

Difference-in-Difference Calculation



Prior to 2011, every low-income renter in the city of Dallas, Texas received a flat Fair Market Rent (FMR) subsidy S_0 that was paid directly to the landlord. (Thus, the renters could not spend less than S_0 on rent.) The city was concerned that many low-income renters chose to live in dangerous neighborhoods with poor-quality schools. Starting in 2011, the city switched to a quality-linked rent subsidy, $S_1(Q)$. For neighborhoods with average quality \bar{Q} , the subsidy remained the same. For high-quality neighborhoods (safer, better schools, etc.) with $Q > \bar{Q}$, it increased the subsidy, and for low-quality neighborhoods (less safe, worse schools) with $Q < \bar{Q}$, it lowered the subsidy. For simplicity, assume that $S_1(Q)$ is a linear subsidy. That is, $\frac{dS_1(Q)}{dQ} = \alpha > 0$ with:

$$S_1(Q) = S_0 \text{ if } Q = \bar{Q}$$

$$S_1(Q) < S_0 \text{ if } Q < \bar{Q}$$

$$S_1(Q) = S_0 \text{ if } Q = \bar{Q}$$

$$S_1(Q) > S_0 \text{ if } Q > \bar{Q}$$

Collinson and Ganong (2018) analyzed the effect of the quality-linked rent subsidy on the quality of neighborhoods that low-income renters chose. Because the policy was not randomly assigned to individual renters, Collinson and Gangong used nearby Fort Worth, Texas as a comparison city for Dallas. The figure below provides key data from their study. The x-axis in this figure is time (in years) and the y-axis is the average quality of neighborhoods that low-income renters in Dallas and Fort Worth chose each year. Higher values correspond to higher-quality neighborhoods.

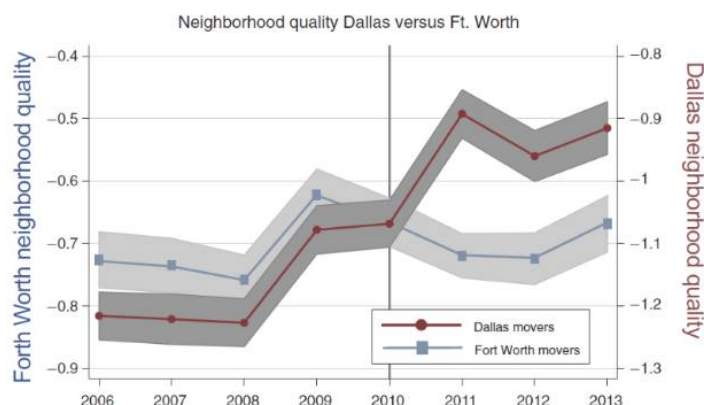


FIGURE 10. IMPACT OF TILTING: NEIGHBORHOOD QUALITY (TIME-SERIES)

Notes: In 2011, Dallas replaced a single, metro-wide FMR with ZIP code-level FMRs, raising rent ceilings in expensive neighborhoods and lowering rent ceilings in cheap neighborhoods. We construct a neighborhood quality index as an equally weighted sum of tract-level poverty rate, test scores, unemployment rate, share of kids with single mothers, and violent crime rate. The index is normalized to have mean zero and unit standard deviation with respect to the entire Dallas metro area. The above figure plots the average neighborhood quality for movers in each year in the Dallas metro area and the Fort Worth metro area. The left vertical axis is the quality level of Fort Worth movers, the right vertical axis reports the quality level of Dallas movers, and both axes share the same scale.

Let Y_{jt} equal the average neighborhood quality of renters in city $j \in (D, F)$ in year $t \in (2010, 2011)$. Let T^* equal the causal effect of the policy on the quality of neighborhoods chosen by Dallas renters, and \hat{T} equal the difference-in-difference estimate of T^* .

Fill in the blanks. $\hat{T} = (E[Y_{D,2011}] - \text{_____}) - (E[Y_{F,2011}] - \text{_____})$

☐ $E[Y_{F,2010}]; E[Y_{D,2010}]$

☐ $E[Y_{D,2010}]; E[Y_{F,2010}]$

I have a problem case as shown in the two pictures.
Please help me to get the answer.



To answer the question, we need to calculate the difference-in-difference estimate \hat{T} for the policy's causal effect. The formula for \hat{T} is given as:

$$\hat{T} = (E[Y_{D,2011}] - E[Y_{D,2010}]) - (E[Y_{F,2011}] - E[Y_{F,2010}])$$

Explanation of Terms:

1. $E[Y_{D,2011}]$: Average neighborhood quality of Dallas renters in 2011.
2. $E[Y_{D,2010}]$: Average neighborhood quality of Dallas renters in 2010.
3. $E[Y_{F,2011}]$: Average neighborhood quality of Fort Worth renters in 2011.
4. $E[Y_{F,2010}]$: Average neighborhood quality of Fort Worth renters in 2010.

Fill in the Blanks:

The correct calculation of \hat{T} involves the following steps:

1. Subtract the average neighborhood quality of Dallas renters in 2010 from that in 2011:
 $E[Y_{D,2011}] - E[Y_{D,2010}]$.
2. Subtract the average neighborhood quality of Fort Worth renters in 2010 from that in 2011:
 $E[Y_{F,2011}] - E[Y_{F,2010}]$.
3. Take the difference between the above two results.

Hence, the blanks should be filled as:

$$\hat{T} = (E[Y_{D,2011}] - E[Y_{D,2010}]) - (E[Y_{F,2011}] - E[Y_{F,2010}])$$

The correct choice from the options is:

- $E[Y_{F,2010}]; E[Y_{D,2010}]$.



Under what assumptions is \hat{T} a valid estimate of T^* ? Select all that apply.

- ☐ Parallel trends assumption.
- ☐ Stable Unit Treatment Value Assumption (SUTVA).
- ☐ Exclusion restriction

Now, please help with the second problem here.



To determine under which assumptions \hat{T} is a valid estimate of T^* , let's analyze each assumption:

1. Parallel Trends Assumption:

- The parallel trends assumption is crucial for the difference-in-differences (DiD) method. It assumes that, in the absence of treatment, the average neighborhood quality for Dallas and Fort Worth would have changed by the same amount. This ensures that any observed differences post-policy are attributable to the policy itself, not pre-existing differences in trends.
- **Answer: Required.**

2. Stable Unit Treatment Value Assumption (SUTVA):

- SUTVA assumes that the treatment effect on any one unit (here, Dallas renters) does not influence the outcome for other units (e.g., Fort Worth renters) and that there are no hidden variations in treatment. This is necessary to isolate the treatment effect.
- **Answer: Required.**

3. Exclusion Restriction:

- The exclusion restriction assumes that the treatment affects the outcome only through the specified mechanism (e.g., subsidy policy affecting neighborhood choice). While this is often needed for causal inference, it is not specific to the DiD framework.
- **Answer: Optional (but often assumed).**

Final Answer:

- Parallel trends assumption
- Stable Unit Treatment Value Assumption (SUTVA)



Fill in the blanks. In the graph above, look again at the average neighborhood quality of rentals in Dallas and Fort Worth. The information from the years 2006-2009 _____ your confidence in the validity of \hat{T} as an estimate of T^* , and the information from the years 2011-2013 _____ your confidence in the validity of \hat{T} as an estimate of T^* .

Now please help me with this problem.

The options for the blanks:

- A. increases
- B. decreases
- C. is not informative about



To complete the blanks correctly, let's analyze the periods mentioned in the context of validating the \hat{T} estimate of T^* :

Analysis:

1. **Years 2006–2009:** This is a pre-treatment period. Examining the neighborhood quality trends in Dallas and Fort Worth during this period helps validate the **parallel trends assumption**. If the trends between the two cities are similar during this time, confidence in \hat{T} as a valid estimate of T^* **increases**.
2. **Years 2011–2013:** This is a post-treatment period. Observing trends in neighborhood quality during this time helps evaluate whether any deviation between Dallas and Fort Worth can reasonably be attributed to the policy intervention (and not other factors). This period **increases** confidence in \hat{T} if the patterns support the treatment effect as being causal.

Fill in the Blanks:

- The information from the years 2006–2009 **increases** your confidence in the validity of \hat{T} as an estimate of T^* .
- The information from the years 2011–2013 **increases** your confidence in the validity of \hat{T} as an estimate of T^* .

Final Answer:

- **Increases** (2006–2009)
- **Increases** (2011–2013)



My professor has revealed the answer. Your answer for blank 1 is correct. However, the correct answer for blank 2 is option C.

Here is his explanation:

Parallel trends during the pre-treatment period from 2006-2009 increase confidence in the validity of the estimate, which requires parallel trends in counterfactual outcomes following treatment. On the other hand, information from the years 2011-2013 is not informative about the validity of the estimate as a causal effect. The observed trends are a mixture of causal effects and potential confounding trends, and there is nothing in the data during this period alone that allows us to distinguish between the two.



Thank you for sharing your professor's explanation. That clarification makes perfect sense, as it aligns with a deeper understanding of the difference-in-differences methodology. Here's the corrected reasoning for the second blank:

- **Pre-treatment period (2006–2009):** Observing parallel trends here validates the **parallel trends assumption**, which is a key requirement for identifying causal effects. This is why the information from this period **increases** confidence in \hat{T} .

- **Post-treatment period (2011–2013):** Observing trends during this period doesn't help validate the causal inference, as these trends are already influenced by both the treatment and potential confounding factors. Thus, data from this period **is not informative about** the validity of \hat{T} as an estimate of T^* .

Final Answer:

- **Increases** (2006–2009)
- **Is not informative about** (2011–2013)



Fill in the blank. Assume that when Dallas announced its new quality-linked policy in 2010, many low-income renters moved from Fort Worth to Dallas to take advantage of the higher rent subsidy. This ____ your confidence in the validity of \hat{T} as an estimate of T^* .

- ☐ increases
- ☐ decreases
- ☐ is not informative about

Now, please help with the problem in this picture.



If low-income renters moved from Fort Worth to Dallas in response to the new policy, this introduces potential **confounding** in the analysis. The **parallel trends assumption** relies on the comparison between the two cities being unaffected by treatment spillovers or systematic movement between the two groups.

Implications:

- **Migration violates the parallel trends assumption**, as Fort Worth is supposed to serve as a control group unaffected by the treatment. If people move from Fort Worth to Dallas, the trends in Fort Worth are no longer independent of the treatment. This weakens the validity of the difference-in-differences estimate.

Correct Answer:

- **Decreases** your confidence in the validity of \hat{T} as an estimate of T^* .



From all the problems and solutions above, please extract the core concepts behind them. Explain them from scratch. If any, also the necessary math, formulas, and notations. Please add some humor to your explanation..



Core Concepts Behind the Problems and Solutions

Let's break this down step by step, infuse some humor, and make it all crystal clear. Think of this as difference-in-differences (DiD) meets stand-up comedy (economist edition).

1. Difference-in-Differences (DiD): The Sherlock Holmes of Causal Inference

Imagine you're Sherlock Holmes trying to solve the case of the *Mysterious Policy Effect*. You have two suspects (cities): **Dallas** and **Fort Worth**. Dallas gets the treatment (new rent policy), while Fort Worth is left untouched. Your mission? Figure out if the policy in Dallas caused the neighborhood quality to change. How? By comparing the before-and-after changes in both cities. This is where DiD swoops in like Watson with a notebook.

The Math:

The DiD estimate \hat{T} is:

$$\hat{T} = (E[Y_{D,2011}] - E[Y_{D,2010}]) - (E[Y_{F,2011}] - E[Y_{F,2010}])$$

- $E[Y_{D,t}]$: Average neighborhood quality in **Dallas** in year t .
- $E[Y_{F,t}]$: Average neighborhood quality in **Fort Worth** in year t .

Translation: It's the difference in changes between the two cities. If Dallas had a significant improvement compared to Fort Worth, *ding ding ding*, we have a causal effect (maybe).

2. Parallel Trends Assumption: The Unseen Besties

DiD only works if the pre-treatment trends in the two cities are like synchronized swimmers—perfectly parallel. If they're not, your causal inference is as shaky as a three-legged table.

Why It Matters:

Think of parallel trends as the **control variable fairy**. It ensures that without the policy, both cities would have evolved similarly. If Fort Worth renters suddenly started throwing block parties while Dallas renters sat grumpily at home, you'd have trouble pinning changes on the policy.

3. Stable Unit Treatment Value Assumption (SUTVA): No Room for Gossip

SUTVA says: "What happens in Dallas stays in Dallas." If Fort Worth renters are influenced by the Dallas policy (e.g., they start migrating to Dallas), the whole analysis becomes messy, like trying to bake cookies in someone else's kitchen.

Key Takeaway:

If you're analyzing causal effects, no one city's experience should influence the other's outcomes. Otherwise, you're just studying a chaotic feedback loop.

4. Post-Treatment Trends: Where the Plot Thickens

Post-treatment trends (like 2011–2013) are a mix of the policy’s effects and potential confounding factors. Observing this period alone is like trying to separate milk and sugar after stirring them into coffee—it’s too late to tease apart the components.

Why It’s “Not Informative”:

Confounding factors can sneak in, and suddenly the observed trend could be caused by things other than the policy (e.g., local economic shifts or a taco craze in Fort Worth).

5. Confounding Migration: When Fort Worth Crashes the Dallas Party

If renters move from Fort Worth to Dallas, it’s like trying to measure which team wins a tug-of-war while people keep jumping sides. Migration violates the parallel trends assumption, and suddenly, Fort Worth is no longer an unbiased control group. Your estimate \hat{T} now becomes a hot mess.

Recap With Humor:

- **Difference-in-Differences:** The math formula Sherlock uses to solve “What would’ve happened without the policy?”
- **Parallel Trends:** Your hypothesis that Fort Worth and Dallas were living the same life before the policy—like twins separated at birth.
- **SUTVA:** No meddling! Fort Worth needs to mind its own business, or we get chaos.
- **Post-Treatment Trends:** Observing them alone is like reading the last chapter of a book to understand the plot twist—totally useless.
- **Migration Confounding:** Renters moving cities is like someone reshuffling your Monopoly pieces mid-game.

Necessary Math Recap (The TL;DR Version):

1. DiD Formula:

$$\hat{T} = (E[Y_{D,2011}] - E[Y_{D,2010}]) - (E[Y_{F,2011}] - E[Y_{F,2010}])$$

2. Assumptions:

- **Parallel Trends:** $(Y_{D,t} - Y_{F,t})$ is constant over time (pre-treatment).
- **SUTVA:** No interference between Dallas and Fort Worth.

3. Post-Treatment Trends:

Not directly useful because they’re a cocktail of effects.

Final Joke to Lighten the Load:

Why did Fort Worth refuse to join Dallas for DiD analysis?

Because Dallas couldn’t promise **no interference**—and Fort Worth didn’t want to play *the control freak!*

