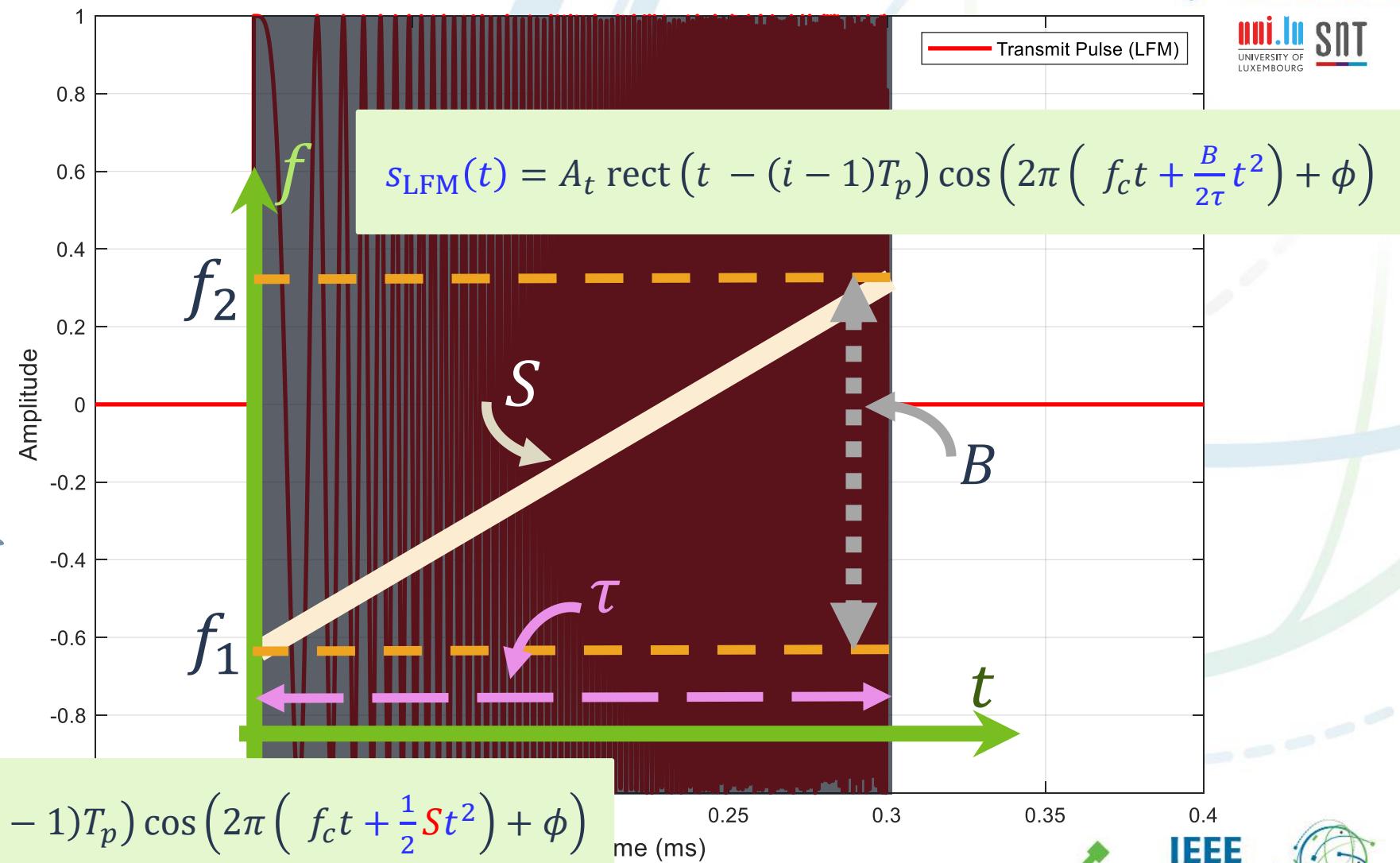
$$\Delta R = \frac{c}{2B} = 150 \text{ m}$$

$$S = \frac{B}{\tau}$$



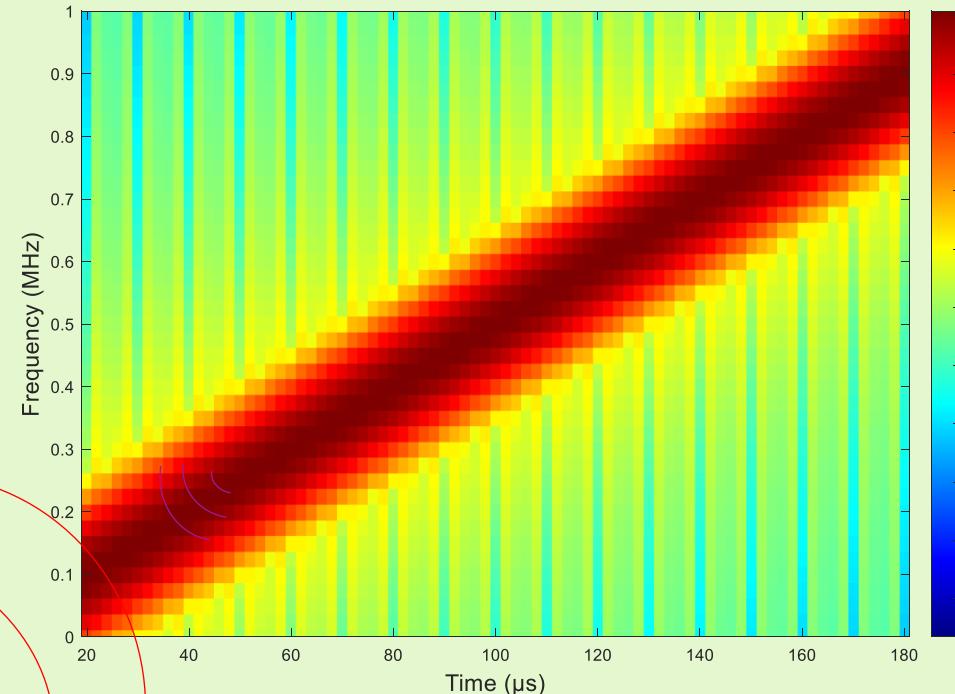
$$s_{\text{LFM}}(t) = A_t \text{rect}(t - (i - 1)T_p) \cos\left(2\pi\left(f_c t + \frac{1}{2}St^2\right) + \phi\right)$$

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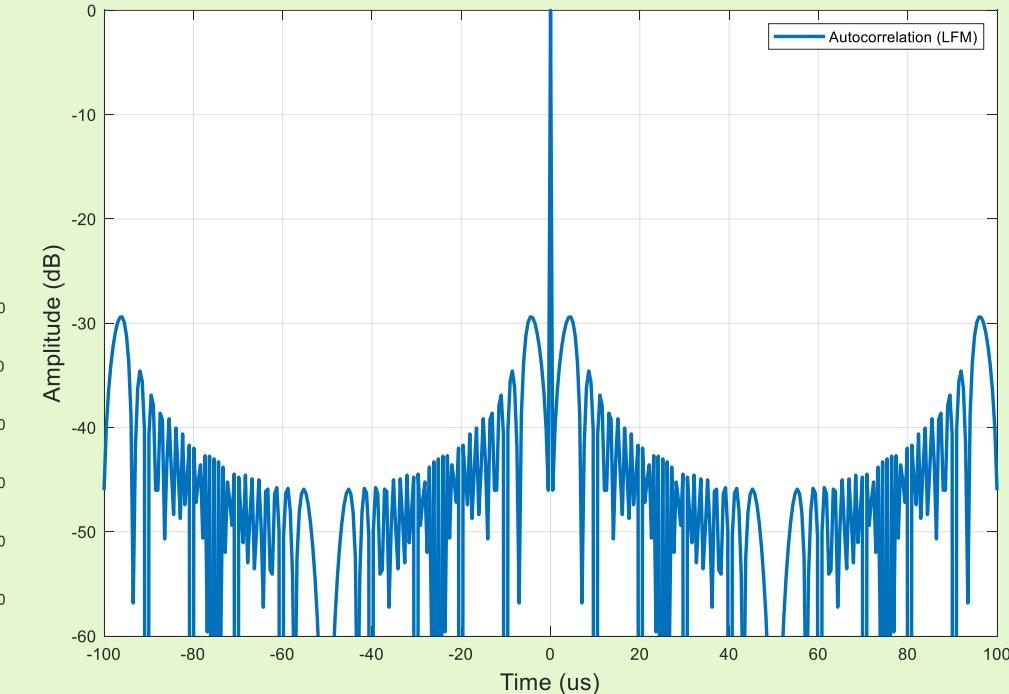


LFM Waveform

Spectrogram and Autocorrelation



Spectrogram



Autocorrelation

Lect1_example7.m

LFM Waveform

Ambiguity function

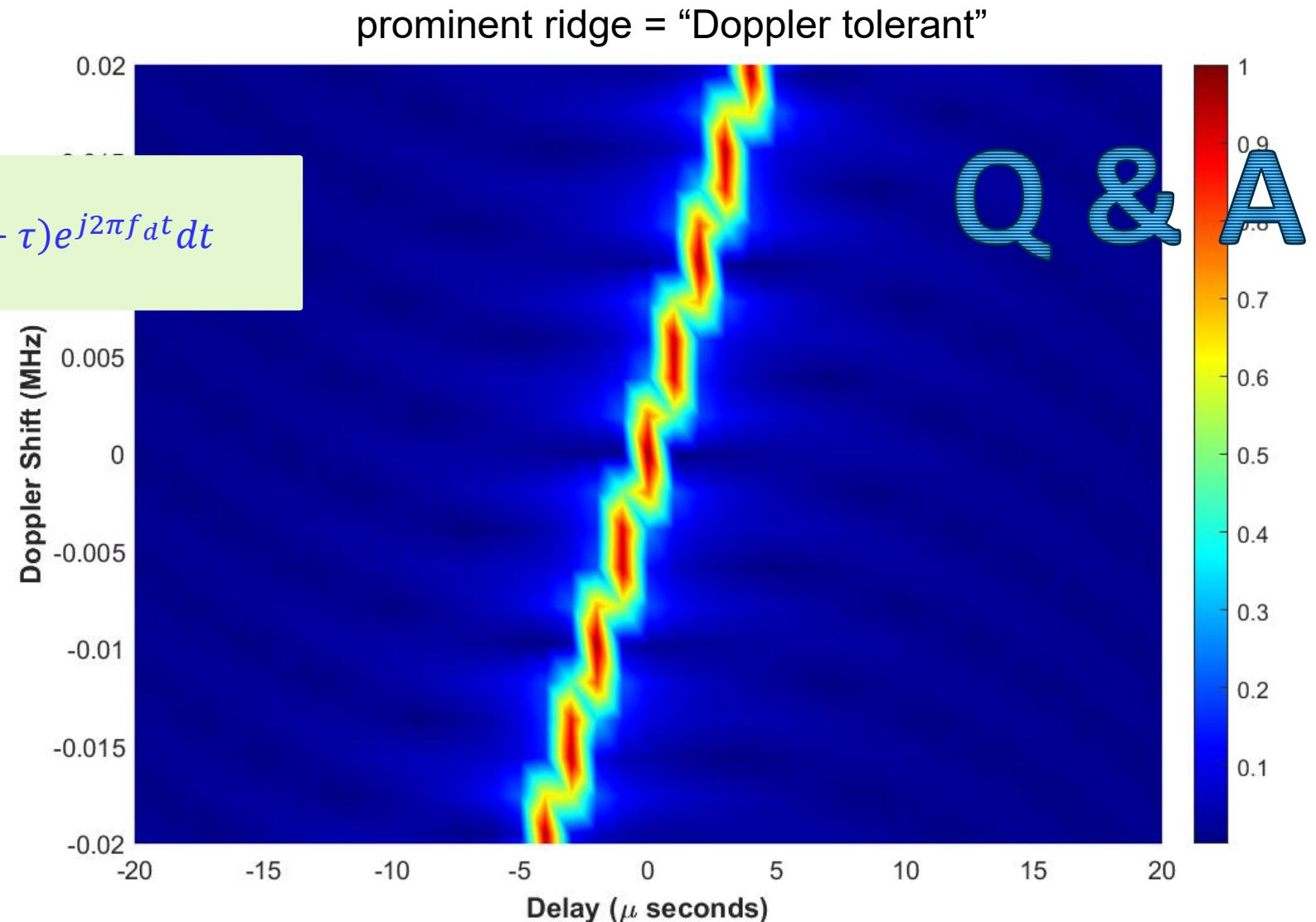
$$\chi(\tau, f_d) = \int_{-\infty}^{+\infty} s_{tx}(t)s^*(t - \tau)e^{j2\pi f_d t} dt$$

$$f_c = 10 \text{ GHz}$$

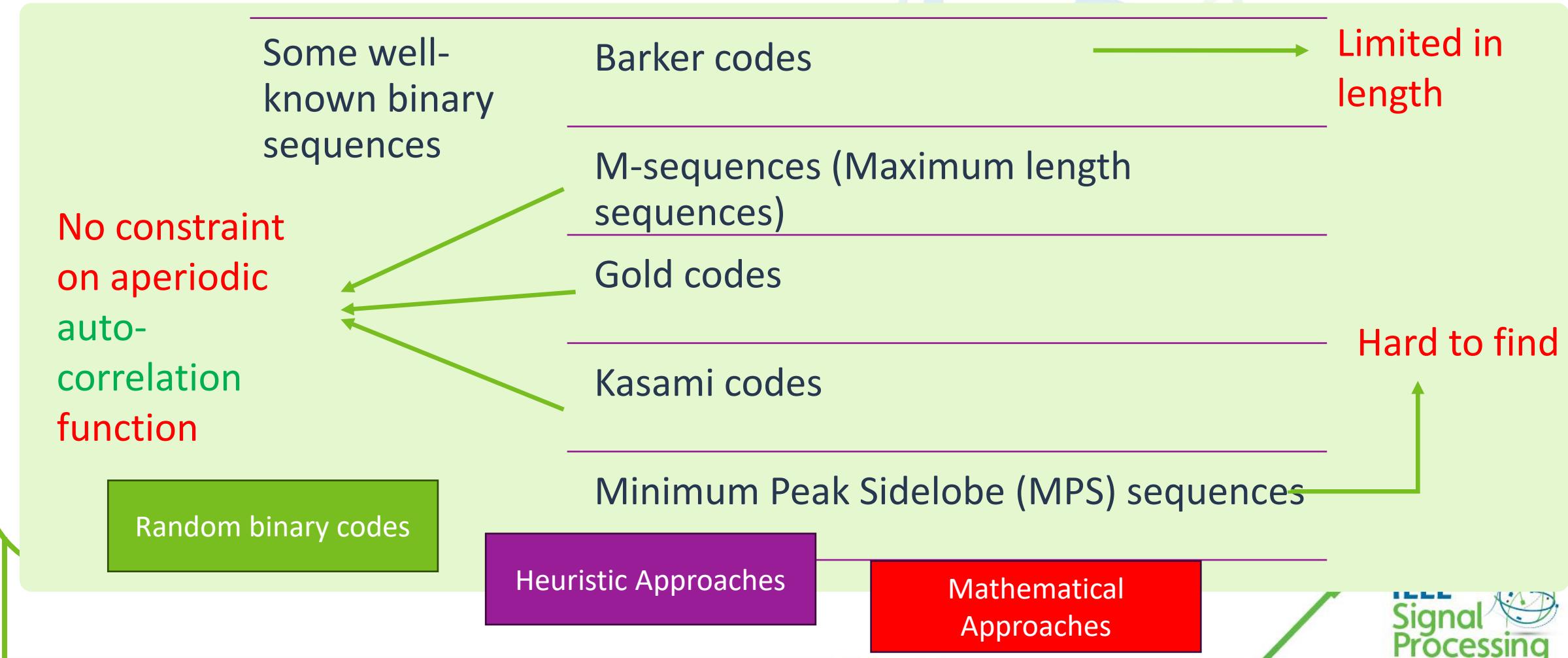
$$\lambda = 0.03 \text{ m}$$

$$v_r = 300 \text{ m/s}$$

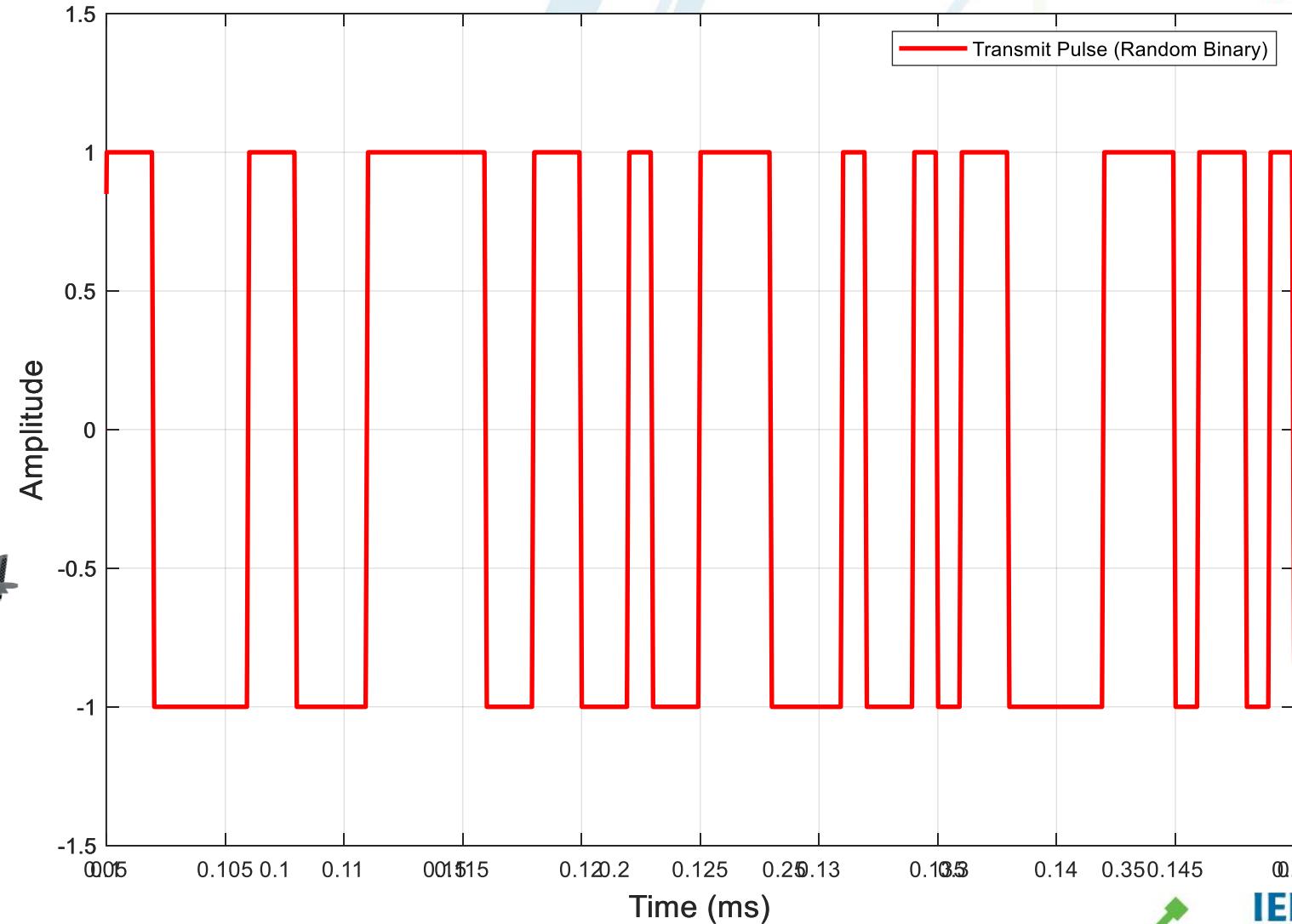
$$f_d = 2 \frac{v_r}{\lambda} = 20 \text{ KHz}$$



Binary Phase Codes

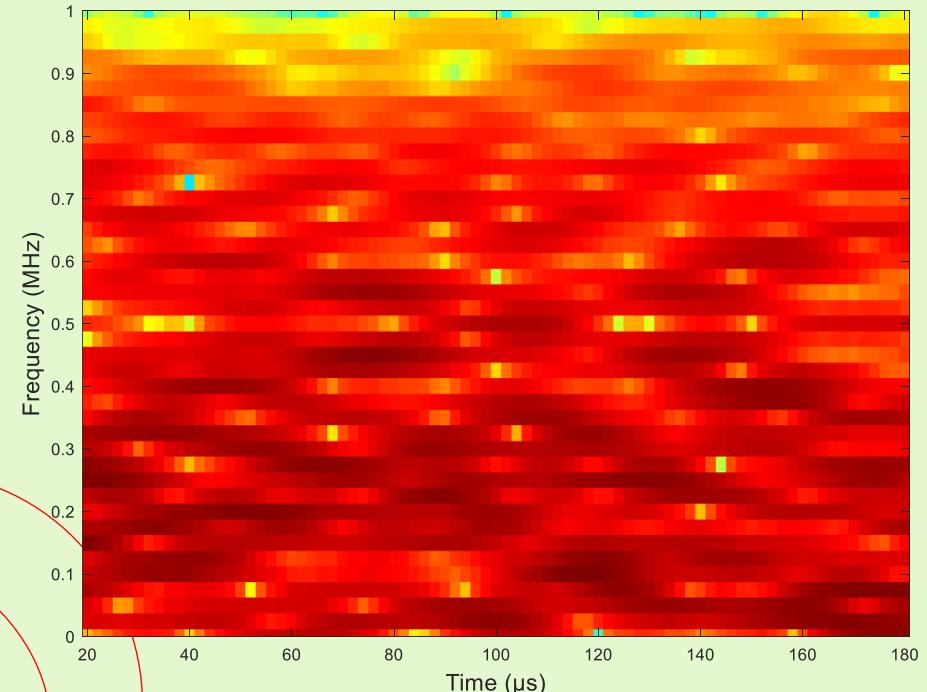


Random Binary Phase Code

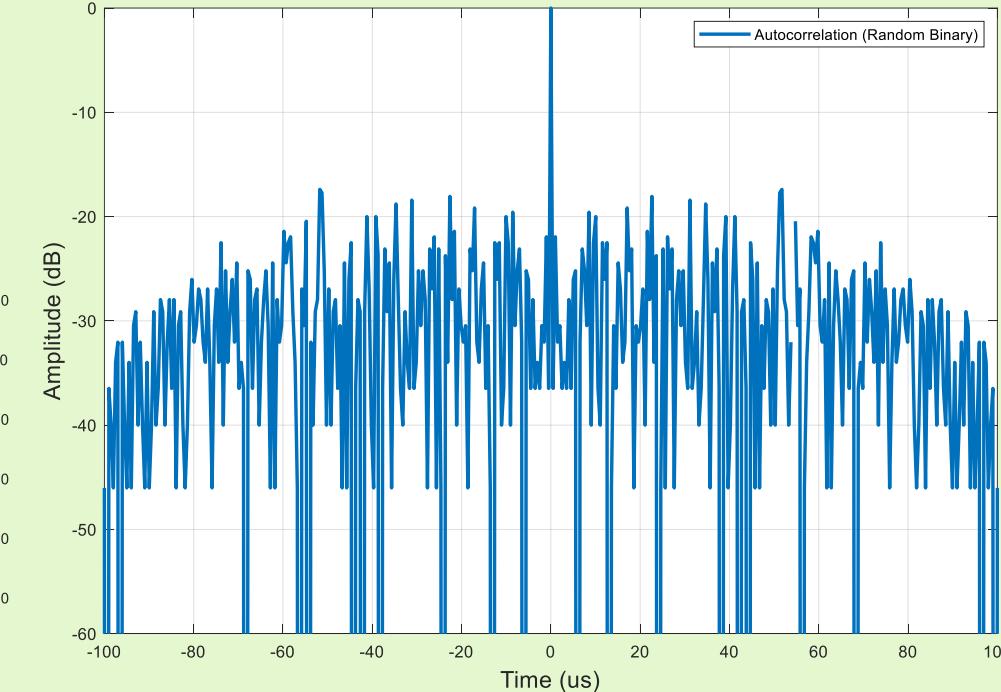


Random Binary Phase Code

Spectrogram and Autocorrelation



Spectrogram



Autocorrelation

Lect1_example8.m

Random Binary Phase Code

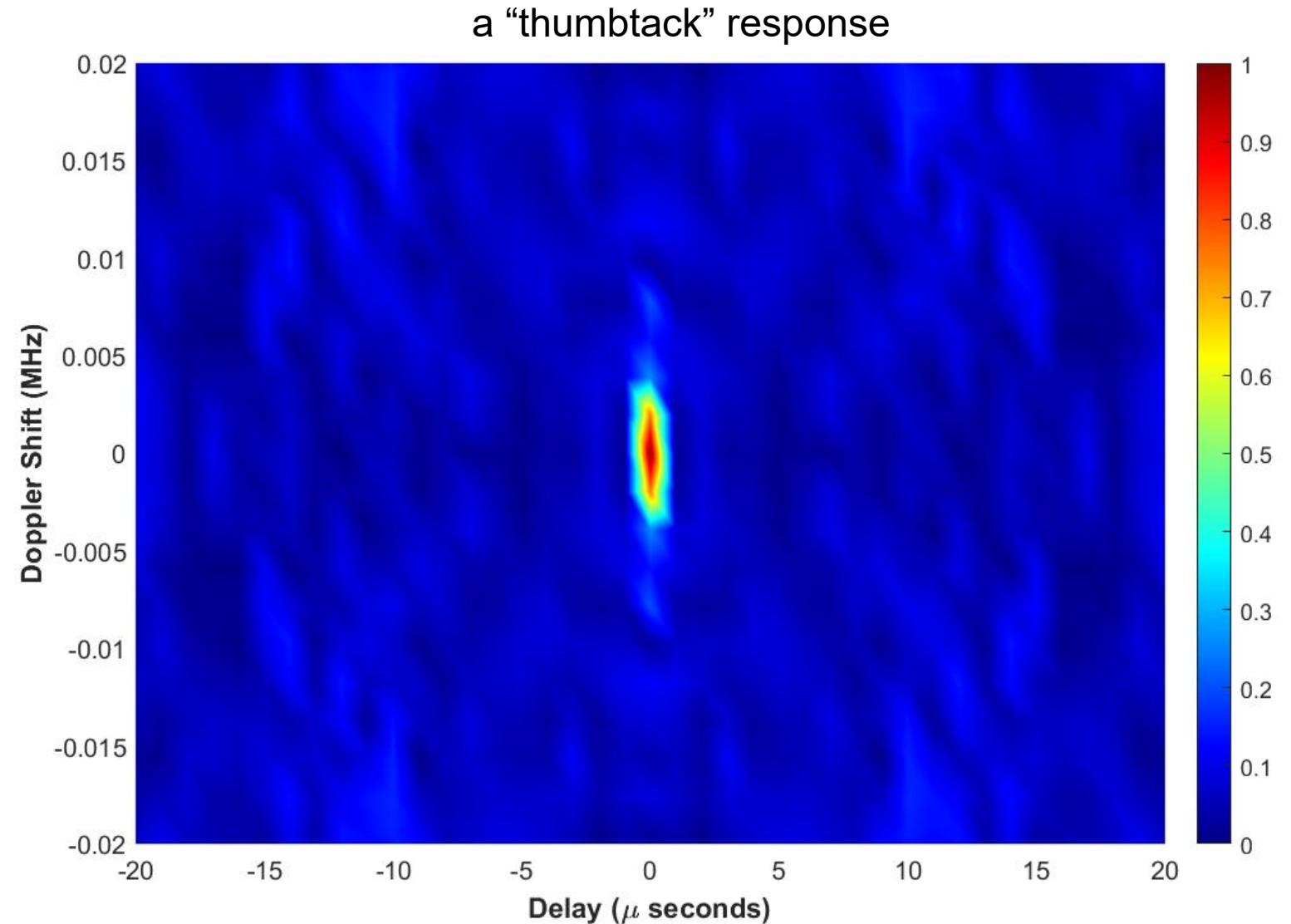
Ambiguity function

$$f_c = 10 \text{ GHz}$$

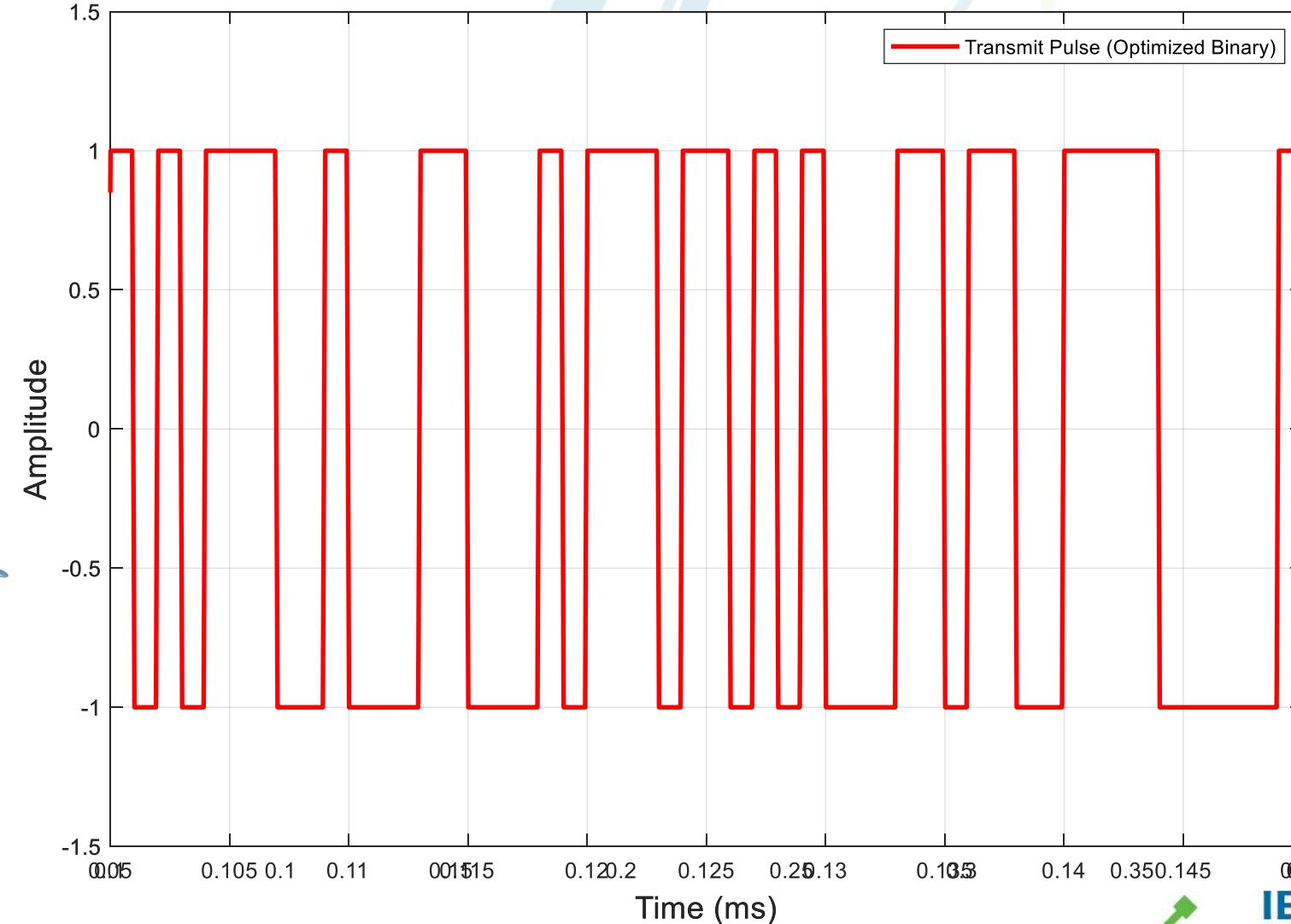
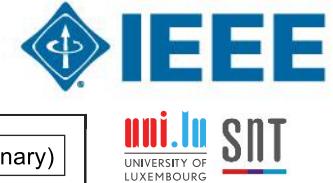
$$\lambda = 0.03 \text{ m}$$

$$v_r = 300 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 20 \text{ KHz}$$



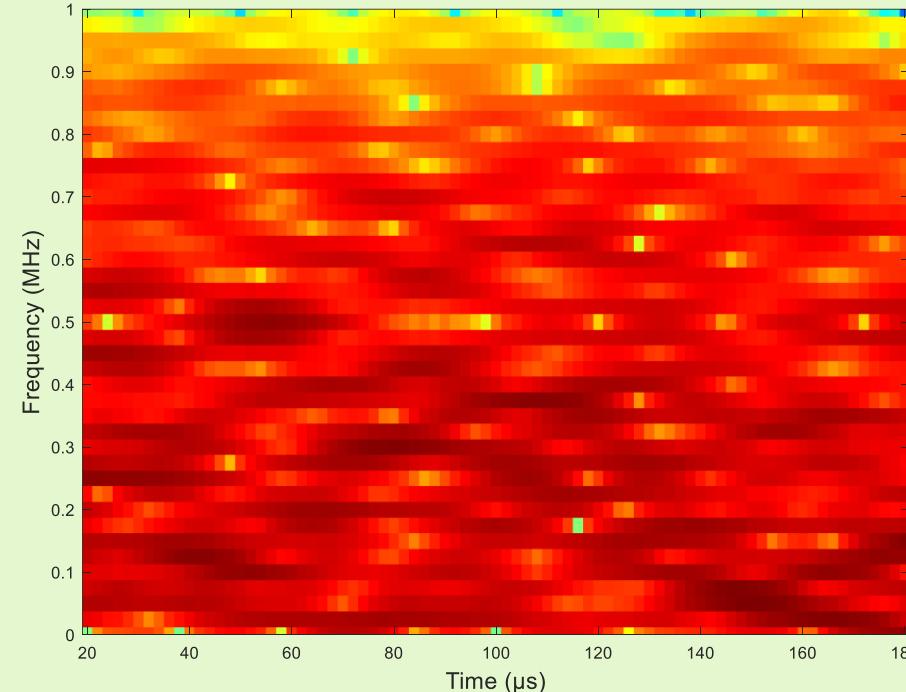
Optimized Binary Code – Mathematical Approaches



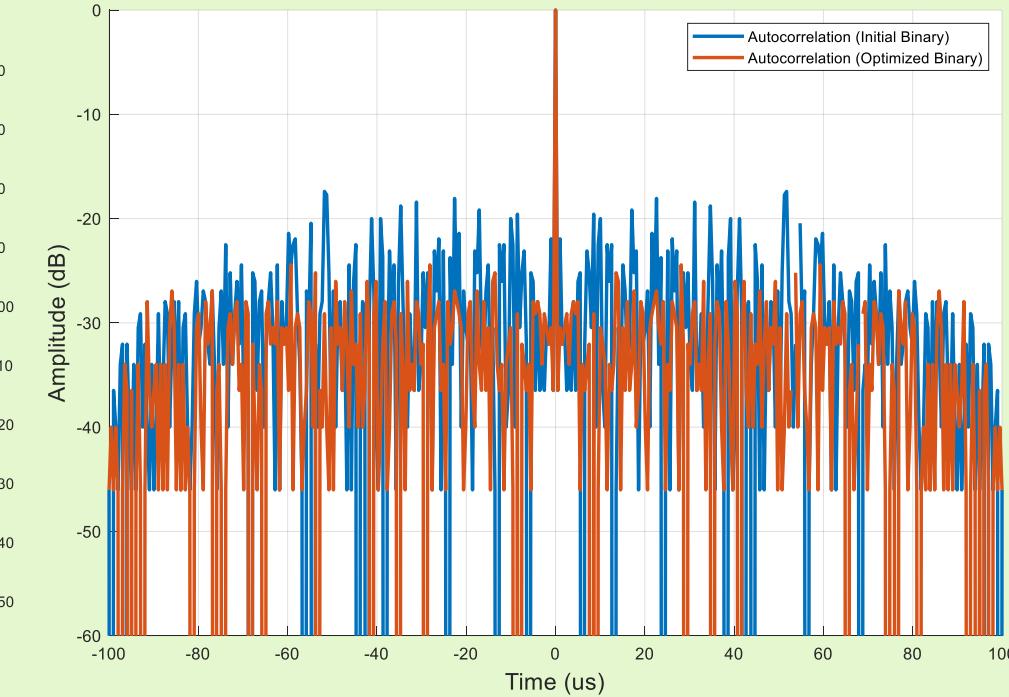
Lect1_example9.m

Optimized Binary Code – Mathematical Approaches

Spectrogram and Autocorrelation



Spectrogram



Autocorrelation

Lect1_example9.m

Optimized Binary Code – Mathematical Approaches

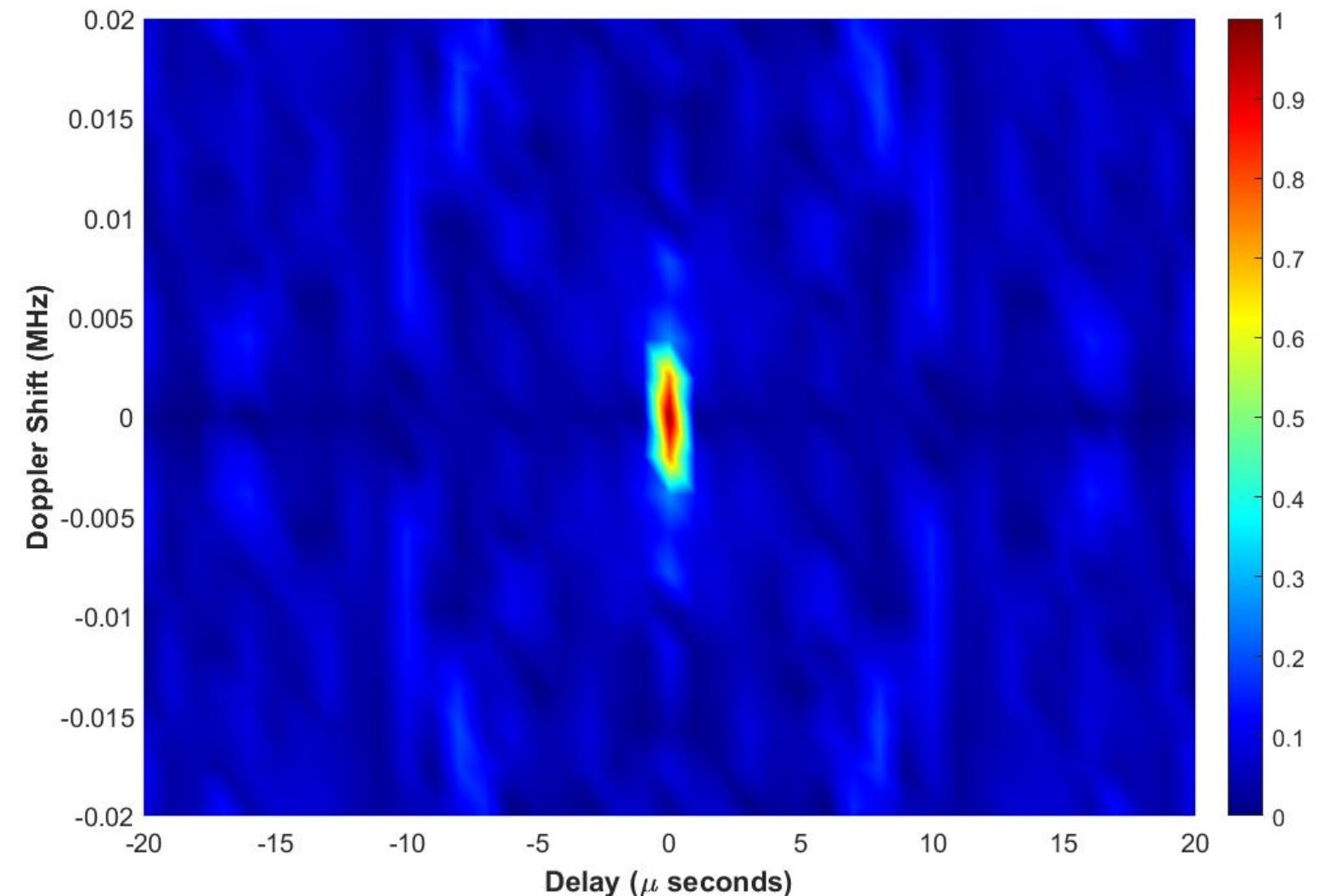
Ambiguity function

$$f_c = 10 \text{ GHz}$$

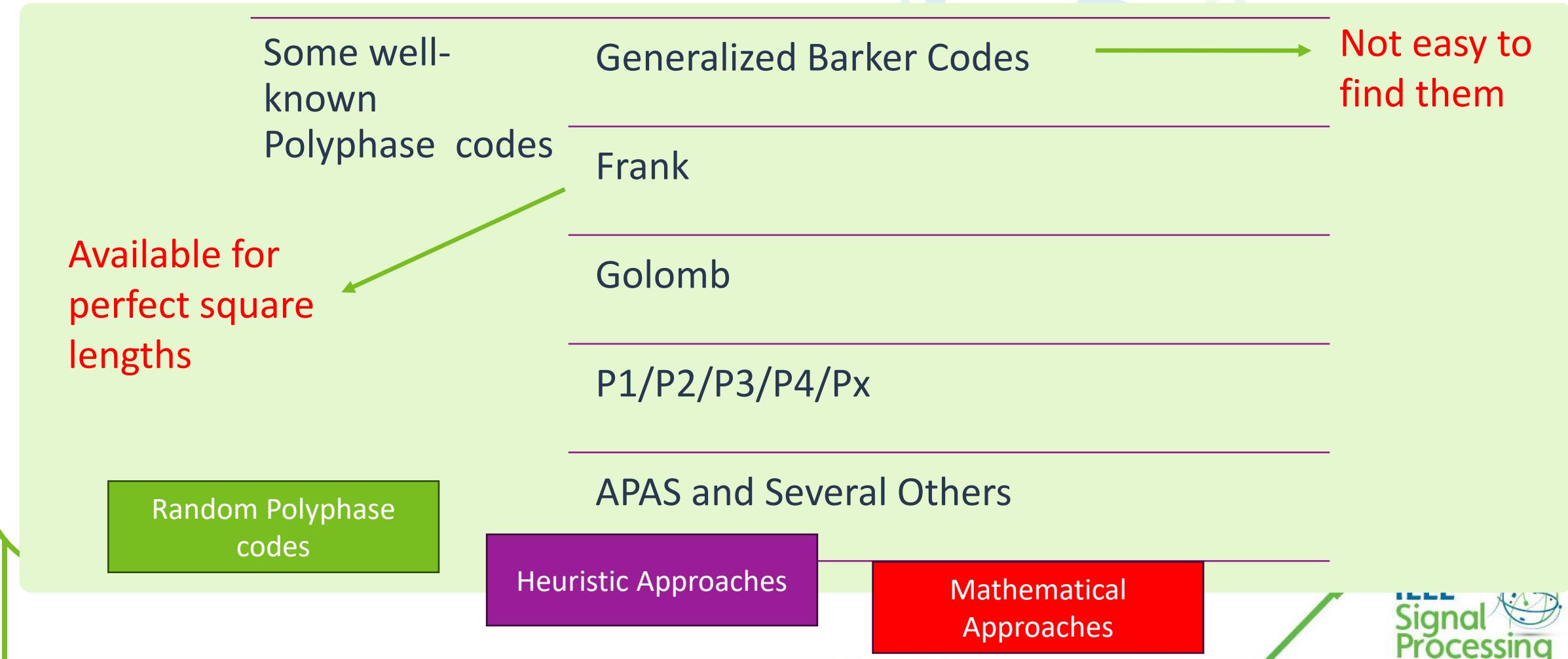
$$\lambda = 0.03 \text{ m}$$

$$v_r = 300 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 20 \text{ KHz}$$



Polyphase Codes

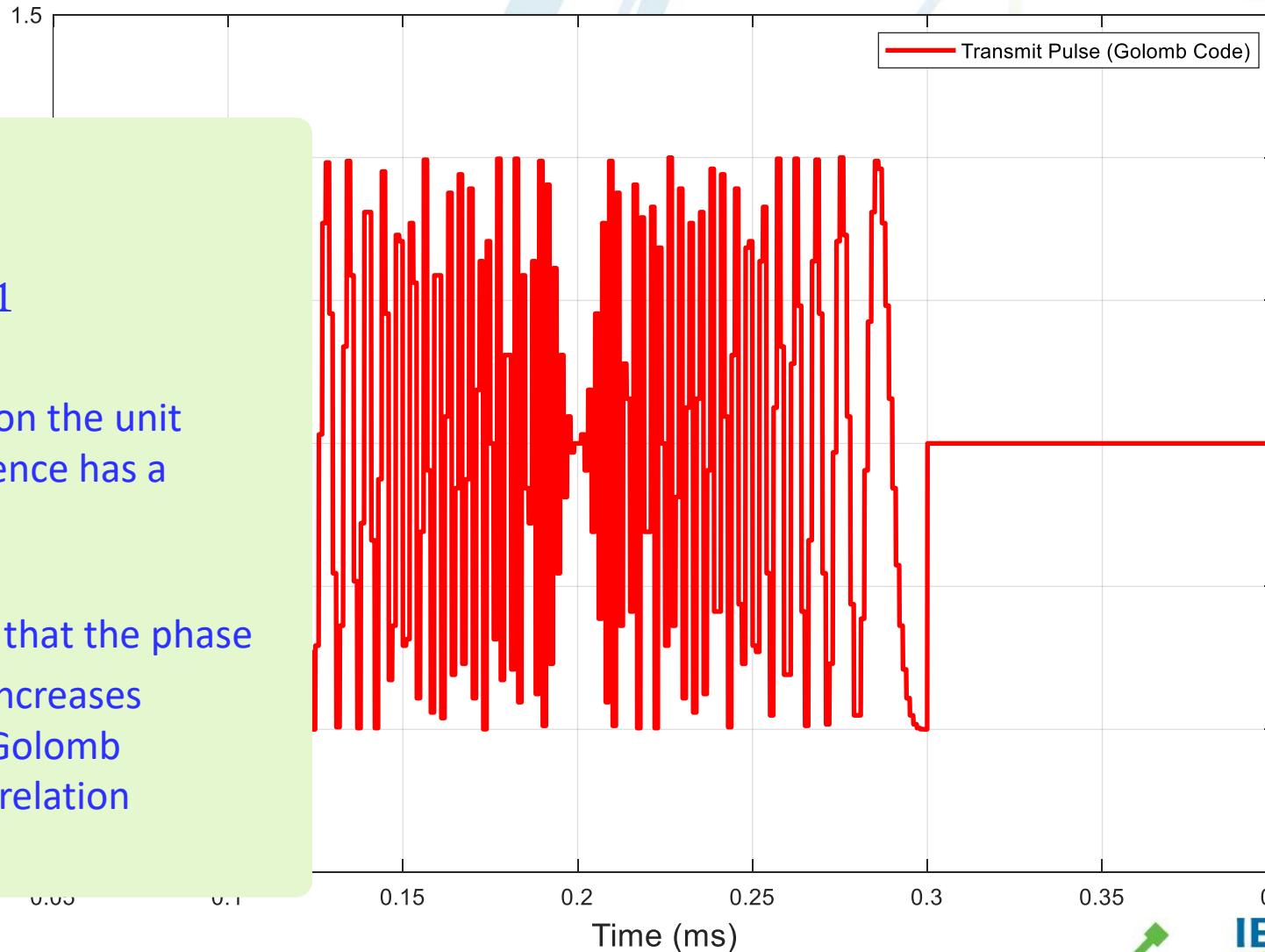


Polyphase Codes – Golomb Sequence

$$s[n] = \alpha^{\frac{n(n-1)}{2}},$$

$$n = 0, 1, \dots, N - 1$$

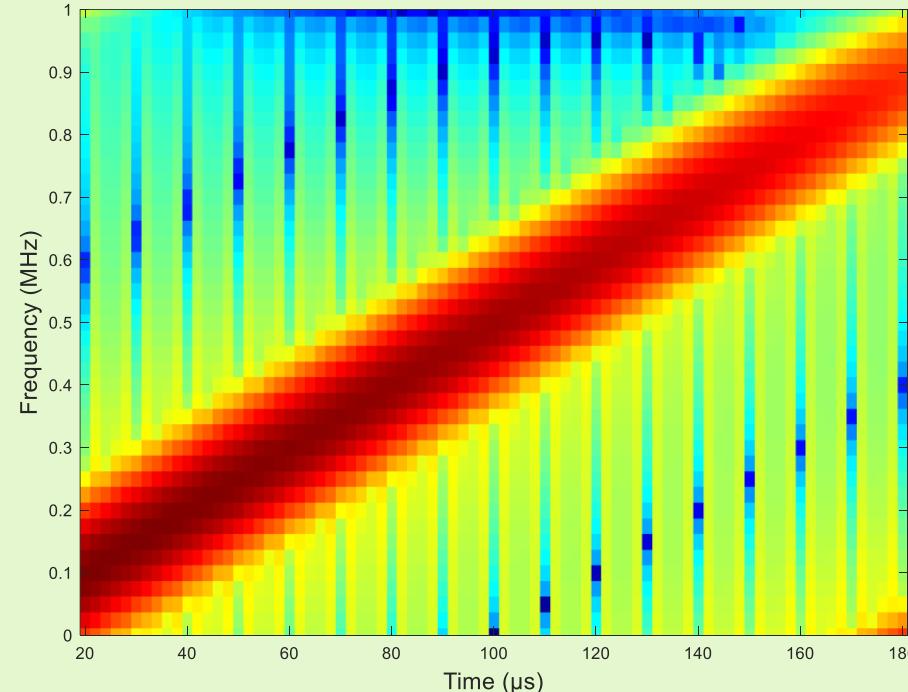
- $\alpha = e^{j\frac{2\pi}{N}}$ is a complex number on the unit circle, which ensures the sequence has a uniform phase progression
- The exponent $\frac{n(n-1)}{2}$ ensures that the phase of each term in the sequence increases quadratically, which gives the Golomb sequence its favorable autocorrelation properties



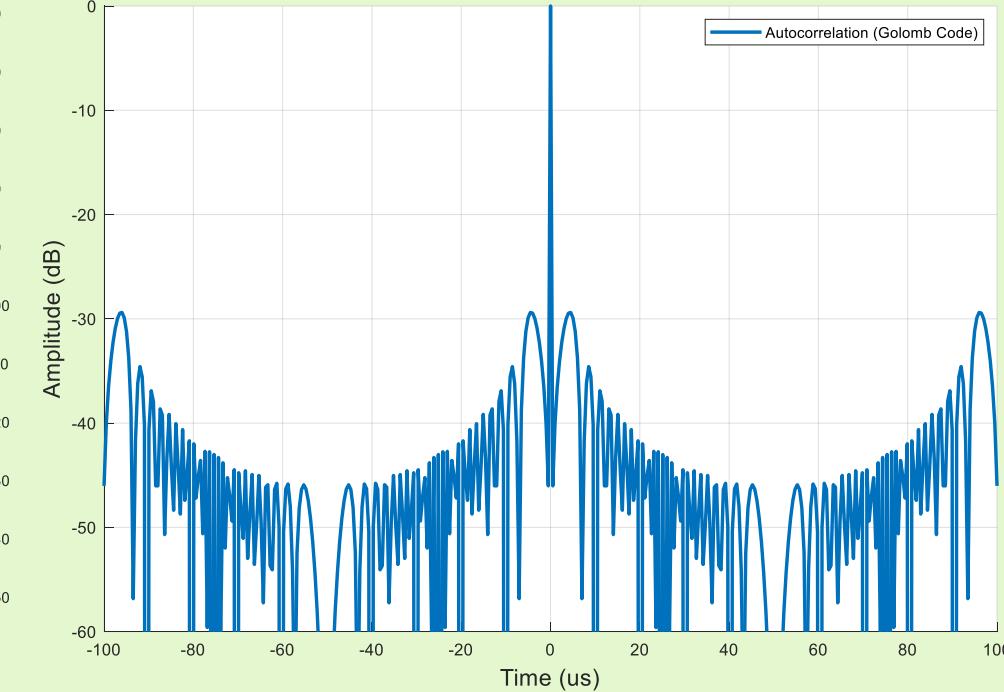
Lect1_example10.m

Polyphase Codes – Golomb Sequence

Spectrogram and Autocorrelation



Spectrogram



Autocorrelation

Lect1_example10.m

Polyphase Codes – Golomb Sequence

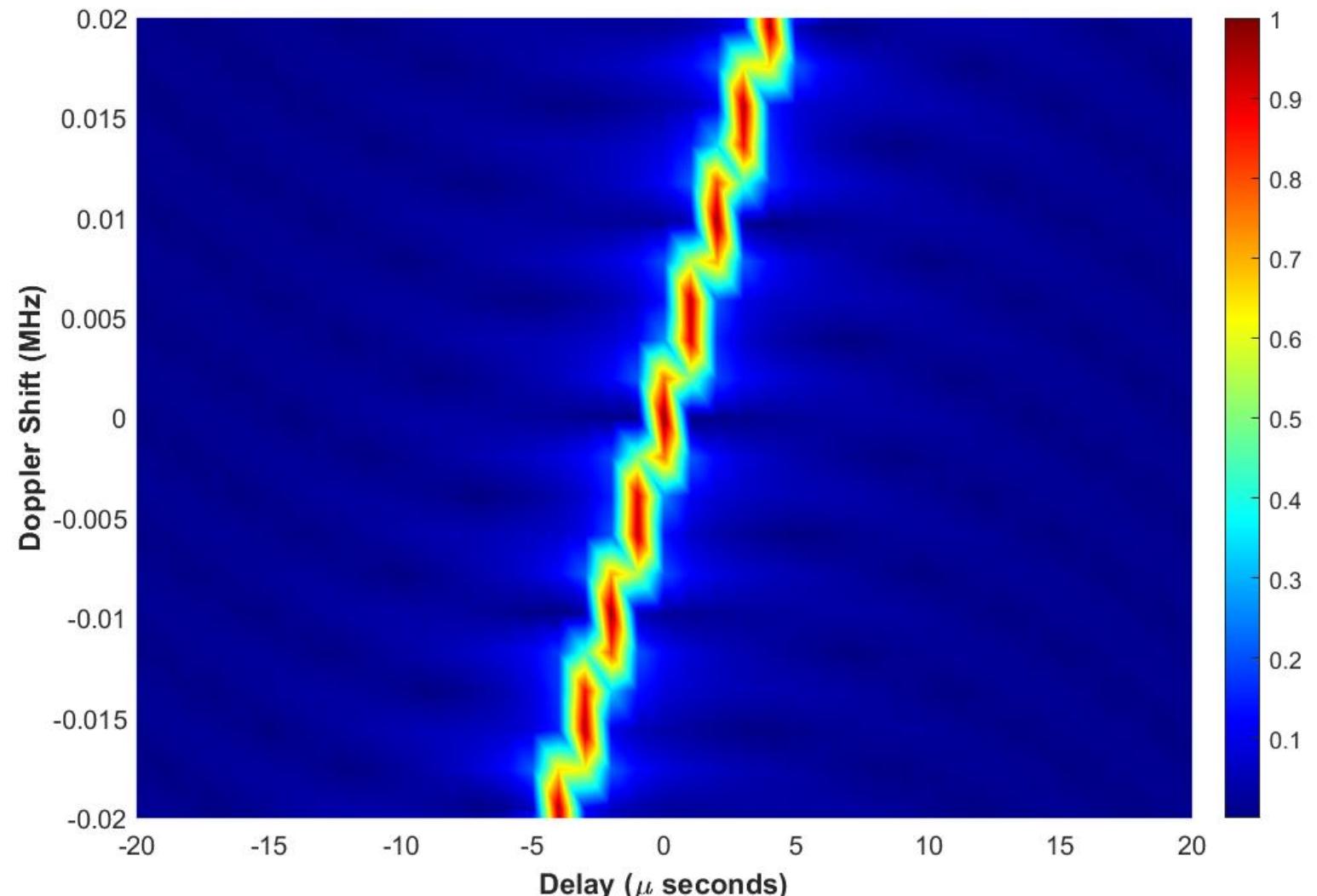
Ambiguity function

$$f_c = 10 \text{ GHz}$$

$$\lambda = 0.03 \text{ m}$$

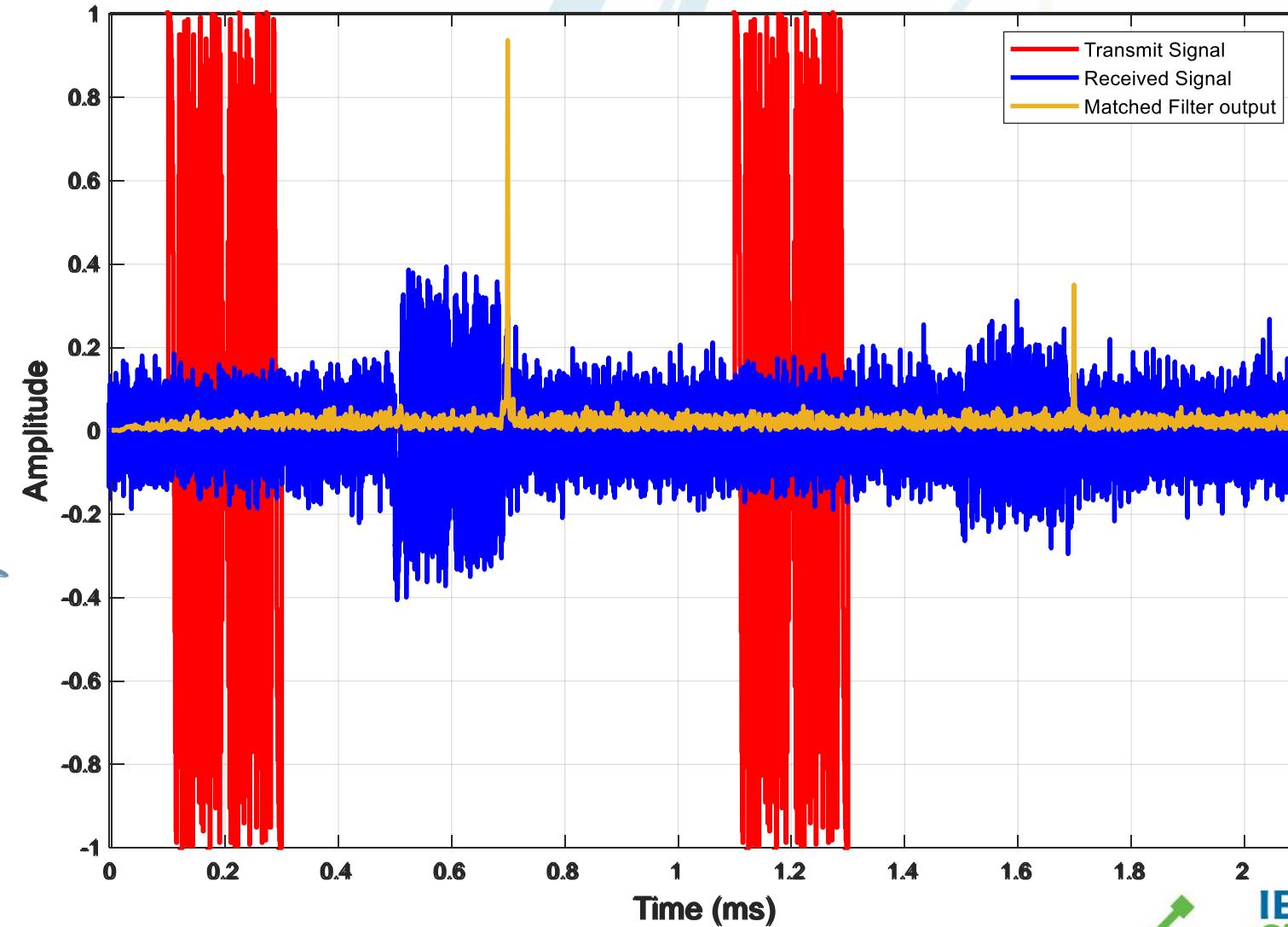
$$v_r = 300 \text{ m/s}$$

$$f_d = 2 \frac{v_r}{\lambda} = 20 \text{ KHz}$$



Doppler Resolution

Can we still separate two targets if they are in a similar distance from radar?



Doppler Resolution in Pulse Doppler Radar

Range-Doppler Processing

Coherent Pulse Interval (CPI) = number of samples in the fast time \times number of pulses in the slow-time

Assume M pulses are available in CPI

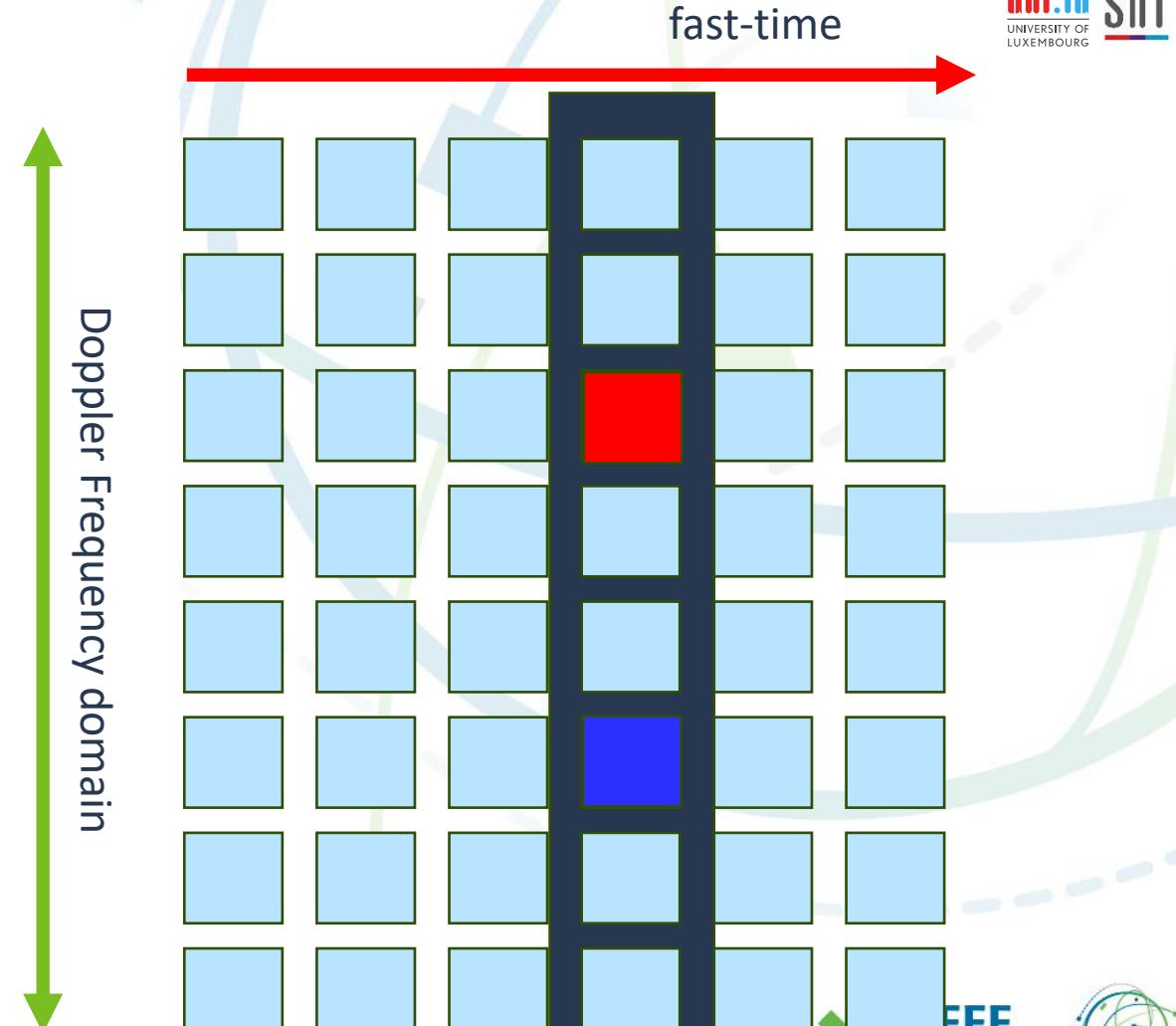
$$T_{\text{CPI}} = M \times \text{PRI}$$

$$\Delta f_d = \frac{1}{T_{\text{CPI}}}$$

$$\text{PRI} = 1 \text{ ms}, \quad M = 128$$

$$\Delta f_d = \frac{1}{T_{\text{CPI}}} = \frac{1}{128 \text{ ms}} = 7.8 \text{ Hz}$$

$$f_c = 10 \text{ GHz}, \quad \Delta v_r = \frac{\lambda \Delta f_d}{2} = 0.03 \times \frac{7.8}{2} = 0.117 \text{ m/s}$$



Doppler Resolution in Pulse Doppler Radar



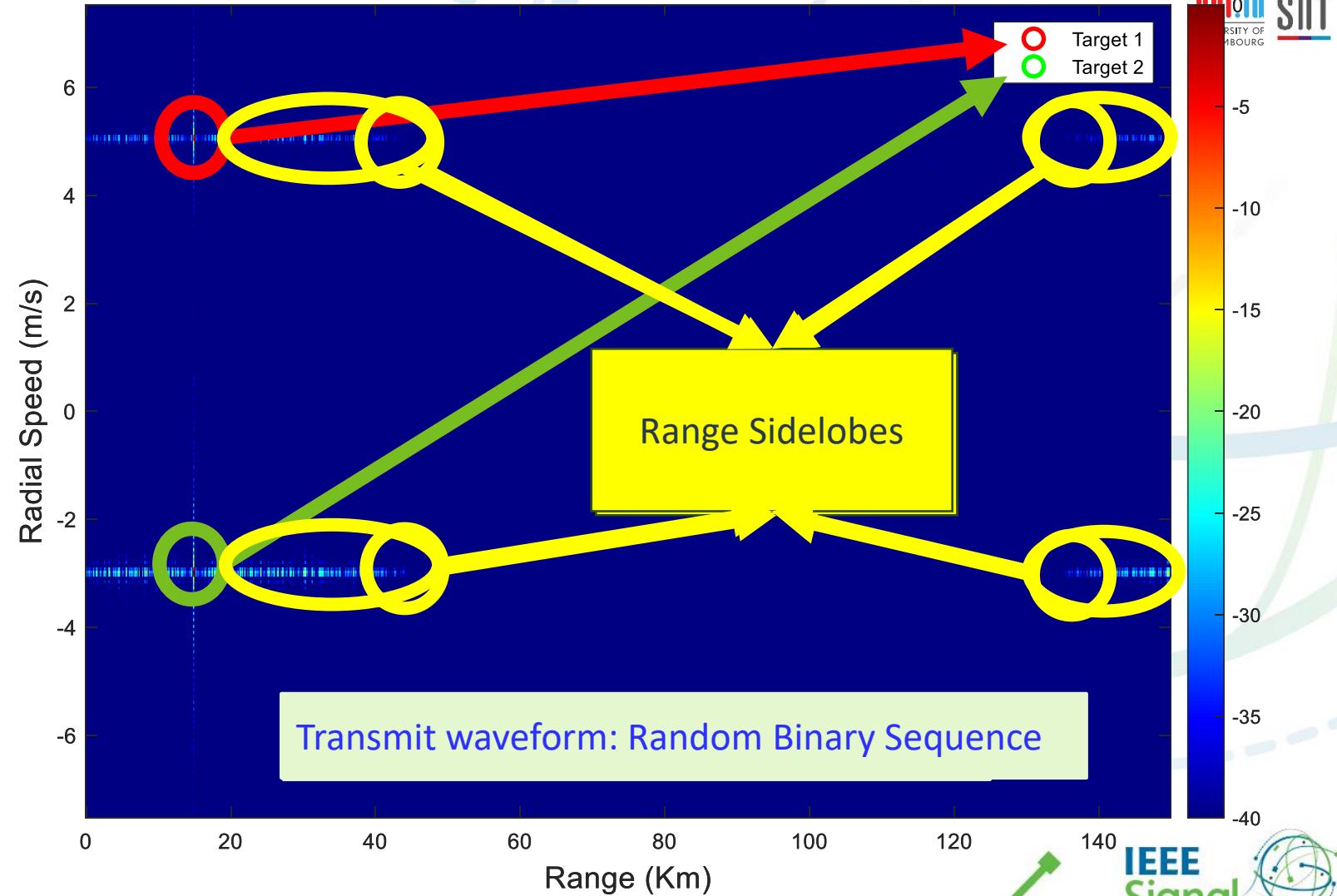
PRI = 1 ms,
 $f_c = 10\text{GHz}$,
 $M = 128$

$$\Delta f_d = \frac{1}{T_{\text{CPI}}} = \frac{1}{128 \text{ ms}} = 7.8 \text{ Hz}$$

$$\Delta v_r = \frac{\lambda \Delta f_d}{2} = 0.117 \frac{\text{m}}{\text{s}}$$

$$v_{r\max} = \frac{\lambda f_p}{4} = 7.5 \frac{\text{m}}{\text{s}}$$

Lect1_example11.m



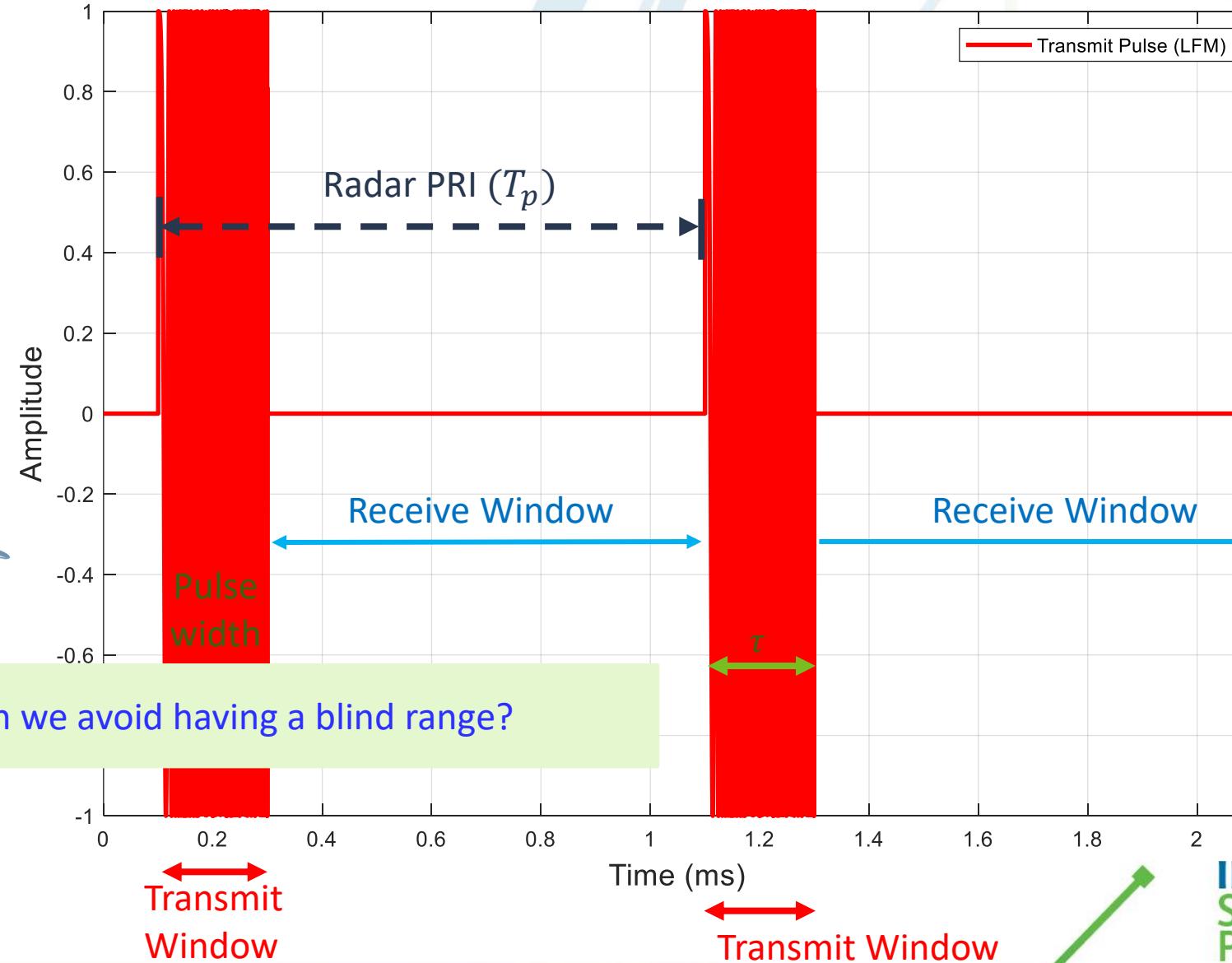
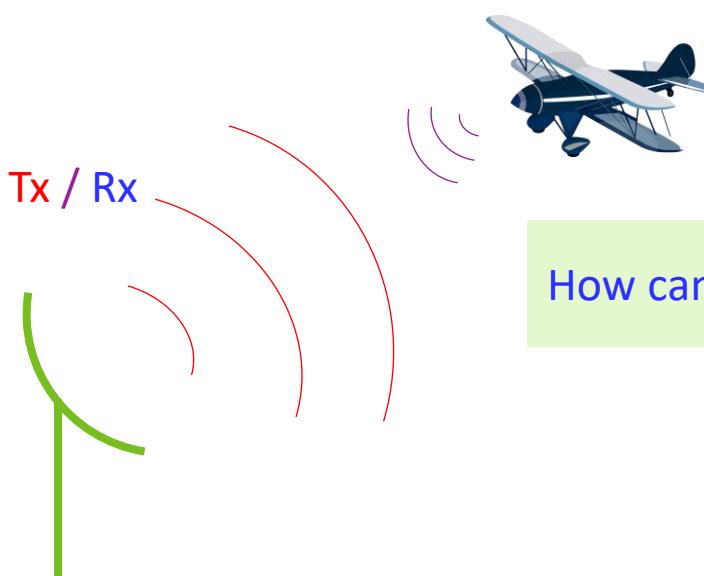
Pulse Compression Doppler Radar

Blind Range

$$\text{Blind Range} = \frac{c\tau}{2}$$

$$\tau = 200 \mu\text{s}$$

$$\text{Blind Range} = 30 \text{ Km}$$

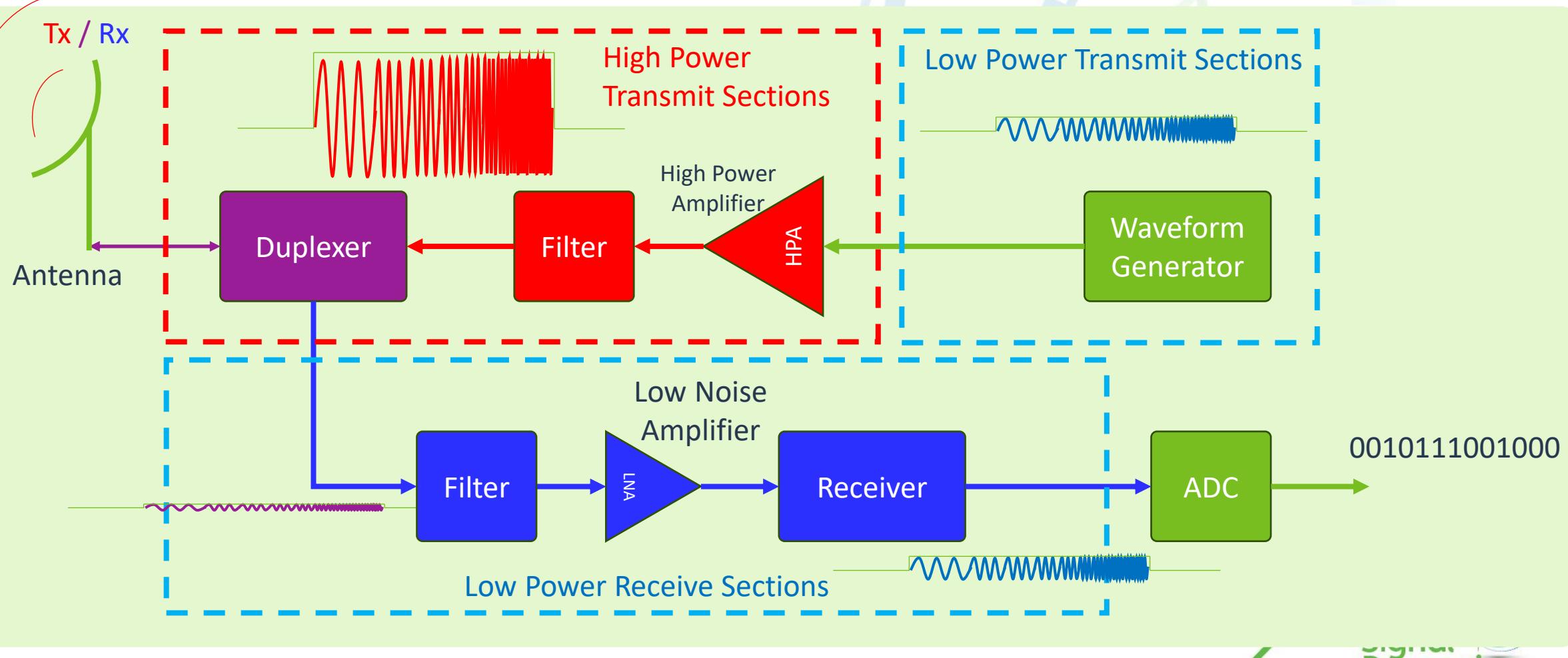


How can we avoid having a blind range?

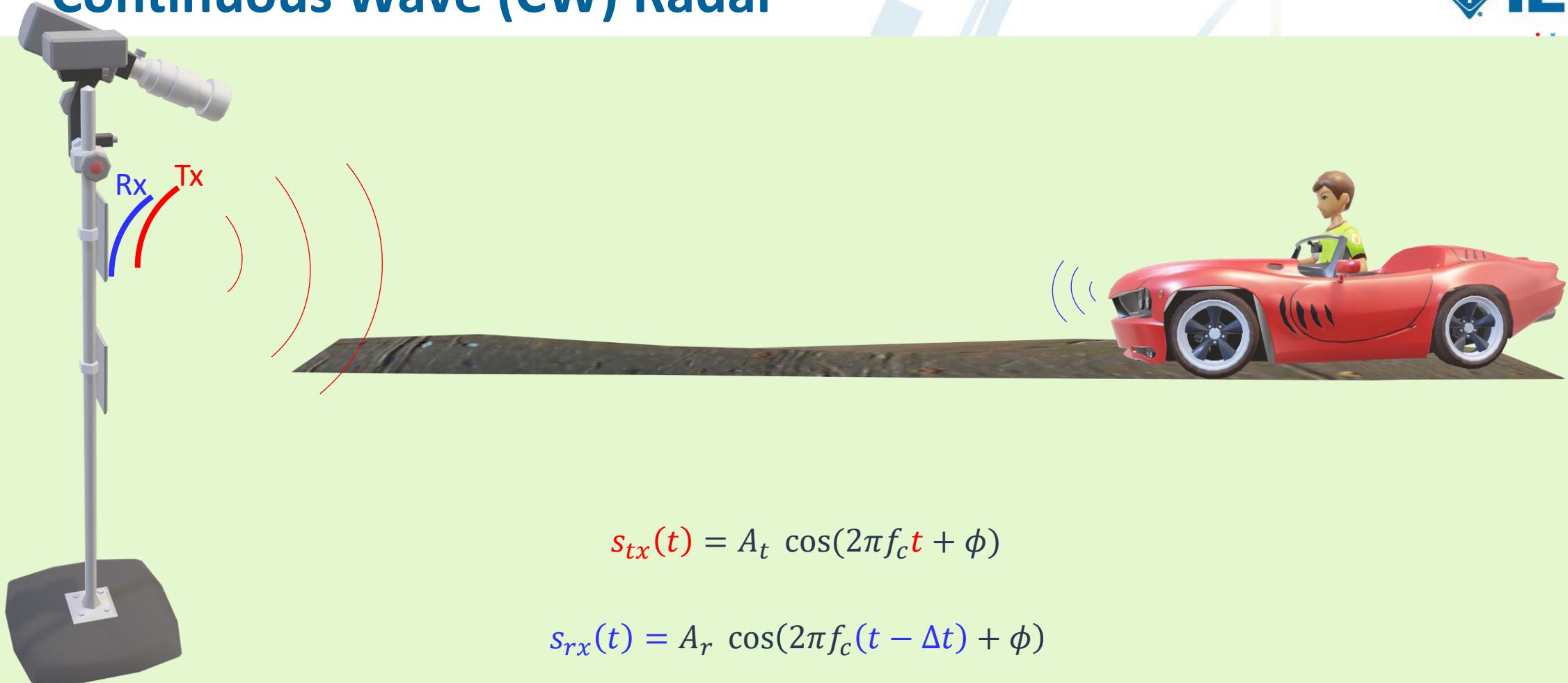
Pulse Compression Doppler Radar

Block Diagram

How can we avoid having a blind range?



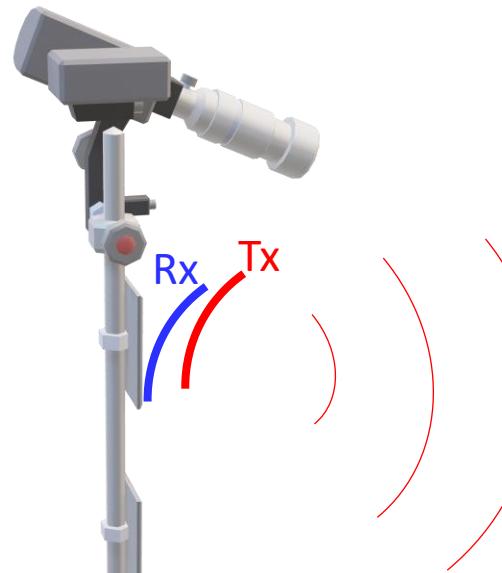
Continuous Wave (CW) Radar



$$s_{tx}(t) = A_t \cos(2\pi f_c t + \phi)$$

$$s_{rx}(t) = A_r \cos(2\pi f_c(t - \Delta t) + \phi)$$

CW Radar

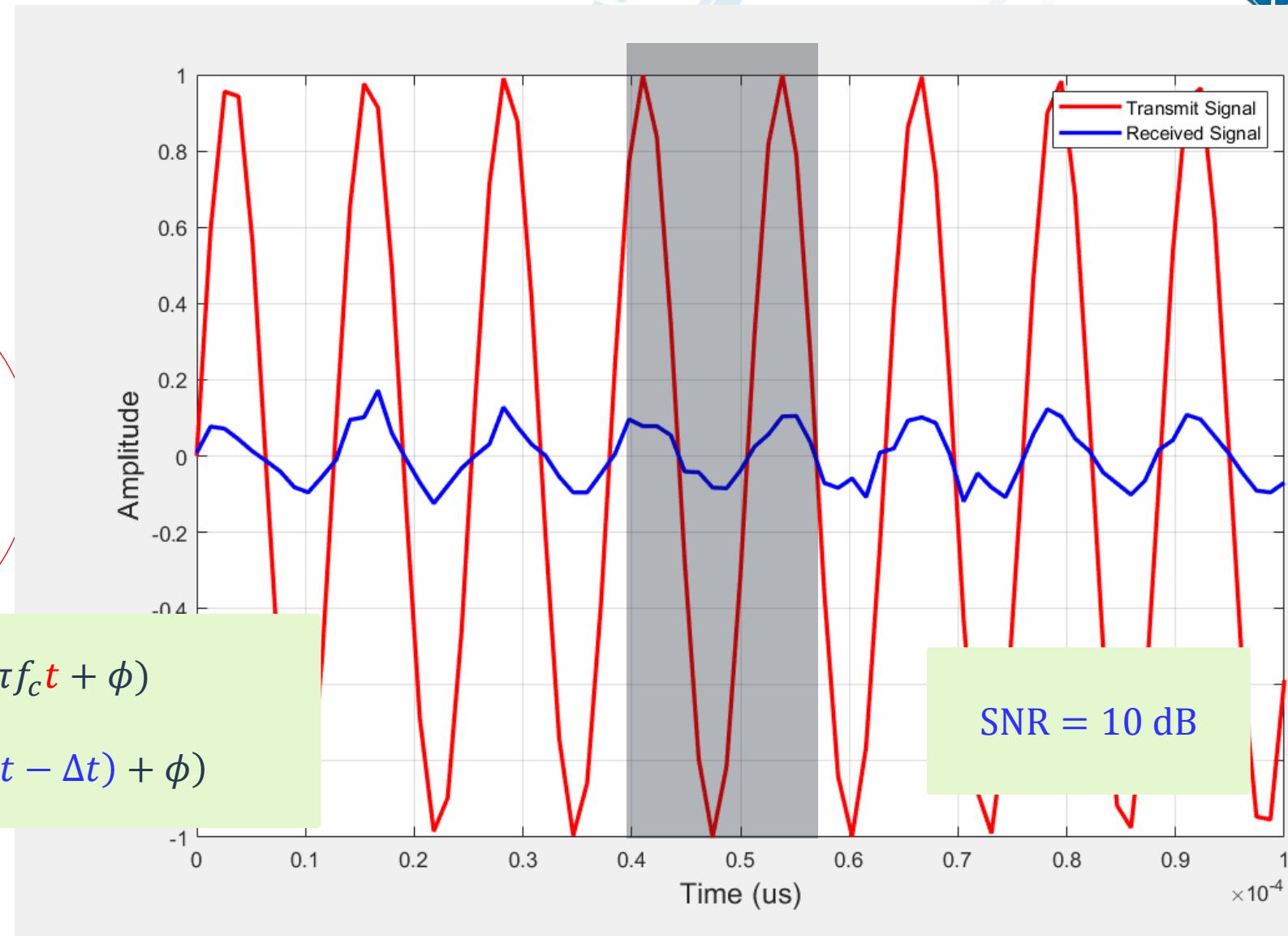


$$s_{tx}(t) = A_t \cos(2\pi f_c t + \phi)$$

$$s_{rx}(t) = A_r \cos(2\pi f_c(t - \Delta t) + \phi)$$

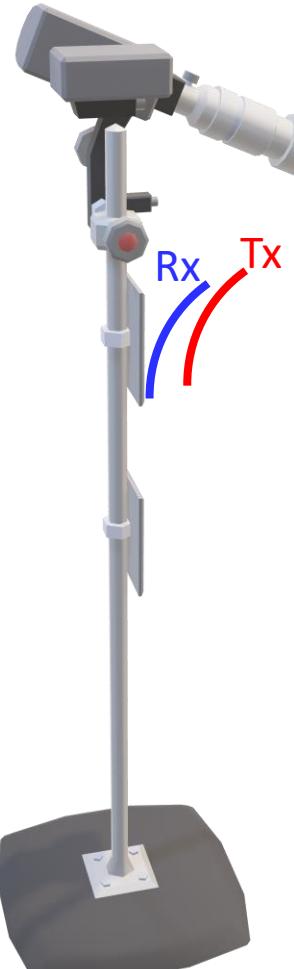


Lect1_example12.m

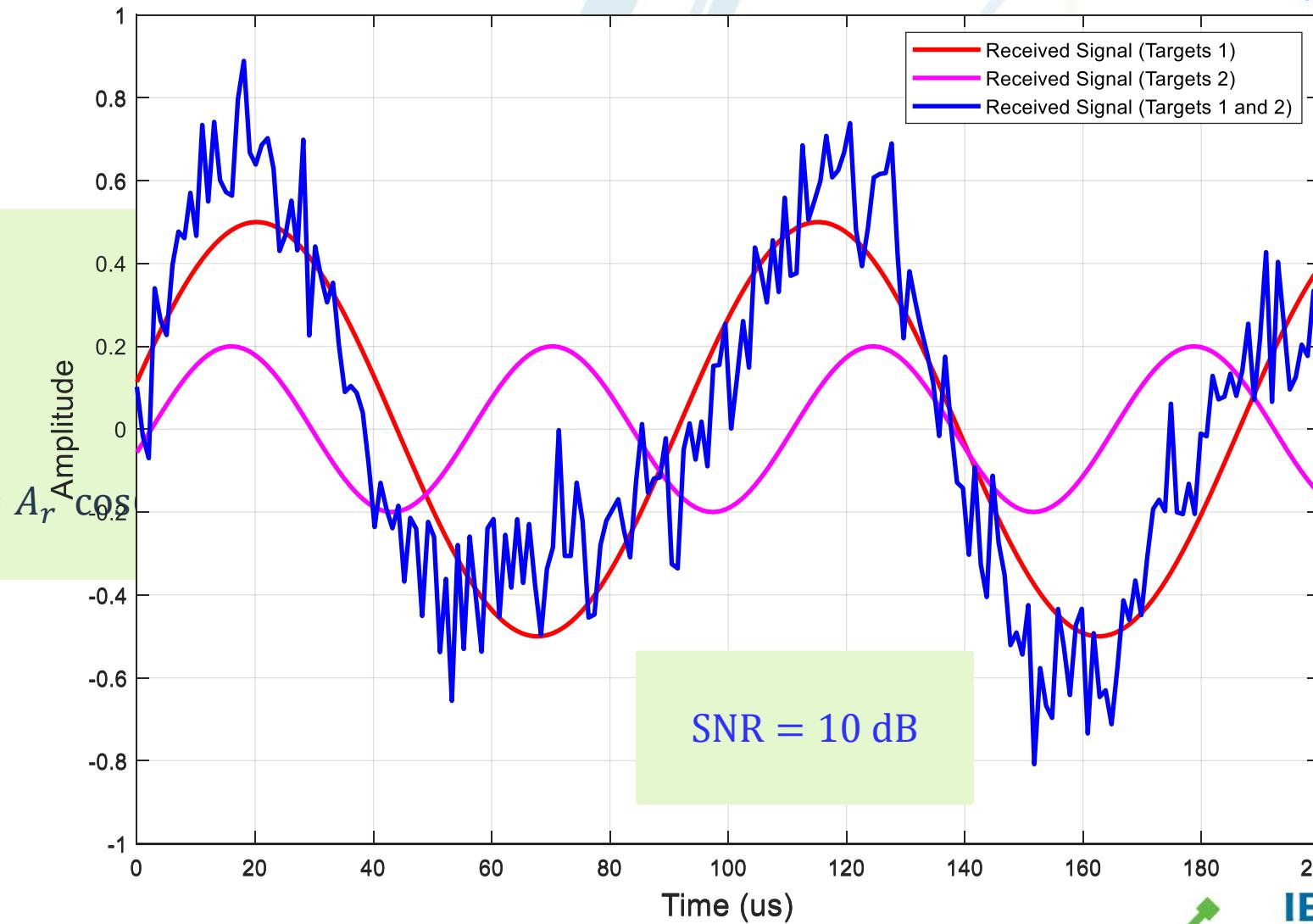


CW Radar

Doppler Effect

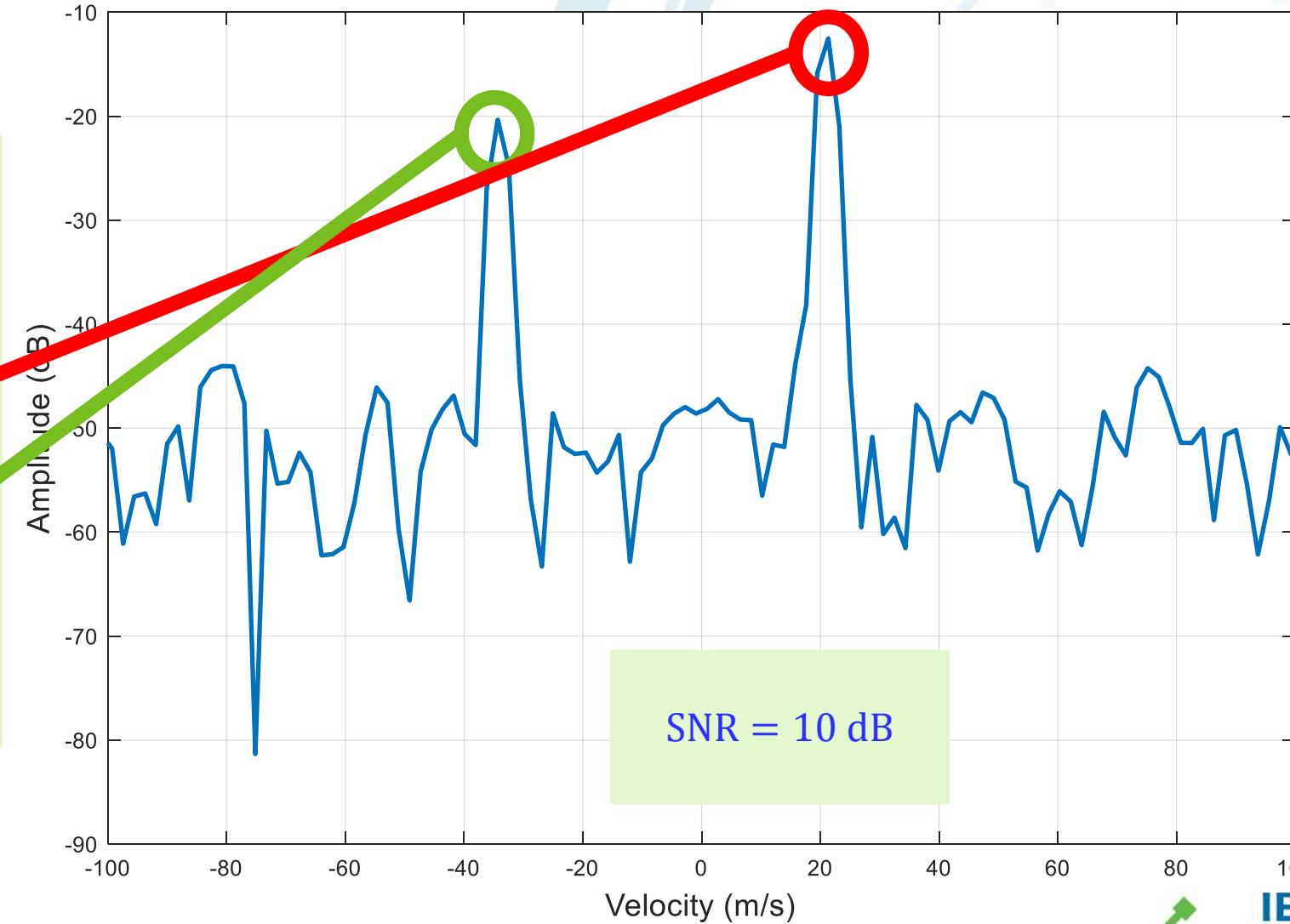
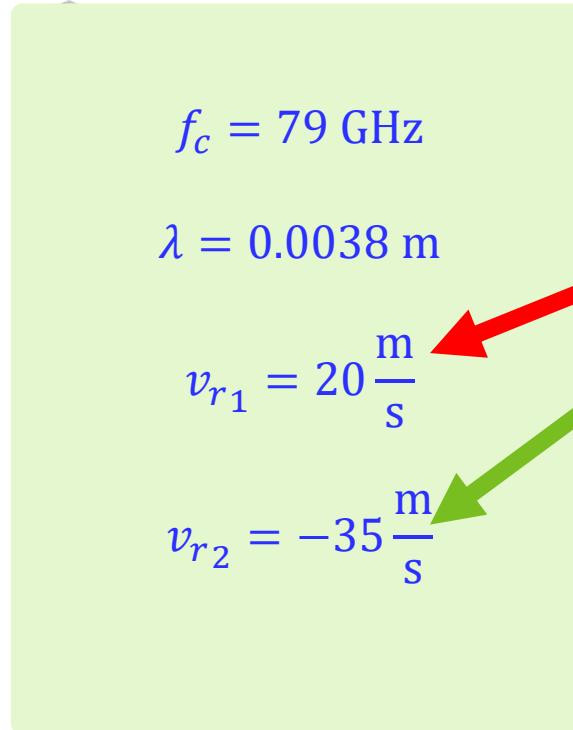


Lect1_example13.m



CW Radar

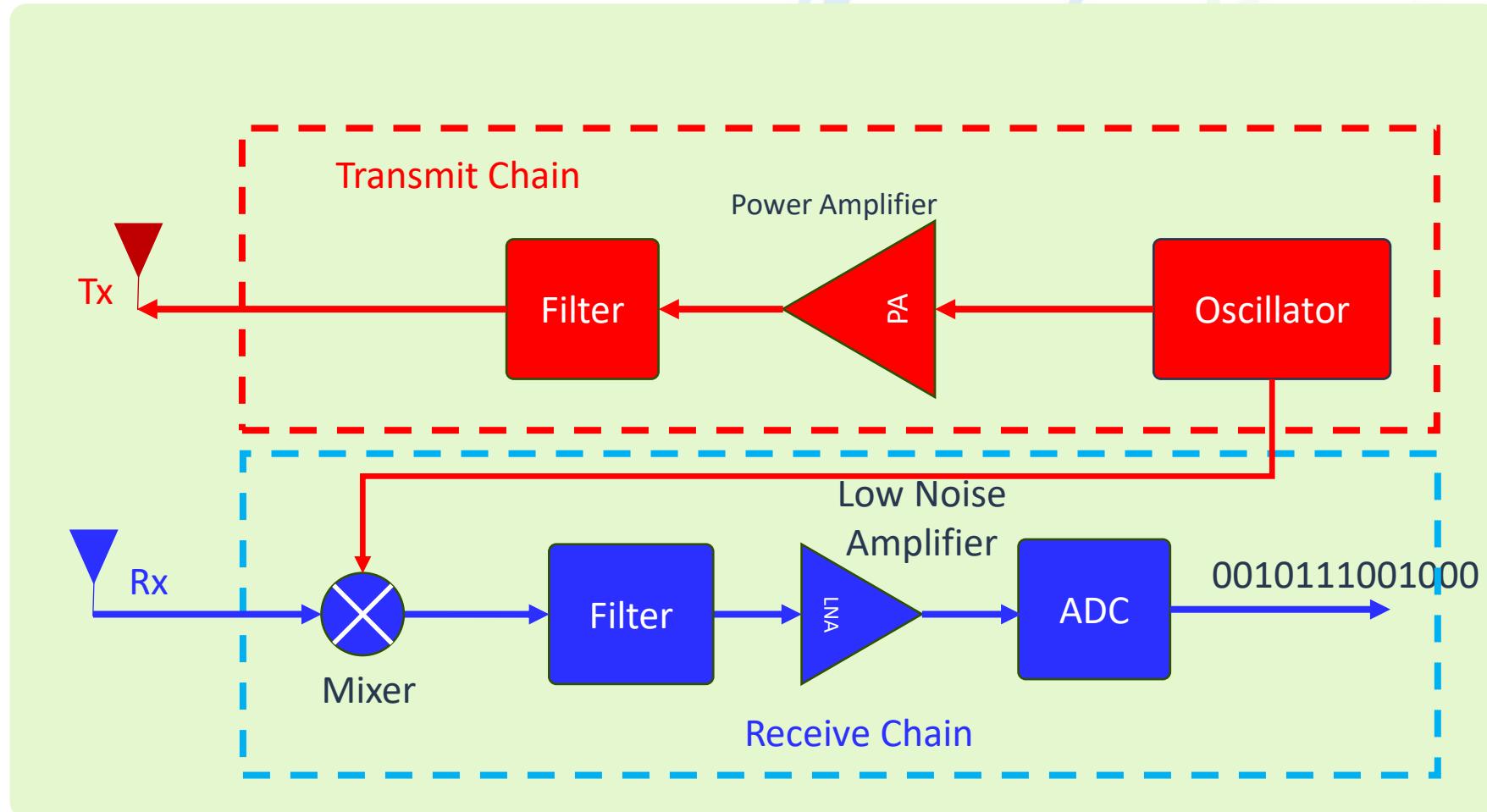
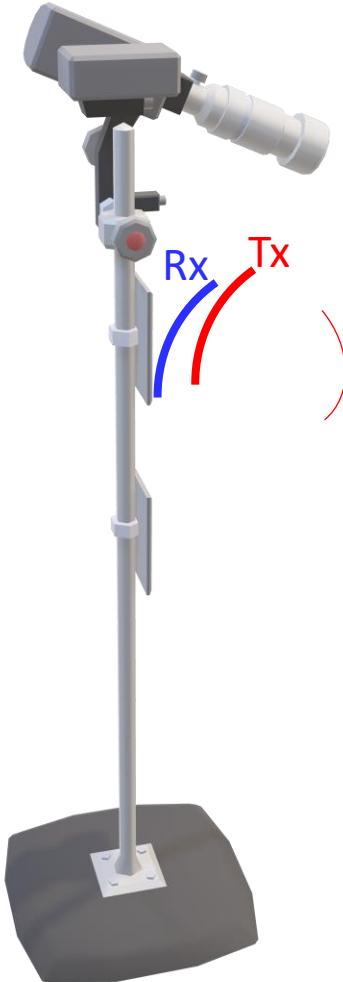
Doppler Effect



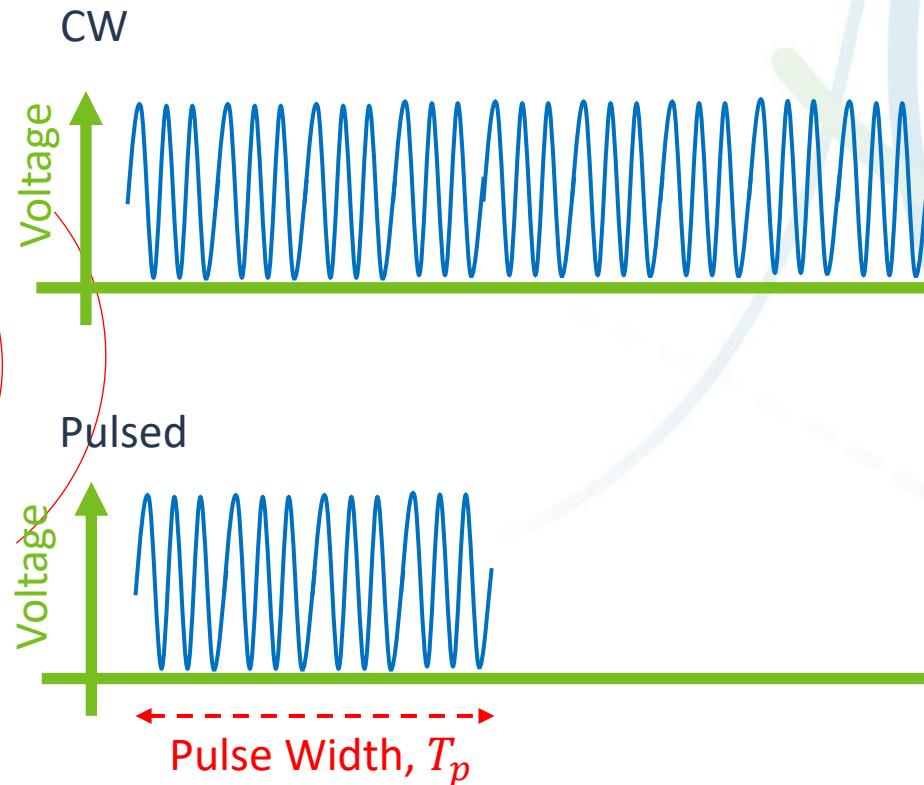
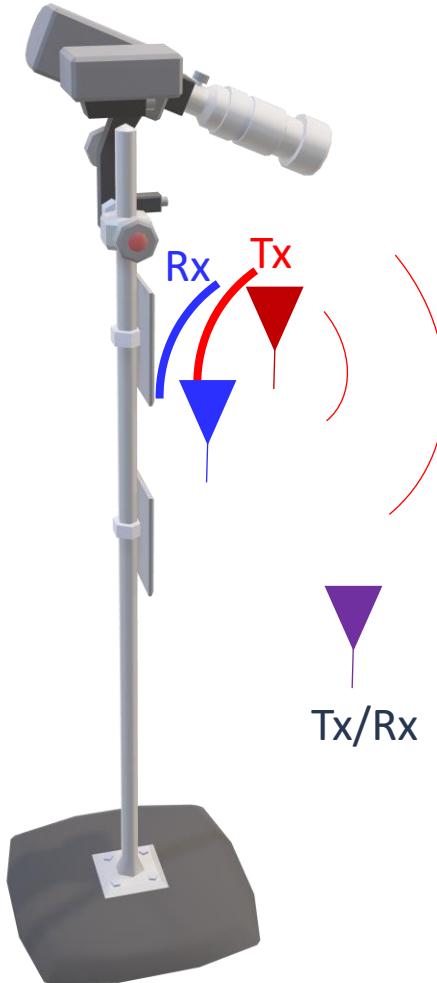
Lect1_example13.m

CW Radar

Block Diagram



Pulsed or CW?

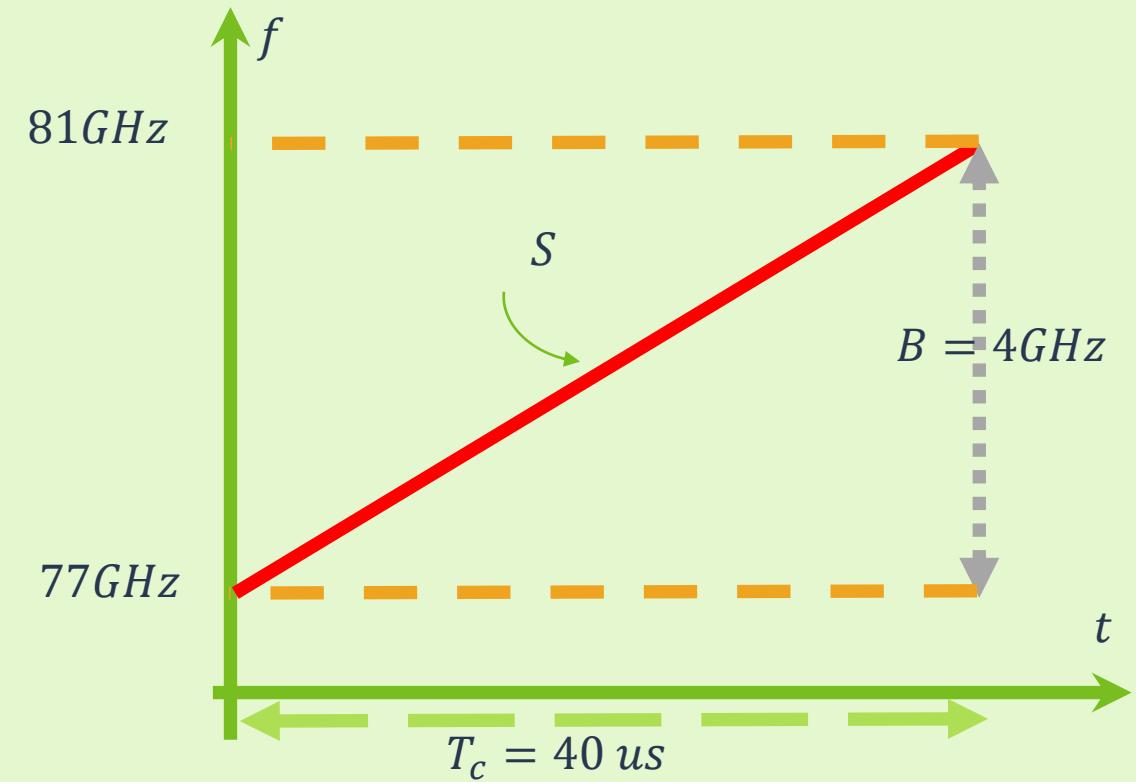
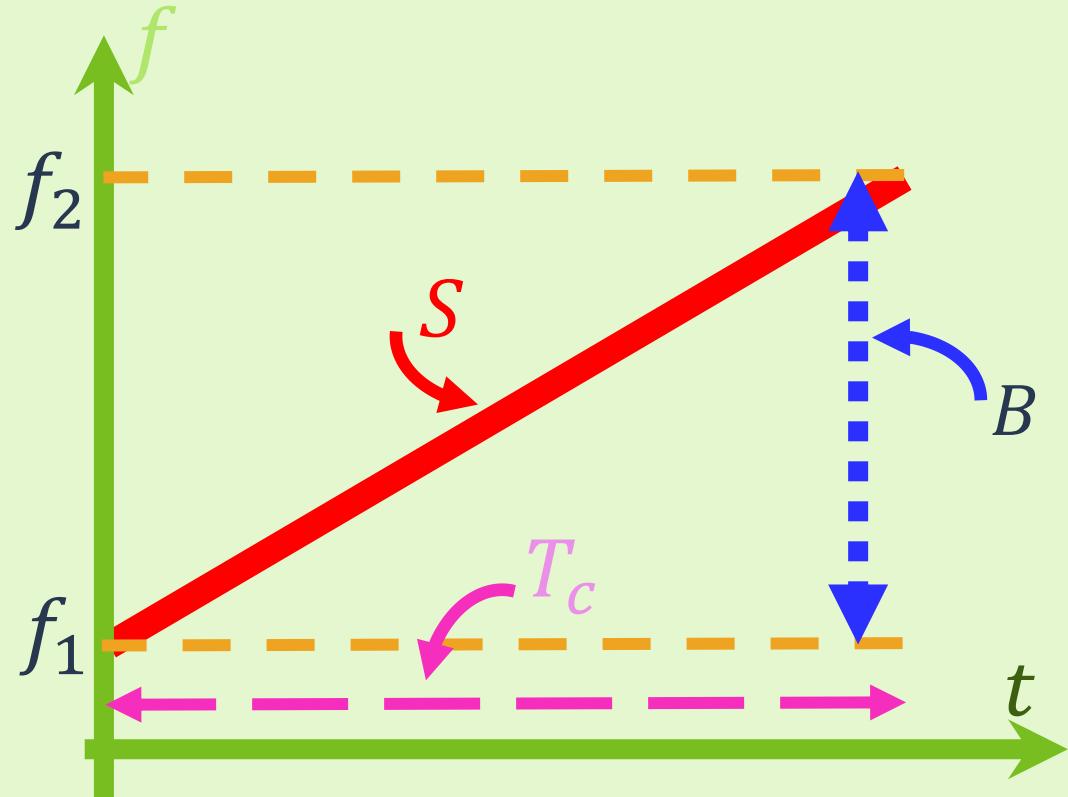


CW	Pulsed
Requires separate transmit and receive antennas.	Same antenna is used for transmit and receive.
Isolation requirements limit transmit power.	Time-multiplexing relaxes isolation requirements to allow high power.
Radar has no blind ranges.	Radar has blind ranges due to "eclipsing" during transmit events.

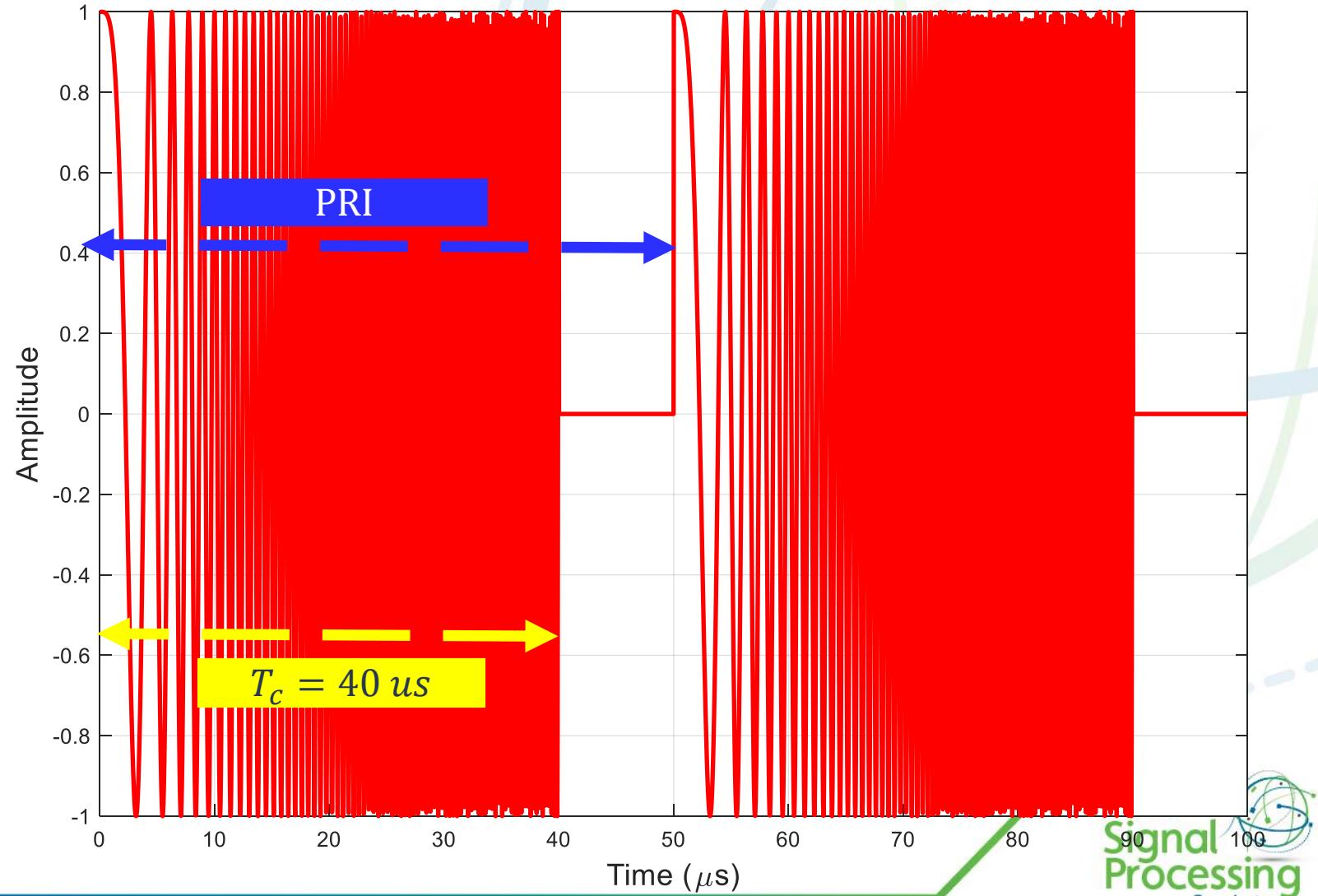
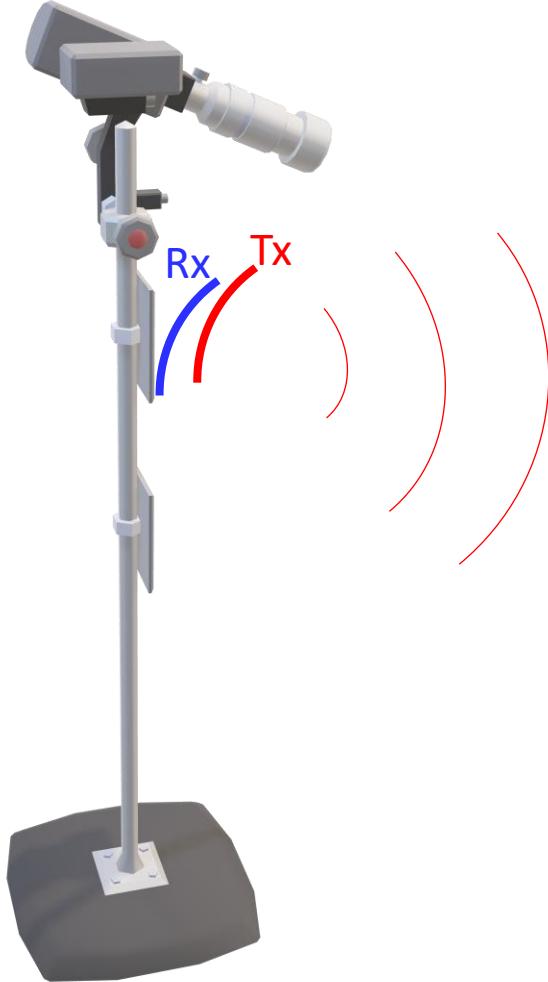
$$\text{Bandwidth} \approx \frac{1}{T_p} \Rightarrow \text{Time} \times \text{Bandwidth} \approx 1$$

Frequency Modulated Continuous Wave (FMCW)

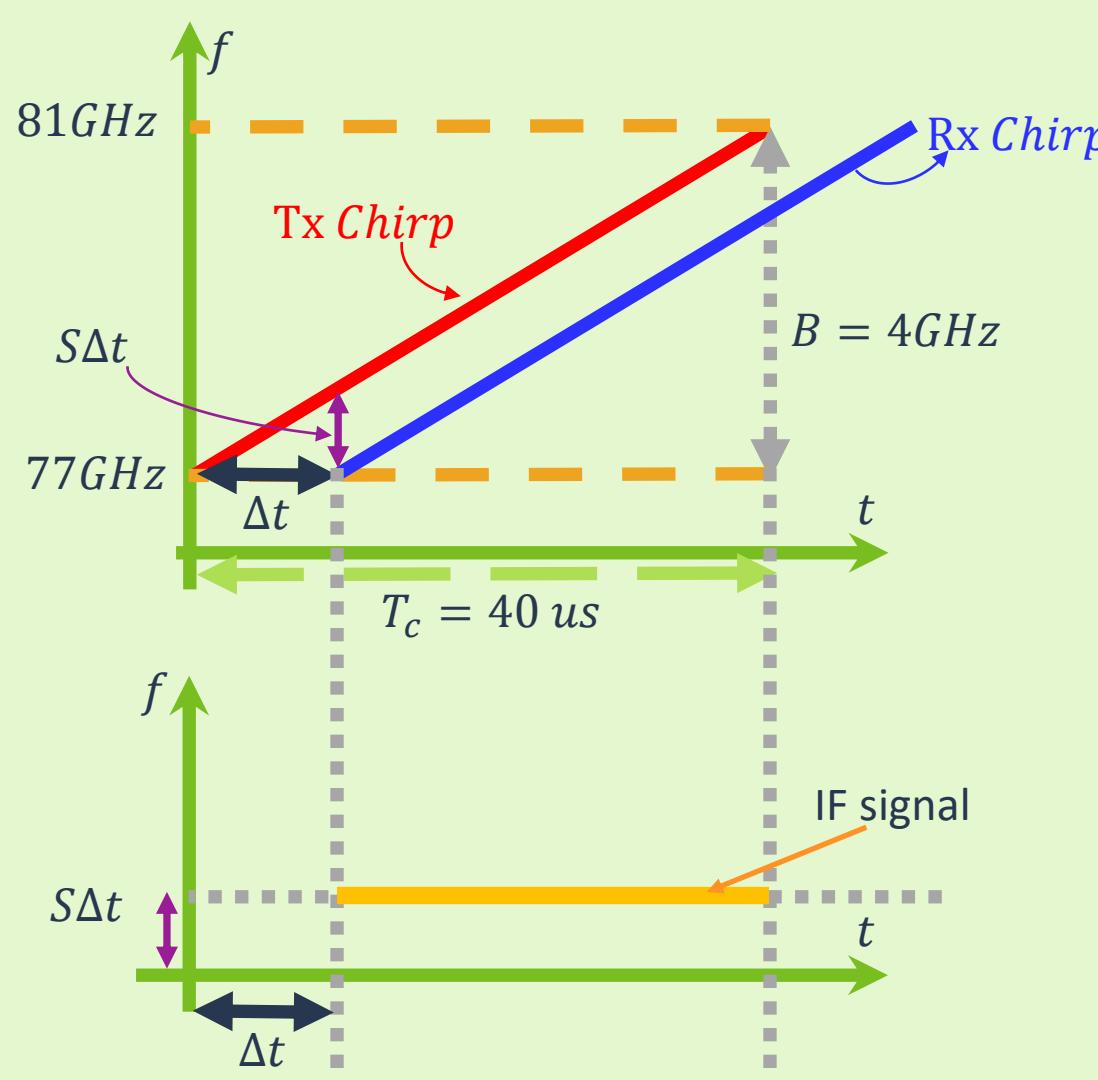
$$s_{\text{FMCW}}(t) = A_t \cos \left(2\pi \left(f_c t + \frac{B}{2\tau} t^2 \right) + \phi \right)$$



Frequency Modulated Continuous Wave (FMCW)



Frequency Modulated Continuous Wave (FMCW)



Example

$$S = \frac{B}{T_c} = \frac{4 \text{ GHz}}{40 \mu\text{s}} = 100 \frac{\text{MHz}}{\mu\text{s}}$$

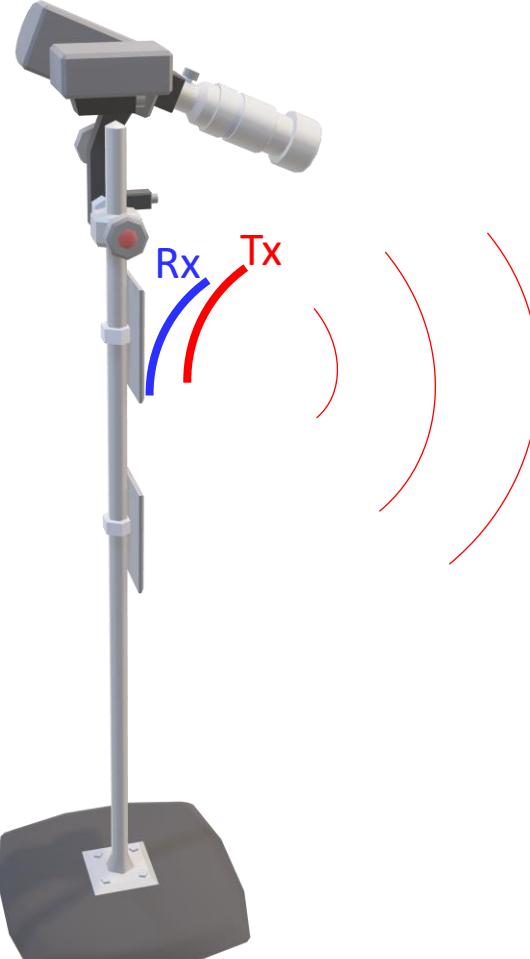
$\Delta t = 0.1 \mu\text{s}$ (object at range of 15 m)

$$f_{\text{IF}} = S\Delta t = 100 \frac{\text{MHz}}{\mu\text{s}} \times 0.1 \mu\text{s} = 10 \text{ MHz}$$

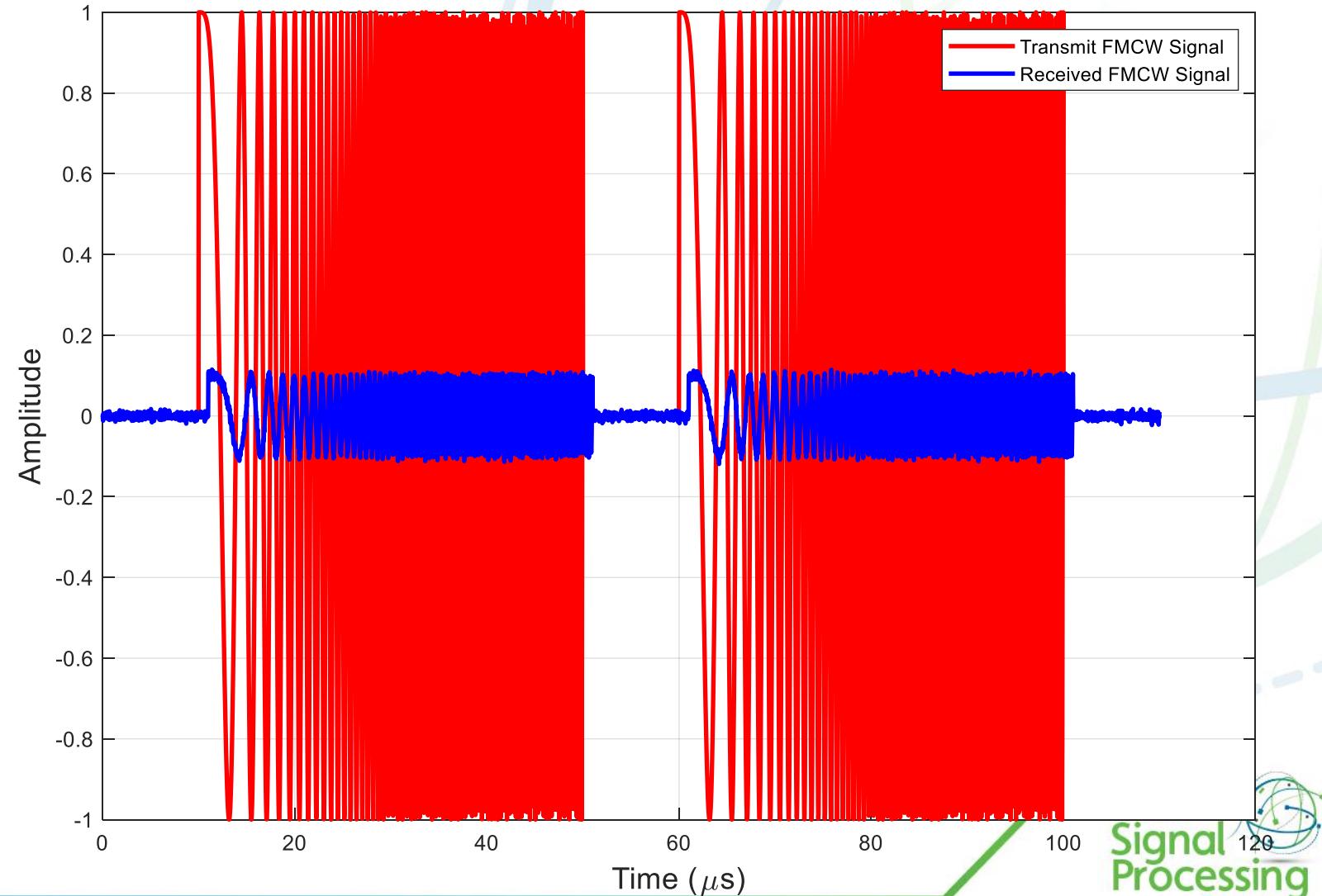
$f_s = 10 \text{ MHz}$ (complex sampling ADCs) instead of 4 GHz (complex sampling ADCs)

This is called “de-chirp”

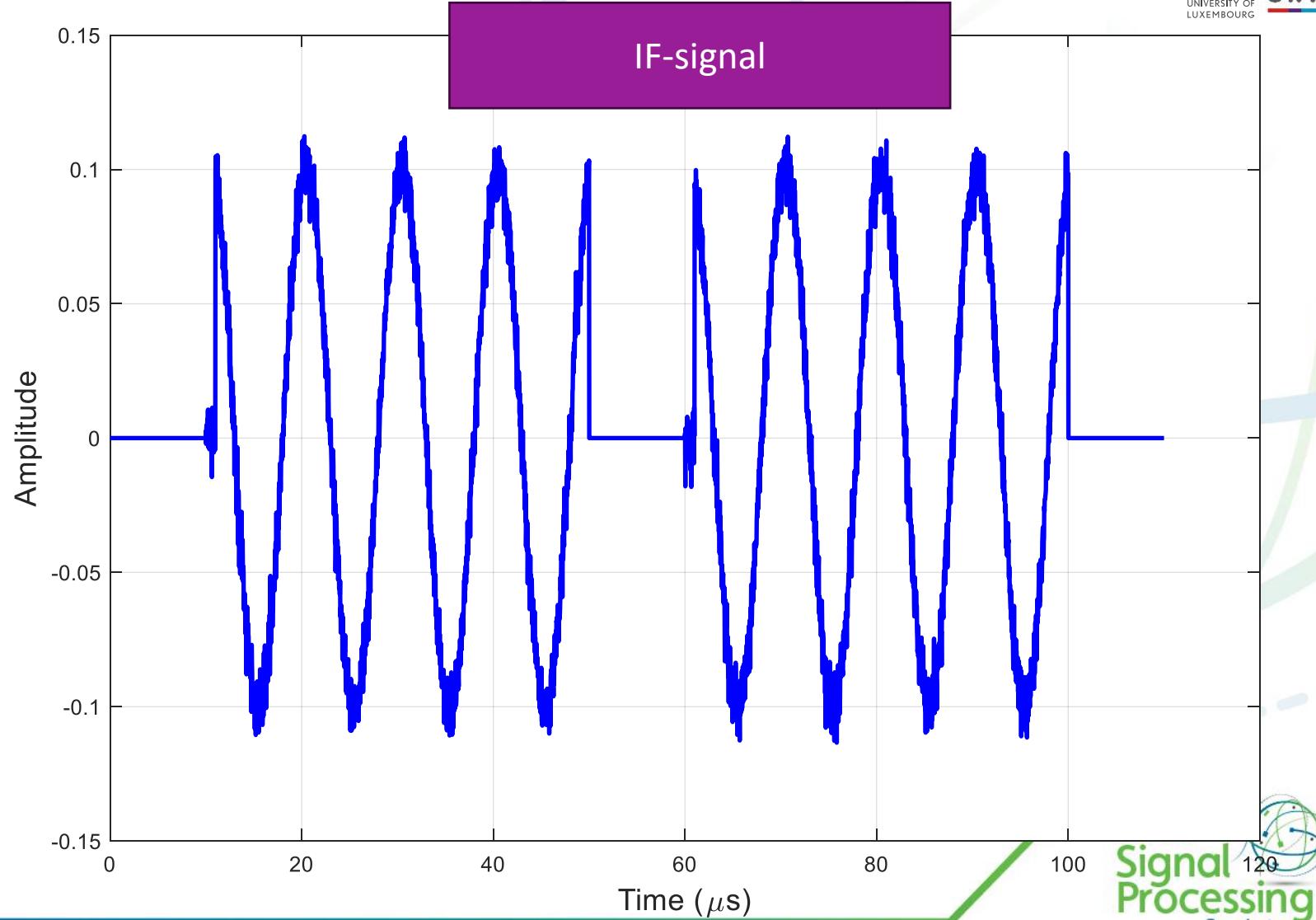
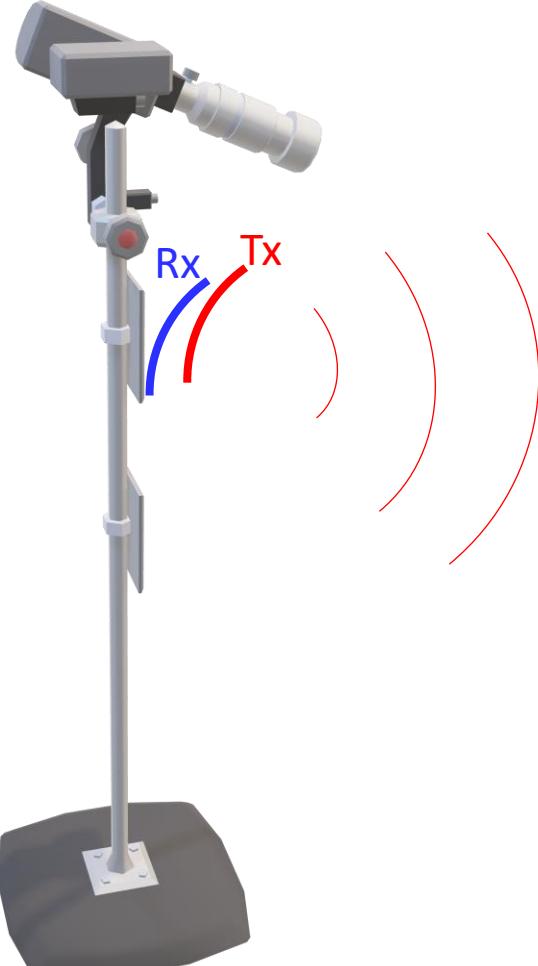
Frequency Modulated Continuous Wave (FMCW)



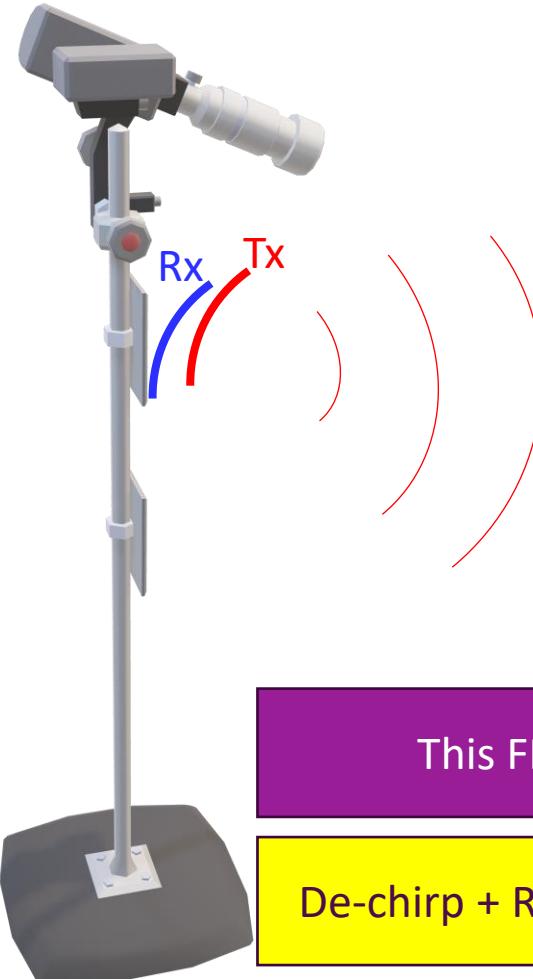
Lect1_example14.m



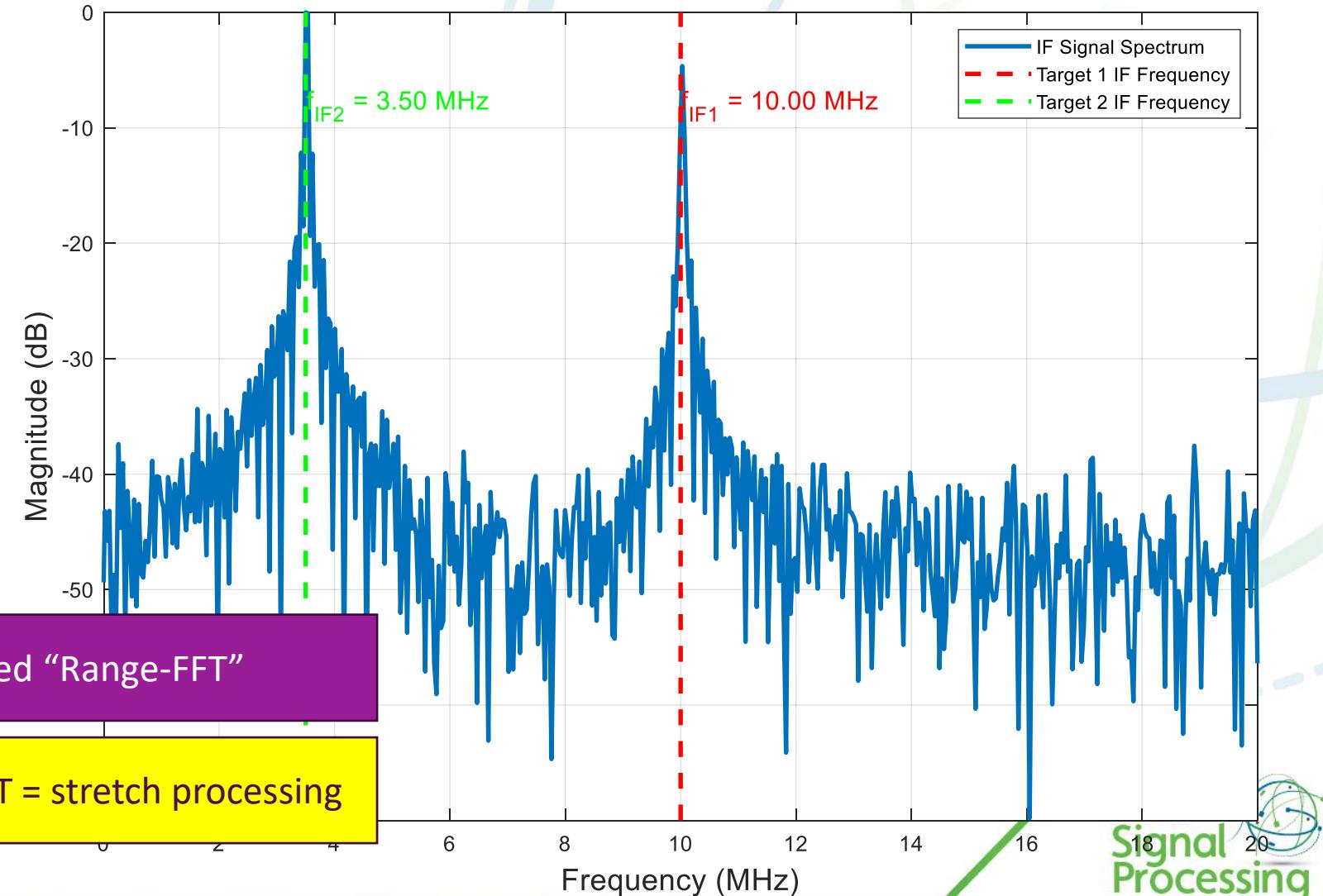
Frequency Modulated Continuous Wave (FMCW)



Frequency Modulated Continuous Wave (FMCW)

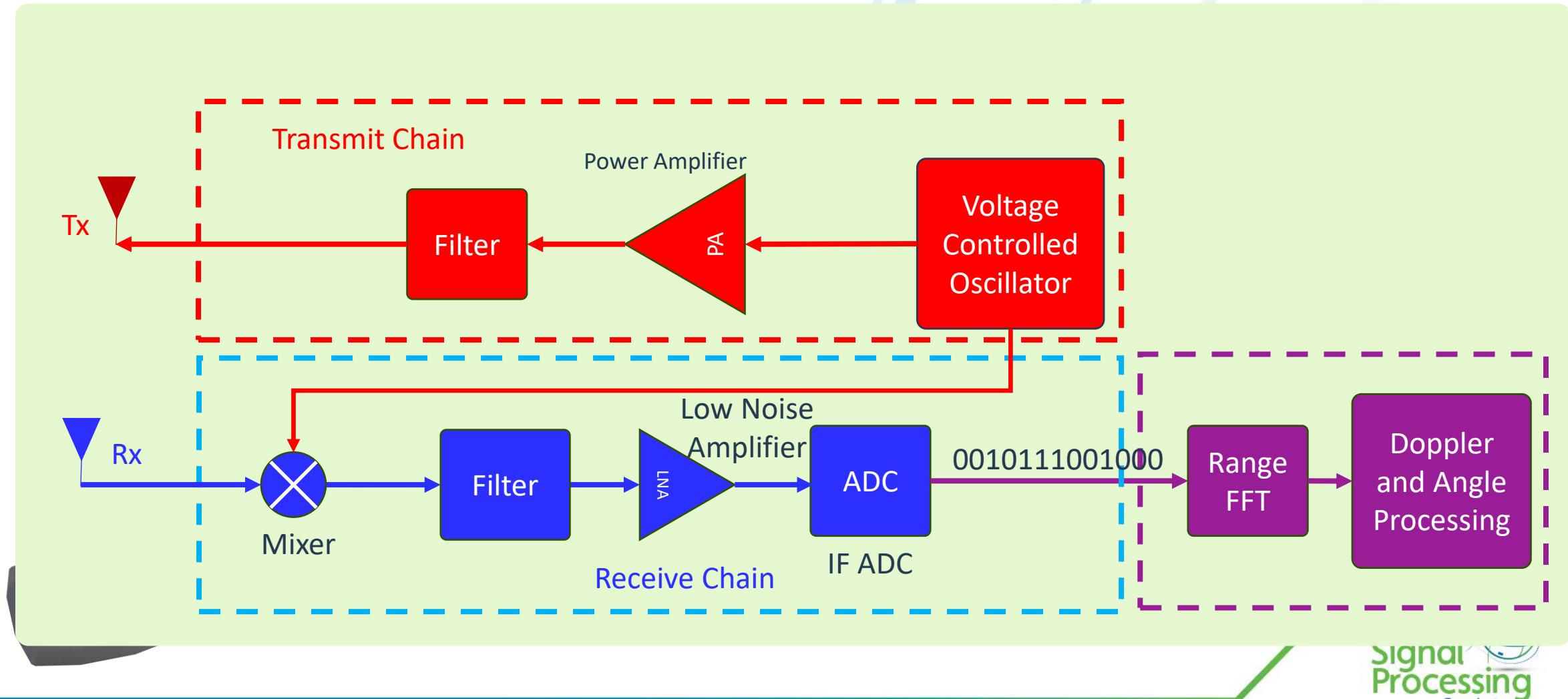


Lect1_example14.m



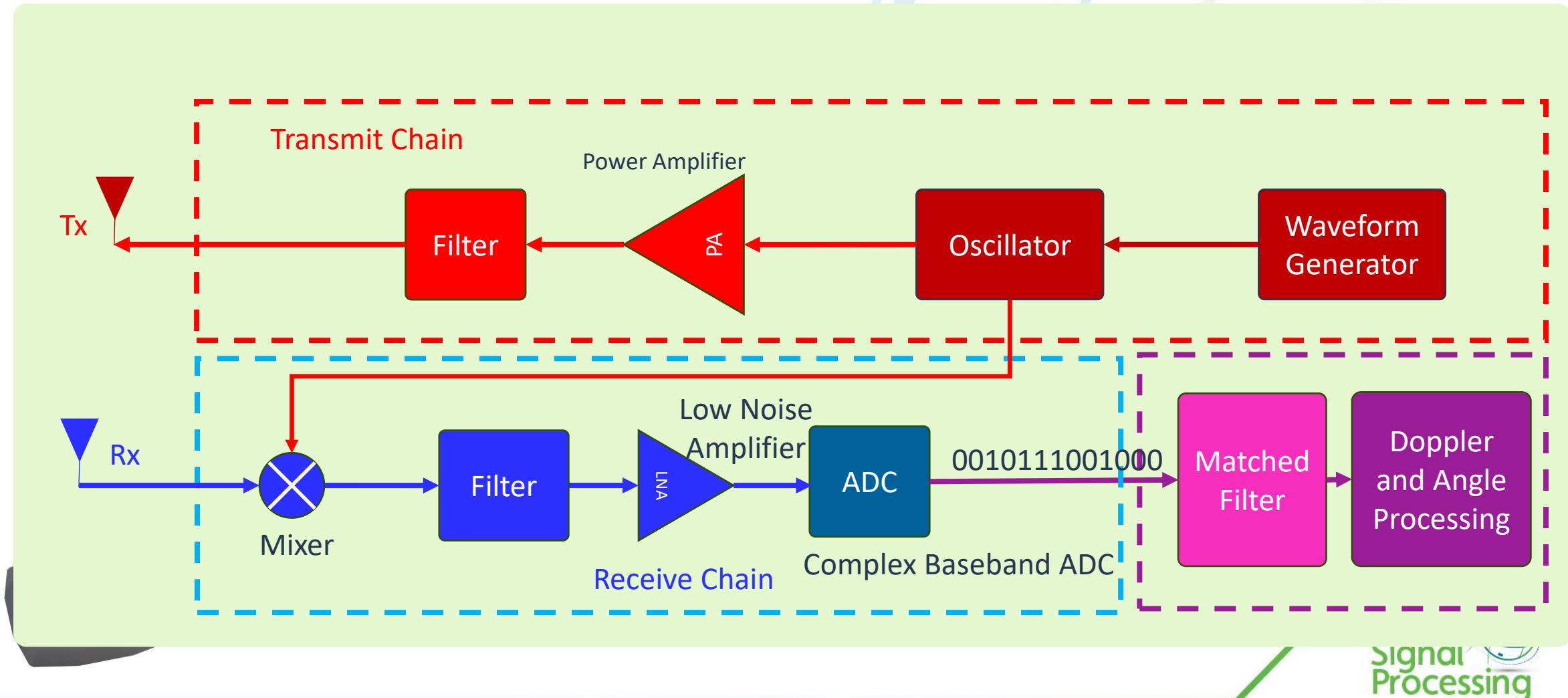
FMCW Radar

Block Diagram



PMCW Radar

Block Diagram



What we learned from Lecture 1

- Lecture 1 laid the groundwork for understanding radar signal processing by covering fundamental concepts such as radar types, key parameters, pulse compression techniques, different waveforms, and range-Doppler processing.



Scan the QR code for
access to the codes

Q & A

Can we still discriminate two targets if they have the same range and Doppler information?