

**Tutorial- 5:****Non linear Programming using Python**

$$\text{Min } x_1 x_4 (x_1 + x_2 + x_3) + x_3$$

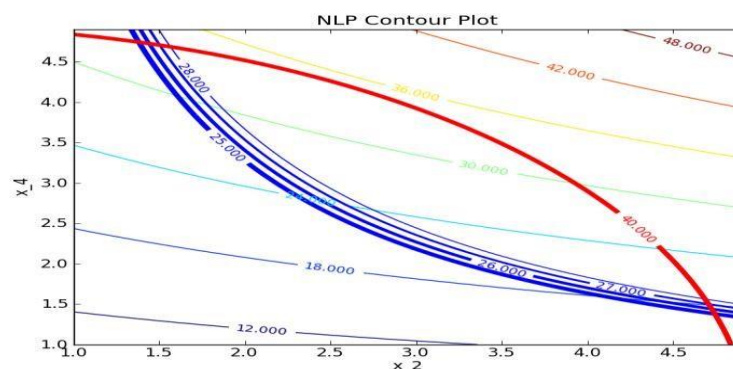
$$\text{s.t. } x_1 x_2 x_3 x_4 \geq 25$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 40$$

$$1 \leq x_1, x_2, x_3, x_4 \leq 5$$

$$x_0 = (1, 5, 5, 1)$$

This problem has a nonlinear objective that the optimizer attempts to minimize. The variable values at the optimal solution are subject to (s.t) both equality(=40) and inequality (>25) constraints. The product of the four variables must be greater than 25 while the sum of squares of the variables must also equal 40. In addition, all variables must be between 1 and 5 and the initial guess is  $x_1 = 1$ ,  $x_2 = 5$ ,  $x_3 = 5$ , and  $x_4 = 1$ .



For this problem determine:

1. A potential feasible solution
2. Identify the constraints on the contour plot
3. Mark the set of feasible solutions on the contour plot
4. Identify the minimum objective feasible solution

5. Identify the maximum objective feasible solution
6. Use a nonlinear programming solver to find a solution

Example 2:

maximise  $2x_1 + x_2 - 5\log_e(x_1)\sin(x_2)$

subject to

$$x_1x_2 \leq 10$$

$$|x_1 - x_2| \leq 2$$

$$0.1 \leq x_1 \leq 5$$

$$0.1 \leq x_2 \leq 3$$

Here we have a nonlinear objective and nonlinear constraints. Using the package we have the input:

Maximize	$2*x1 + x2 - 5*\log[x1]*\sin[x2]$
C1	$x1*x2 \leq 10$
C2	$\text{abs}[x1-x2] \leq 2$
X1	$>=0.1, <=5$
X2	$>=0.1, <=3$

and it clear from this there are values of  $x_1$  and  $x_2$  that exceed the supposed maximum objective function value of 8.8166 given by the package.

### CODE

```
import numpy as np
from scipy.optimize import minimize

def objective(x):
    return x[0]*x[3]*(x[0]+x[1]+x[2])+x[2]
def constraint1(x,sign=1.0):
    return x[0]*x[1]*x[2]*x[3]-25.0
def constraint2(x,sign=1.0):
    sum_eq = 40.0
    for i in range(4):
        sum_eq = sum_eq - x[i]**2
    return sum_eq

#initial guess
n = 4
x0 = np.zeros(n)
```

```
x0[0] = 1.0
x0[1] = 5.0
x0[2] = 5.0
x0[3] = 1.0
```

```
print('Initial SSE Objective:'+str(objective(x0)))
```

**#output:-** Initial SSE Objective:16.0

```
b=(1.0,5.0)
bnds=(b,b,b,b)
con1={'type':'ineq','fun': constraint1}
con2={'type':'eq','fun': constraint2}
cons=([con1,con2])
solution=minimize(objective,x0,method='SLSQP',bounds=bnds,constraints=cons)
x=solution.x
print('Final SSE Objective:' + str(objective(x)))
```

**#output:-** Final SSE Objective:17.01401724563517

```
print('Solution')
print('x1='+str(x[0]))
print('x2='+str(x[1]))
print('x3='+str(x[2]))
print('x4='+str(x[3]))
```

**#output:-**

```
Solution
x1=1.0
x2=4.742996096883977
x3=3.8211546234095715
x4=1.379407645075325
```

```
+ Code + Text
def constraint1(x):
    return x[0]*x[1]*x[2]*x[3]-25.0
def constraint2(x):
    sum_eq=40.0
    for i in range(4):
        sum_eq=sum_eq+x[i]**2
    return sum_eq

n=4
x0=np.zeros(n)
x0[0]=1.0
x0[1]=5.0
x0[2]=5.0
x0[3]=1.0

print("Initial SSE objective:"+str(objective(x0)))

b=(1.0,5.0)
bnds=(b,b,b,b)
con1={'type':'ineq','fun':constraint1}
con2={'type':'eq','fun':constraint2}
cons=([con1,con2])
solution=minimize(objective,x0,method='SLSQP',\
    bounds=bnds,constraints=cons)
x=solution.x
print("Final SSE objective:"+str(objective(x)))

print("Solution")
print("x1 = "+str(x[0]))
print("x2 = "+str(x[1]))
print("x3 = "+str(x[2]))
print("x4 = "+str(x[3]))

Initial SSE objective:16.0
Final SSE objective:17.01401724563517
Solution
x1 = 1.0
x2 = 4.742996096883977
x3 = 3.8211546234095715
x4 = 1.379407645075325
```