

Ordinal vs Cardinal Utility



You are an expert in Microeconomics and Public Policy.

Please help to solve the following problem:

True or False? In consumer theory, we tend to use the concept of ordinal but not cardinal utility functions. Although in practice, we build cardinal utility functions as functions of goods X and Y , we are only concerned with rankings but not the cardinality.



True.

Explanation:

In consumer theory, the concept of **ordinal utility** is primarily used. Here's why:

1. Ordinal Utility:

- Ordinal utility focuses on the **ranking** of preferences. Consumers can say they prefer one bundle of goods over another or that they are indifferent between two bundles, but they do not assign specific numerical values to their level of satisfaction.
- This is sufficient for analyzing most consumer choices, as decisions are based on **comparisons** of utility levels, not their magnitude.

2. Cardinal Utility:

- Cardinal utility assigns specific numerical values to satisfaction, implying the degree or intensity of preference (e.g., "Bundle A gives twice as much utility as Bundle B").
- In modern microeconomic theory, this is generally unnecessary because consumer preferences and choices can be adequately represented with ordinal utility.

3. Utility Functions of Goods X and Y :

- While utility functions $U(X, Y)$ are often written and studied in consumer theory, these functions are interpreted ordinally. That is, they are used to rank combinations of X and Y , not to measure exact differences in utility.

4. Concern with Rankings, Not Cardinality:

- What matters for decision-making is how one bundle compares to another (ranking), not the specific numerical values (cardinality).

Conclusion:

The statement is **True** because consumer theory relies on ordinal utility, and the cardinal representation is used only as a tool to derive and analyze preferences, focusing solely on rankings.



Please explain the following terms from scratch with an analogy:
bundle,

cardinal utility functions,
cardinality.

Please add some humor to your explanation.



1. Bundle

- **Analogy:** Think of a **bundle** as your grocery bag. Inside, you might have apples, bananas, and maybe a chocolate bar (because you “deserve it”). In consumer theory, a **bundle** is just a combination of goods, like your grocery haul.
- **Funny Twist:** If someone says, “This bundle contains apples and bananas,” they’re basically trying to sound fancy while describing your shopping bag. Imagine someone saying, “I’m optimizing my utility by consuming the apple-banana bundle,” when they really mean, “I’m having a snack.”

2. Cardinal Utility Functions

- **Analogy:** Imagine your taste buds have a numerical scoreboard. When you eat chocolate, it’s like they flash a “100,” while broccoli gets a “10” (unless you’re one of those weird broccoli fans). A **cardinal utility function** assigns numbers to how much satisfaction (utility) you get from consuming stuff.
- **Funny Twist:** It’s like a food critic who scores every dish they eat—but instead of saying, “This soup has a delightful hint of basil,” they go, “Soup = 85 utility points.” Sounds ridiculous, right? Economists realized this too, which is why cardinal utility isn’t used much these days. Ranking (ordinal utility) makes more sense—who cares if chocolate is 100 and broccoli is 10, as long as chocolate clearly wins?

3. Cardinality

- **Analogy:** Cardinality is the ability to assign specific numbers to things. Imagine you’re at a pizza party and someone asks, “How good is this pizza?” Instead of saying, “Better than last week’s cardboard,” you say, “This pizza is a solid 9 out of 10.”
- **Funny Twist:** Economists with a love for numbers thought: “Why just enjoy pizza when you can quantify how much you enjoy it?” So, they came up with cardinality. But here’s the kicker—do we really need to know if your pizza joy is 9.3 or 9.8? No one’s comparing it to a scientific experiment, so ordinal utility (just saying you like pizza better than broccoli) is usually good enough.

In a Nutshell:

- **Bundle:** Your grocery bag—no need for complex math, just apples, bananas, and chocolate.
- **Cardinal Utility Function:** A nerdy scoreboard for your happiness—but who needs it when we can just rank things?
- **Cardinality:** Measuring happiness with numbers, because economists secretly wish they were statisticians.

Life Tip: Focus on ranking your chocolate above broccoli. Leave the cardinality for people who argue about the “utility” of pineapple on pizza. 🍕



Let x be defined on the real numbers (\mathbb{R}), which of the following $g(x)$ is a positive monotone transformation of x ? Select all that apply.

☐ $g(x) = 10x + 5$

☐ $g(x) = \text{abs}(x)$

☐ $g(x) = x^2$

☐ $g(x) = x^3$

☐ $g(x) = \frac{-1}{x}$ for $x > 0$

now, please help to solve the problem shown in the picture.



To solve the problem, let's analyze **positive monotone transformations** for each given $g(x)$.

Definition: Positive Monotone Transformation

A function $g(x)$ is a **positive monotone transformation** if it is strictly increasing. This means:

- The derivative $g'(x) > 0$ for all x in its domain.
- As x increases, $g(x)$ also increases.

Now, evaluate each option:

1. $g(x) = 10x + 5$

- Derivative: $g'(x) = 10 > 0$.
- $g(x)$ is a linear, strictly increasing function.
- **This is a positive monotone transformation.**

2. $g(x) = \text{abs}(x)$

- $g(x) = |x|$:
 - For $x > 0$, $g(x) = x$, which is increasing.
 - For $x < 0$, $g(x) = -x$, which is decreasing.
- $g(x)$ is **not strictly increasing** over \mathbb{R} .
- **This is not a positive monotone transformation.**

3. $g(x) = x^2$

- Derivative: $g'(x) = 2x$.
 - For $x > 0$, $g'(x) > 0$.
 - For $x < 0$, $g'(x) < 0$.
- $g(x)$ is not strictly increasing over \mathbb{R} (it decreases for $x < 0$).
- **This is not a positive monotone transformation.**

4. $g(x) = x^3$

- Derivative: $g'(x) = 3x^2$.
 - $g'(x) > 0$ for all $x \neq 0$.
 - $g(x)$ is strictly increasing over \mathbb{R} .
- **This is a positive monotone transformation.**

5. $g(x) = -\frac{1}{x}$ for $x > 0$
- Derivative: $g'(x) = \frac{1}{x^2} > 0$ for $x > 0$.
 - $g(x)$ is strictly increasing for $x > 0$.
 - **This is a positive monotone transformation for $x > 0$.**

Final Answer:

The following functions are positive monotone transformations:

- $g(x) = 10x + 5$
- $g(x) = x^3$
- $g(x) = -\frac{1}{x}$ (for $x > 0$)



True or False? It is not always true that monotone transformations preserve the MRS of a utility function.



False.

Explanation:

Monotone transformations **always preserve the Marginal Rate of Substitution (MRS)** of a utility function. Here's why:

1. Marginal Rate of Substitution (MRS):

- MRS is the ratio of the marginal utilities of two goods:

$$\text{MRS} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}$$

- MRS represents the trade-off a consumer is willing to make between two goods to maintain the same level of utility.

2. Effect of Monotone Transformations:

- A monotone transformation $g(U)$, where $g'(U) > 0$, changes the utility function but does not alter the relative **marginal utilities** of the goods.
- Since the MRS depends only on the ratio of marginal utilities, it remains unchanged under monotone transformations.

3. Example:

- If $U(x_1, x_2) = x_1 + x_2$, the MRS is -1 (constant).

- Applying a monotone transformation $g(U) = \ln(U)$ still results in the same MRS.

Conclusion:

The statement is **false** because monotone transformations **do** preserve the MRS of a utility function.



can you provide a tangible example of monotone transformations ?



Tangible Example of Monotone Transformations

Let's work through a concrete example step by step:

1. Original Utility Function

Suppose the utility function is:

$$U(x_1, x_2) = x_1 + 2x_2$$

This function represents the consumer's preferences over two goods, x_1 and x_2 .

2. Marginal Rate of Substitution (MRS)

The Marginal Rate of Substitution (MRS) is:

$$\text{MRS} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}$$

For $U(x_1, x_2) = x_1 + 2x_2$:

$$\frac{\partial U}{\partial x_1} = 1, \quad \frac{\partial U}{\partial x_2} = 2$$

$$\text{MRS} = -\frac{1}{2}$$

3. Apply a Monotone Transformation

Now, apply a monotone transformation to the utility function. For example:

$$g(U) = \ln(U) \quad (\text{logarithmic transformation, strictly increasing})$$

The transformed utility function becomes:

$$\tilde{U}(x_1, x_2) = \ln(x_1 + 2x_2)$$

4. Check MRS After Transformation

For the transformed utility function $\tilde{U}(x_1, x_2) = \ln(x_1 + 2x_2)$:

$$\frac{\partial \tilde{U}}{\partial x_1} = \frac{1}{x_1 + 2x_2} \cdot 1, \quad \frac{\partial \tilde{U}}{\partial x_2} = \frac{1}{x_1 + 2x_2} \cdot 2$$

The MRS is:

$$\text{MRS} = -\frac{\frac{1}{x_1+2x_2} \cdot 1}{\frac{1}{x_1+2x_2} \cdot 2} = -\frac{1}{2}$$

5. Conclusion

The MRS remains the same ($-1/2$) before and after applying the monotone transformation. This confirms that **monotone transformations preserve the MRS**, even though the utility function itself looks different.

Intuitive Analogy:

- Imagine your preferences are like a mountain. A monotone transformation is like **changing the measurement units** of the mountain's height (e.g., switching from feet to meters, or applying a logarithmic scale). The shape (preferences) doesn't change—only the scale does.



what you show above is the explanation of mathematically.mathematical.
but, I can't imagine it in real life cases.



You're absolutely right! Let me bring monotone transformations into a **real-life, relatable scenario** instead of sticking to just the math.

Real-Life Case: Dessert Preferences

Imagine you go to an **ice cream shop** that offers two flavors: **Chocolate** (x_1) and **Vanilla** (x_2). Here's the setup:

Step 1: Your Original Utility Function

Your happiness (utility) from these flavors is based on a simple rule:

$$U(x_1, x_2) = x_1 + 2x_2$$

This means:

- Every scoop of Chocolate gives you **1 point of happiness**.
- Every scoop of Vanilla gives you **2 points of happiness** (you love Vanilla more!).

For example:

- 2 scoops of Chocolate + 1 scoop of Vanilla: $U = 2(1) + 1(2) = 4$.
- 1 scoop of Chocolate + 2 scoops of Vanilla: $U = 1(1) + 2(2) = 5$.

Step 2: A Monotone Transformation: Happiness Reimagined

Now, let's say someone asks you not just "How happy are you?" but "**How happy are you on a logarithmic scale?**" This is like changing the way you think about your satisfaction.

Using the monotone transformation $g(U) = \ln(U)$, your utility changes:

$$\tilde{U}(x_1, x_2) = \ln(U(x_1, x_2)) = \ln(x_1 + 2x_2)$$

- Original: 2 scoops of Chocolate + 1 scoop of Vanilla $\rightarrow U = 4$.
- Transformed: $\ln(4) \approx 1.39$.
- Original: 1 scoop of Chocolate + 2 scoops of Vanilla $\rightarrow U = 5$.
- Transformed: $\ln(5) \approx 1.61$.

Step 3: Real-Life Impact on Preferences

Does the transformation change your preferences?

No! Whether you're looking at raw points ($U = x_1 + 2x_2$) or the transformed utility ($\ln(U)$), **you still prefer the bundle with 2 scoops of Vanilla and 1 scoop of Chocolate** because it gives higher happiness in both cases.

Real-Life Analogy for "Monotone Transformation"

- A monotone transformation is like **changing how you describe your happiness**:
 - Original: "This ice cream gives me 4 happiness points."
 - Transformed: "This ice cream gives me 1.39 happiness on the log scale."

Your ranking of preferences doesn't change, just the **scale** of how you describe your happiness.

Fun Takeaway:

It's like watching a movie in **2D** or **3D**—the story stays the same, but the perspective changes. A monotone transformation changes the "view" but keeps your **taste for ice cream** exactly the same. 🍦