

# Confidence Interval Analysis

ChatGPT said:



The correct answer is "True" for the problem in the picture for the following reasons:

## 1. Uniform Distribution on $[0, \theta]$

- The problem states that the  $X_i$ s have a uniform distribution on the interval  $[0, \theta]$ . In a uniform distribution, every value between 0 and  $\theta$  is equally likely, and the mean of this distribution is  $\frac{\theta}{2}$ .

## 2. Sample Mean as an Estimator

- The mean of a uniform distribution on  $[0, \theta]$  is  $\frac{\theta}{2}$ . Given the sample mean  $\bar{X}$ , you can estimate  $\theta$  using  $\hat{\theta} = 2\bar{X}$ . This makes  $2\bar{X}$  a reasonable point estimate for the true value of  $\theta$ .

## 3. Sample Variance is Known

- The problem mentions that the sample variance was computed and retained. Even though the original data is lost, the sample mean and variance are sufficient statistics for constructing a confidence interval in this context.

## 4. Confidence Interval for the Mean

- A confidence interval can still be constructed using the sample mean  $\bar{X}$  and the fact that the sample comes from a uniform distribution. The key is that the mean of the distribution is  $\frac{\theta}{2}$ , and with  $\hat{\theta} = 2\bar{X}$ , you can establish a confidence interval around  $\frac{\hat{\theta}}{2} = \bar{X}$ .

## 5. Uniform Distribution Properties

- In a uniform distribution, the variance  $\text{Var}(X)$  can be related to the range  $\theta$  as  $\text{Var}(X) = \frac{\theta^2}{12}$ . Knowing the sample variance allows for further refinement of the estimate of  $\theta$ , if needed, although in this context, the sample mean  $\bar{X}$  alone is sufficient for estimating  $\theta$  and constructing the interval.

## 6. Sufficient Statistics

- In statistical theory, sufficient statistics (like the sample mean in this case) contain all the information needed about a parameter (here,  $\theta$ ) that can be gleaned from the sample. Therefore, even though the original data is lost, the sample mean provides all the necessary information to estimate  $\theta$  and construct the confidence interval.

## Summary

Given the uniform distribution on  $[0, \theta]$ , the retained sample mean is a sufficient statistic for estimating the population mean. Despite losing the original data, you can use the sample mean to

construct a 95% confidence interval for the mean of the  $X_i$ s, making the statement "True." The key reasons are the properties of the uniform distribution, the sufficiency of the sample mean, and the relationship between the sample mean and the parameter  $\theta$ .