

John's Budgeting Happiness Challenge



John lives in a world of two goods, A and B , and has a utility function - which denotes how much John likes each bundle with combinations of good A and B - given by ¹

$$U_J = A^{1/2} B^{1/2}$$

John earns an income of 2. The price of good A is \$1 and of good B is \$2.

1. Write down John's budget constraint and maximization problem.

Solution.

1. John's budget constraint is given by all of the combinations of goods A and B he can afford with his income. The amount he spends on good A is given by $P_A \times A$

$$P_A A + P_B B \leq I \Rightarrow A + 2B \leq 2$$

His optimization problem is to maximize his utility subject to his budget constraint. We write the budget constraint as an equality instead of an inequality due to fact that John likes both goods (the utility is strictly increasing in both), which means that John will spend all of his money.

$$\max_{A,B} A^{1/2} B^{1/2} \quad \text{subject to} \quad A + 2B = 2$$

You are an expert in microeconomics.

I have a problem with the solution as shown in the picture.

However, I can't get the points.

Please help to re-explain with a beginner-friendly style.

Include some humor.



Alright, let's break this down in a beginner-friendly way, with some humor thrown in to lighten things up!

John's Situation: Imagine John as a guy who loves two things in life: eating **Apples** (let's call them good A 🍏) and reading **Books** (good B 📖). His happiness, or utility, comes from a mix of these two, described by the formula:

$$U_J = A^{1/2} B^{1/2}$$

Now, don't freak out about the math—this just means John gets happier when he has more apples **and** books, but he also likes to keep a balance. No one wants 100 apples and no books, or 100 books and no apples, right?

The Budget Reality: Poor John only has \$2 to spend. (He's on a tight budget, maybe saving up for Netflix?)

- **Price of an apple (A)** = \$1 each
- **Price of a book (B)** = \$2 each

So, how much can John buy? He has to make sure:

Total spending \leq Income

Translated, this is:

$$A + 2B \leq 2$$

This is John's **budget constraint**, aka the "Don't blow all your cash" rule. He's like, "I can't spend more than \$2, or I'll have to wash dishes at the bookstore."

The Game Plan (Optimization): John isn't just winging it. He wants to maximize his happiness (utility) while staying within budget. So, the problem becomes:

Maximize $A^{1/2}B^{1/2}$, **subject to** $A + 2B = 2$.

(Why equality instead of \leq ? Because John loves both apples and books. He'll always use every cent to maximize happiness!)

Simple Takeaway (with Humor):

1. John's budget is like your bank account on a Friday night—limited but packed with potential.
2. He wants to stretch his \$2 to buy the **perfect mix** of apples and books. Think of it as creating the ultimate combo meal 🍏+📖.
3. The formula for his happiness is just a fancy way of saying, "John loves variety but hates extremes."

If this still feels tricky, remember: economists just like making things sound complicated so they can feel smart (but you're totally getting it now!).

Let me know if you'd like me to go deeper into solving this or explaining anything else!



From the solution in the picture, I still don't understand the following statement:
"We write the budget constraint as an equality instead of an inequality due to fact that John likes both goods"

My question: how and where can I obtain the fact?



Great question! Let's address this step-by-step with a touch of clarity and a sprinkle of humor.

Where does the statement come from?

The fact that we use an **equality** ($A + 2B = 2$) instead of an **inequality** ($A + 2B \leq 2$) comes directly from understanding two key points about John and his preferences:

1. Utility Function Reveals All the Secrets

The utility function $U_J = A^{1/2}B^{1/2}$ shows that John's happiness depends on having **both goods**.

- $A^{1/2}$: His happiness increases if he has more apples.
- $B^{1/2}$: His happiness increases if he has more books.

Since John likes **both goods**, he's not going to leave any money unspent. Why leave cash sitting idle when he could buy more apples or books and make himself happier? (Imagine John saying, "Why save money when I can be happier right now? YOLO!")

Key insight from utility function:

John's utility is **strictly increasing** in A and B . This means the more he spends on these goods (while staying within his budget), the happier he gets.

2. Optimization Is About Using Every Penny

If we wrote the budget constraint as an **inequality** ($A + 2B \leq 2$), there would be cases where John doesn't spend all his \$2. But that's illogical because:

- John can always get more happiness by spending the rest of his money.
- The whole idea of optimization here is to maximize happiness, so leaving money unspent means he's not optimizing.

For example:

- If John spends \$1 on 1 apple and nothing on books, $A + 2B = 1$, and he has \$1 left. But his happiness (utility) would increase if he spends that remaining \$1 to buy more apples or books. So, he'll **always** spend exactly \$2.

How to deduce this fact?

Here's the thought process for deducing it:

1. Look at the utility function: $U_J = A^{1/2}B^{1/2}$.
 - It increases when A or B increases.
 - Conclusion: John doesn't stop spending until he's used all his money to increase A and B .
2. Think about the optimization:
 - The goal is to maximize utility subject to a budget constraint.
 - Leaving money unspent would mean he's not maximizing utility.
3. Check the context:
 - Since John "likes both goods" (utility increases in both A and B), he'll never leave money unspent. Hence, the **inequality becomes an equality**.

Humor Takeaway:

Think of John as a foodie with FOMO (Fear of Missing Out). He loves both apples and books, so he'll always spend every last dollar on them. Economists know this because the utility function is like a "happiness detector," revealing John's obsession with getting the most out of life (and his \$2 budget).

Let me know if you'd like me to simplify this further or walk through another example!



Please explain how we can deduce the following statement:
"the utility is strictly increasing in both"

Should I deduce it from the utility function?



Yes, you can deduce that the utility is strictly increasing in both goods A and B directly from the utility function! Let's break it down step by step and make it crystal clear.

Utility Function: $U_J = A^{1/2}B^{1/2}$

This function tells us how happy John is based on the amounts of A (apples 🍏) and B (books 📖). To figure out if **utility increases** as A or B increases, we need to look at **how utility behaves** when we adjust A and B .

Step 1: Partial Derivatives

The mathematical tool to analyze whether utility increases with A or B is to compute the **partial derivatives** of U_J with respect to A and B .

1. Derivative with respect to A :

$$\frac{\partial U_J}{\partial A} = \frac{1}{2}A^{-1/2}B^{1/2}$$

- The derivative is **positive** as long as $A > 0$ and $B > 0$.
- This means U_J increases as A increases (holding B constant).

2. Derivative with respect to B :

$$\frac{\partial U_J}{\partial B} = \frac{1}{2}A^{1/2}B^{-1/2}$$

- Similarly, this derivative is **positive** as long as $A > 0$ and $B > 0$.
- This means U_J increases as B increases (holding A constant).

Step 2: Strictly Increasing

Now that we know both derivatives are **positive**, we can conclude that:

- U_J is **strictly increasing in A** because increasing A while keeping B constant makes U_J bigger.
- U_J is **strictly increasing in B** because increasing B while keeping A constant makes U_J bigger.

Strictly increasing means that **more of any good always increases utility**, no matter how much you already have.

Step 3: Why This Matters

Because utility is strictly increasing in both A and B , John will **always want to spend his entire budget**. Why? Because spending more money on apples or books will always make him happier. Leaving money unspent would mean missing out on potential happiness, and John isn't about that life!

How to Deduce This Intuitively Without Math

Even without derivatives, you can see that the **utility function** $U_J = A^{1/2}B^{1/2}$ has these properties:

1. $A^{1/2}$ gets bigger as A gets bigger (e.g., if A grows from 1 to 4, $A^{1/2}$ grows from 1 to 2).
2. $B^{1/2}$ gets bigger as B gets bigger (e.g., if B grows from 1 to 9, $B^{1/2}$ grows from 1 to 3).

Since both parts of the function increase with A and B , the utility function as a whole increases when A or B increases. This is why we say **utility is strictly increasing in both**.

Humor Takeaway

Think of John as a fan of apples and books. No matter how many apples or books he already has, **adding more always makes him happier**. Economists figured this out by taking derivatives and realizing, "Yep, John's happiness keeps climbing as long as he keeps buying more apples or books."

Let me know if you'd like me to walk through this again in another way!



2. Calculate John's MRS between goods A and B .

Solution.

2. John's MRS represents the rate at which John is willing to trade the two goods. Visually, it is the slope of the indifference curve. We can compute the MRS as the ratio of marginal utilities, as in the following equation

$$MRS(A, B) = -\frac{MU_A}{MU_B}$$

For more mathematically inclined students, the theorem that provides this result is the *Implicit Function Theorem*. In our example:

$$MRS(A, B) = -\frac{A^{-.5}B^{.5}}{A^{.5}B^{-.5}} = -\frac{B}{A}$$

At a point (A, B) , the slope of the indifference curve is $-\frac{B}{A}$, which is the rate at which John is willing to trade goods A and B . For instance, if $A = 1$ and $B = .25$, John is willing to trade X units of A for $.25X$ extra units of B .

I have a second question with the solution as shown in the picture.
Please help to re-explain.



Let's break this down into an easy-to-understand explanation—no heavy math jargon, just clear logic and a bit of humor to keep it fun.

What Is MRS (Marginal Rate of Substitution)?

The **MRS** is like John's personal trading rate between apples A 🍏 and books B 📖. It answers this question:

"If I take away some of John's apples, how many books do I need to give him to keep him just as happy?"

Visually, this is the slope of the **indifference curve**—a line that shows all the combinations of A and B that give John the same level of happiness.

How Do We Calculate It?

The formula for the MRS is:

$$MRS(A, B) = -\frac{MU_A}{MU_B}$$

Here:

- MU_A : Marginal utility of apples (how much happiness one more apple brings).
- MU_B : Marginal utility of books (how much happiness one more book brings).
- The negative sign just reflects the slope—if you want more of one good, you have to give up some of the other.

Step-by-Step Example

Step 1: Compute Marginal Utilities

The marginal utilities are like happiness detectors. From John's utility function $U_J = A^{1/2}B^{1/2}$, we calculate:

1. $MU_A = \frac{\partial U}{\partial A} = \frac{1}{2}A^{-1/2}B^{1/2}$
 - This says, "How much happier does John get when he eats one more apple?"
2. $MU_B = \frac{\partial U}{\partial B} = \frac{1}{2}A^{1/2}B^{-1/2}$
 - This says, "How much happier does John get when he reads one more book?"

Step 2: Calculate MRS

Using the formula $MRS(A, B) = -\frac{MU_A}{MU_B}$, we plug in the marginal utilities:

$$MRS(A, B) = -\frac{\frac{1}{2}A^{-1/2}B^{1/2}}{\frac{1}{2}A^{1/2}B^{-1/2}}$$

Simplify the fraction:

$$MRS(A, B) = -\frac{B}{A}$$

So, the **MRS** is $-\frac{B}{A}$, meaning John's willingness to trade apples for books depends on how much of each good he has.

What Does $MRS(A, B) = -\frac{B}{A}$ Mean?

The value of the MRS tells us:

- $\frac{B}{A}$: How many books John is willing to trade for one apple (absolute value).
- $-B/A$: The slope of the indifference curve.

Example:

If $A = 1$ apple and $B = 0.25$ books:

$$MRS(1, 0.25) = -\frac{0.25}{1} = -0.25$$

This means:

- John is willing to **give up 0.25 books** to get **1 more apple**.

Why? Because he has more apples relative to books, so books are more precious to him at that moment.

Why Is MRS Important?

The MRS shows us how much John values apples vs. books at any given moment. It changes depending on his current mix of apples and books:

- If John has **lots of apples** and **few books**, books are more valuable, and he'll trade more apples for fewer books.
- If John has **lots of books** and **few apples**, apples become more valuable.

It's like how someone who's been eating pizza for a week would happily trade a slice for a salad—variety matters!

Humor Takeaway

Think of John as a picky shopper at a flea market. If he has a ton of apples 🍏, he'll say, "Sure, I'll trade a few apples for one book 📖!" But if he's already buried in books, he's like, "You want my apples? You better give me LOTS of books!" The MRS is just John's negotiating tactic, based on how much of each good he already has.

Let me know if this helps or if you'd like another example!



In computing MRS, I always confuse, which one should be the numerator and which one should be placed on denominator. How should I determine?
Which one act as the vertical and which one is the horizontal?



This confusion is super common, so let's simplify it with clear rules, an easy trick, and even a bit of humor to make it stick!

Rule for Determining the Numerator and Denominator

1. **Numerator:** The good that you're measuring on the **vertical axis** of the graph (in this case, B).
2. **Denominator:** The good that you're measuring on the **horizontal axis** of the graph (in this case, A).

So, in the standard indifference curve setup:

$$MRS(A, B) = -\frac{MU_A}{MU_B}$$

- A (the horizontal good) is in the denominator.
- B (the vertical good) is in the numerator.

Why This Rule Works

The Marginal Rate of Substitution (MRS) tells you the slope of the **indifference curve**, which is the rate at which you're willing to substitute one good for the other.

- **Slope** in math = Rise over Run = Change in Vertical / Change in Horizontal.
- That's why:
 - **Vertical** = B (good on the y-axis).
 - **Horizontal** = A (good on the x-axis).

So, the numerator (top) comes from the vertical good (B), and the denominator (bottom) comes from the horizontal good (A).

Intuitive Trick to Avoid Confusion

Here's an easy way to remember:

1. Graph the Indifference Curve in Your Head

- Think of the x-axis as apples (A) and the y-axis as books (B).
- When you're calculating MRS , ask:
 - "How many **books (vertical)** am I willing to give up for one more **apple (horizontal)**?"
- The answer will be $-\frac{B}{A}$, with B on top because it's the vertical good.

2. Remember the Negative Slope

- MRS is the slope of the indifference curve, which is **negative** because as you get more of one good, you must give up some of the other. So just take the slope formula:

$$MRS = -\frac{\text{Change in Vertical Good}}{\text{Change in Horizontal Good}}$$

Quick Example

Suppose John is trading apples (A) for books (B). If:

- $A = 2$ and $B = 4$,
then:

$$MRS(A, B) = -\frac{B}{A} = -\frac{4}{2} = -2$$

This means John is willing to trade **2 books for 1 apple**.

Humor Takeaway

Think of it as directions: if you're driving uphill (vertical), you care about how steep the hill is (books lost). If you're driving horizontally (apples gained), you care about how far you're going sideways. Slope = vertical over horizontal = B over A !

Let me know if this clears it up or if you'd like more examples!



so, in $MRS(\text{item1}, \text{item2})$, the second item inside the parentheses will always be the vertical good?



Actually, no—the **order of items in** $MRS(\text{item1}, \text{item2})$ does not directly determine which good is vertical or horizontal. Instead, it depends on how the goods are assigned to the **axes of the graph** in your analysis. Here's how to think about it:

Key Concept:

1. The good on the **vertical axis** (y-axis) is the "numerator" in the MRS calculation.
 2. The good on the **horizontal axis** (x-axis) is the "denominator" in the MRS calculation.
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How to Avoid Confusion

To decide which is vertical and which is horizontal:

1. Look at the **graph**:
 - If the good you're giving up is on the **y-axis**, it's the numerator.
 - If the good you're gaining is on the **x-axis**, it's the denominator.
2. Don't rely on the order in $MRS(\text{item1}, \text{item2})$:
 - Economists sometimes write MRS in terms of the **good being substituted for**, not the graph axes. For example:
 - $MRS(A, B)$: Might mean "the rate of substitution of A for B " (apples for books).
 - But mathematically, the good on the vertical axis still determines the numerator.
3. Use the **definition**:

$$MRS = - \frac{\text{Marginal Utility of Vertical Good}}{\text{Marginal Utility of Horizontal Good}}$$

Practical Example

Let's take two cases:

Case 1: Apples on x-axis, Books on y-axis

- A (apples) is the horizontal good.
- B (books) is the vertical good.

$$MRS(A, B) = - \frac{MU_B}{MU_A}$$

Here, MU_B is the numerator because books are on the vertical axis.

Case 2: Books on x-axis, Apples on y-axis

- B (books) is the horizontal good.
- A (apples) is the vertical good.

$$MRS(B, A) = -\frac{MU_A}{MU_B}$$

Now, MU_A is the numerator because apples are on the vertical axis.

Why Does This Matter?

The **axes of the graph** always decide the numerator (vertical good) and denominator (horizontal good) for MRS. The order inside $MRS(\text{item1}, \text{item2})$ is just how we name it; it doesn't dictate the calculation.

Humor Takeaway

Think of the graph as the boss: it decides who's vertical (y-axis) and who's horizontal (x-axis). The order in $MRS(A, B)$ is just a name—like deciding whether to call your pet "Fluffy" or "Spot." The graph makes the real rules about who's on top (literally, in the numerator)!

Let me know if you want more examples or clarification!



But from the problem, there is no graph visualized. So, how can you deduce that B will be vertical and A will be horizontal good?



Excellent point! When there is **no graph provided**, you can still deduce which good is vertical (numerator) and which is horizontal (denominator) by following these **logical conventions** and clues from the problem.

1. Check the Common Convention

In economics, when no graph is explicitly drawn:

- The **first good** mentioned is usually treated as the **horizontal good** (x-axis).
- The **second good** mentioned is usually treated as the **vertical good** (y-axis).

In this case:

- Apples (A) are listed first, so they're typically treated as the horizontal good.
 - Books (B) are listed second, so they're typically treated as the vertical good.
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2. Look at the Utility Function

The utility function $U_J = A^{1/2}B^{1/2}$ does not specify an axis directly, but it hints at **symmetry between the goods**. Since the problem doesn't suggest any deviation from convention:

- **Horizontal good:** A (apples).
- **Vertical good:** B (books).

This assumption is valid unless the problem explicitly says otherwise.

3. Examine the Question Language

The problem asks you to compute the **MRS of apples for books**, which is conventionally interpreted as:

- "How many **books** (y-axis, vertical) are you willing to trade for one more **apple** (x-axis, horizontal)?"

This supports the idea that:

- B is vertical.
 - A is horizontal.
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4. Follow the Economic Logic

The MRS formula:

$$MRS = -\frac{\text{MU of Vertical Good}}{\text{MU of Horizontal Good}}$$

tells us that the denominator represents the good you gain **one more unit of**, and the numerator represents the good you give up.

In this case:

- "How many books (B) will John give up for one more apple (A)?"
This question aligns with B (books) being the numerator and A (apples) being the denominator.
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Why Does the Problem Assume B is Vertical and A is Horizontal?

The problem implicitly assumes standard conventions:

- Apples (A) are treated as horizontal (x-axis).
 - Books (B) are treated as vertical (y-axis).
- And it sticks to these conventions throughout the calculations.
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What If the Problem Doesn't Follow Convention?

If a problem wants you to flip the axes (e.g., make apples vertical and books horizontal), it will **explicitly tell you** or provide context, such as:

- A graph with axes labeled differently.
 - A question phrased as: "How many apples (A) would John give up for one more book (B)?"
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Summary Rules to Deduce Vertical and Horizontal Goods Without a Graph

1. Follow **convention**: The first good listed (A) is horizontal, and the second good (B) is vertical.
2. Pay attention to the **utility function**: If it's symmetric, stick with convention unless explicitly told otherwise.

3. Interpret the **language**: "How many books for one more apple?" suggests books are vertical.
4. Use **economic logic**: The good being given up (denominator) is the horizontal axis, and the good being gained (numerator) is the vertical axis.

Humor Takeaway

When in doubt, treat economics like a recipe: apples come first, books second. So apples go on the x-axis (horizontal) and books on the y-axis (vertical). If the problem flips it, they'll tell you explicitly—economists aren't trying to bake a surprise cake here!

Let me know if you'd like further clarification!



is $MRS(A, B)$ different from $MRS(B, A)$?



Yes, **$MRS(A, B)$** is different from **$MRS(B, A)$** because they represent the marginal rates of substitution in **opposite directions**, and the numerators and denominators are swapped.

Let's break this down clearly.

1. What Does $MRS(A, B)$ Mean?

- $MRS(A, B)$ is the rate at which a person is willing to **give up good B** (vertical) to get **one more unit of good A** (horizontal), while keeping the same level of utility.
- Mathematically:

$$MRS(A, B) = -\frac{MU_B}{MU_A}$$

Here:

- MU_B : Marginal utility of B .
- MU_A : Marginal utility of A .

Example:

If $MRS(A, B) = -2$, it means John is willing to **give up 2 units of B** to get **1 more unit of A** .

2. What Does $MRS(B, A)$ Mean?

- $MRS(B, A)$ is the rate at which a person is willing to **give up good A** (vertical) to get **one more unit of good B** (horizontal), while keeping the same level of utility.
- Mathematically:

$$MRS(B, A) = -\frac{MU_A}{MU_B}$$

Here:

- MU_A : Marginal utility of A .
- MU_B : Marginal utility of B .

Example:

If $MRS(B, A) = -0.5$, it means John is willing to **give up 0.5 units of A to get 1 more unit of B** .

3. How Are They Related?

$MRS(A, B)$ and $MRS(B, A)$ are **reciprocals** of each other in absolute value:

$$MRS(A, B) = -\frac{1}{MRS(B, A)}$$

Why? Because one measures the rate of substitution in one direction (trading B for A), while the other measures the reverse direction (trading A for B).

4. Visualizing the Difference

Imagine an indifference curve where A is on the horizontal axis and B is on the vertical axis. At any point on the curve:

- $MRS(A, B)$ is the **slope** of the curve (how steep it is).
- $MRS(B, A)$ is the **inverse of the slope** (how flat it is).

If $MRS(A, B) = -2$:

- You give up 2 units of B to get 1 unit of A .
If $MRS(B, A) = -0.5$:
 - You give up 0.5 units of A to get 1 unit of B .
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5. Summary

- $MRS(A, B)$: Measures the trade-off of B for A . Numerator: B , Denominator: A .
 - $MRS(B, A)$: Measures the trade-off of A for B . Numerator: A , Denominator: B .
 - They are reciprocals in absolute value and represent substitution rates in **opposite directions**.
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Humor Takeaway

Think of $MRS(A, B)$ and $MRS(B, A)$ like two people arguing over pizza and soda:

- $MRS(A, B)$: "How much soda would you give up for one more slice of pizza?"
- $MRS(B, A)$: "How much pizza would you give up for one more soda?"

The arguments are connected, but they're not the same—they depend on who's willing to give up what!

Let me know if you need further clarification! 😊