DRUBBLE

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ABSTRACT. asdf

1. Introduction

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2. The History of Drubble

A game we used to play while drinking in college.

2.1. Single Player Drubblei.
Throw it up, bounce it on the stool, try to keep it bouncing.

2.2. Two Player Drubble.
First person throws it, second person tries to bounce it as far as they can on the stool.

2.3. Three Player Drubble.
First person throws it, second person bounces it to the third person, who bounces it to the first person who has run behind them.

3. Dynamics Model

The generalized coordinate vector is

$$\boldsymbol{q} = (x, y, l, \theta)^T \tag{3.1}$$

Person is a point mass that can translate in two directions, their position is

$$\boldsymbol{r}_c = \begin{bmatrix} x \\ y \end{bmatrix} \tag{3.2}$$

and their velocity is

$$\boldsymbol{v}_c = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}, \tag{3.3}$$

and the matrix containing derivatives of $\dot{\boldsymbol{r}}_c$ with respect to $\boldsymbol{q}$ is

$$\mathbf{V}_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{3.4}$$

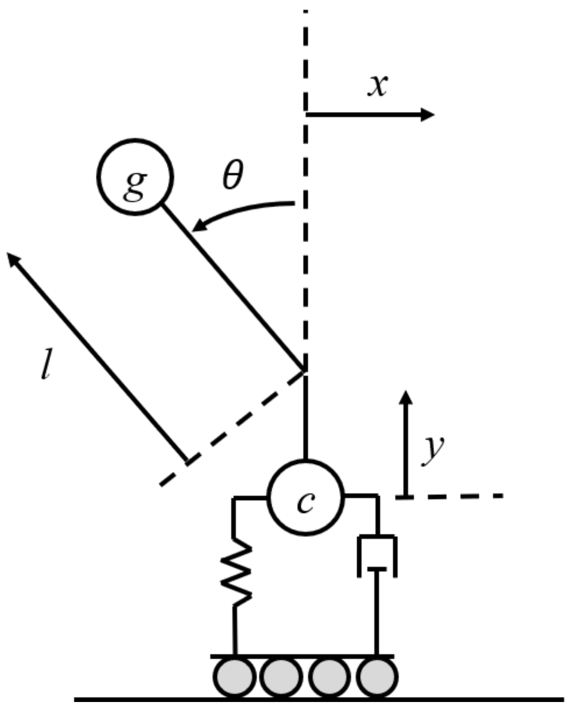
The position of the stool is

$$\boldsymbol{r}_g = \begin{bmatrix} x - l\sin\theta \\ y + a + l\cos\theta \end{bmatrix} \tag{3.5}$$

and its velocity is

$$\boldsymbol{v}_g = \begin{bmatrix} \dot{x} - \dot{l}\sin\theta - l\dot{\theta}\cos\theta \\ \dot{y} + \dot{l}\cos\theta - l\dot{\theta}\sin\theta \end{bmatrix}, \tag{3.6}$$

Key words and phrases. Dynamics.



and the matrix containing derivatives of $\dot{\boldsymbol{r}}_g$ with respect to $\boldsymbol{q}$ is

$$\mathbf{V}_g = \begin{bmatrix} 1 & 0 & -\sin\theta & -l\cos\theta \\ 0 & 1 & \cos\theta & -l\sin\theta \end{bmatrix}. \tag{3.7}$$

Lagranges method used to define equations of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right) - \frac{\partial L}{\partial \boldsymbol{q}} = \boldsymbol{Q} \tag{3.8}$$

where $L = T - V$. Kinetic energy is

$$\begin{aligned} T &= \frac{1}{2}\dot{\boldsymbol{q}}^T\mathbf{M}\dot{\boldsymbol{q}} \\ &= \frac{1}{2}\Big((m_c + m_g)\dot{x}^2 + (m_c + m_g)\dot{y}^2 + m_g\dot{l}^2 + m_g l^2\dot{\theta}^2 \\ &\qquad - 2m_g\dot{x}\dot{l}\sin\theta - 2m_g\dot{x}\dot{\theta}l\cos\theta + 2m_g\dot{y}\dot{l}\cos\theta - 2m_g\dot{y}\dot{\theta}l\sin\theta\Big) \end{aligned} \tag{3.9}$$

where the mass matrix

$$\mathbf{M} = m_c\mathbf{V}_c^T\mathbf{V}_c + m_g\mathbf{V}_g^T\mathbf{V}_g = \begin{bmatrix} m_c + m_g & 0 & -m_g\sin\theta & -m_g l\cos\theta \\ 0 & m_c + m_g & m_g\cos\theta & -m_g l\sin\theta \\ -m_g\sin\theta & m_g\cos\theta & m_g & 0 \\ -m_g l\cos\theta & -m_g l\sin\theta & 0 & m_g l^2 \end{bmatrix}. \tag{3.10}$$

These can be used to derive

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\boldsymbol{q}}}\right) = \mathbf{M}\ddot{\boldsymbol{q}} + \dot{\mathbf{M}}\dot{\boldsymbol{q}} \tag{3.11}$$

where
(3.12)

$$\dot{\mathbf{M}} = m_g \begin{bmatrix} 0 & 0 & -\dot{\theta}\cos\theta & -\dot{l}\cos\theta + l\dot{\theta}\sin\theta \\ 0 & 0 & -\dot{\theta}\sin\theta & -\dot{l}\sin\theta - l\dot{\theta}\cos\theta \\ -\dot{\theta}\cos\theta & -\dot{\theta}\sin\theta & 0 & 0 \\ -\dot{l}\cos\theta + l\dot{\theta}\sin\theta & -\dot{l}\sin\theta - l\dot{\theta}\cos\theta & 0 & 2l\dot{l} \end{bmatrix}$$

$$\dot{\mathbf{M}}\dot{\boldsymbol{q}} = m_g \begin{bmatrix} -2\dot{l}\dot{\theta}\cos\theta + l\dot{\theta}^2\sin\theta \\ -2\dot{l}\dot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta \\ -\dot{x}\dot{\theta}\cos\theta - \dot{y}\dot{\theta}\sin\theta \\ -\dot{x}\dot{l}\cos\theta + \dot{x}l\dot{\theta}\sin\theta - \dot{y}\dot{l}\sin\theta - \dot{y}l\dot{\theta}\cos\theta \end{bmatrix} \tag{3.13}$$

The derivatives

$$\begin{aligned} \frac{\partial T}{\partial \boldsymbol{q}} &= \frac{1}{2}m_g \begin{bmatrix} 0 \\ 0 \\ \dot{\boldsymbol{q}}^T\left([\partial\mathbf{V}_g^T/\partial l]^T\mathbf{V}_g + \mathbf{V}_g^T[\partial\mathbf{V}_g/\partial l]\right) \\ \dot{\boldsymbol{q}}^T\left([\partial\mathbf{V}_g^T/\partial\theta]^T\mathbf{V}_g + \mathbf{V}_g^T[\partial\mathbf{V}_g/\partial\theta]\right) \end{bmatrix} \dot{\boldsymbol{q}} \\ &= m_g \begin{bmatrix} 0 \\ 0 \\ -\dot{x}\dot{\theta}\cos\theta - \dot{y}\dot{\theta}\sin\theta \\ -\dot{x}\dot{l}\cos\theta + \dot{x}\dot{\theta}l\sin\theta - \dot{y}\dot{l}\sin\theta - \dot{y}\dot{\theta}l\cos\theta \end{bmatrix} \end{aligned} \tag{3.14}$$

where

$$\frac{\partial\mathbf{V}_g}{\partial l} = \begin{bmatrix} 0 & 0 & 0 & -\cos\theta \\ 0 & 0 & 0 & -\sin\theta \end{bmatrix} \tag{3.15}$$

and

$$[\partial\mathbf{V}_g^T/\partial l]^T\mathbf{V}_g = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\cos\theta & -\sin\theta & 0 & l \end{bmatrix} \tag{3.16}$$

and

$$\frac{\partial\mathbf{V}_g}{\partial\theta} = \begin{bmatrix} 0 & 0 & -\cos\theta & l\sin\theta \\ 0 & 0 & -\sin\theta & -l\cos\theta \end{bmatrix}. \tag{3.17}$$

Define

$$\boldsymbol{D} = \dot{\mathbf{M}}\dot{\boldsymbol{q}} - \frac{\partial T}{\partial\boldsymbol{q}} = \begin{bmatrix} -2\dot{l}\dot{\theta}\cos\theta + l\dot{\theta}^2\sin\theta \\ -2\dot{l}\dot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta \\ 0 \\ 0 \end{bmatrix} \tag{3.18}$$

Potential energy is

$$V = \frac{1}{2}k_y(y-y_0)^2 + \frac{1}{2}k_l(l-l_0)^2 + \frac{1}{2}k_\theta\theta^2 + g(m_c+m_g)y + gm_g l\cos\theta \tag{3.19}$$

and derivatives

$$\frac{\partial V}{\partial\boldsymbol{q}} = \mathbf{K}\boldsymbol{q} - \mathbf{K}\boldsymbol{q}_0 + \boldsymbol{G} \tag{3.20}$$

where

$$\mathbf{K} = \mathrm{diag}(0, k_y, k_l, k_\theta) \tag{3.21}$$

and

$$\boldsymbol{G} = g \begin{bmatrix} 0 \\ m_c + m_g \\ m_g \cos\theta \\ -m_g l \sin\theta \end{bmatrix} \tag{3.22}$$

and $\partial V / \partial \dot{\boldsymbol{q}} = (0, 0, 0, 0)^T$.

Combining the above gives the equations of motion

$$\mathbf{M}\ddot{\boldsymbol{q}} = -\mathbf{C}\dot{\boldsymbol{q}} - \mathbf{K}\boldsymbol{q} + \mathbf{K}\boldsymbol{q}_0 - \boldsymbol{D} - \boldsymbol{G} + \boldsymbol{Q} \tag{3.23}$$

4. Application

Built the app using ...

4.1. Controls. Left hand controls stool, right hand controls person.

5. Conclusions

Made a dollar.

References

1. D. T. Greenwood, "Principles of Dynamics"

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