

Causal Inference Strategies



Please help to solve the following problem, related to Microeconomics Theory and Public Policy:

With the fundamental problem of causal inference, which work-around can we use to answer causal questions for human subjects? Select all that apply.

- A. Postulate temporal stability.
- B. Postulate reversibility.
- C. Postulate unit homogeneity.
- D. Estimate causal effects for populations rather than for individuals.



To address the **fundamental problem of causal inference**, which is the inability to observe both the treated and untreated outcomes for the same unit at the same time, we often rely on certain assumptions or approaches. Here's the analysis for each option:

A. Postulate temporal stability

Temporal stability assumes that the outcomes of interest do not change over time in a way unrelated to the treatment. This assumption is not commonly used as a standalone solution for the fundamental problem of causal inference but may be part of a broader set of assumptions.

B. Postulate reversibility

Reversibility refers to the idea that if the treatment is reversed, the outcome will return to its original state. While theoretically appealing, reversibility is not a standard work-around for the fundamental problem of causal inference in human studies, as it's often not practically feasible.

C. Postulate unit homogeneity

Unit homogeneity assumes that units with the same characteristics will respond identically to treatment. This assumption helps generalize causal effects across similar units but is not a perfect work-around. However, it is a plausible assumption in many causal inference approaches.

D. Estimate causal effects for populations rather than for individuals

This is a key strategy in causal inference. While individual-level causal effects are unobservable due to the fundamental problem, researchers can estimate **average treatment effects (ATE)** for populations using methods like randomized experiments, matching, or instrumental variables.

Correct Answer:

- C. Postulate unit homogeneity

- **D. Estimate causal effects for populations rather than for individuals**

These two approaches are practical and commonly used to address the fundamental problem of causal inference in human subjects.



However, my professor said that the correct answer is option D only.
His explanation: for human subjects, neither (1) temporal stability and causal transience nor (2) unit homogeneity are plausible. However, we can instead measure causal effects for populations rather than for individuals.



Your professor's explanation aligns with a nuanced understanding of the fundamental problem of causal inference. Let's revisit the options with this specific perspective:

A. Postulate temporal stability

- **Temporal stability** assumes that outcomes remain unchanged over time when conditions do not vary. While appealing in controlled environments, it is often implausible for human subjects because external factors or natural changes over time can influence outcomes.

B. Postulate reversibility

- **Reversibility** assumes that outcomes can return to their original state if the treatment is reversed. For human subjects, this is often not realistic, as treatments (e.g., educational interventions, medical procedures) may have irreversible or long-term effects.

C. Postulate unit homogeneity

- **Unit homogeneity** assumes that individuals with identical observable characteristics will respond in the same way to a treatment. For human subjects, this is generally implausible because of unobservable individual differences (e.g., genetics, preferences, psychological factors) that can lead to varying responses to the same treatment.

D. Estimate causal effects for populations rather than for individuals

- Estimating **causal effects for populations** rather than individuals is the key work-around for the fundamental problem of causal inference. This approach focuses on **average treatment effects** (ATE) across a group, which is measurable and meaningful, even when individual-level causal effects are not directly observable. Techniques like randomized controlled trials (RCTs), matching, or instrumental variable estimation are designed for this purpose.

Correct Answer: D

Your professor is correct in emphasizing that **option D is the only valid work-around for human subjects** because:

1. The assumptions of **temporal stability** and **unit homogeneity** are generally unrealistic for human populations.
2. Estimating causal effects at the population level avoids reliance on these problematic assumptions and provides actionable insights for policy and interventions.



What does the following formula represent?

$$T_* = E[Y_1 - Y_0 | X = 1]$$

Average Treatment Effect (ATE)

Average Treatment Effect for the Treated (ATT)

Average Treatment Effect for the Untreated (ATU)

Treatment effect for individual x

Now please help with the problem in this picture.



The formula provided in the image is:

$$T_* = E[Y_1 - Y_0 | X = 1]$$

Let's break it down step by step:

1. Interpretation of terms:

- Y_1 : The potential outcome if the individual **receives the treatment**.
- Y_0 : The potential outcome if the individual **does not receive the treatment**.
- $X = 1$: Indicates that we are conditioning on individuals who **actually received the treatment**.
- $E[Y_1 - Y_0 | X = 1]$: The **expected difference** between the treated and untreated potential outcomes, **for those who actually received the treatment**.

2. **Key concept:** This formula represents the **Average Treatment Effect for the Treated (ATT)**, as it focuses on estimating the causal effect specifically for individuals who received the treatment.

3. **Answer:** The correct choice is:

Average Treatment Effect for the Treated (ATT)



Fill in the blank. It is a ____ idea to compare $E[Y | X = 1]$ and $E[Y | X = 0]$ by convenience sampling ((i.e., treated and untreated units are selected for the study because they are the easiest to access)) to estimate the average treatment effect for the treated (ATT).

- A. good
- B. bad



The correct answer is:

B. bad

Explanation:

When using **convenience sampling**, the treated ($X = 1$) and untreated ($X = 0$) units are selected based on ease of access rather than random assignment. This introduces **selection bias**, as the treated and untreated groups may differ in ways that affect their outcomes (Y), aside from the treatment itself.

Why it's a bad idea:

1. **Non-random selection:** Convenience sampling often results in systematic differences between the groups being compared.
2. **Confounding variables:** Differences in characteristics unrelated to the treatment but affecting Y make it impossible to isolate the treatment effect.
3. **Biased ATT estimates:** Comparing $\mathbb{E}[Y \mid X = 1]$ and $\mathbb{E}[Y \mid X = 0]$ without addressing these biases will likely result in invalid or misleading estimates of the ATT.

To estimate the ATT accurately, it's necessary to use methods that account for potential confounders, such as **randomized experiments**, **matching methods**, or **instrumental variables**.



Based on the problems above, please construct an engaging explanation that explains the fancy concepts from scratch behind the problems.
Please include the necessary fancy math tools and symbol involved.
Please also add some quantitative tangible examples and some humor.



Sure! Let's dive into the fancy world of causal inference and treatment effects, breaking it down step by step. We'll use some math, relatable examples, and a touch of humor to make it stick!

The Fundamental Problem of Causal Inference

Imagine you're trying to find out if eating ice cream makes people happier. You hand a cone to Alice and ask, "Did this make you happy?" Alice says yes. Great! But how do you know she wouldn't have been just as happy if she had skipped the ice cream? Maybe Alice is just naturally cheerful (or maybe she got a raise that day). This is the **fundamental problem of causal inference**:

We can't observe both the "ice-cream Alice" (treated) and the "no-ice-cream Alice" (untreated) at the same time.

Enter the Math:

Causal inference is about estimating the difference in outcomes for the same person **if treated** versus **if not treated**:

$$T_i = Y_1 - Y_0$$

Where:

- Y_1 = Alice's happiness if she eats the ice cream.
- Y_0 = Alice's happiness if she doesn't.
- T_i = Alice's "treatment effect" (how much happier the ice cream made her).

But here's the kicker: we **never** observe Y_1 and Y_0 for the same person. It's like Schrödinger's ice cream—you only get one outcome per person. To work around this, we get creative!

Population-Level Thinking to the Rescue

Instead of obsessing over Alice, we think big. Instead of asking, "How happy did the ice cream make Alice?" we ask, "On average, how happy does ice cream make people?" The math now becomes:

$$ATE = \mathbb{E}[Y_1 - Y_0]$$

Fancy talk for: "What's the average treatment effect for everyone?"

What About ATT?

Sometimes we're only interested in the people who **actually ate the ice cream** (e.g., Alice and her ice-cream-loving friends). This is called the **Average Treatment Effect for the Treated (ATT)**:

$$ATT = \mathbb{E}[Y_1 - Y_0 \mid X = 1]$$

This says: "Among people who ate ice cream ($X = 1$), what's the average happiness boost?"

Why Convenience Sampling is a Bad Idea

Let's imagine you want to compare happiness for ice-cream eaters ($X = 1$) versus non-eaters ($X = 0$). So, you just grab whoever's easiest to study. Bad move. Why? Because convenience sampling is like only interviewing people in an ice-cream shop and comparing them to people standing in line at the DMV. Of course, ice-cream eaters are happier! The DMV folks aren't sad because they skipped dessert—they're sad because their number is "B196" and the counter is on "B12."

Statistically speaking, convenience sampling introduces **selection bias**: the groups you're comparing are fundamentally different in ways unrelated to the treatment.

A Tangible Example:

Let's say you're studying a new tutoring program. You want to estimate the ATT: how much did the program improve scores for students who participated? Here's the data:

Group	Average Exam Score
Treated ($X = 1$)	85
Untreated ($X = 0$)	70

If you simply compare scores ($\mathbb{E}[Y \mid X = 1] - \mathbb{E}[Y \mid X = 0]$), you might think the tutoring program adds **15 points**. But hold on—what if the treated group consisted of motivated, wealthy kids with private tutors, and the untreated group was... well, me on a lazy Sunday? That's **selection bias**! You can't trust this estimate.

Why Randomization Works

To eliminate bias, we need a **randomized controlled trial (RCT)**. Randomly assign students to either get the tutoring or not. This ensures the treated and untreated groups are statistically identical—like flipping a coin to decide who eats the ice cream.

Key Assumptions: Why Your Professor's Right

Your professor says **D (estimate population effects)** is the only viable solution for human subjects. Why? Because other assumptions, like **temporal stability** and **unit homogeneity**, are just unrealistic:

- **Temporal stability:** Humans aren't stable! Alice's happiness changes by the minute (coffee or traffic can ruin your ice-cream experiment).
- **Unit homogeneity:** No two people are identical. Alice may adore chocolate, while Bob thinks it's "meh."

Instead, we rely on **population-level effects** to answer causal questions.

A Dash of Humor

Think of causal inference like being a detective at a messy crime scene. Sure, you'd love to interrogate every suspect individually, but instead, you have to piece together clues from the whole neighborhood. Sometimes, this means comparing ice-cream eaters to DMV survivors and realizing... maybe life isn't fair after all. 🍦

The Moral of the Story

- Don't trust convenience sampling—it's lazy and biased.
- Embrace the big picture: Population-level estimates (ATE and ATT) are your friends.
- Randomized experiments are the gold standard, but when you can't randomize, get creative (e.g., matching or instrumental variables).

Causal inference might seem like wizardry at first, but once you master the tools, you're basically a stats superhero. 💪



To reinforce the understanding, please generate some similar problems and show the solutions.



Here are some similar problems to reinforce your understanding of causal inference, treatment effects, and the pitfalls of biased estimation methods. Each problem is followed by a detailed solution.

Problem 1: Estimating ATT

A new fitness program is introduced to help participants lose weight. You are tasked with estimating the **Average Treatment Effect for the Treated (ATT)**. Here's the data:

Group	Average Weight Loss (kg)
Treated ($X = 1$)	5.5
Untreated ($X = 0$)	3.0

The untreated group didn't participate in the program but had access to general weight-loss advice. Can you compute the ATT directly from this data? Why or why not?

Solution:

- To compute $ATT = \mathbb{E}[Y_1 - Y_0 \mid X = 1]$, we need to know:
 - Y_1 : Weight loss for treated participants if they joined the program (observed as 5.5).
 - Y_0 : Weight loss for treated participants if they had **not** joined the program (counterfactual, unobserved).

Here's the problem: we only know the untreated group's weight loss (3.0) **on average**, not what the treated group would have lost if they hadn't participated. We can't directly compute the ATT unless:

- Randomization:** If participants were randomly assigned, we could assume the untreated group is a valid comparison for the counterfactual Y_0 .
- Matching or Statistical Adjustments:** We could use observational methods to estimate Y_0 for treated participants.

Without these, simply comparing 5.5 (treated) and 3.0 (untreated) is **biased** because the groups might differ in other ways (e.g., motivation levels, demographics).

Problem 2: Selection Bias

You are analyzing the impact of a new scholarship program on students' test scores. The data shows:

Group	Average Test Score
Scholarship Recipients	90
Non-Recipients	70

You estimate the treatment effect as $90 - 70 = 20$. Is this a valid estimate of the **Average Treatment Effect (ATE)**? Why or why not?

Solution:

No, this is not a valid estimate of the **ATE** because of **selection bias**:

1. Students who received the scholarship (recipients) are likely different from those who didn't:
 - They may already be higher achievers or more motivated.
 - They might have access to better resources.
2. The test score difference may not be due solely to the scholarship program.

To compute the true ATE, you need a method that ensures treated and untreated groups are comparable:

- **Randomization:** Assign students to receive the scholarship or not randomly.
- **Propensity Score Matching:** Match recipients with non-recipients who have similar characteristics.
- **Instrumental Variables:** Use a variable that influences scholarship assignment but not test scores directly.

In short, the observed difference (20) is likely **biased**.

Problem 3: Confounding Variables

A company launches a new training program to improve employee productivity. Here's the data:

Group	Productivity Increase (%)
Trained ($X = 1$)	25
Untrained ($X = 0$)	15

You also know that employees in the trained group worked 5 extra hours per week on average, while those in the untrained group didn't. How might this extra work affect your estimate of the treatment effect?

Solution:

The extra work hours act as a **confounding variable**, meaning they affect productivity but are not part of the treatment (training program). Without controlling for this, the observed difference ($25 - 15 = 10$) might be **overstating the true effect** of the training program.

To address this:

1. **Adjust for Confounders:** Use regression analysis to account for the extra work hours:

$$Y = \beta_0 + \beta_1 \cdot \text{Training} + \beta_2 \cdot \text{Work Hours} + \epsilon$$

Here, β_1 would give the **true causal effect** of the training.

2. **Randomization:** If possible, randomize the assignment to the training program, ensuring that work hours are balanced across groups.

Without these adjustments, the observed treatment effect is **biased**.

Problem 4: Instrumental Variables

A researcher is studying the effect of college education on income. However, they realize that factors like family wealth and motivation also influence both education and income, creating endogeneity. The

researcher proposes using **distance to the nearest college** as an instrumental variable. Is this a valid approach?

Solution:

Using **distance to the nearest college** as an instrumental variable can work if:

1. **Relevance:** Distance significantly influences whether someone attends college ($Z \rightarrow X$).
2. **Exclusion Restriction:** Distance affects income only through its effect on education, not directly or through other pathways.

If these conditions hold, the researcher can use the two-stage least squares (2SLS) method to estimate the causal effect of education on income:

1. First Stage:

$$X = \pi_0 + \pi_1 Z + \epsilon$$

Estimate the effect of distance (Z) on college attendance (X).

2. Second Stage:

$$Y = \beta_0 + \beta_1 \hat{X} + \nu$$

Use the predicted college attendance (\hat{X}) to estimate its effect on income (Y).

If the instrument is valid, this approach avoids bias from confounders like family wealth or motivation.

Problem 5: Randomized Controlled Trials

A pharmaceutical company is testing a new drug to reduce blood pressure. They randomly assign participants to the treatment group ($X = 1$) or the placebo group ($X = 0$). After the trial, the data shows:

Group	Average Blood Pressure Reduction (mmHg)
Treated ($X = 1$)	15
Untreated ($X = 0$)	5

Estimate the **Average Treatment Effect (ATE)** and explain why this estimate is unbiased.

Solution:

The ATE is:

$$ATE = \mathbb{E}[Y_1 - Y_0] = \mathbb{E}[Y | X = 1] - \mathbb{E}[Y | X = 0]$$

$$ATE = 15 - 5 = 10 \text{ mmHg}$$

This estimate is **unbiased** because the study uses randomization:

1. Random assignment ensures that treated and untreated groups are comparable in all aspects (e.g., age, health status, lifestyle) except the treatment.

2. Any observed difference in outcomes is attributable to the treatment, not confounders.

These problems cover different aspects of causal inference and treatment effect estimation. By practicing these concepts and thinking critically about bias and assumptions, you'll become a pro at solving causal inference puzzles!