

Cost Minimization Simplified



Suppose $q = \sqrt{K} + \sqrt{L}$, $r = 5$, and $w = 50$. How much labor will the firm use to produce $q = 110$, if it minimizes costs?

100 ✓ Answer: 100

100

Explanation

$$MP_L = \frac{1}{2\sqrt{L}}, MP_K = \frac{1}{2\sqrt{K}}, MRTS = \frac{MP_L}{MP_K} = \frac{\sqrt{K}}{\sqrt{L}}.$$

Setting $MRTS = \frac{r}{w}$, we have $\frac{\sqrt{K}}{\sqrt{L}} = \frac{50}{5} \Rightarrow K = 100L$.

Solving simultaneous equations $K = 100L$ and $110 = \sqrt{K} + \sqrt{L}$, and substituting for K in the latter equation, we have

$$110 = \sqrt{100L} + \sqrt{L}$$

$$110 = 11\sqrt{L}$$

$$10 = \sqrt{L}$$

$$L = 100.$$

You are an expert in microeconomics. I have a problem with the solution as shown in the picture. However, I feel the solution explanation is hard to digest. I still feel unfamiliar with the core concepts behind it, the specific terms, and the math. Please help to re-explain it in beginner-friendly style and include some humor.



Of course! Let's break this down **step by step** in simple, fun terms. Imagine we're working at a factory where robots (capital K) and humans (labor L) produce goods, and we want to do it **as cheaply as possible**.

1. The Problem:

You're given the equation for production:

$$q = \sqrt{K} + \sqrt{L}$$

This equation means the total production q comes from the square roots of capital K and labor L .

We know:

- $q = 110 \rightarrow$ We want to produce 110 units.
- $r = 5 \rightarrow$ Cost of capital (robots) per unit.
- $w = 50 \rightarrow$ Cost of labor (humans) per unit.

Our goal: **Find the amount of labor L that minimizes costs.**

2. Key Concept: Marginal Products and MRTS

To find the cheapest way to produce, we need to balance how **useful** each input (robots vs. humans) is at the margin.

- **Marginal Product of Labor (MPL)** = Extra production you get by using a bit more labor.

$$MPL = \frac{1}{2\sqrt{L}}$$

- **Marginal Product of Capital (MPK)** = Extra production you get by using a bit more capital.

$$MPK = \frac{1}{2\sqrt{K}}$$

Why do we care?

Because firms like to “trade” between labor and capital to find the cheapest combination. This trade-off is called the **Marginal Rate of Technical Substitution (MRTS)**:

$$MRTS = \frac{MPL}{MPK} = \frac{\sqrt{K}}{\sqrt{L}}$$

3. The Cost Balancing Rule:

To minimize costs, the MRTS (how inputs trade off) **must match the input price ratio** $\frac{w}{r}$ (wage divided by cost of robots):

$$MRTS = \frac{w}{r}.$$

Here:

$$\frac{\sqrt{K}}{\sqrt{L}} = \frac{50}{5} = 10.$$

Simplify this to:

$$\sqrt{K} = 10\sqrt{L}.$$

Now square both sides to get rid of the square roots:

$$K = 100L.$$

Translation: You need 100 times more robots than humans.

4. Plug Back Into the Production Equation:

We know the production equation is:

$$110 = \sqrt{K} + \sqrt{L}.$$

Substitute $K = 100L$:

$$110 = \sqrt{100L} + \sqrt{L}.$$

Simplify the square roots:

$$110 = 10\sqrt{L} + \sqrt{L}.$$

Combine terms:

$$110 = 11\sqrt{L}.$$

Solve for \sqrt{L} :

$$\sqrt{L} = \frac{110}{11} = 10.$$

Square both sides to find L :

$$L = 10^2 = 100.$$

5. Final Answer:

The firm will use **100 units of labor** to minimize costs.

A Bit of Humor to Wrap It Up:

Imagine robots and humans having a conversation:

- Robots: "We're expensive but efficient!"
- Humans: "You need 100 of me to balance you out!"
- Firm Manager: "Perfect. Let's keep the robots happy but not too many—100 humans it is!"

Moral of the story: Math is like a robot—it balances things out, even if it feels intimidating at first! 🎉



$$MP_L = \frac{1}{2\sqrt{L}} \cdot MP_K = \frac{1}{2\sqrt{K}} \cdot MRTS = \frac{MP_L}{MP_K} = \frac{\sqrt{K}}{\sqrt{L}}.$$

I still don't understand this part.
can you explain more?



Sure! Let's slow this down and explain step by step, as if I'm teaching a friend who's new to this.

Step 1: What are MPL and MPK ?

- MPL (Marginal Product of Labor): How much extra production you get if you add one more unit of labor L , keeping everything else constant.
- MPK (Marginal Product of Capital): How much extra production you get if you add one more unit of capital K .

The equations for MPL and MPK in this case are:

$$MPL = \frac{1}{2\sqrt{L}} \quad \text{and} \quad MPK = \frac{1}{2\sqrt{K}}.$$

- The square root \sqrt{L} and \sqrt{K} show **diminishing returns**.
If you add more labor or capital, the extra production you get becomes smaller.

Why?

Imagine baking cookies. At first, adding more workers (labor) makes things faster. But if you keep adding workers without increasing ovens (capital), the kitchen gets crowded, and the workers bump into each other. That's diminishing returns—each worker adds less and less value.

Step 2: What is *MRTS*?

The **Marginal Rate of Technical Substitution (MRTS)** tells you **how much capital K you can trade for labor L** while keeping production constant.

Mathematically:

$$MRTS = \frac{MPL}{MPK}.$$

Substitute MPL and MPK :

$$MRTS = \frac{\frac{1}{2\sqrt{L}}}{\frac{1}{2\sqrt{K}}}.$$

Simplify the fractions:

$$MRTS = \frac{\sqrt{K}}{\sqrt{L}}.$$

Translation:

To balance labor L and capital K , you compare their square roots. If \sqrt{K} is larger, it means capital is contributing **more** at the margin compared to labor.

Why Is *MRTS* Important?

The firm wants to **balance costs** by choosing the optimal mix of K and L . To minimize costs:

$$MRTS = \frac{w}{r},$$

where w is the wage (cost of labor) and r is the cost of capital.

This equation ensures the firm doesn't waste money.

If labor is too expensive relative to capital, the firm will use **more capital** and **less labor** to save costs, and vice versa.

Summary:

1. MPL : Extra output from 1 more unit of labor.

$$MPL = \frac{1}{2\sqrt{L}}.$$
2. MPK : Extra output from 1 more unit of capital.

$$MPK = \frac{1}{2\sqrt{K}}.$$
3. $MRTS$: Trade-off between labor and capital:

$$MRTS = \frac{\sqrt{K}}{\sqrt{L}}.$$

4. **Optimal Rule:** Balance $MRTS$ with $\frac{w}{r}$ to minimize costs.

Think of it like this:

If you're running a pizza shop, you balance between hiring workers to knead dough (labor) and buying more ovens (capital). If labor gets too expensive, you buy more ovens instead. $MRTS$ helps you find the sweet spot!

Let me know if this part still feels confusing! 😊



where the MPL and MPK formula come from?
are they basic formula that I have to memorize?



Great question! The formulas for MPL and MPK are not something you just memorize blindly—they actually come from **calculus**! Let me explain how they are derived and why they look the way they do.

1. What Are MPL and MPK Again?

- MPL : The additional output produced when you slightly increase labor L , keeping capital K constant.
Mathematically, this is the **partial derivative of the production function** with respect to L .
- MPK : The additional output produced when you slightly increase capital K , keeping labor L constant.
This is the **partial derivative of the production function** with respect to K .

2. The Production Function

Here, the production function is:

$$q = \sqrt{K} + \sqrt{L}.$$

This function tells you how much output q you get from capital K and labor L .

Now let's compute the partial derivatives to find MPL and MPK .

3. Deriving MPL (Marginal Product of Labor)

To find MPL , take the partial derivative of q with respect to L (treat K as constant):

$$q = \sqrt{K} + \sqrt{L}.$$

The derivative of \sqrt{L} is $\frac{1}{2\sqrt{L}}$ (this is a basic calculus rule for square roots).

So:

$$MPL = \frac{\partial q}{\partial L} = \frac{1}{2\sqrt{L}}.$$

4. Deriving MPK (Marginal Product of Capital)

To find MPK , take the partial derivative of q with respect to K (treat L as constant):

$$q = \sqrt{K} + \sqrt{L}.$$

The derivative of \sqrt{K} is $\frac{1}{2\sqrt{K}}$.

So:

$$MPK = \frac{\partial q}{\partial K} = \frac{1}{2\sqrt{K}}.$$

5. Why Are These Useful?

The MPL and MPK formulas tell you **how productive each input is** at the margin. They help firms decide how to allocate resources efficiently.

For example:

- If $MPL > MPK$, labor is relatively more productive, so you should use more labor.
- If $MPL < MPK$, capital is relatively more productive, so you should use more capital.

6. Do You Need to Memorize Them?

You don't need to **memorize** these formulas, but you should:

1. **Understand where they come from:** Take the derivative of the production function.
2. **Know how to apply them:** For a production function involving square roots, the derivatives will look like $\frac{1}{2\sqrt{L}}$ or $\frac{1}{2\sqrt{K}}$.

If you see other production functions (like Cobb-Douglas $q = K^\alpha L^\beta$), the process is similar—just take the derivatives to find MPL and MPK .

Summary:

- The formulas for MPL and MPK come from **calculus** by taking the partial derivatives of the production function.
- They aren't arbitrary formulas to memorize—you can always derive them if you know the production function.
- Focus on understanding **how they're derived** and why they're useful for decision-making.

Let me know if you want me to explain this with another production function example! 😊



what about MRTS formula and w/r , are they the basic formula that I need to memorize?



Great follow-up question! Let's discuss the **MRTS** formula and the $\frac{w}{r}$ ratio.

1. What is MRTS?

The **Marginal Rate of Technical Substitution (MRTS)** tells you **how much capital K you can reduce if you add a bit more labor L** while keeping production constant.

Think of it like this:

- MRTS is the slope of the **isoquant curve** (a curve showing combinations of K and L that produce the same output).
- It shows the trade-off between K and L **at the margin**.

2. The MRTS Formula

By definition:

$$MRTS = \frac{MPL}{MPK}.$$

Here:

- MPL = Marginal Product of Labor (additional output from one more unit of L).
- MPK = Marginal Product of Capital (additional output from one more unit of K).

Do You Need to Memorize This?

Yes, but it's more important to **understand** why this formula works.

- If labor is productive (high MPL) compared to capital (low MPK), the MRTS will be large. This means you can reduce a lot of K if you add a bit more L .
- If labor is not productive (low MPL) compared to capital (high MPK), the MRTS will be small.

So:

$$MRTS = \frac{MPL}{MPK}$$

just captures the relative productivity of labor and capital.

3. The $\frac{w}{r}$ Rule

The $\frac{w}{r}$ term represents the **cost ratio**:

- w : Wage rate (cost of hiring labor).

- r : Cost of capital (e.g., price of machines or rent for using them).

Why Does It Matter?

To minimize costs, a firm wants to balance the trade-off between labor and capital. The condition for cost minimization is:

$$MRTS = \frac{w}{r}.$$

Intuition:

- If labor is too expensive relative to capital (w is high), the firm will use more capital.
- If capital is too expensive relative to labor (r is high), the firm will use more labor.

This rule ensures the firm is getting the **biggest bang for its buck** for every dollar spent.

4. Do You Need to Memorize This?

Yes, but focus on understanding **why**:

1. $MRTS = \frac{MPL}{MPK}$ is the **rate at which inputs can substitute** for each other.
2. $\frac{w}{r}$ is the **rate at which inputs cost** relative to each other.

To minimize costs, the firm aligns these two rates:

$$\frac{MPL}{MPK} = \frac{w}{r}.$$

If they're not equal, the firm is not using its money efficiently:

- If $\frac{MPL}{MPK} > \frac{w}{r}$, labor is relatively cheap, so the firm should use **more labor** and less capital.
 - If $\frac{MPL}{MPK} < \frac{w}{r}$, capital is relatively cheap, so the firm should use **more capital** and less labor.
-

5. Big Picture

Here's what to remember:

- **$MRTS = \frac{MPL}{MPK}$** : Trade-off between labor and capital at the margin.
- **$\frac{w}{r}$** : Cost ratio of labor to capital.
- To minimize costs:
Set $MRTS = \frac{w}{r}$.

Once you understand this, the formulas will feel much more natural.

A Fun Analogy:

Imagine you're cooking with **eggs** (capital K) and **flour** (labor L).

- **$MRTS$** : How many grams of flour can you trade for one egg without changing the recipe's output?
- **$\frac{w}{r}$** : If eggs are way more expensive than flour, you'll use more flour and fewer eggs.

The cost minimization rule balances the recipe (MRTS) and your grocery budget ($\frac{w}{r}$) perfectly! 🔍



is MRS and MRTS similar?



Ah, great question! The **MRS (Marginal Rate of Substitution)** and **MRTS (Marginal Rate of Technical Substitution)** sound similar, and they are conceptually related, but they apply in **different contexts**. Let's break it down.

1. MRS: Marginal Rate of Substitution

- **Where it's used:** In **consumer theory** (how individuals make choices).
- **What it measures:** The **rate at which a consumer is willing to trade one good for another** while keeping the same level of utility (satisfaction).
- **Formula:**

$$MRS = \frac{\text{Marginal Utility of Good X (MU}_X\text{)}}{\text{Marginal Utility of Good Y (MU}_Y\text{)}}.$$

- **Intuition:**
 - Imagine you're trading **coffee** for **donuts**.
 - If you're willing to give up 2 cups of coffee for 1 donut, your MRS = 2.
- **Key takeaway:** MRS is about **consumer preferences** and their willingness to substitute goods to remain equally happy (same utility).

2. MRTS: Marginal Rate of Technical Substitution

- **Where it's used:** In **production theory** (how firms produce goods).
- **What it measures:** The **rate at which a firm can trade capital (K) for labor (L)** while keeping the same level of output.
- **Formula:**

$$MRTS = \frac{\text{Marginal Product of Labor (MPL)}}{\text{Marginal Product of Capital (MPK)}} = \frac{\sqrt{K}}{\sqrt{L}}.$$

- **Intuition:**
 - If you're running a factory, the MRTS tells you how much **capital** you can reduce when you add 1 more unit of **labor** to produce the same output.
- **Key takeaway:** MRTS is about the **trade-off between inputs (K and L)** in production while maintaining the same output.

The Big Difference:

Feature	MRS	MRTS
Used in	Consumer theory	Production theory
What it compares	Goods (e.g., donuts vs. coffee)	Inputs (e.g., labor vs. capital)
Goal	Maintain the same utility	Maintain the same output
Formula	$MRS = \frac{MU_X}{MU_Y}$	$MRTS = \frac{MPL}{MPK}$

Analogy to Make It Crystal Clear

Imagine you're:

1. **A consumer** at a carnival with **tickets** to trade for rides (MRS):
 - You trade a roller coaster ride for a Ferris wheel ride to stay equally happy.
2. **A factory manager** adjusting inputs (MRTS):
 - You swap machines (capital) for more workers (labor) to keep producing the same number of goods.

Conclusion

MRS and MRTS are **similar** because both measure trade-offs while keeping something constant:

- MRS: Trade-offs between goods → same **utility**.
- MRTS: Trade-offs between inputs → same **output**.

They live in different worlds (consumers vs. firms), but their logic is **nearly identical!** 😊