

T-statistic Significance Level



Help me to solve the following problem:

Fill in the blank. In practice, we can say that a point estimate is statistically significant at the 5% level if the t-statistic (absolute value) is greater than ____.

- A. 0.05
- B. 0.95
- C. 1.0
- D. 2.0

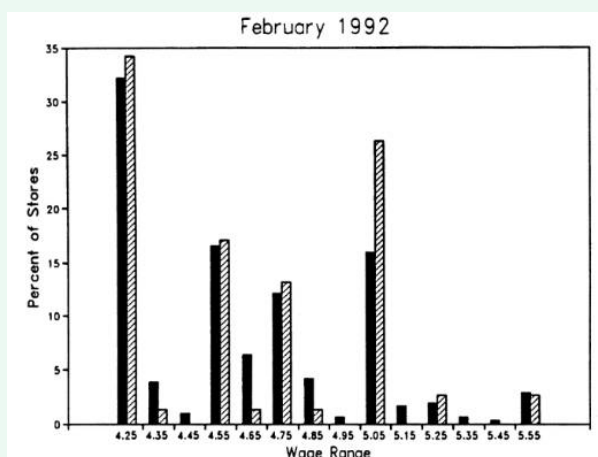


To determine statistical significance at the 5% level using a **t-statistic**, we compare the **absolute value of the t-statistic** to a critical value determined by the t-distribution. Here's how to fill in the blank:

1. **Significance Level (α):** 5% (0.05), which means there is a 5% chance of rejecting the null hypothesis when it is true.
2. **Critical Value:** The critical value of the t-statistic depends on the degrees of freedom (df), but for most practical purposes with a large sample size, the critical value for a **two-tailed test** is approximately **2.0**.
3. **Rule of Thumb:** If the absolute value of the t-statistic is greater than **2.0**, the result is typically considered statistically significant at the 5% level.

Correct Answer:

D. 2.0



The chart in the picture shows the distribution of wage rates for fast food restaurants in New Jersey and Pennsylvania prior to the rise in New Jersey's minimum wage. What information in this chart

best supports the idea of using New Jersey and Pennsylvania as treatment and comparison groups in the Card and Krueger (1994) study?

- A. It shows that in both states, more than 30% of stores paid minimum wage.
- B. It shows that the wage rate structures are similar in both states.
- C. It shows that the wage rate structures are very different in both states.
- D. There is no relevant information in this chart.



The correct answer is:

B. It shows that the wage rate structures are similar in both states.

Explanation:

The chart illustrates the distribution of wage rates in fast-food restaurants in New Jersey (treatment group) and Pennsylvania (comparison group) before the minimum wage increase in New Jersey. For the study by Card and Krueger (1994) to effectively analyze the impact of the minimum wage increase, it's essential that the two groups are similar in characteristics (e.g., wage structures) before the policy change. This similarity ensures that any differences observed after the policy change can be attributed to the policy itself, rather than pre-existing differences.

In the chart, the wage distributions for New Jersey and Pennsylvania appear quite similar, with overlapping patterns and wage ranges, supporting the validity of using these two states as treatment and comparison groups.



	Before	After	Δ
NJ	20.44	21.03	$\Delta Y_n = +0.59$
PA	23.33	21.17	$\Delta Y_p = -2.16$

The table in this picture shows employment in fast food restaurants in New Jersey (NJ) and Pennsylvania (PA) before and after the rise in New Jersey's minimum wage. Use Difference-in-Difference estimation to calculate the treatment effect.



To calculate the **treatment effect** using the **Difference-in-Difference (DiD)** estimation, follow these steps:

Data from the table:

- **New Jersey (NJ):**
 - Before: $Y_{NJ,before} = 20.44$
 - After: $Y_{NJ,after} = 21.03$
 - Change: $\Delta Y_{NJ} = 21.03 - 20.44 = +0.59$
- **Pennsylvania (PA):**

- Before: $Y_{PA,before} = 23.33$
- After: $Y_{PA,after} = 21.17$
- Change: $\Delta Y_{PA} = 21.17 - 23.33 = -2.16$

Difference-in-Difference Formula:

The treatment effect is calculated as:

$$\text{Treatment Effect (DiD)} = \Delta Y_{NJ} - \Delta Y_{PA}$$

Substitute the values:

$$\text{Treatment Effect (DiD)} = (+0.59) - (-2.16)$$

$$\text{Treatment Effect (DiD)} = 0.59 + 2.16 = 2.75$$

Final Answer:

The treatment effect estimated using Difference-in-Difference is **2.75**. This indicates that the rise in New Jersey's minimum wage is associated with an increase in employment of 2.75 (units of measurement not provided, but likely employees per store).



Based on all of the problems above, please construct an engaging explanation that explains the fancy concepts from scratch behind them.

In the explanation, please include the following:

- all necessary fancy math tools and symbols (if any),
- some tangible or quantitative examples,
- some humor.



Sure! Let's dive into these seemingly fancy ideas step by step. We'll use just enough math to feel clever, add some tangible examples to make things relatable, and throw in humor to keep it fun.

1. Statistical Significance and the Mighty t-Statistic

Ah, the **t-statistic**, the superhero of statistics! When we're trying to figure out whether something is *statistically significant* at the 5% level, the t-statistic tells us, "Hey, is this result likely just dumb luck?"

The Rule of Thumb: If the absolute value of your t-statistic is bigger than **2**, congratulations—you've just impressed the stats gods at the 5% level.

Math Moment

If you're scratching your head, here's the formula for the t-statistic:

$$t = \frac{\text{estimate} - \text{hypothesized value}}{\text{standard error}}$$

Translation? Take your result, subtract what you expect under the boring "null hypothesis," and divide by how wobbly your data is (standard error).

Tangible Example

Let's say you think eating pizza improves math scores by 5 points. But what if it's just a coincidence? If your t-statistic is, say, **2.5**, you can confidently claim: *Pizza boosts math genius!*

Humor

Remember, though, t-statistics can't fix your love life. If someone rejects you, you can't say, "But I'm significant at the 5% level!"

2. Card and Krueger's Tale: Wage Wars of NJ vs. PA

Imagine New Jersey (NJ) and Pennsylvania (PA) as two rival kingdoms. In 1992, NJ's King Minimum Wage said, "Let's raise wages for the serfs in fast food!" The big question was: Would this wage hike slay the economy or make it flourish?

The **Difference-in-Difference (DiD)** method is like a royal court investigation—it compares NJ and PA to see what happens after the policy change.

Math Moment

The treatment effect is:

$$\text{Treatment Effect} = (\text{After}_{NJ} - \text{Before}_{NJ}) - (\text{After}_{PA} - \text{Before}_{PA})$$

This sounds fancy, but it's just comparing changes.

Tangible Example

Imagine NJ was selling 20 burgers per hour before the wage hike, then sold 21 afterward. Meanwhile, PA went from 23 to 21. DiD tells us:

$$(21 - 20) - (21 - 23) = 1 - (-2) = +2$$

Voilà! NJ's minimum wage hike increased burger sales by 2 (okay, we're really talking jobs, but burgers are more fun).

Humor

Think of PA like a jealous sibling: "Ugh, NJ raised wages, but now they're stealing all our workers!" And NJ just smirks, "DiD, bro. DiD."

3. Statistical Twins: NJ and PA Wage Structures

Before NJ raised its minimum wage, the wage structures of fast-food workers in NJ and PA looked **eerily similar**. Why does this matter? If you want to test the effect of a policy, you need your comparison

group (PA) to look like your treatment group (NJ). Otherwise, it's like comparing apples to...a cheeseburger.

Tangible Example

Let's say NJ and PA both had about **30% of stores paying \$4.25/hour** (minimum wage at the time) before the hike. This similarity is crucial because it means any changes we see after the policy can be blamed on the wage hike, not some pre-existing difference.

Humor

If PA had been paying \$10/hour already, comparing it to NJ would be like saying, "Hey, let's see how a workout routine affects two people"—except one is a couch potato, and the other is already an Olympic athlete. Spoiler: It's not a fair fight.

4. Difference-in-Difference: Why Subtract Twice?

DiD might sound like a tongue twister, but it's actually super logical. Imagine you want to find the effect of NJ's wage hike on jobs. First, you look at the **change in NJ's jobs** (before vs. after). But wait! What if jobs were declining everywhere, even in PA? That's where the second subtraction comes in—it accounts for trends that would've happened anyway.

Math Moment

The DiD formula:

$$\text{DiD Effect} = (\Delta Y_{NJ}) - (\Delta Y_{PA})$$

Where:

- $\Delta Y_{NJ} = Y_{NJ,after} - Y_{NJ,before}$
- $\Delta Y_{PA} = Y_{PA,after} - Y_{PA,before}$

Tangible Example

If NJ adds 0.59 jobs per store, but PA loses 2.16 jobs, the **treatment effect** is:

$$0.59 - (-2.16) = 2.75$$

So, NJ's minimum wage hike "created" 2.75 more jobs per store than PA.

Humor

Think of DiD as the gossip of statistics: "Sure, NJ gained jobs, but PA lost jobs, so NJ looks even better by comparison!"

Wrapping It Up: Stats for the Real World

To recap, these tools—t-statistics, wage distributions, and Difference-in-Difference—help us make sense of the chaos in the world:

1. **The t-statistic** is your friend when you want to know if something's statistically legit or just a fluke. If it's above 2, pop the champagne!
2. **Comparing groups like NJ and PA** before a policy change ensures fairness—like starting a race with everyone at the same line.
3. **Difference-in-Difference** is your detective tool for teasing out what really caused a change.

Final Humor

Remember: Statistics is like dating. Just because it looks good on paper doesn't mean it's true love—always check your assumptions!



To reinforce the understanding, please generate some similar problems and show the solutions.



Problem 1: Statistical Significance and t-Statistic

A researcher is studying whether eating chocolate improves memory. The average improvement in memory score (out of 100) after eating chocolate is 10 points, with a standard error of 4 points. Is this result statistically significant at the 5% level?

Solution:

We calculate the **t-statistic** using the formula:

$$t = \frac{\text{estimate} - \text{hypothesized value}}{\text{standard error}}$$

Here:

- Estimate = 10 (improvement in memory score),
- Hypothesized value = 0 (no effect),
- Standard error = 4.

$$t = \frac{10 - 0}{4} = 2.5$$

The absolute value of t is **2.5**, which is greater than **2.0**. Therefore, the result is statistically significant at the 5% level. Eating chocolate seems to improve memory!

Problem 2: Wage Distribution Comparison

In a hypothetical study, two states—"Chocoland" and "Candyland"—are compared. Before Chocoland raised its minimum wage, the wage distribution was as follows:

- Chocoland: 30% of workers earned \$5/hour, 50% earned \$6/hour, and 20% earned \$7/hour.
- Candyland: 35% of workers earned \$5/hour, 45% earned \$6/hour, and 20% earned \$7/hour.

What feature of the wage distribution supports using these states as treatment and comparison groups?

Solution:

The wage distributions in Chocoland and Candyland are **very similar** before the policy change (only small differences in percentages). This similarity ensures that any differences observed after the wage hike can be attributed to the policy, not pre-existing differences between the states. This is a foundational assumption for the **Difference-in-Difference** approach.

Problem 3: Difference-in-Difference Estimation

The following table shows the number of chocolate bars sold per week (in thousands) in Chocoland (treatment group) and Candyland (comparison group) before and after Chocoland's new advertising campaign:

State	Before	After	Change (Δ)
Chocoland	50	60	$\Delta C = 10$
Candyland	40	38	$\Delta K = -2$

Use **Difference-in-Difference (DiD)** estimation to calculate the effect of the advertising campaign.

Solution:

The treatment effect is:

$$\text{Treatment Effect (DiD)} = (\Delta C) - (\Delta K)$$

Substitute the values:

$$\text{Treatment Effect (DiD)} = (10) - (-2) = 10 + 2 = 12$$

The DiD estimation suggests that the advertising campaign increased chocolate sales by **12,000 bars per week**.

Problem 4: Testing the Assumptions of DiD

In a study, the treatment group (State A) saw a significant wage increase, while the comparison group (State B) did not. However, you observe the following changes in employment before the policy:

- State A: Employment was decreasing by 5% annually.
- State B: Employment was increasing by 2% annually.

Does this violate the assumptions of **Difference-in-Difference**? Why?

Solution:

Yes, this violates a key assumption of **Difference-in-Difference**: the **parallel trends assumption**. This assumption states that in the absence of treatment (the policy), the changes in the outcome (employment) would have been the same for both groups. Since State A was already experiencing declining employment and State B was seeing increasing employment before the policy, the two states were not following parallel trends. This makes it harder to attribute post-policy differences to the treatment alone.

Problem 5: Hypothetical Policy Impact on Test Scores

A city implements a new teaching method in public schools. The following table summarizes average math scores before and after the policy change:

Group	Before	After	Change (Δ)
Schools with Policy	65	75	$\Delta T = 10$
Schools without Policy	70	72	$\Delta C = 2$

Calculate the treatment effect using Difference-in-Difference (DiD).

Solution:

The treatment effect is:

$$\text{Treatment Effect (DiD)} = (\Delta T) - (\Delta C)$$

Substitute the values:

$$\text{Treatment Effect (DiD)} = (10) - (2) = 8$$

The DiD estimation suggests that the new teaching method improved math scores by **8 points**.

Problem 6: Parallel Trends Check

To use DiD for a policy impact study, we check if the pre-treatment trends are parallel. The math scores in two groups are shown for two years before the policy:

Year	Group A	Group B
Year 1	50	55
Year 2	52	57

Are the trends parallel?

Solution:

Calculate the annual changes:

- Group A: $52 - 50 = +2$
- Group B: $57 - 55 = +2$

The pre-treatment changes are identical, indicating that the groups exhibit **parallel trends**, which supports the use of DiD for this study.

Wrap-Up

These problems reinforce the core ideas behind **statistical significance**, **wage distribution comparisons**, and **Difference-in-Difference**:

- Use **t-statistics** to determine if results are just noise or something real.
- Ensure groups are **similar before treatment** to make meaningful comparisons.

- Leverage **Difference-in-Difference** to isolate policy impacts while checking key assumptions like **parallel trends**.

Humor Recap

- Don't let t -statistics fool you—they may look fancy, but they're just math's version of a reality check.
- Wage comparisons aren't just for economics—they're also great for office debates about who's buying lunch!
- And remember, if your trends aren't parallel, DiD will crash harder than a poorly coded program.