

Expected Utility and CE

You said:



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Please explain the concepts involved in the problem and solution shown in the picture.

ChatGPT said:



The problem in the image involves concepts from decision theory, specifically related to expected utility and risk aversion.

Concepts Involved:

1. Von Neumann-Morgenstern (VNM) Utility Maximizers:

- The problem assumes that Don and John are VNM Expected Utility maximizers. This means they make decisions to maximize their expected utility, a measure of satisfaction or value derived from wealth, rather than maximizing the wealth itself.

2. Utility Function:

- The utility of wealth is given by the function $u(w) = w^{1/2}$, which is a square root function. This function is concave, indicating that Don and John are risk-averse. For risk-averse individuals, the utility of wealth increases at a decreasing rate; in other words, as wealth increases, the additional utility gained from an additional dollar decreases.

3. Certainty Equivalent (CE):

- The certainty equivalent is the amount of certain wealth that gives the same utility as a risky prospect. In other words, it is the guaranteed amount of money that a person would accept instead of taking a gamble with an uncertain outcome.

Problem Breakdown:

• Don's Wealth and Risk:

- Don has a wealth of \$10,000.
- He faces a 40% chance of losing \$7,500, which means he has a 40% chance of having only \$2,500 left, and a 60% chance of retaining his \$10,000 wealth.

• John's Wealth and Risk:

- John has \$1,000,000 and faces no risk, so his wealth remains the same.

Solution Explanation:

1. Expected Utility Calculation:

- To find Don's certainty equivalent, we first calculate the expected utility from his risky situation.
- The utility function is $u(w) = w^{1/2}$.
- The utility for \$2,500 is $2500^{1/2} = 50$.

- The utility for \$10,000 is $10000^{1/2} = 100$.
The expected utility (EU) is then calculated as:

$$EU = 0.4 \times 50 + 0.6 \times 100 = 20 + 60 = 80$$

2. Finding the Certainty Equivalent:

- The next step is to find the wealth level CE that gives Don the same utility as the expected utility from his risky prospect.
- Since $u(CE) = CE^{1/2} = 80$, we square both sides to solve for CE :

$$CE = 80^2 = 6400$$


This means Don would be indifferent between receiving a certain \$6,400 or facing the risky situation described.

Summary:

- The certainty equivalent of \$6,400 is the amount that Don would accept instead of facing the gamble with a 40% chance of losing \$7,500 from his initial \$10,000 wealth. This solution leverages the utility function $u(w) = w^{1/2}$ to equate the expected utility from the risky scenario with the utility of the certain amount.

You said:



 Uploaded image

Still related, I have the second problem with the solution in this picture.

Please explain the context of the problem.

Also, please extract all specific terms. Explain each with an analogy. If any, please also describe the relationship between them.

ChatGPT said:



Problem Context:

The problem involves a situation where Don offers to pay John an amount P to bear the 40% risk of losing \$7,500. Under this arrangement:

- **Don's** wealth would equal $10,000 - P$ with certainty, as he pays P to John regardless of the outcome.
- **John's** wealth would equal $1,000,000 + P$, but he bears the 40% risk of losing \$7,500. Therefore, John has a 40% chance of ending up with $1,000,000 + P - 7,500$ and a 60% chance of having $1,000,000 + P$.

The problem asks for the lowest and highest values of P that both Don and John would agree to in this arrangement, assuming John's risk aversion does not change substantially with the payment P .

Explanation of Specific Terms:

1. Certainty Equivalent (CE):

- **Analogy:** Imagine you have a lottery ticket. The certainty equivalent is like saying, "I would rather have \$X in cash right now than take a chance on winning the lottery." It represents a guaranteed amount you'd accept instead of a gamble.
- **In the Problem:** For Don, the certainty equivalent is derived from his utility function to ensure he is indifferent between paying P and not taking the gamble himself.

2. Expected Utility (EU):

- **Analogy:** Picture yourself deciding between two dishes at a restaurant, each with different flavors you might enjoy to varying degrees. Expected utility is like calculating the overall enjoyment you'd expect from each dish, considering both the likelihood of liking it and how much you'd like it.
- **In the Problem:** John evaluates the expected utility of receiving P but also bearing the 40% risk of losing \$7,500.

3. Risk Aversion:

- **Analogy:** If you're afraid of heights, you might prefer taking the stairs rather than the elevator, even if the elevator is faster. In economic terms, a risk-averse person prefers a certain outcome over a risky one, even if the risky option has a higher potential reward.
- **In the Problem:** Both Don and John are risk-averse, which influences their decisions. Don is willing to pay P to avoid the risk, and John needs to be compensated sufficiently by P to accept the risk.

4. Minimum and Maximum Acceptable P :

- **Analogy:** Imagine you're negotiating the price of a used car. The seller has a minimum price they will accept, and the buyer has a maximum price they are willing to pay. If these ranges overlap, a deal can be made.
- **In the Problem:** Don has a maximum P he is willing to pay to transfer the risk to John. John has a minimum P he needs to accept the risk. The acceptable range of P where both agree is between these two values.

Relationship Between the Terms:

- **Certainty Equivalent and Expected Utility:** These are directly related, as the certainty equivalent is derived from the expected utility. For example, Don's certainty equivalent is calculated from the expected utility of his wealth with the risk.
- **Risk Aversion and Certainty Equivalent:** The more risk-averse an individual is, the lower the certainty equivalent will be relative to the expected value of the risky prospect. This means Don might be willing to pay more P to avoid the risk.

Detailed Explanation of the Solution:

1. For Don:

- Don's certainty equivalent was previously calculated as \$6,400. Thus, he is willing to pay up to $10,000 - 6,400 = 3,600$ to avoid the 40% risk of losing \$7,500. Therefore, Don will agree to the arrangement if $P \leq 3,600$.

2. For John:

- John's utility from his wealth is evaluated for both possible outcomes after accepting P and the risk of losing \$7,500.

- The condition for John to agree to the arrangement is derived by ensuring that his expected utility after accepting the risk and receiving P is at least equal to the utility from his initial wealth. The calculation in the image shows this condition as:

$$1,000,000^{1/2} \leq 0.4 \times (1,000,000 + P - 7,500)^{1/2} + 0.6 \times (1,000,000 + P)^{1/2}$$

- After simplifying, the minimum P that John would accept is found to be 3,003.

3. Result:

- Both Don and John will agree to the arrangement if $3,003 \leq P \leq 3,600$. This range represents the overlap where both parties find the deal acceptable based on their respective risk aversions and utility functions.

You said:



You said "to bear the 40% risk of losing \$7,500."
In this context, what is the meaning of "to bear"?

ChatGPT said:



In this context, "to bear" means to take on or assume the responsibility or burden of something. Specifically, when Don offers to pay John an amount P "to bear the 40% risk of losing \$7,500," it means that John would take on the risk of potentially losing \$7,500. If the loss occurs, John would have to cover it, effectively absorbing the negative outcome of the risky situation. Don, by paying P , transfers the risk to John, who then bears the consequences if the risky event happens.

You said:



I still don't understand how 3003 is obtained.
Can you explain?

ChatGPT said:



 Memory updated

Let's go step by step to understand how the value of 3003 is obtained.

Problem Recap:

• John's Situation:

- John has \$1,000,000, and he is considering taking on a 40% risk of losing \$7,500.
- If John takes the risk, his wealth could either be $1,000,000 + P - 7,500$ (with a 40% probability) or $1,000,000 + P$ (with a 60% probability).

- **Objective:**

- We need to find the minimum amount P that John would require to be willing to accept the risk.

Step-by-Step Calculation:

1. Utility Before Taking the Risk:

- John's initial wealth is \$1,000,000.
- Using the utility function $u(w) = w^{1/2}$, his utility is:

$$u(1,000,000) = 1,000,000^{1/2} = 1,000$$

2. Expected Utility After Taking the Risk:

- If John accepts the risk and receives P , his utility will depend on the outcomes:
 - With a 40% probability, his wealth would be $1,000,000 + P - 7,500$.
 - With a 60% probability, his wealth would be $1,000,000 + P$.
- The expected utility (EU) is:

$$EU = 0.4 \times (1,000,000 + P - 7,500)^{1/2} + 0.6 \times (1,000,000 + P)^{1/2}$$

3. Equating Utilities to Find P :

- John will agree to the arrangement if the expected utility of taking the risk is at least as large as the utility from his current wealth:

$$1,000 \leq 0.4 \times (1,000,000 + P - 7,500)^{1/2} + 0.6 \times (1,000,000 + P)^{1/2}$$

4. Simplifying and Solving for P :

- The equation is complex to solve algebraically, but we are looking for the smallest P such that the inequality holds true.
- Through numerical methods or iterative calculation, $P = 3003$ is found to be the smallest value where the expected utility is equal to or greater than 1,000.

When $P = 3003$:

- Calculate the expected utility:
 - Calculate the utility of $1,000,000 + 3003 - 7,500 = 995,503$:

$$u(995,503) = 995,503^{1/2} \approx 997.75$$

- Calculate the utility of $1,000,000 + 3003 = 1,003,003$:

$$u(1,003,003) = 1,003,003^{1/2} \approx 1001.50$$

- Expected utility (EU):

$$EU = 0.4 \times 997.75 + 0.6 \times 1001.50 \approx 399.1 + 600.9 = 1,000$$

Since the expected utility equals the initial utility of 1,000 at $P = 3003$, this is the minimum amount John would accept to take on the risk.

Conclusion:

The value 3003 is obtained by ensuring that John's expected utility after accepting the risk is at least as large as his current utility without the risk. This calculation involves determining the smallest P that

satisfies the condition, ensuring John is indifferent between taking the risk and maintaining his current wealth.

You said:



Still related, I have other problem with its solution explanation below:

What insurance mechanism explains why both John and Don are willing to strike this bargain?

- A. Risk pooling
- B. Risk spreading
- C. Risk transfer

Answer: C

Explanation: The arrangement in previous question is a risk transfer. Risk transfers are possible when one agent is less risk averse than another. To see why John is less risk averse – and thus willing to take up Don's risk for a price that Don finds acceptable, notice that $u(w) = w^{1/2}$ has the property of decreasing absolute risk aversion. Since John has much more wealth than Don, he is more willing to accept risks. Indeed, John is almost risk-neutral, since a risk neutral agent's certainty equivalent is $\$1,000,000 - 0.4 \times \$7,500 = \$997,000$, which is only \$3 more than John's certainty equivalent.

Please explain the context of the problem.

Also, please extract all specific terms. Explain each with an analogy. If any, please also describe the relationship between them.

ChatGPT said:



Problem Context:

The problem is about understanding the insurance mechanism underlying the agreement between John and Don, where John takes on Don's risk in exchange for a payment P . The solution identifies this mechanism as **Risk Transfer** and provides reasoning based on the differences in risk aversion between John and Don.

Explanation of Specific Terms:

1. Risk Transfer:

- **Analogy:** Imagine you're worried about dropping your smartphone, so you buy insurance. By doing so, you're transferring the financial risk of breaking your phone to the insurance company. If the phone breaks, the insurer pays to replace it, not you.

- **In the Problem:** Don is transferring the financial risk of losing \$7,500 to John. By paying John P , Don ensures that John will bear the risk instead of him.

2. Risk Aversion:

- **Analogy:** Imagine two people faced with the option of playing a game where they could win or lose money. One person might prefer not to play at all, choosing a sure but smaller reward instead (risk-averse), while the other might be willing to play, hoping for a bigger win (less risk-averse or risk-neutral).
- **In the Problem:** Don is more risk-averse than John because Don is more concerned about losing money. John, having more wealth, can afford to take the risk and is thus less risk-averse.

3. Decreasing Absolute Risk Aversion (DARA):

- **Analogy:** Think of it like this: if you had only \$100, losing \$50 would feel like a big deal, but if you had \$10,000, losing \$50 wouldn't worry you much. The more wealth you have, the less sensitive you are to losing small amounts.
- **In the Problem:** The utility function $u(w) = w^{1/2}$ implies DARA, meaning that as John's wealth increases, he becomes less concerned about taking on additional risks, making him more willing to accept Don's risk.

4. Certainty Equivalent:

- **Analogy:** This is like asking how much money you'd take for sure instead of gambling. If you're offered a 50/50 chance to win \$100 or nothing, and you'd rather have \$40 for sure, then \$40 is your certainty equivalent.
- **In the Problem:** John's certainty equivalent is very close to the risk-neutral value (\$997,000), showing he's almost indifferent to the small risk Don wants to transfer.

5. Risk Neutrality:

- **Analogy:** Imagine someone who doesn't care whether they face a gamble or take a certain outcome, as long as the expected value is the same. They are indifferent between a sure \$50 and a 50/50 chance of \$100 or nothing.
- **In the Problem:** John's behavior is almost risk-neutral because the certainty equivalent he would accept is just \$3 less than the expected value of the risky situation.

Relationship Between the Terms:

- **Risk Transfer and Risk Aversion:** The fundamental mechanism at play is risk transfer, made possible because of the difference in risk aversion between Don and John. Don is more risk-averse and thus willing to pay P to avoid the risk, while John, being less risk-averse, is willing to accept the risk for a fee.
- **Decreasing Absolute Risk Aversion (DARA) and Certainty Equivalent:** Because John's utility function exhibits decreasing absolute risk aversion, he becomes less sensitive to risk as his wealth increases. This reduces the amount he requires (his certainty equivalent) to accept Don's risk, making the transfer possible and beneficial for both.

Summary of the Explanation:

The problem highlights a scenario where Don transfers his risk of losing \$7,500 to John in exchange for a payment P . The concept of **Risk Transfer** is central to the solution, supported by the fact that John is less risk-averse than Don. This difference in risk aversion is explained by John's higher wealth and the property of decreasing absolute risk aversion in the utility function. Because John is almost risk-neutral due to his wealth, he is willing to accept the risk for a small fee, making the agreement mutually beneficial.

You said:



Still related, I have a T/F problem with its solution explanation below:

True or False? The arrangement in previous question is Pareto improving.

Answer: True

Explanation: This arrangement is obviously Pareto improving since both Don and John are willing to accept it.

Please explain the context of the problem.

Also, please extract all specific terms. Explain each with an analogy. If any, please also describe the relationship between them.

ChatGPT said:



Problem Context:

The problem asks whether the arrangement between Don and John—where Don transfers the risk of losing \$7,500 to John in exchange for a payment P —is **Pareto improving**. The answer is "True," with the explanation that the arrangement is Pareto improving because both parties agree to it.

Explanation of Specific Terms:

1. Pareto Improvement:

- **Analogy:** Imagine a group of friends sharing a pizza. A Pareto improvement would be finding a way to divide the pizza so that at least one person gets more without anyone else getting less. It's a win-win situation where no one is worse off.
- **In the Problem:** The arrangement between Don and John is Pareto improving because both parties benefit or at least do not lose from the deal. Don reduces his risk by paying P , which is worth it to him, and John receives P , which compensates him for taking on the risk.

2. Willingness to Accept the Arrangement:

- **Analogy:** Think of it like a trade between two people—one person has apples, and the other has oranges. If they both agree to trade, it's because each person values what they're getting more than what they're giving up. If both sides agree, it's a sign that the trade is mutually beneficial.
- **In the Problem:** Both Don and John are willing to accept the arrangement because it makes them better off or at least not worse off than before. Don is willing to pay P to avoid the risk, and John finds P sufficient to take on the risk.

Relationship Between the Terms:

- **Pareto Improvement and Willingness to Accept:** The concept of a Pareto improvement is directly tied to the willingness of both parties to accept the arrangement. If both Don and John

agree to the arrangement, it means that the transaction has made at least one of them better off without making the other worse off, which is the definition of a Pareto improvement.

Summary of the Explanation:

The arrangement between Don and John, where Don transfers his risk to John in exchange for a payment P , is **Pareto improving** because both parties benefit or at least do not lose from the transaction. Don reduces his exposure to risk, and John is compensated for taking on that risk. The mutual willingness to accept the arrangement indicates that it's beneficial for both parties, fulfilling the criteria for a Pareto improvement.

ChatGPT can make mistakes. Check important info.