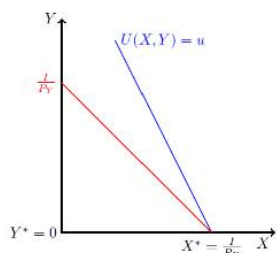# Candy Economics 101

Example 2. Suppose now that we maximize:

$$\max_{X,Y} \ X + Y \ \text{s.t.} \ X + 4Y \leq 100 \qquad (20)$$

In this case $MRS(X,Y) = -1$ is not a function of (X,Y), so we cannot equalize $MRS(X,Y) = -\frac{P_x}{P_Y}$. This is the case where X,Y are perfect substitutes. In this case, the consumer will buy all of his income in good X (since it is the cheaper one in this case.). Graphically:

You are an expert in microeconomics.
I read the text as shown in the picture.
However, I don't understand.
Can you help to re-explain with a more beginner-friendly tone?
Please include some humor in your explanation.

Absolutely! Let me break it down for you in a simpler way—along with a touch of humor.

So, imagine you're at a candy shop (because economics is sweeter when candy is involved). You've got two types of treats:

- **Candy X**: Costs $1 per piece.
- **Candy Y**: Costs $4 per piece.

You love candy so much that you don't care which one you get. For you, they're **perfect substitutes**—as long as you get a sugar rush, you're happy.

Now, you've got $100 in your pocket. What's your game plan? The goal is to **maximize your happiness** (or "utility") by buying as much candy as you can without going over budget.

The Math Behind the Sweet Decisions

The shopkeeper's rule (budget constraint) is:

$$X + 4Y \leq 100$$

This means:

- 1 piece of Candy X costs the same as 4 pieces of Candy Y.

So, how do you maximize happiness? You buy whichever candy gives you **the most bang for your buck.**

The Twist: Marginal Rate of Substitution (MRS)

The MRS is like asking: "How many Y's would you give up for 1 more X and still feel just as happy?" Since X and Y are perfect substitutes, your MRS is **always -1.**

Translation: "I'll trade 1 Candy X for 1 Candy Y without any drama."

But here's the catch: MRS doesn't care about prices! And economics is all about the **prices**—the cold, hard truth of what you can actually afford.

The Big Decision

Since Candy X is cheaper (just $1 compared to $4 for Candy Y), you'll spend all your money on Candy X. That's like spotting a "Buy 1 Get 4 Free" deal and saying, "Why would I buy the expensive stuff?"

So, you buy:

- **X*** = 100 pieces of Candy X
- **Y*** = 0 pieces of Candy Y (sorry, Y lovers)

Graphically, this means you sit on the **X-axis** (all your money on X) instead of sharing the budget with Y.

Why So Serious, Utility?

If utility functions were people, this one would be your overly chill friend who says, "Hey, just get whatever's cheaper—I don't care." It's the ultimate laid-back situation in economics. You're not stuck agonizing over choices. Just follow the price tag.

The Moral of the Story

Economics isn't always about deep trade-offs. Sometimes it's about being frugal and grabbing the cheaper candy. And let's be honest: if all candies taste the same, why spend more? Save that extra cash for a soda!

Does that sweeten the explanation for you? 😊

What does the blue line represent?

Ah, the **blue line**—that mysterious slant on the graph! Let me demystify it for you.

The blue line represents the **indifference curve** for your utility $U(X, Y)$. In simpler terms, it's the line that shows all the combinations of X and Y (the two "candies") that give you the **same level of happiness** or "utility."

What's Special About It?

For **perfect substitutes** like in this example, the indifference curve is **straight**. Why? Because you're equally happy swapping 1 unit of X for 1 unit of Y without caring (as long as your total happiness stays the same). That's why the blue line slopes downward at a 45° angle.

Why Does It Matter Here?

While the blue line shows the combinations of X and Y that keep you equally happy, your **budget constraint** (red line) dictates what you can actually afford. Since the blue line keeps sliding outward to higher happiness levels, you'll stop at the highest point where the **red line (budget)** meets the blue line.

In this case, since X is cheaper, the blue line touches the red line at the point where $Y = 0$ and $X = 100$. Translation: you spend all your money on X.

How to Think About It

Imagine the blue line as your **wishlist** of candy combinations that make you equally happy. But your budget (red line) is like your strict parent saying, "You only get what you can afford!" So you maximize your happiness at the tangency point—or, in this case, by spending all your money on the cheaper treat, X.

Fun analogy: The blue line is like your dream vacation spots, but the red line is your travel budget. Sure, you'd love to visit every destination on the blue line, but you'll settle for the one where your wallet agrees!

how to read the equation (20) in english?

Great question! Let's break down how to read **Equation (20)** in plain English.

The equation is:

$$\max_{X,Y} X + Y \quad \text{s.t.} \quad X + 4Y \leq 100$$

Here's how you'd say it in English:

First Part: "max_{X, Y} X + Y"

- "Maximize $X + Y$"
  - This means we want to find the highest possible value of $X + Y$.
  - X and Y are the two things we're working with (like two types of goods).

Second Part: "s.t. X + 4Y ≤ 100"

- "Subject to $X + 4Y$ being less than or equal to 100."
  - The **s.t.** stands for **subject to** and introduces a constraint.

- $X + 4Y \leq 100$ means you can't spend more than 100 (e.g., your budget). For every 1 unit of Y, it costs 4 times as much as 1 unit of X.

Put It Together:

"Maximize $X + Y$ subject to the condition that $X + 4Y$ is less than or equal to 100."

Think of it as your mission: Get the most total candy (the sum of $X + Y$) without breaking your budget (100). Pretty straightforward, right?

is the indifference curve has a utility value of 100?

Great observation! However, **the indifference curve in this example does not have a utility value of 100**. Let me explain why.

Why Not 100?

The equation being maximized is:

$$\max_{X,Y} X + Y$$

Here, $X + Y$ represents the **utility function**, which reflects your happiness or satisfaction from consuming X and Y. The value of the utility function depends on the combination of X and Y you choose.

Since $X + Y$ can vary depending on how much X and Y you pick (within the budget constraint), the blue line shows all combinations of X and Y that yield the **same utility level**, but it doesn't specify the exact utility value in this case.

What Utility Value Does the Indifference Curve Represent?

It could represent any utility level u, depending on where the curve is drawn. For example:

- If the indifference curve is closer to the origin, the utility u is smaller (fewer total goods consumed).
- If the indifference curve shifts outward, the utility u is larger (more total goods consumed).

The graph doesn't explicitly label a utility value for the blue curve. It just shows that all points on the curve yield the **same level of happiness**—whether it's 50, 80, 100, or something else, we don't know for sure unless the graph specifies.

What About the Budget Constraint (Red Line)?

The red line limits what you can afford:

$$X + 4Y \leq 100$$

At the optimal point, you spend all your money on X, so:

$$X = 100, \quad Y = 0$$

Here, the utility is:

$$U(X, Y) = X + Y = 100 + 0 = 100$$

So yes, at the point where $X = 100$ and $Y = 0$, the utility is 100—but that's the **optimal point**, not the entire indifference curve.

Summary

The **blue indifference curve** represents some utility level u, but it's not necessarily 100. The utility value is only 100 at the point $(X = 100, Y = 0)$, where you maximize your happiness under the budget constraint.

Hope that clears it up! 😊

> From the reading, I don't understand the following statement:
> "In this case MRS(X,Y) = -1 is not a function of (X, Y)"
>
> Please help to re-explain.

Of course! Let's break down that mysterious statement:

"In this case, $MRS(X, Y) = -1$ is not a function of (X, Y)."

Here's what it means, step by step:

---

1. What Is $MRS(X, Y)$?

The **Marginal Rate of Substitution (MRS)** is a fancy way of saying:
"How much of good Y are you willing to give up to get one more unit of good X, while staying just as happy?"

Mathematically:

$$MRS(X, Y) = -\frac{\partial U / \partial X}{\partial U / \partial Y}$$

For most utility functions, MRS depends on the quantities of X and Y you have. For example, if you're really low on Y, you might value it more and be willing to trade more X to get Y. In those cases, MRS changes with X and Y.

---

2. What's Special About This Case?

In this example, X and Y are **perfect substitutes**, meaning you don't care whether you consume X or Y. You're just as happy trading 1 unit of X for 1 unit of Y, no matter how much of each you have.

- The utility function is $U(X, Y) = X + Y$, which shows that you're equally happy as long as the total $X + Y$ is the same.
- For perfect substitutes, $MRS(X, Y)$ is always **constant** because your willingness to trade between X and Y doesn't change.

In this case, $MRS(X, Y) = -1$, meaning:

- You're always happy to give up **1 unit of X** to get **1 unit of Y**, or vice versa.
- This does **not** depend on how much X or Y you currently have.

---

3. Why Isn't $MRS(X, Y)$ a Function of (X, Y)?

When $MRS(X, Y)$ is **not a function of X and Y**, it means MRS doesn't vary based on the amounts of X or Y you have. In this example:

$$MRS(X, Y) = -1$$

is a **constant**, so it's independent of X or Y.

Contrast this with other utility functions where MRS depends on X and Y. For example:

$$U(X, Y) = X \cdot Y$$

In this case, $MRS(X, Y)$ would depend on the ratio of X to Y:

$$MRS(X, Y) = -\frac{Y}{X}$$

So MRS changes as your quantities of X and Y change. That's what we'd expect in many cases—but **not here**.

---

4. Why Does This Matter?

Since $MRS(X, Y) = -1$ is constant, it doesn't give us enough information to find the optimal X and Y based on the usual **MRS = price ratio** rule. Instead, the decision is entirely driven by the **prices** of X and Y:

- If X is cheaper, buy only X.
- If Y is cheaper, buy only Y.
- If they cost the same, you can buy any mix.

In this case, X is cheaper, so you spend all your money on X.

Rephrased Explanation:

In this example, $MRS(X, Y) = -1$ is constant, meaning it doesn't change no matter how much X or Y you have. It's not a "function of (X, Y)" because it doesn't depend on those values—it's just always

-1. This reflects the fact that X and Y are perfect substitutes, so you're equally willing to trade between them at a constant rate.

Did that make things clearer? 😊

> can you show the steps of how the "MRS(X,Y)=−1" was obtained?

Sure! Let's go through the steps to derive $MRS(X, Y) = -1$ for the given example.

1. Understand the Utility Function

The utility function is given as:

$$U(X, Y) = X + Y$$

This is a **perfect substitutes** utility function, meaning the consumer values 1 unit of X exactly the same as 1 unit of Y.

2. Definition of MRS

The **Marginal Rate of Substitution (MRS)** is defined as:

$$MRS(X, Y) = -\frac{\partial U/\partial X}{\partial U/\partial Y}$$

This represents the slope of the **indifference curve**, or how much of Y the consumer is willing to give up for an additional unit of X, while keeping utility constant.

3. Calculate the Partial Derivatives

From the utility function $U(X, Y) = X + Y$:

- The **marginal utility of X** (how utility changes when X increases by 1 unit) is:

$$\frac{\partial U}{\partial X} = 1$$

- The **marginal utility of Y** (how utility changes when Y increases by 1 unit) is:

$$\frac{\partial U}{\partial Y} = 1$$

4. Substitute into the MRS Formula

Substitute these partial derivatives into the formula for $MRS(X, Y)$:

$$MRS(X,Y) = -\frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial Y}}$$

$$MRS(X,Y) = -\frac{1}{1}$$

5. Simplify

$$MRS(X,Y) = -1$$

6. Interpretation

This result tells us that the consumer is always willing to trade **1 unit of** Y for **1 unit of** X (and vice versa) without any change in happiness. It's constant and does **not depend on the quantities of** X **and** Y—hence why $MRS(X,Y)$ is not a function of (X,Y).

Key Takeaway

The $MRS(X,Y) = -1$ arises directly from the structure of the utility function $U(X,Y) = X + Y$, where the marginal utilities of both goods are equal ($\frac{\partial U}{\partial X} = \frac{\partial U}{\partial Y} = 1$).

Does that clarify things? 😊

---

we cannot equalize $MRS(X,Y) = -\frac{P_X}{P_Y}$.

I also don't understand the statement shown in the picture.
Please help to re-explain.

Certainly! Let me break down the statement:

"We cannot equalize $MRS(X,Y) = -\frac{P_X}{P_Y}$."

1. What Does This Mean?

In economics, the condition $MRS(X,Y) = -\frac{P_X}{P_Y}$ is called the **optimality condition for utility maximization**. It states that:

> To maximize utility, the rate at which you're willing to trade X for Y (the MRS) should equal the rate at which the market allows you to trade X for Y (the price ratio $\frac{P_X}{P_Y}$).

For example:

- If $P_X = 1$ and $P_Y = 4$, the price ratio $\frac{P_X}{P_Y} = \frac{1}{4}$.

- To achieve optimality, you should adjust your consumption of X and Y so that $MRS(X,Y) = -\frac{1}{4}$.

---

2. Why Can't We Equalize Them Here?

In this example, X and Y are **perfect substitutes**, and the $MRS(X,Y)$ is constant:

$$MRS(X,Y) = -1$$

However, the price ratio $\frac{P_X}{P_Y}$ depends on the prices of X and Y. For instance:

- If $P_X = 1$ and $P_Y = 4$, then $\frac{P_X}{P_Y} = \frac{1}{4}$, which is **not equal to** -1.
- Because $MRS(X,Y)$ is constant (-1) and doesn't change based on how much X or Y you consume, you **cannot "adjust" your consumption** to make $MRS(X,Y)$ equal to $\frac{P_X}{P_Y}$.

In other words, the MRS is **fixed** at -1, while the price ratio $\frac{P_X}{P_Y}$ could be any value depending on the prices. They cannot match unless the price ratio also happens to be -1, which isn't the case here.

---

3. What Happens Instead?

Since $MRS(X,Y)$ cannot equal $\frac{P_X}{P_Y}$, the usual optimality condition breaks down. Instead, you make your decision based on **which good is cheaper**:

- If $P_X < P_Y$ (as in this case, $P_X = 1$ and $P_Y = 4$), you spend all your money on X.
- If $P_Y < P_X$, you spend all your money on Y.
- If $P_X = P_Y$, any combination of X and Y is optimal because they're perfect substitutes.

This is why the consumer buys **only** X in this example: it's the cheaper good.

---

4. Summary

The statement means that the usual rule of matching $MRS(X,Y)$ to the price ratio $\frac{P_X}{P_Y}$ doesn't work here because:

1. The $MRS(X,Y) = -1$ is constant and doesn't depend on X and Y.
2. The price ratio $\frac{P_X}{P_Y}$ is different (e.g., $\frac{1}{4}$) and cannot equal -1.

Instead, the consumer maximizes utility by choosing the cheaper good (all X in this case).

Does that clarify it? 😊

> How can I know that X, Y are perfect substitutes? From the utility function?

Yes, you can identify that X and Y are **perfect substitutes** directly from the **utility function**! Let me explain how.

1. The Utility Function for Perfect Substitutes

The utility function in this example is:

$$U(X, Y) = X + Y$$

This tells us that:

- The total utility U depends **only on the sum of X and Y**.
- One unit of X gives you the exact same increase in utility as one unit of Y.

---

2. Key Characteristics of Perfect Substitutes

Goods are **perfect substitutes** if:

1. **Utility depends on a linear combination of X and Y:**
    - The utility function takes the form:

    $$U(X, Y) = aX + bY$$

    where a and b are constants. In this example, $a = 1$ and $b = 1$.
    - This means the consumer is indifferent between exchanging b units of Y for a units of X, maintaining the same utility.
2. **Marginal utilities are constant:**
    - The **marginal utility of X** $(\frac{\partial U}{\partial X})$ is constant:

    $$\frac{\partial U}{\partial X} = 1$$

    - The **marginal utility of Y** $(\frac{\partial U}{\partial Y})$ is also constant:

    $$\frac{\partial U}{\partial Y} = 1$$

    - This indicates that your additional satisfaction from consuming one more unit of X or Y never changes, no matter how much of X or Y you already have.
3. **The Marginal Rate of Substitution (MRS) is constant:**
    - The MRS is defined as:

    $$MRS(X, Y) = -\frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial Y}}$$

    For this utility function:

    $$MRS(X, Y) = -\frac{1}{1} = -1$$

    - A constant MRS means you're always willing to trade the same amount of Y for X, regardless of how much you have. This is a hallmark of perfect substitutes.

---

3. Contrast with Other Utility Functions

If X and Y were **not perfect substitutes**, the utility function would not be linear. For example:

- **Perfect complements**:

$$U(X,Y) = \min(aX, bY)$$

Here, X and Y must be consumed in fixed proportions (like left and right shoes).

- **Typical Cobb-Douglas preferences**:

$$U(X,Y) = X^a Y^b$$

Here, X and Y are neither perfect substitutes nor perfect complements, and the MRS depends on the ratio of X to Y.

4. How to Recognize Perfect Substitutes

When you see a utility function like:

$$U(X,Y) = aX + bY$$

you immediately know:

- X and Y are **perfect substitutes**.
- You'll always trade b units of Y for a units of X at a constant rate.

In this example, $U(X,Y) = X + Y$, so X and Y are perfect substitutes, and you're indifferent between 1 unit of X and 1 unit of Y.

Does that make sense? 😊

besides perfect substitutes case, is there any other case where we cannot equalize MRS(X,Y) = -Px/Py?

Yes, there are other cases besides **perfect substitutes** where we **cannot equalize $MRS(X,Y) = -\frac{P_X}{P_Y}$**. Here are the key scenarios:

1. Perfect Complements

Utility Function:

$$U(X,Y) = \min(aX, bY)$$

In the case of perfect complements (e.g., left shoes and right shoes), the consumer only cares about consuming X and Y in fixed proportions (like 1 left shoe for 1 right shoe).

Why $MRS(X,Y) = -\frac{P_X}{P_Y}$ Doesn't Apply:

- For perfect complements, there's no meaningful MRS on most of the indifference curve because the consumer only consumes at the "kink" of the indifference curve (where $aX = bY$).
- You can't "trade" X for Y and remain on the same indifference curve because $U(X,Y)$ depends on the **minimum** of aX and bY, not their sum.
- The optimal consumption bundle is determined by **the proportionality rule** ($aX = bY$) and the budget constraint, not by MRS.

---

2. Corner Solutions (Non-Interior Optima)

Utility Function:

Any utility function can lead to a corner solution if the price ratio $\frac{P_X}{P_Y}$ is too extreme relative to preferences. For example:

$$U(X,Y) = X^{0.5} + Y^{0.5}$$

Here, X and Y are not perfect substitutes, but they are "good enough substitutes" for corner solutions to arise.

Why $MRS(X,Y) = -\frac{P_X}{P_Y}$ Doesn't Apply:

- If one good is far cheaper than the other, the consumer might spend all their income on the cheaper good.
- In these cases, the $MRS(X,Y)$ **at the chosen point** does not equal $-\frac{P_X}{P_Y}$ because the consumer doesn't consume both goods.
- Instead, the budget constraint "forces" consumption to one of the axes (e.g., all X or all Y).

---

3. Non-Differentiable Utility Functions

Utility Function:

For example:

$$U(X,Y) = |X - Y|$$

This utility function is non-differentiable where $X = Y$, and the indifference curves have sharp "kinks."

Why $MRS(X,Y) = -\frac{P_X}{P_Y}$ Doesn't Apply:

- At the kink (e.g., $X = Y$), the $MRS(X,Y)$ is undefined because there's no single slope for the indifference curve.
- The consumer might still reach an optimal bundle, but you cannot use $MRS = -\frac{P_X}{P_Y}$ to determine it.

4. Non-Convex Preferences

Utility Function:

For example:

$$U(X, Y) = \max(X, Y)$$

Here, the consumer only cares about consuming **as much as possible of the good they prefer more**.

Why $MRS(X, Y) = -\frac{P_X}{P_Y}$ Doesn't Apply:

- Non-convex preferences lead to "jumps" in consumption rather than smooth trade-offs.
- The consumer might spend all their income on X or Y depending on the price ratio, without satisfying the MRS condition.

---

5. Leontief Preferences (Similar to Perfect Complements)

Utility Function:

$$U(X, Y) = \min(X, Y)$$

Here, the consumer values X and Y in a **one-to-one ratio**, consuming them in fixed proportions.

Why $MRS(X, Y) = -\frac{P_X}{P_Y}$ Doesn't Apply:

- Similar to perfect complements, the optimal bundle occurs at the "kink" of the indifference curve, where the consumer consumes $X = Y$.
- MRS is undefined along the flat or vertical segments of the indifference curve.

Summary of When $MRS(X, Y) = -\frac{P_X}{P_Y}$ Doesn't Apply:

1. **Perfect substitutes** (MRS constant and doesn't match price ratio).
2. **Perfect complements** (no meaningful MRS except at the kink).
3. **Corner solutions** (optimal bundle is at one of the axes, not an interior point).
4. **Non-differentiable utility functions** (e.g., sharp kinks in indifference curves).
5. **Non-convex preferences** (e.g., max-type utility functions).

General Rule of Thumb:

The condition $MRS(X, Y) = -\frac{P_X}{P_Y}$ applies **only when the optimal solution involves consuming positive amounts of both X and Y**, and the preferences are **smooth, convex, and differentiable.**

Let me know if you'd like more details on any of these cases! 😊

tell me some cases when MRS(X,Y) = -Px/Py apply.

Great question! The condition $MRS(X, Y) = -\frac{P_X}{P_Y}$ applies in many common scenarios of utility maximization. Here are some cases where this condition works perfectly:

---

1. Cobb-Douglas Utility

Utility Function:

$$U(X, Y) = X^a Y^b, \quad a > 0, b > 0$$

This is one of the most common utility functions in economics. It represents preferences where X and Y are both essential and consumed in proportions determined by a and b.

Why $MRS(X, Y) = -\frac{P_X}{P_Y}$ Applies:

- The Cobb-Douglas utility function generates **smooth, convex indifference curves**.
- Consumers balance X and Y so that the rate at which they are willing to substitute X for Y (the MRS) equals the market trade-off (the price ratio $\frac{P_X}{P_Y}$).
- The optimal bundle is always an **interior solution** where both $X > 0$ and $Y > 0$.

Example:

If $U(X, Y) = X^{0.5} Y^{0.5}$, then:

$$MRS(X, Y) = -\frac{Y}{X}$$

At the optimal point:

$$-\frac{Y}{X} = -\frac{P_X}{P_Y}$$

You can solve this equation to find the optimal quantities of X and Y.

---

2. Quasi-Linear Utility

Utility Function:

$$U(X, Y) = X + \ln(Y)$$

In this case, the utility function has a linear term (X) and a non-linear term ($\ln(Y)$).

Why $MRS(X, Y) = -\frac{P_X}{P_Y}$ Applies:

- The indifference curves are smooth and convex, so the optimal point is determined by balancing $MRS(X, Y)$ and $-\frac{P_X}{P_Y}$.

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- Quasi-linear preferences are common in cases where X and Y have diminishing marginal utilities but the consumer still values both goods.

Example:

For $U(X, Y) = X + \ln(Y)$:

$$MRS(X, Y) = -\frac{1}{Y}$$

At the optimal point:

$$-\frac{1}{Y} = -\frac{P_X}{P_Y}$$

Solve for Y, then use the budget constraint to find X.

---

3. CES Utility (Constant Elasticity of Substitution)

Utility Function:

$$U(X, Y) = (aX^r + bY^r)^{1/r}, \quad r < 1$$

This is a flexible utility function that allows for different degrees of substitutability between X and Y. The parameter r controls the elasticity of substitution.

Why $MRS(X, Y) = -\frac{P_X}{P_Y}$ Applies:

- CES utility functions generate smooth, convex indifference curves (as long as $r < 1$).
- The consumer balances $MRS(X, Y)$ and $-\frac{P_X}{P_Y}$ to maximize utility.

Example:

For $U(X, Y) = (X^{0.5} + Y^{0.5})^2$, the MRS depends on the ratio of X to Y:

$$MRS(X, Y) = -\frac{X^{-0.5}}{Y^{-0.5}} = -\frac{Y^{0.5}}{X^{0.5}}$$

At the optimal point:

$$-\frac{Y^{0.5}}{X^{0.5}} = -\frac{P_X}{P_Y}$$

---

4. Linear Utility with Interior Solutions

Utility Function:

$$U(X, Y) = aX + bY, \quad a, b > 0$$

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This is the utility function for **perfect substitutes**, but when the prices are such that $\frac{P_X}{P_Y} = \frac{a}{b}$, the consumer can choose any combination of X and Y along the budget line.

Why $MRS(X,Y) = -\frac{P_X}{P_Y}$ Applies:

- The $MRS(X,Y) = -\frac{a}{b}$ is constant.
- If $\frac{P_X}{P_Y} = \frac{a}{b}$, the consumer is indifferent between all combinations of X and Y that satisfy the budget constraint.

---

5. Cobb-Douglas with a Binding Budget Constraint

Utility Function:

$$U(X,Y) = X^a Y^{1-a}, \quad 0 < a < 1$$

If the consumer has a binding budget constraint, the MRS condition determines the optimal bundle.

Why $MRS(X,Y) = -\frac{P_X}{P_Y}$ Applies:

- The $MRS(X,Y) = -\frac{aY}{(1-a)X}$ balances the trade-off between X and Y, which equals the price ratio.

---

6. Additive Preferences with Positive Marginal Utilities

Utility Function:

$$U(X,Y) = f(X) + g(Y)$$

where $f(X)$ and $g(Y)$ are increasing and differentiable.

Why $MRS(X,Y) = -\frac{P_X}{P_Y}$ Applies:

- As long as the indifference curves are smooth and convex, the optimal consumption point satisfies $MRS(X,Y) = -\frac{P_X}{P_Y}$.

Summary of Cases Where $MRS(X,Y) = -\frac{P_X}{P_Y}$ Applies:

1. **Cobb-Douglas utility**: $U(X,Y) = X^a Y^b$
2. **Quasi-linear utility**: $U(X,Y) = X + \ln(Y)$
3. **CES utility**: $U(X,Y) = (aX^r + bY^r)^{1/r}$
4. **Linear utility with interior solutions**: $U(X,Y) = aX + bY$ when prices are proportional to marginal utilities.
5. **Smooth, additive utility functions**: $U(X,Y) = f(X) + g(Y)$.

The key condition is that the utility function must produce **smooth, convex indifference curves** with an **interior solution** where both $X > 0$ and $Y > 0$. Let me know if you'd like an example for any of these

cases! 😊