

# Microeconomics Problem Explanation

You said:



 Uploaded image

I have a problem case with the solution, related to Microeconomics, as shown in the picture.  
Please explain the context of the problem.  
Please extract all specific terms and math notations.  
Then explain each with an analogy and aquantitative example.  
If any, please also describe the relationship between them.

ChatGPT said:



## Problem Context

The problem involves optimizing the production of homemade hot chocolate using two ingredients: milk (denoted as  $m$ ) and cocoa powder (denoted as  $c$ ). The production function is given by:

$$h(m, c) = (\min\{m, c\})^{\frac{1}{2}}$$

Here,  $h$  represents the number of cups of hot chocolate produced. The cost structure is given as 40 cents per cup of milk and 10 cents per tablespoon of cocoa powder. The goal is to determine the number of cups of hot chocolate  $q^*$  that should be produced to maximize profit, given that each cup of hot chocolate can be sold for \$4.

## Specific Terms and Math Notations

### 1. Production Function $h(m, c)$ :

- This function describes the relationship between the inputs (milk  $m$  and cocoa powder  $c$ ) and the output (cups of hot chocolate  $h$ ).
- The function uses the  $\min$  operator, meaning it depends on the smaller of the two inputs,  $m$  or  $c$ .
- The expression is then raised to the power of  $\frac{1}{2}$ , indicating that the production of hot chocolate increases with inputs but at a decreasing rate.

### 2. $\min\{m, c\}$ :

- This notation represents the minimum of  $m$  and  $c$ . The production is constrained by the lesser of the two inputs.

### 3. $q^*$ :

- This represents the optimal quantity of hot chocolate that maximizes profit.

### 4. Marginal Cost (MC):

- The additional cost of producing one more unit of output (here, one more cup of hot chocolate).

### 5. Price (p):

- The selling price per unit of output (here, \$4 per cup of hot chocolate).

### 6. Total Cost (TC):

- The sum of the costs associated with the inputs used to produce the hot chocolate.

### 7. Profit Maximization Condition:

- The condition  $MC = p$ , where marginal cost equals the price, is used to find the profit-maximizing output level.

## Explanation with Analogy and Quantitative Example

Imagine you are making a smoothie that requires two main ingredients: bananas and strawberries. The recipe tells you that the number of smoothies you can make depends on whichever ingredient you have less of. For example, if you have 3 bananas and 5 strawberries, you can only make 3 smoothies because you run out of bananas first.

In mathematical terms, this is captured by the  $\min$  function, which selects the smaller number between the two. If we assume that each smoothie is made using equal parts of bananas and strawberries, the output (smoothies) is directly tied to the limiting factor.

The formula for hot chocolate production is similar:

$$h(m, c) = (\min\{m, c\})^{\frac{1}{2}}$$

This means that if you have 4 cups of milk and 4 tablespoons of cocoa, you can produce  $\sqrt{4} = 2$  cups of hot chocolate. However, since the recipe requires equal amounts of both ingredients, if you had only 3 tablespoons of cocoa but 4 cups of milk, you could only produce  $\sqrt{3} \approx 1.73$  cups of hot chocolate.

## Relationship Between Terms

### 1. Marginal Cost and Price:

- The marginal cost is calculated by determining the change in total cost when producing one additional cup of hot chocolate. The price is the amount you receive per cup sold. To maximize profit, the number of cups produced should be where the cost of making the last cup (marginal cost) equals the price at which it can be sold.

### 2. Production Function and Cost:

- The production function defines how the input quantities translate into the output, while the cost function depends on the prices of these inputs. The interplay between these functions determines the optimal production level.

## Quantitative Example

Suppose you have the ability to buy ingredients. Milk costs 40 cents per cup, and cocoa powder costs 10 cents per tablespoon. You want to produce the number of cups of hot chocolate that maximizes your profit.

Given that the selling price per cup is \$4, the optimal production occurs when:

$$MC = p$$

Using the information provided:

1. The total cost  $TC$  of producing  $h$  cups is:

$$TC = 0.4m + 0.1c = 0.4h^2 + 0.1h^2 = 0.5h^2$$

2. The marginal cost  $MC$  is the derivative of  $TC$  with respect to  $h$ :

$$MC = \frac{d(TC)}{dh} = h$$

3. To maximize profit, set  $MC = p$ :

$$h = 4$$

Thus, the optimal number of cups to produce is  $q^* = 4$ , as this is where the additional cost of producing one more cup (marginal cost) equals the price you receive per cup.

You said:



What core concepts are involved to solve the problem?  
Please show detailed steps for obtaining the final solution.

ChatGPT said:



## Core Concepts Involved

To solve the problem, the following core microeconomic concepts are crucial:

### 1. Production Function:

- This defines how inputs (milk  $m$  and cocoa powder  $c$ ) are converted into outputs (cups of hot chocolate  $h$ ). The production function used here is  $h(m, c) = (\min\{m, c\})^{\frac{1}{2}}$ .

### 2. Perfect Complements:

- Milk and cocoa powder are perfect complements in this production process, meaning that the amount of output is constrained by the smaller of the two input quantities.

### 3. Cost Minimization:

- This concept involves choosing the combination of inputs that minimizes the cost of producing a given level of output.

### 4. Profit Maximization:

- To maximize profit, the producer needs to set the quantity produced where marginal cost ( $MC$ ) equals the price ( $p$ ).

### 5. Marginal Cost (MC):

- This is the cost of producing one additional unit of output.

## Detailed Steps for Obtaining the Final Solution

### Step 1: Understand the Production Function

The production function is:

$$h(m, c) = (\min\{m, c\})^{\frac{1}{2}}$$

This means that the output  $h$  is determined by the smaller of the two inputs,  $m$  or  $c$ . The output increases as you increase either  $m$  or  $c$ , but the increase is at a diminishing rate because of the square root.

## Step 2: Determine the Cost Structure

Given:

- Cost of milk ( $m$ ): 40 cents per cup
- Cost of cocoa powder ( $c$ ): 10 cents per tablespoon

Total cost ( $TC$ ) of producing  $h$  cups of hot chocolate can be expressed as:

$$TC = 0.4m + 0.1c$$

Since  $m = c = h^2$  (because of the perfect complement nature), substitute into the cost function:

$$TC = 0.4h^2 + 0.1h^2 = 0.5h^2$$

## Step 3: Calculate the Marginal Cost

The marginal cost is the derivative of the total cost with respect to the output  $h$ :

$$MC = \frac{d(TC)}{dh} = \frac{d}{dh}(0.5h^2) = h$$

## Step 4: Set Marginal Cost Equal to Price

In a perfectly competitive market, to maximize profit, the producer sets the marginal cost equal to the price:

$$MC = p$$

Given  $p = 4$  (the price per cup of hot chocolate):

$$h = 4$$

This means the profit-maximizing quantity of hot chocolate to produce is 4 cups.

## Step 5: Verify the Optimal Solution

Substitute  $h = 4$  back into the cost function to find the total cost:

$$TC = 0.5(4)^2 = 0.5 \times 16 = 8 \text{ cents}$$

Revenue from selling 4 cups:

$$\text{Revenue} = 4 \times 4 = 16 \text{ dollars}$$

Profit:

$$\text{Profit} = \text{Revenue} - \text{Total Cost} = 16 - 8 = 8 \text{ dollars}$$

This verifies that producing 4 cups maximizes profit, as increasing production further would increase the marginal cost above the selling price, reducing profit.

## Summary

- **Production Function** dictates the output based on the smaller input.
- **Cost Function** is derived from the input prices.
- **Marginal Cost** is calculated to determine how cost changes with additional output.
- **Profit Maximization** occurs where marginal cost equals price, resulting in  $h = 4$ , which is the optimal quantity to produce.

You said:



I still don't understand the concept of marginal cost.  
From your explanation "This is the cost of producing one additional unit of output".  
In this context, the the term "additional" is additional to what? to existing previous output?

ChatGPT said:



Yes, you're correct! Let's clarify this concept.

## Understanding Marginal Cost (MC)

**Marginal Cost (MC)** is the additional cost incurred to produce one more unit of output. In simpler terms, it's the cost of increasing production by one unit beyond the current level of production.

### "Additional" Means Adding to the Existing Output

To illustrate, let's consider this hot chocolate problem:

- Suppose you're already producing 3 cups of hot chocolate.
- The marginal cost is the cost of making 1 more cup (so, the cost of producing the 4th cup).

## Example with Quantitative Details

Imagine you are producing hot chocolate and you have already produced 3 cups. To understand marginal cost, you need to compare the total cost of producing 3 cups with the total cost of producing 4 cups.

### Step 1: Total Cost for 3 Cups

From the previous discussion, the total cost function is:

$$TC = 0.5h^2$$

For 3 cups ( $h = 3$ ):

$$TC(3) = 0.5 \times 3^2 = 0.5 \times 9 = 4.5 \text{ dollars}$$

### Step 2: Total Cost for 4 Cups

Now, calculate the total cost for 4 cups:

$$TC(4) = 0.5 \times 4^2 = 0.5 \times 16 = 8 \text{ dollars}$$

### Step 3: Calculate Marginal Cost

Marginal cost is the difference in total cost between producing 4 cups and 3 cups:

$$MC = TC(4) - TC(3) = 8 - 4.5 = 3.5 \text{ dollars}$$

So, the **marginal cost** of producing the 4th cup of hot chocolate is 3.5 dollars.

### General Concept

- **Before Producing 4 Cups:** You had already made 3 cups.
- **"Additional" Cost:** Refers to the extra cost of producing that one more cup (the 4th cup) on top of the 3 cups already produced.

Marginal cost gives you a way to think about how much it costs to ramp up production slightly. It's crucial for decision-making in a business because it helps you determine whether producing more units is profitable (if the marginal cost is lower than the price you can sell that unit for).

### Conclusion

- In this context, "additional" refers to the cost of producing one more unit beyond the units you have already produced.
- If you're producing 3 units, the marginal cost tells you how much it would cost to produce the 4th unit. If you're producing 4 units, it tells you the cost of producing the 5th unit, and so on.

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