Non-Convex Optimization for Practical Signal Design in Radar Systems with Emerging Applications

Part II-A:

Coordinate Descent (CD) framework for waveform optimization in radar systems

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CD/BCD FRAMEWORK

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Coordinate Descent (Ascent) Methods

Coordinate Descent

- Successively minimizes along coordinate directions
  – Optimize each parameter separately, holding all the others fixed.
- Why is it used?
  - ✓ Very simple and easy to implement
  - ✓ Careful implementations can attain state-of-the-art
  - ✓ Scalable, don’t need to keep data in memory, low memory requirements
  - ✓ Faster than gradient descent if iterations are N times cheaper

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Coordinate Descent

$$\boldsymbol{x} = [x_1, x_2, \dots, x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & f(\boldsymbol{x}) \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

idea: optimize over individual coordinates

Coordinate Descent – Steps

$$x_1^{(k)} \in \arg\min_{x_1} f(x_1, x_2^{(k-1)}, x_3^{(k-1)}, \dots, x_N^{(k-1)})$$

$$x_2^{(k)} \in \arg\min_{x_2} f(x_1^{(k)}, x_2, x_3^{(k-1)}, \dots, x_N^{(k-1)})$$

$$x_3^{(k)} \in \arg\min_{x_3} f(x_1^{(k)}, x_2^{(k)}, x_3, \dots, x_N^{(k-1)})$$

$$\vdots$$

$$x_N^{(k)} \in \arg\min_{x_N} f(x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \dots, x_N)$$

Note:

1- After we solve for $x_i^{(k)}$, we use its new value from then on

2- Can everywhere replace individual coordinates with blocks of coordinates (Block Coordinate Descent)

Coordinate Descent – Algorithm

- Start from initial guess $\boldsymbol{x}^{(0)} = [x_1, x_2, \dots, x_N]^T$
- For $k = 0, 1, \dots$
  - Pick an index i from $\{1, \dots, N\}$
  - Optimize the i-th coordinate

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f(\underbrace{x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}}_{\text{done}}, \underset{\text{current}}{\zeta}, \underbrace{x_{i+1}^{(k)}, \dots, x_N^{(k)}}_{\text{To do}})$$

- Decide when/how to stop; return $\boldsymbol{x}^{(k+1)}$

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Gauss-Seidel and Jacobi

Gauss-Seidel style (One-at-a-time)

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f\left(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)}\right)$$

Jacobi style (all-at-once ; easy to parallelize)

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f\left(x_1^{(k)}, \dots, x_{i-1}^{(k)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)}\right)$$

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Which Coordinate? (One-at-a-time)

- **Greedy or Gauss-Southwell (Maximum Block Improvement)**
  - If f is differentiable, at iteration k, pick the index that minimizes $\nabla f(x_i^k)$
- **Derivative free rules**
  - **Cyclic** order $1, 2, \dots, N, 1, \dots$
  - **Double sweep**, $1, 2, \dots, N$, then $N - 1, \dots, 1$, repeat
  - **Cyclic with permutation**, random order each cycle
  - **Random sampling**, pick random index at each iteration

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Advantages

- Each iteration is usually cheap (single variable optimization)
- No extra storage vectors needed
- No stepsize tuning
- No other parameters that must be tuned
- In general, “derivative free”
- Simple to implement
- Works well for large-scale problems
- Currently quite popular; parallel version exist

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Disadvantages

- Each sub-problem needs to be easily solvable. Tricky if single variable optimization is hard
- Can be “slow” if sub-problems cannot be solved efficiently
- Convergence theory can be complicated
  - “One-at-a-time” update scheme is critical, and “all-at-once” scheme does not necessarily converge
- Non-differentiable cases are more tricky

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Convergence (One-at-a-time)

- The objective function values are non-decreasing, i. e.,

$$f\left(\boldsymbol{x}^{(0)}\right) \geq f\left(\boldsymbol{x}^{(1)}\right) \geq \; \ldots .$$

- If f is strictly convex and smooth, the algorithm converges to a global minimum (optimal solution).
- If f is strictly convex -> unique minimum -> local minimum = global minimum
  a. continuously differentiable over the feasible set,
  b. has separable constraints,
  c. has unique minimizer at each step,

  then CD method will converge to stationary points

[1] - Dimitri P, et al.. Nonlinear programming. Athena Scientific; 1999.

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Other Alternating methods - Alternating Minimization

2 blocks is called alternative optimization

$$\boldsymbol{x} = [x_1, x_2]^T$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x & w \end{array}$$

$$\mathcal{P}_{x,w} \begin{cases} \underset{x,w}{\text{minimize}} & f(x, w) \\ \text{subject to} & x \in \psi_1, w \in \psi_2 \end{cases}$$

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Other Alternating methods - BSUM

Block successive upper-bound minimization (BSUM)

$$\boldsymbol{x} = [x_1, x_2, \dots., x_N]^T$$

$$\mathcal{P}_{\boldsymbol{x}} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & f(\boldsymbol{x}) \\ \text{subject to} & x_n \in \psi_n \end{cases}$$

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} u_i(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$$

Local approximation of the objective function

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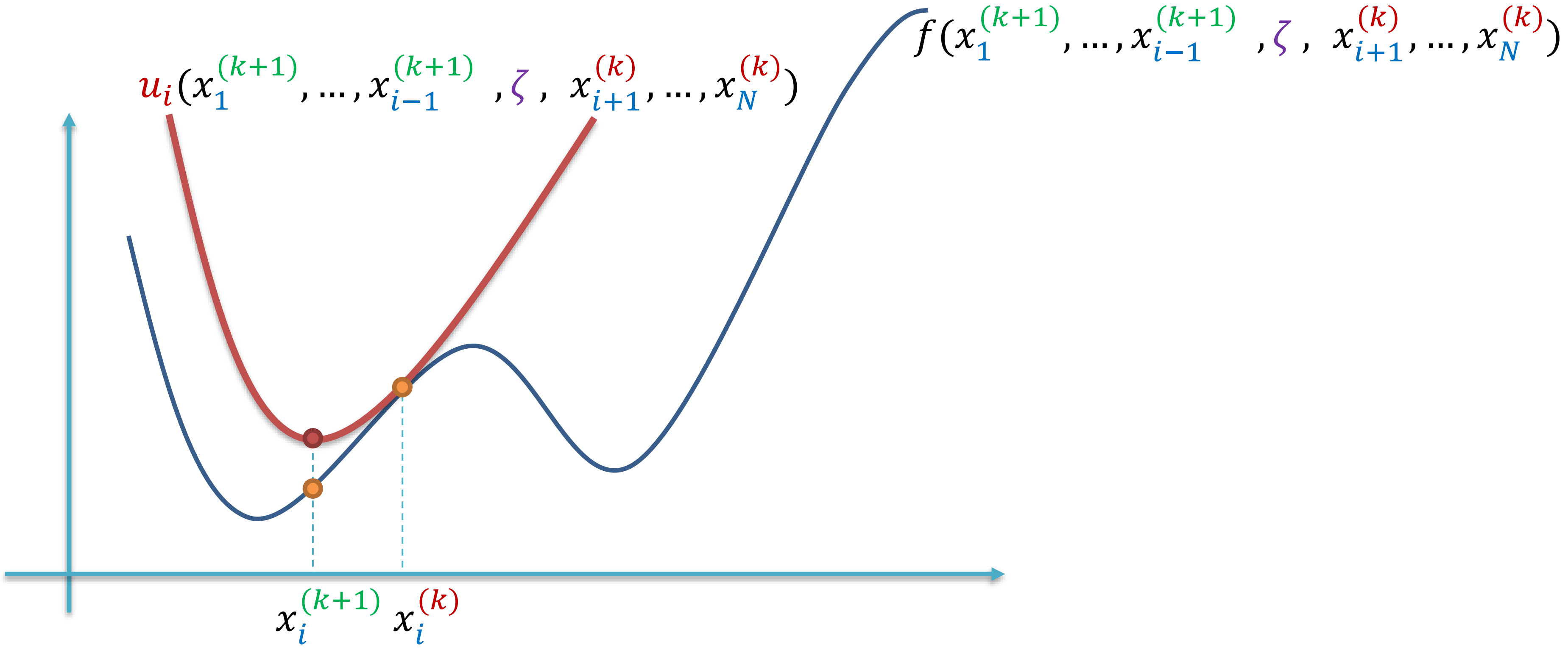
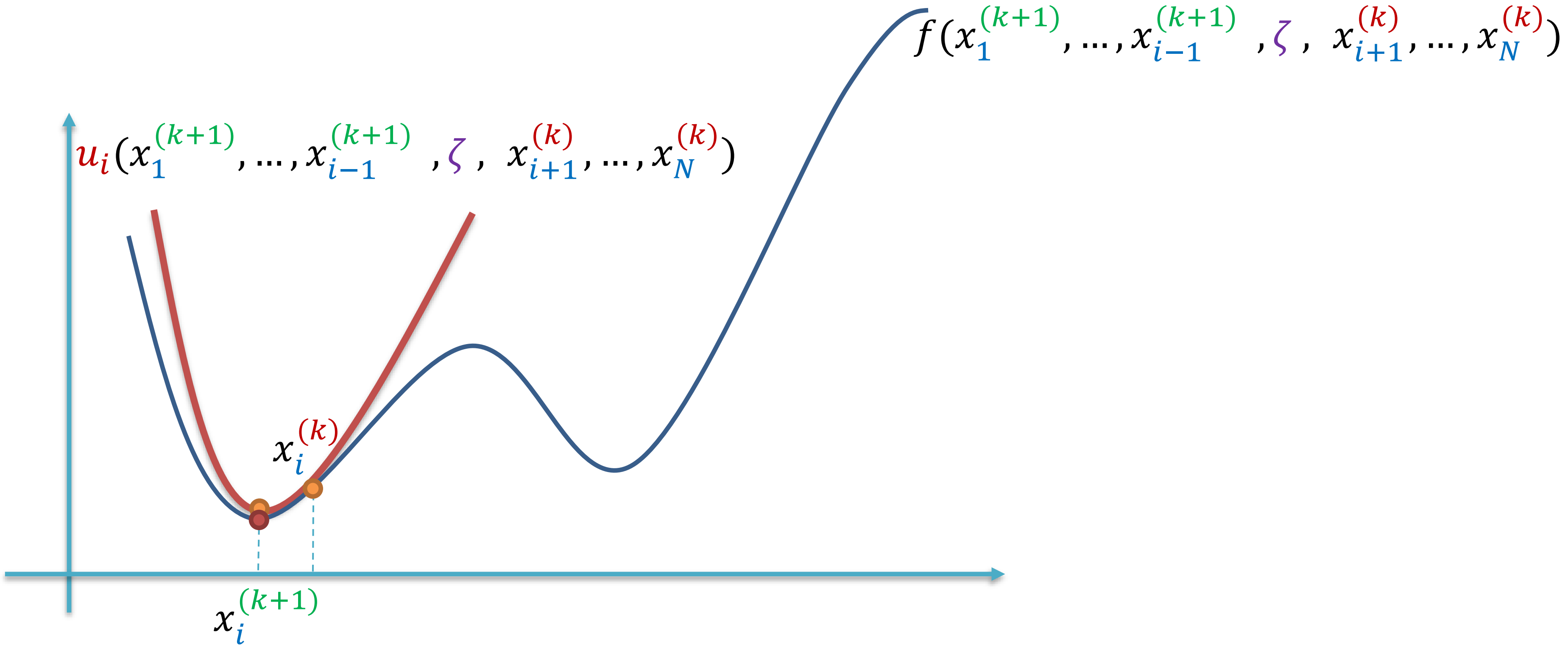
Other Alternating methods - BSUM



Upper-bound $\quad u_i\left(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)}\right) \geq f(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)})$

Other Alternating methods - BSUM



Block successive upper-bound minimization, block successive convex approximation, convex-concave procedure, majorization-minimization, dc-programming, BCGD,…

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Other Alternating methods

- Alternating direction method of multipliers (ADMM)
- Several others...

$$\begin{cases} \underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} & f(\boldsymbol{x}) + g(\boldsymbol{z}) \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} = \boldsymbol{c} \end{cases}$$

$$L_\rho(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{y}) = f(\boldsymbol{x}) + g(\boldsymbol{z}) + y^T(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} - \boldsymbol{c}) + \left(\frac{\rho}{2}\right) \|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{z} - \boldsymbol{x}\|_2^2$$

$$\boldsymbol{x}^{(k+1)} \leftarrow \arg\min_{\boldsymbol{x}} L_\rho(\boldsymbol{x}, \boldsymbol{z}^{(k)}, \boldsymbol{y}^{(k)})$$

$$\boldsymbol{z}^{(k+1)} \leftarrow \arg\min_{\boldsymbol{z}} L_\rho(\boldsymbol{x}^{(k+1)}, \boldsymbol{z}, \boldsymbol{y}^{(k)})$$

$$\boldsymbol{y}^{(k+1)} \leftarrow \boldsymbol{y}^{(k)} + \rho(\boldsymbol{A}\boldsymbol{x}^{(k+1)} + \boldsymbol{B}\boldsymbol{z}^{(k+1)} - \boldsymbol{c})$$

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WAVEFORM DESIGN WITH GOOD CORRELATION PROPERTIES IN RADAR SYSTEMS

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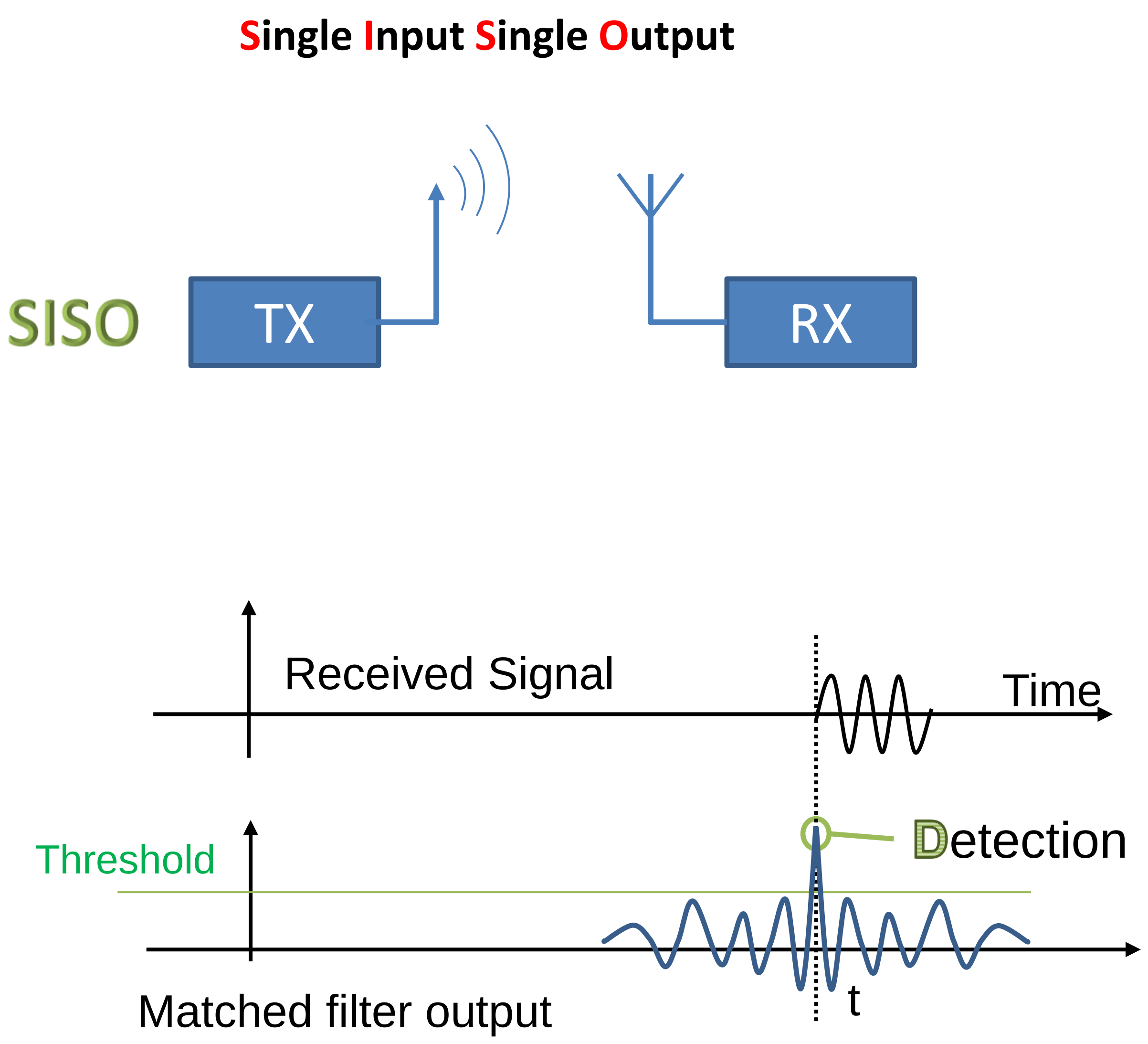
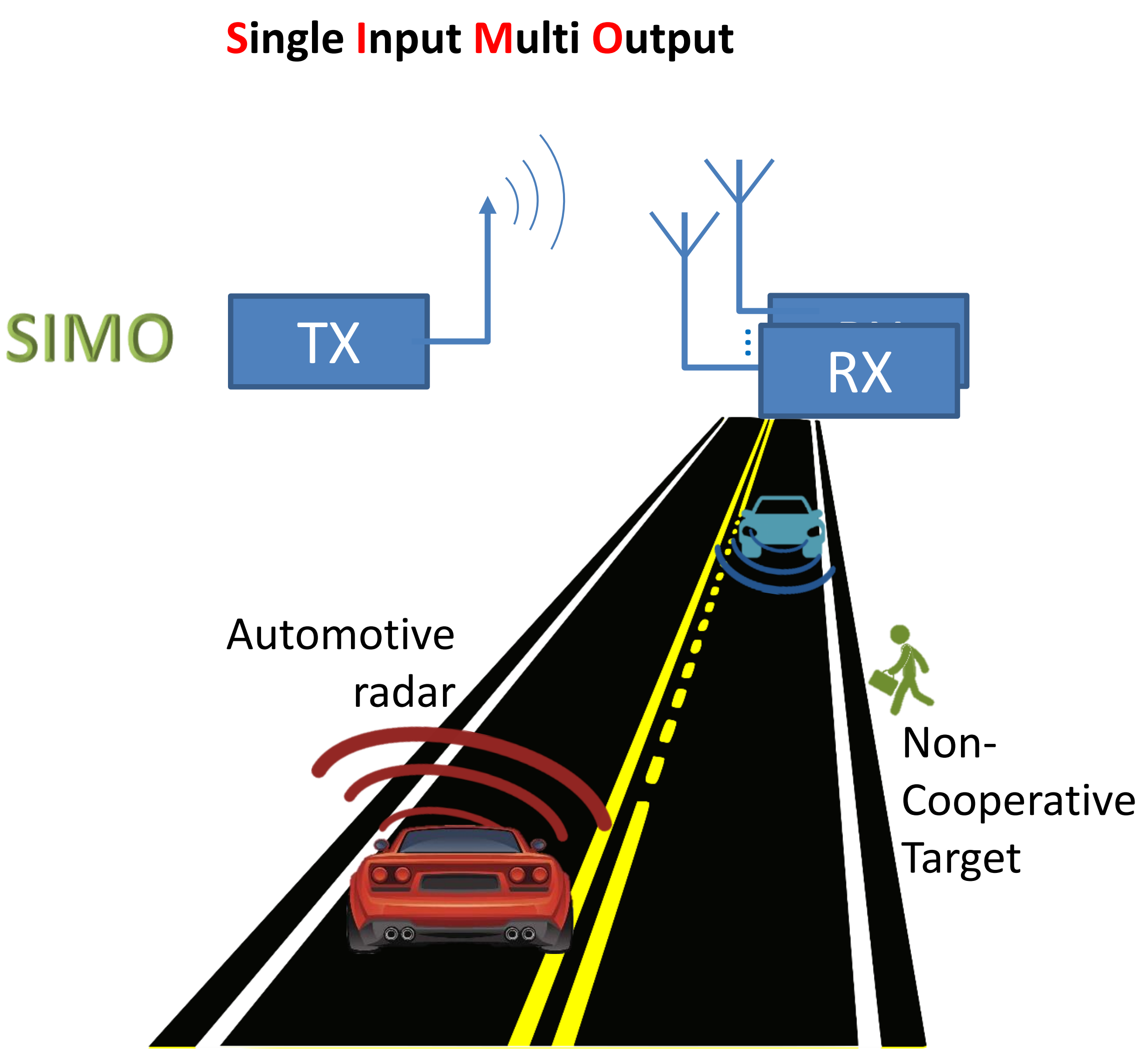
Waveforms in SISO/SIMO Radar Systems





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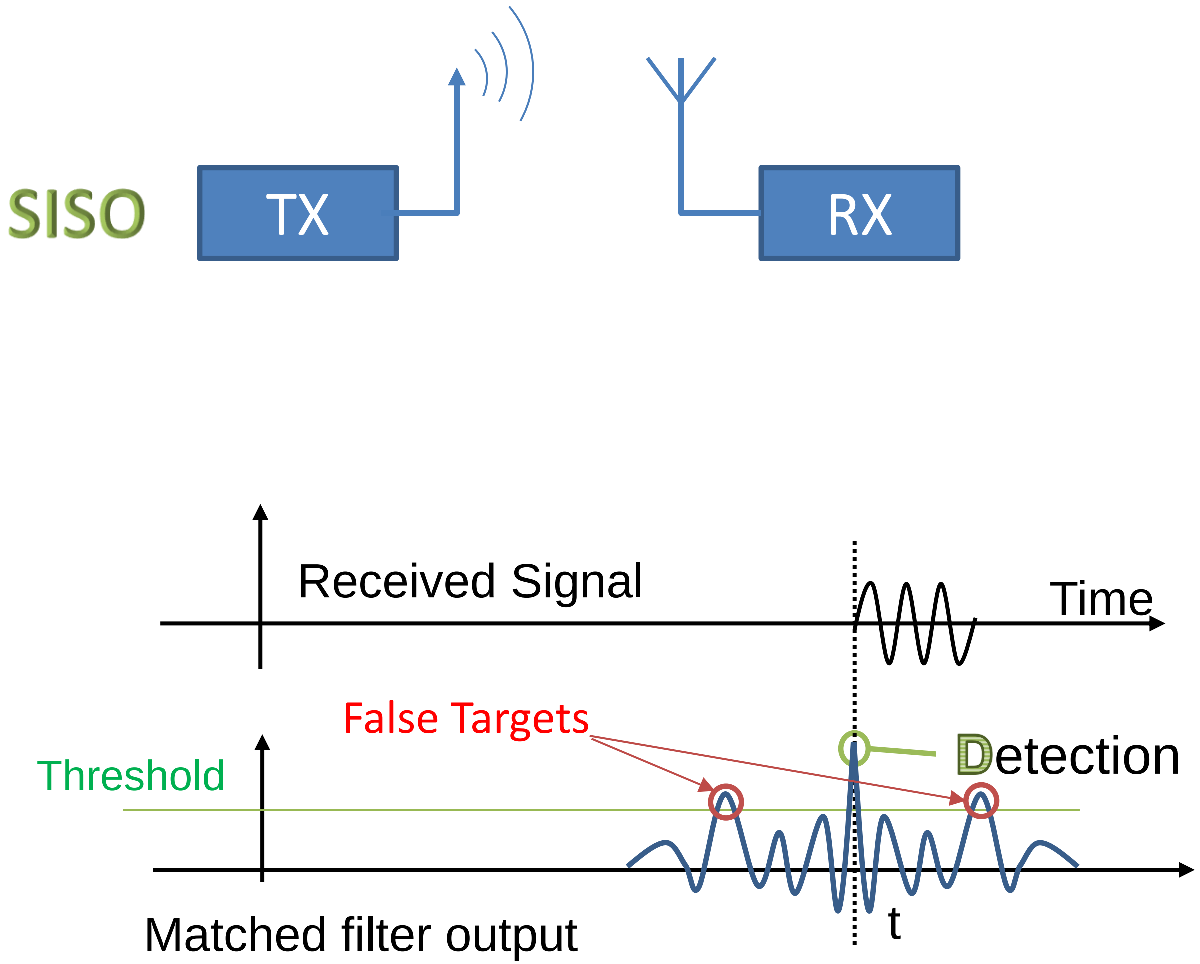
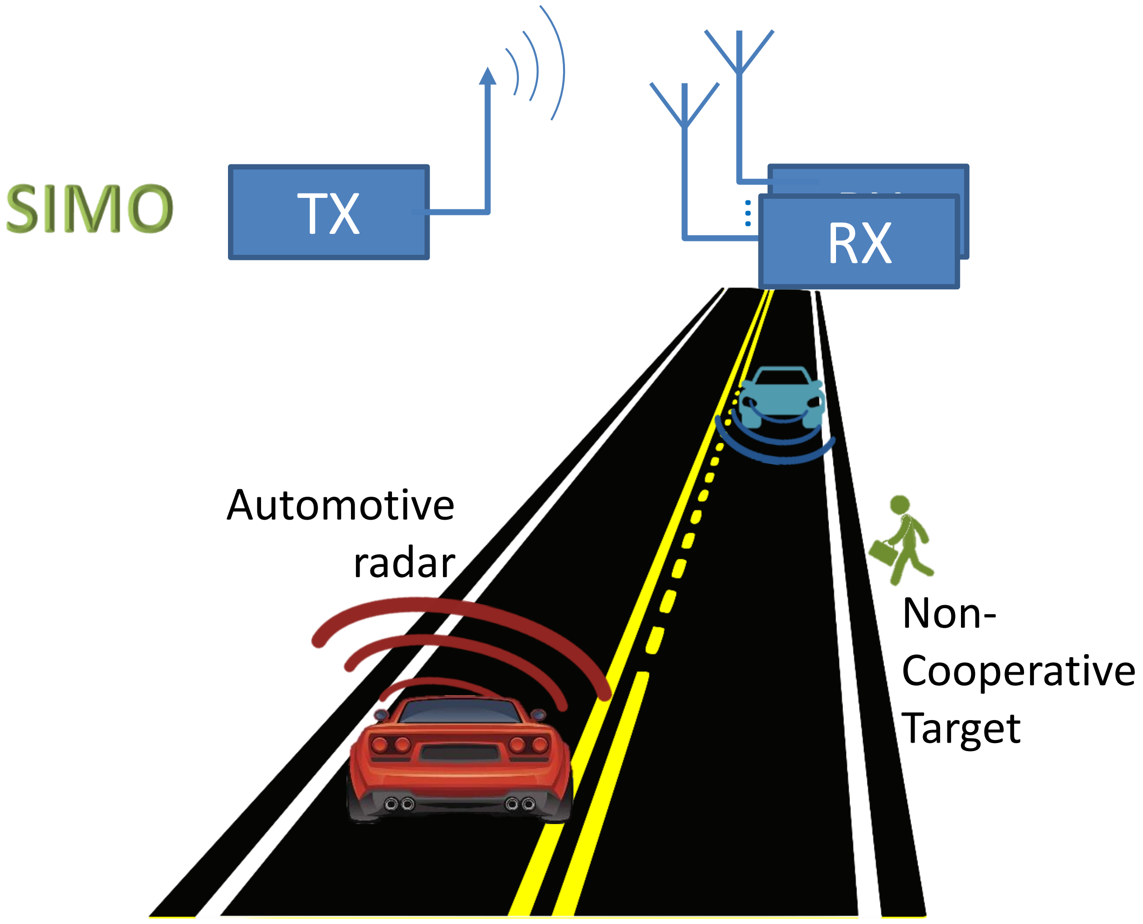
Waveforms in SISO/SIMO Radar Systems

Single Input Single Output



Single Input Multi Output



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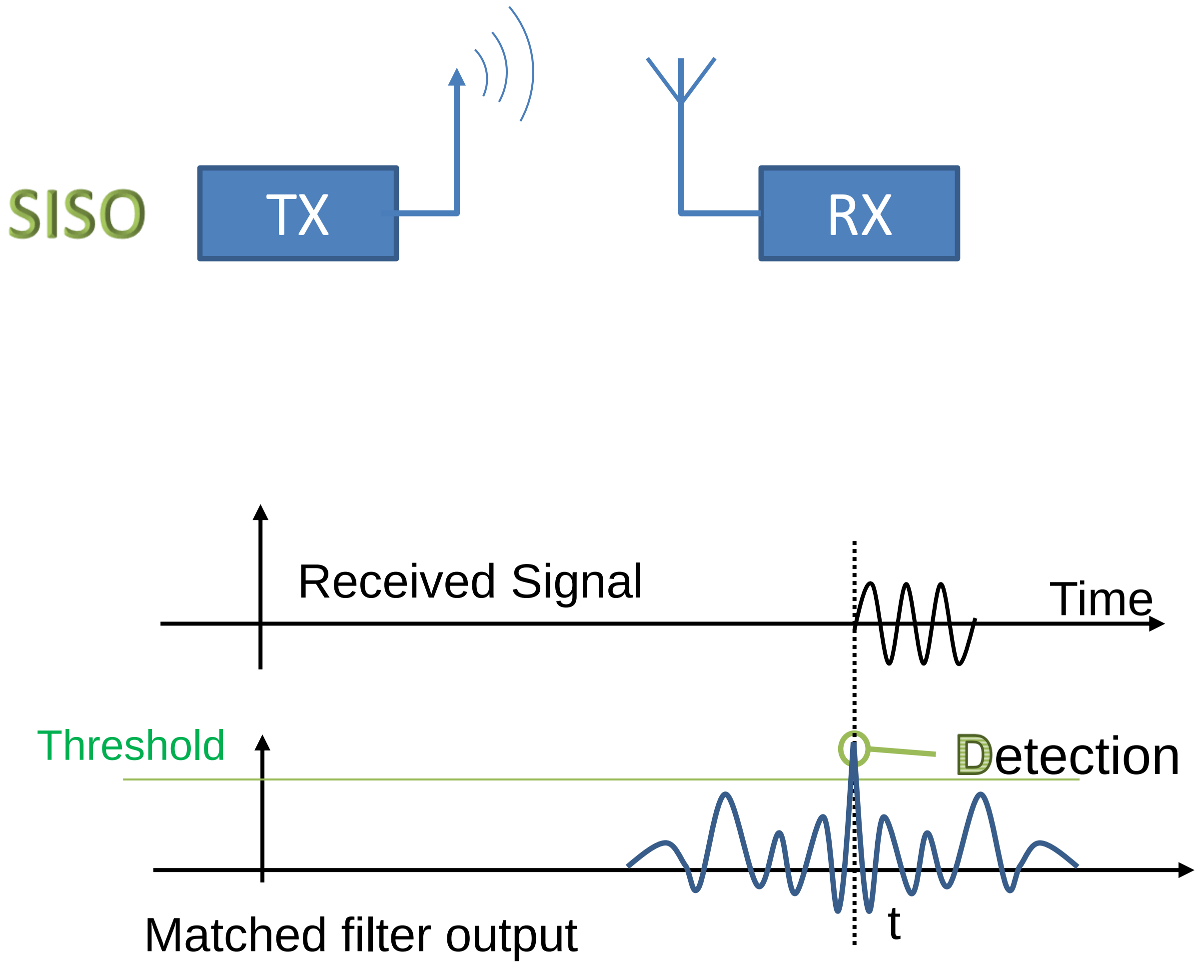
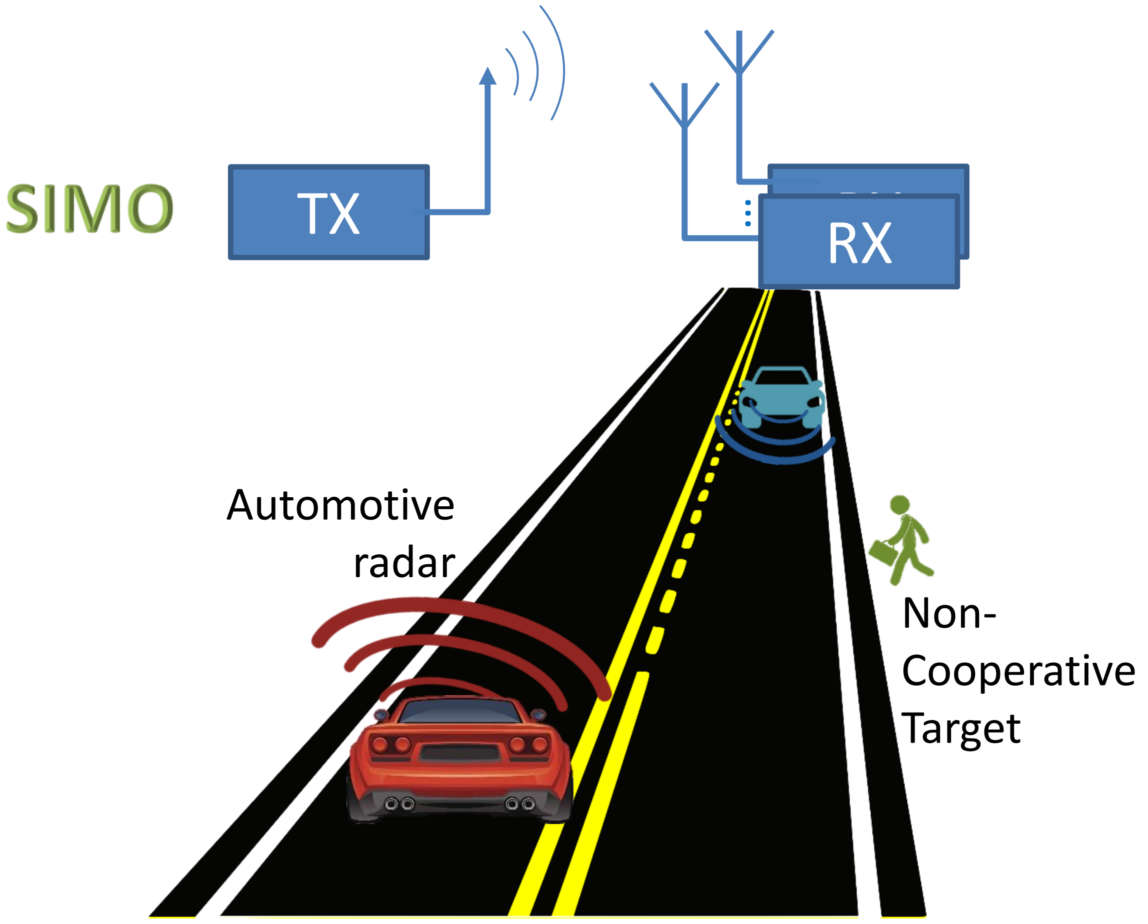
Waveforms in SISO/SIMO Radar Systems

Single Input Single Output

Single Input Multi Output





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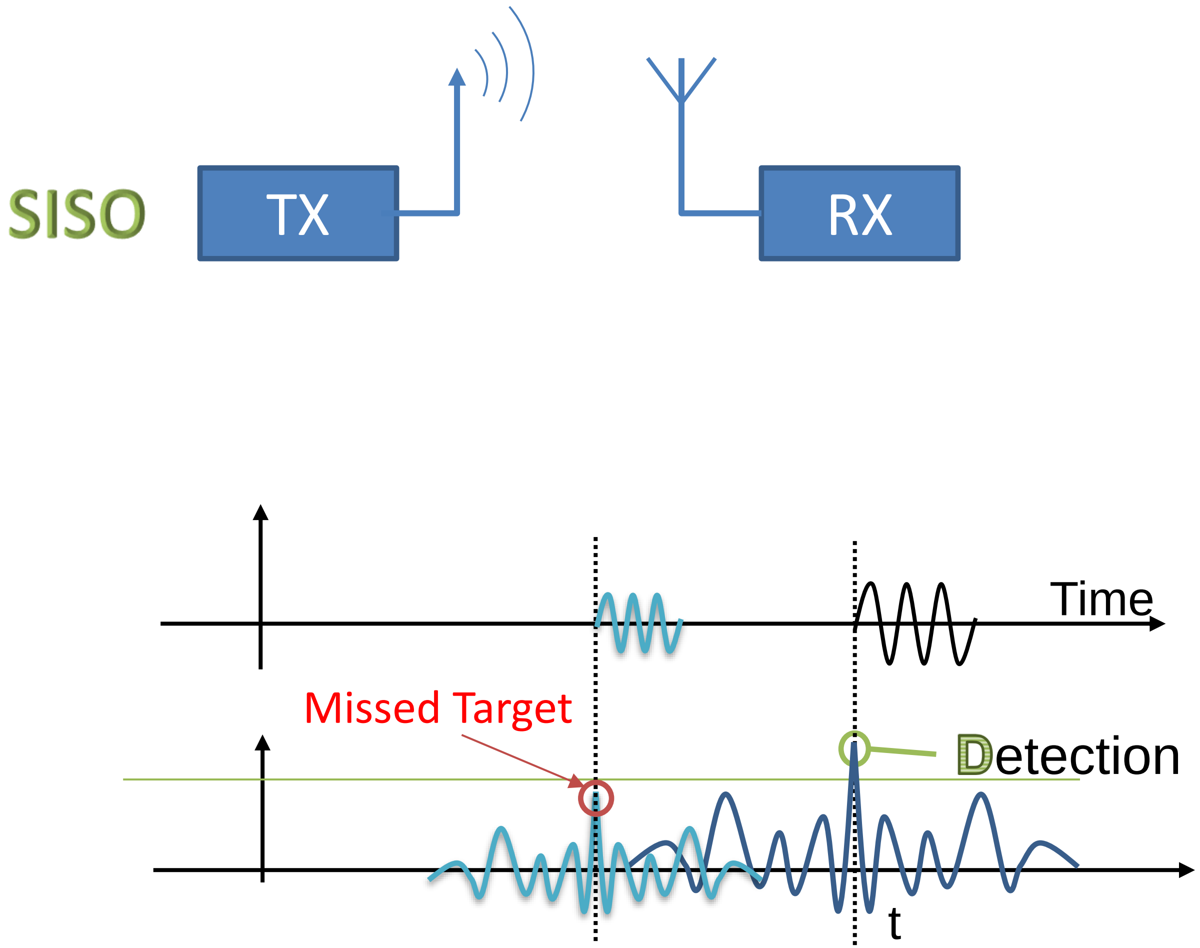
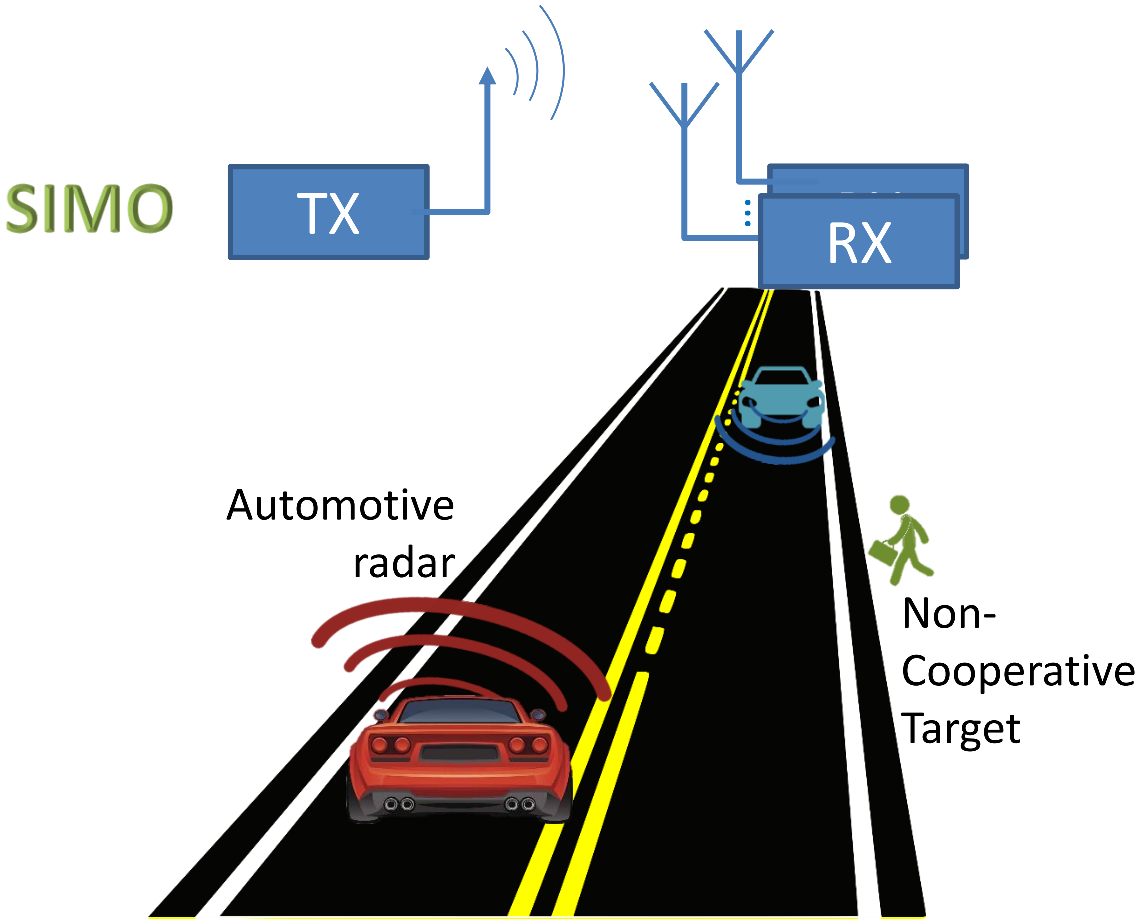
Waveforms in SISO/SIMO Radar Systems

Single Input Single Output



Single Input Multi Output



Metrics for Good Waveforms

- Small
  - Peak Sidelobe Level (PSL)
  - Integrated Sidelobe Level (ISL)

Small PSL	• avoid masking of weak targets in range sidelobes of a strong return
Low ISL	• mitigate the deleterious effects of distributed clutter echoes which are close to the target of interest

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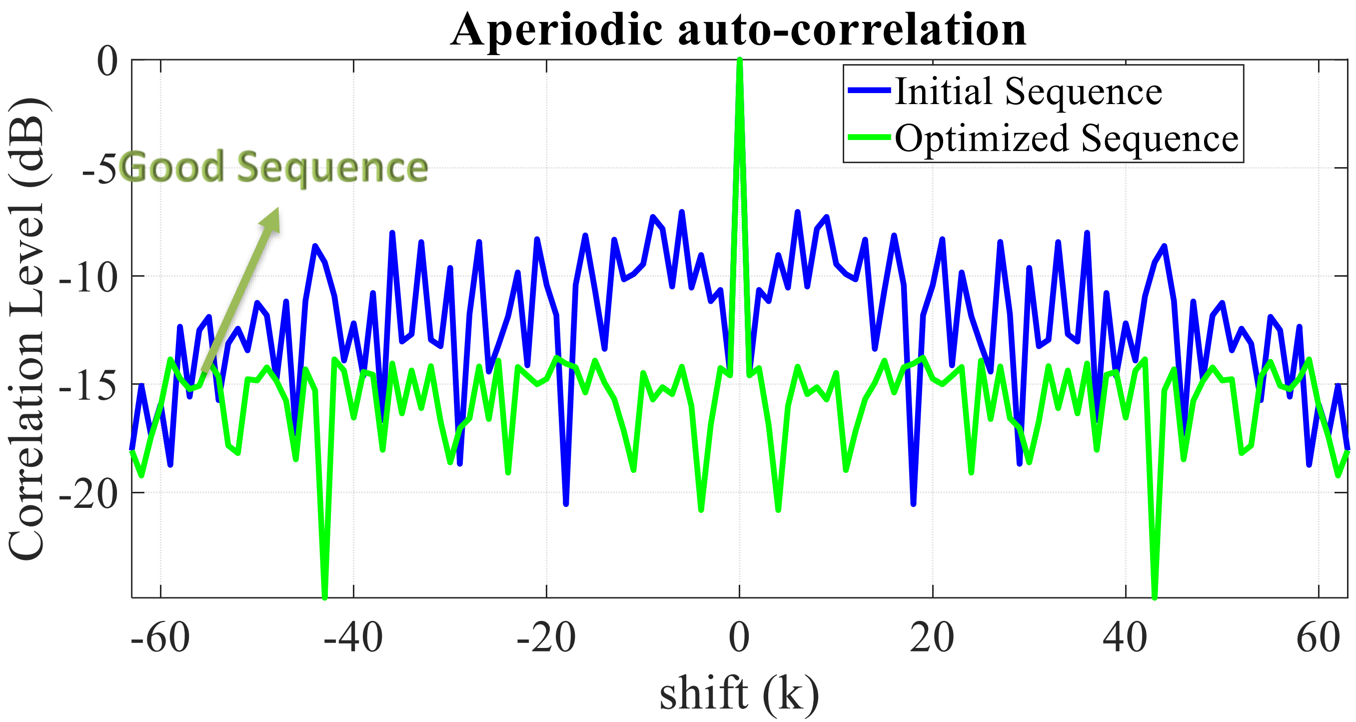
Example



$$\boldsymbol{x} = [x_1 . x_2 \dots . x_N]^T \in \mathbb{C}^N,$$

$$r_k = \sum_{n=1}^{N-k} x_n^* x_{n+k} . \quad k = 0. \dots . N-1$$

$$r_k^P = \sum_{n=1}^{N-k} x_n \, x_{n+k \bmod (N)}^* = r_{-k}^P$$

$$\text{PSL} = \max_{k \neq 0} |r_k| \qquad \text{ISL} = \sum_{k=1}^{N-1} r_k^2$$

How to design a sequence with small PSL / ISL ?

Example: PSL Minimization in SISO/SIMO Radar Systems

$$\boldsymbol{x} = [x_1, x_2, \ldots., x_N]^T \in \mathbb{C}^N,$$

$$\mathrm{f}(\mathbf{x}) = \max\{|r_k|\}_{k=1}^{N-1}$$

$$\mathcal{P}_{\boldsymbol{x}}^{M} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max\{|r_k|\}_{k=1}^{N-1} \\ \text{subject to} & x_{\mathrm{n}} \in \Omega_M \end{cases}$$

$$\mathcal{P}_{\boldsymbol{x}}^{\infty} \begin{cases} \underset{\boldsymbol{x}}{\text{minimize}} & \max\{|r_k|\}_{k=1}^{N-1} \\ \text{subject to} & |x_n| = 1 \end{cases}$$

$$\underbrace{\Omega_M}_{\text{Alphabet size}} = \left\{1, e^{\frac{j2\pi}{M}}, \ldots, e^{\frac{j2\pi(M-1)}{M}}\right\}$$

Non-Convex Multi-variable Constrained min-max optimization problems

Example: PSL Minimization in SISO/SIMO Radar Systems

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \max\{|r_k|\}_{k=1}^{N-1}$$

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad f(\boldsymbol{x})$$

$$x_i^{(k+1)} \leftarrow \arg\min_{\zeta} f\left(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)}\right)$$

$$r_k\left(x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, \zeta, x_{i+1}^{(k)}, \dots, x_N^{(k)}\right) = a_{ki}\,\zeta + b_{ki}\zeta^* + c_{ki}$$

$$\zeta = e^{j\phi}$$

Example: PSL Minimization – Discrete Phase

$$\mathcal{P}_{\phi}^{(k+1)} \begin{cases} \underset{\phi}{\text{minimize}} & \max\left\{\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|\right\}_{k=1}^{N-1} \\ \text{subject to} & \phi \in \left\{0, \frac{2\pi}{M}, \dots, \frac{2\pi(M-1)}{M}\right\} \end{cases}$$

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Example: PSL Minimization – Constant Modulus

$$\mathcal{P}_{\phi}^{(k+1)} \begin{cases} \underset{\phi}{\text{minimize}} & \max\left\{\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|^2\right\}_{k=1}^{N-1} \\ \text{subject to} & \phi \in [0, 2\pi) \end{cases}$$

$$\beta = \tan\frac{\phi}{2}$$

$$\left|a_{ki}e^{j\phi} + b_{ki}e^{-j\phi} + c_{ki}\right|^2 = \frac{\mu\beta^4 + \kappa\beta^3 + \delta\beta^2 + \eta\beta + \rho}{(1+\beta^2)^2}$$

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Example: PSL Minimization – Constant Modulus

$$\underset{\beta}{\text{minimize}} \qquad \max\left\{\frac{\mu\beta^4+\kappa\beta^3+\delta\beta^2+\eta\beta+\rho}{(1+\beta^2)^2}\right\}_{k=1}^{N-1}$$

$$\begin{cases} \text{find} & \beta \\ \text{subject to} & \dfrac{\mu\beta^4+\kappa\beta^3+\delta\beta^2+\eta\beta+\rho}{(1+\beta^2)^2} \le \gamma \end{cases}$$

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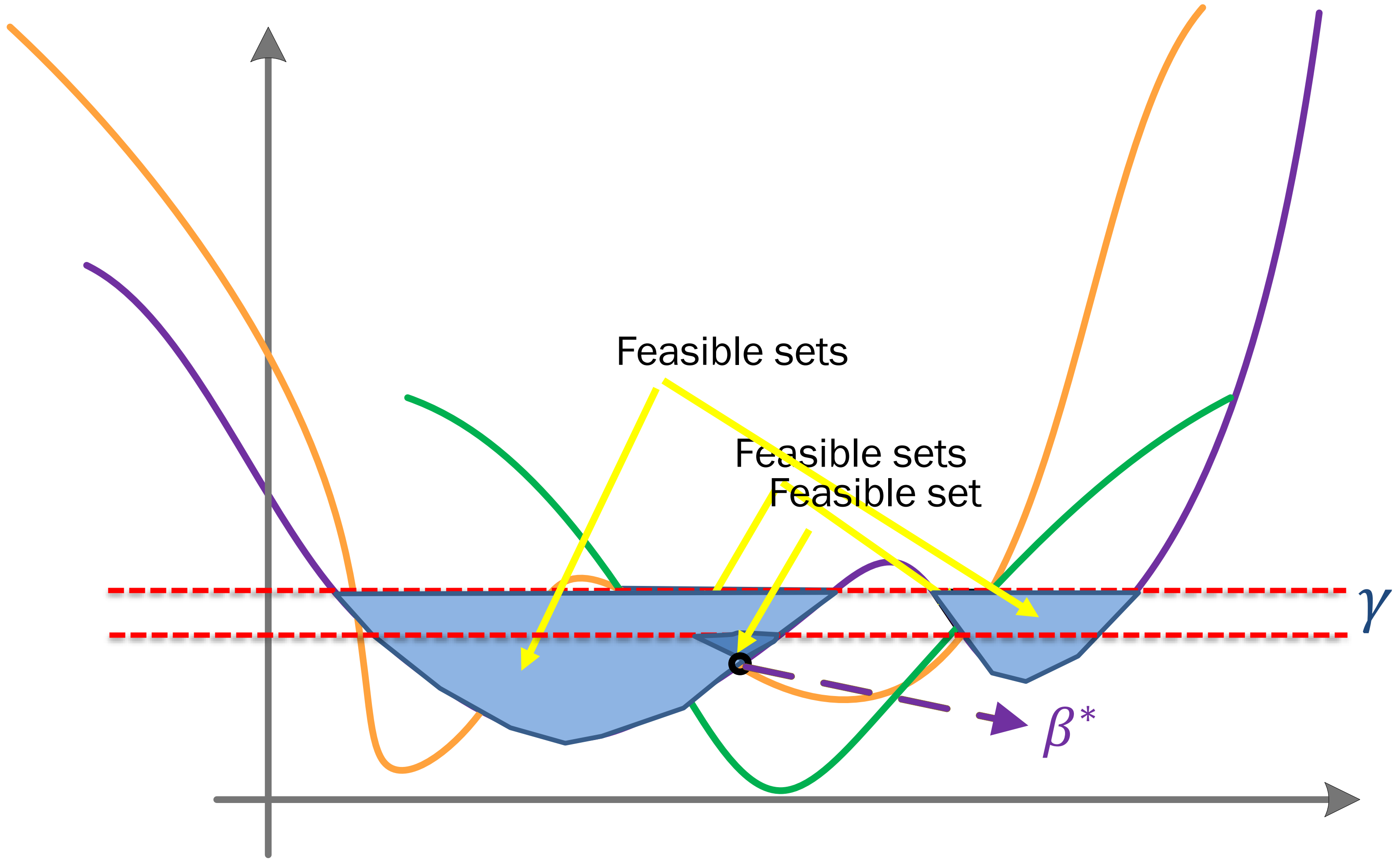
Example: PSL Minimization in SISO/SIMO Radar Systems



Example: PSL Minimization in SISO/SIMO Radar Systems

N	Sample Binary Codes with Good PSL (in HEX)
320	3398D83F635CC5A0D5727CB53A97D39896CFD7C6F1EF86C9AEDE20400F546DF8AB49D7D0879C21BB
360	6D4A71524C40837C9DA7F101F7580E457FFE23696BFA3B7DB9A957CA7923E185985396572CCB9AAD7347A38682
400	B6686A4E6FEA1CF29CBFE6ECA477E2A5D7A8F448A108A5F3F593E63ABC7917D84CA736F15C447BD2072CABA99F127CA5185C
440	73B8B3397676BB952A97A519AEB64C7C544D00242B2A8180BFCB610F4AE6D1C0740F1D8904DE217F4F79248D054B2C7FB490C3CE10BC67
480	64A83F1A672F6E4A4CF5824F5FDD9FBE73FC48322A4D930E17702F859E67911CFD2E12415ACBB55159C229E8ACFF70C25227A379A92CAA17A712B91D

Matlab source codes can be downloaded from :

https://radarmimo.com/how-to-design-binary-codes-for-radar-systems/

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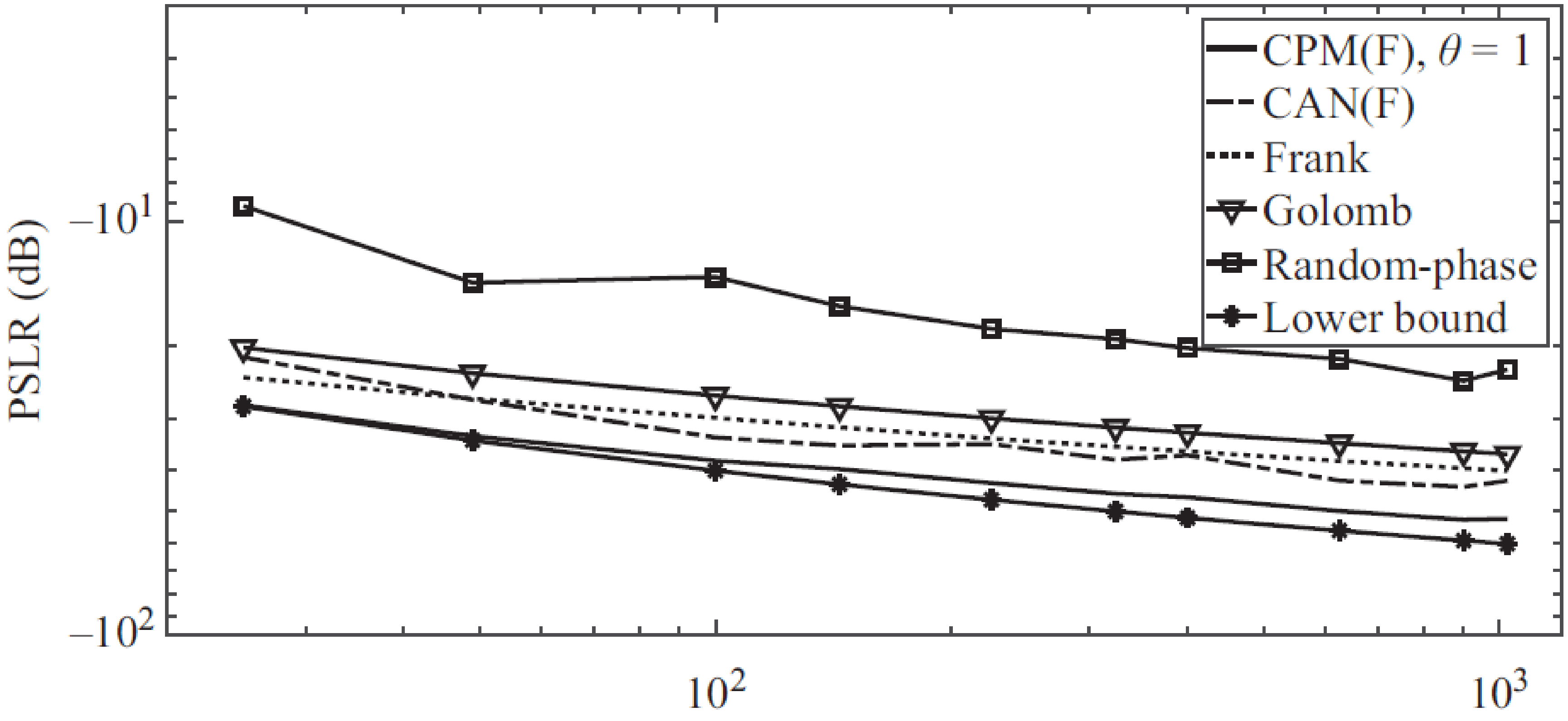
Example: PSL Minimization in SISO/SIMO Radar Systems



Matlab source codes can be downloaded from :

https://radarmimo.com/how-to-design-binary-codes-for-radar-systems/

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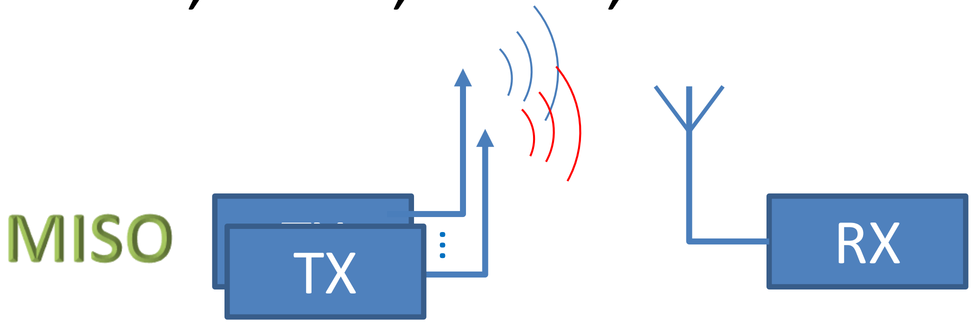
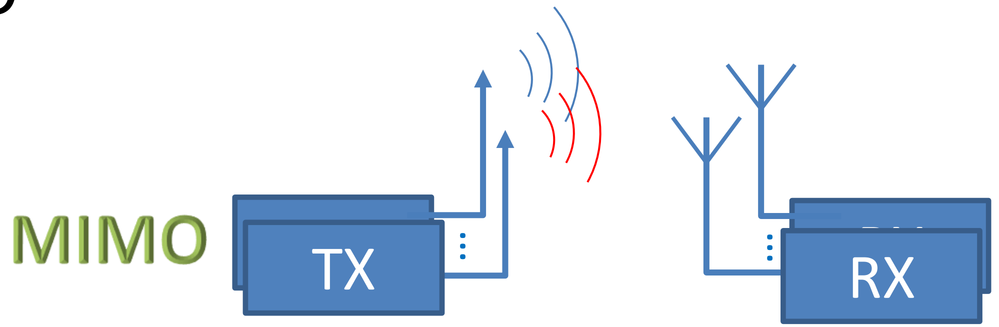
Waveforms in (Colocated/Widely Separated) MISO/MIMO Radar Systems

- ✓ Transmitters should be observable at each receiver
- ✓ Enabled by Orthogonal Waveforms
  - Limit mutual interference
  - Enable cooperative operation
  - Provide visibility into paths between transmitter and receivers
  - Determines spatial distribution of energy
  - Orthogonality achieved by division in time, frequency or code
- ✓ FDM-, TDM-, DDM-, and CDM-MIMO



Multi Input Single Output



Multi Input Multi Output

CDM-MIMO Waveform Design Problem

$$\boldsymbol{x}_m = [x_m(1), x_m(2), \ldots, x_m(N)]^T \in \mathbb{C}^N,$$

$$\boldsymbol{X} = [\boldsymbol{x}_1, \quad \boldsymbol{x}_2, \quad \ldots, \boldsymbol{x}_{N_T}] \in \mathbb{C}^{N \times N_T}$$

$$r_{ml}(k) = \sum_{n=1}^{N-k} x_m(n)\, x_l^*(n+k) = r_{lm}^*(-k)$$

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CDM-MIMO Waveform Design Problem

$$\mathrm{PSL} = \max\left\{\max_{m}\max_{k\neq 0}|r_{mm}(k)|\ ,\max_{\substack{m,l\\ m\neq l}}\max_{k}|r_{ml}(k)|\right\}$$

$$\mathrm{ISL} = \sum_{m=1}^{N_T}\sum_{\substack{k=-N+1\\ k\neq 0}}^{N-1}|r_{mm}(k)|^2 + \sum_{\substack{m,l=1\\ m\neq l}}^{N_T}\sum_{k=-N+1}^{N-1}|r_{ml}(k)|^2$$

How to design set of sequences with small PSL / ISL ?

[2] - M. Alaee-Kerahroodi, M. Modarres-Hashemi and M. M. Naghsh, "Designing Sets of Binary Sequences for MIMO Radar Systems," in *IEEE Transactions on Signal Processing*, vol. 67, no. 13, pp. 3347-3360, 1 July1, 2019.

Get in touch for more info

Thank you!

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