

Bayesian Inference Explained

ChatGPT said:



To solve the problem, the core concepts involved are:

1. **Bayesian Inference:** Updating the probability of a hypothesis based on new evidence.
2. **Prior Probability:** The initial belief about the likelihood of an event before considering new evidence.
3. **Conditional Probability:** The probability of an event given that another event has occurred.
4. **Complementary Events:** Events that are mutually exclusive and exhaustive, meaning one event happening excludes the other.

Detailed Steps to Obtain the Final Solution

Let's go through the problem step by step, applying these concepts.

1. Identify the Events and Probabilities

- Let **O** represent the event that you left your wallet in the office.
- Let **F** represent the event that you left your wallet at your friend's place.
- Let **Call** represent the event that you receive a call from someone who found your wallet.

Given:

- $P(O)$: Prior probability that you left your wallet in the office.
- $P(F)$: Prior probability that you left your wallet at your friend's place.
- $P(Call^C|O)$: Probability that you do **not** receive a call given that your wallet is in the office.
- $P(Call^C|F)$: Probability that you do **not** receive a call given that your wallet is at your friend's place.

You want to find the updated probability that your wallet is in the office given that you have **not** received a call, i.e., $P(O|Call^C)$.

2. Apply Bayes' Theorem

Bayes' Theorem states:

$$P(O|Call^C) = \frac{P(Call^C|O) \cdot P(O)}{P(Call^C)}$$

Where $P(Call^C)$ is the total probability of not receiving a call, which can be computed using the law of total probability:

$$P(Call^C) = P(Call^C|O) \cdot P(O) + P(Call^C|F) \cdot P(F)$$

So, substituting $P(Call^C)$ into Bayes' Theorem:

$$P(O|Call^C) = \frac{P(Call^C|O) \cdot P(O)}{P(Call^C|O) \cdot P(O) + P(Call^C|F) \cdot P(F)}$$

3. Plug in the Given Values

Let's assume the following values based on the problem description:

- $P(O) = 0.5$: You initially think it's equally likely that your wallet is in the office or at your friend's place.
- $P(F) = 0.5$: Thus, $P(F) = 1 - P(O) = 0.5$.
- $P(Call^C|O) = 0.95$: There's a 95% chance that you won't receive a call if your wallet is in the office.
- $P(Call^C|F) = 0.8$: There's an 80% chance that you won't receive a call if your wallet is at your friend's place.

4. Calculate the Posterior Probability

Now, substitute the values into the Bayes' formula:

$$P(O|Call^C) = \frac{0.95 \times 0.5}{(0.95 \times 0.5) + (0.8 \times 0.5)}$$

Simplify the expression:

$$P(O|Call^C) = \frac{0.475}{0.475 + 0.4} = \frac{0.475}{0.875} \approx 0.543$$

5. Interpret the Result

The final result, $P(O|Call^C) \approx 0.543$, indicates that after considering the fact that you have not received a call, there is about a 54.3% chance that your wallet is in the office. This is an update from your prior belief (which was 50%) based on the new evidence (not receiving a call).

Summary

- **Bayesian Inference** allows you to update your belief about where your wallet is based on the evidence (no call).
- The **Prior Probability** represents your initial belief before the evidence.
- The **Conditional Probability** accounts for how likely the evidence is depending on where your wallet is.
- By applying **Bayes' Theorem**, you obtain an updated (posterior) probability that your wallet is in the office, incorporating the fact that no one has called you.

Is this conversation helpful so far?