wednesday
17-6-2020

DAA Assignment-5
UNIT 5

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17[illegible]
3rd B.Tech CSE-C

2 Marks

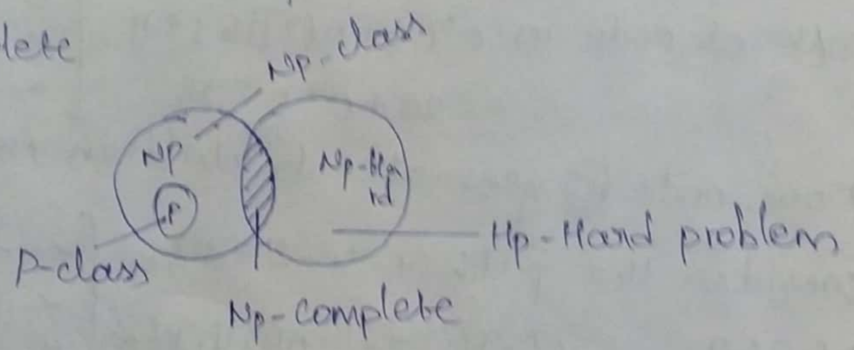
1A) cook's Theorem:

The scientist stephen cook in 1971 stated that boolean statisfiability problem is Np-complete problem

cook's theorem states that the satisfiability is in p iff p=Np.

2A) Relation b/w p, Np, Np-Hard, Np-complete

Relation b/w p and Np and Np Hard & Np-complete



3A) class p and Np:-

p-class : A problem which can be solved in polynomial time.

Np-class: A problem which can't be solved in polynomical time but is verified in non-deterministic polynomial time

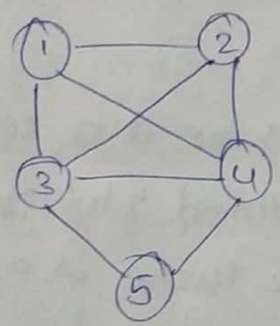
It uses non-deterministic Algorithm to find the solutions.

4A) A Boolean Formula is in 3CNF if it consists of each clause has exactly three direct variables/literals

5A) vertex cover:-

A vertex cover of a graph $G=(V,E)$ is a set of vertices that touches every edge in the graph.

Ex:



$\therefore\ C_1=(V,E)$

$V=\{1,2,3,4,5\}$

$V'=\{1,3,4\}$

$V'\leq V$

10 marks

1a) Boolean satisfiability:-

This problem is based on boolean formula, consists of various operations as OR, AND, NOT.

SAT = { <p> : p is a satisfiable propositional formula}

* This SAT problem is to find out for which truth assignment for variables for a Boolean Formula that produces a result "True".

Ex:- $F=(x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$

$\therefore$ Some Assignments as $x_1=1;\ x_2=0;\ x_3=1$

$\Rightarrow (1 \vee 0 \vee 0) \wedge (0 \vee 1 \vee 0)$

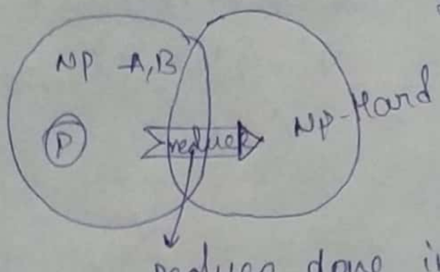
$\Rightarrow 1 \wedge 1$

$\Rightarrow 1$ (True)

1b A) **Np-Hard problem:-**

A problem is Np-Hard if every problem in Np can be polynomial reduced to it.

A problem 'L' is Np-Hard, iff SAT problem reduc to L.

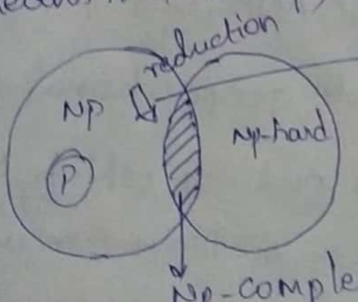


* If there is a solution to one Np-Hard problem in polynomial time there is a solution of al Np-Hard problem.

reduce done in polynomial time.

Np-complete problem:-

A problem 'L' is Np-complete, if it belongs to Np (means it is in Np) & Np-Hard.



* The class of Np-complete proble is insertion of Np & Np-Hard class.

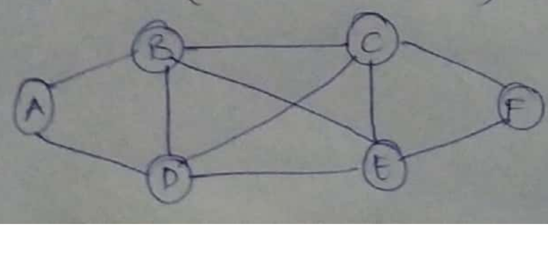
* Np-complete problems are Decision problem (yes/no)
* Np-Hard problem are optimization problem (max/m

2A) **clique decision problem in Np-complete:-**

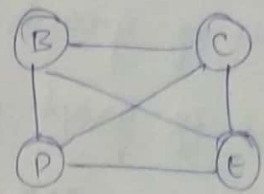
clique of graph (or) clique Decision problem:

A complete subgraph of a graph is a clique of size 'k' (∵ k-vertices)

Ex:-



clique of size $k=4$ → $V' = \{B, C, D, E\}$



NP-completeness of clique problem:

1. clique is in NP: check if $V' \subset V$ with $|V'| = k$ vertices of G. check if each pair (u,v) in V' has an edge of G. Thus verification is done in polynomial time.

2. 3CNF-SAT reduces to clique: It means clique problem is NP-Hard. we give a reduction from 3-SAT problem to clique problem.

The reduction function F for any 3CNF formula P:

1. create a vertex for each variable. so that $|V| = 3n$
2. For any two variables x_1 and x_2. create an edge (x_1, x_2) if they are not in same clause & not negation of each other.

3CNF-SAT: $P = (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)$

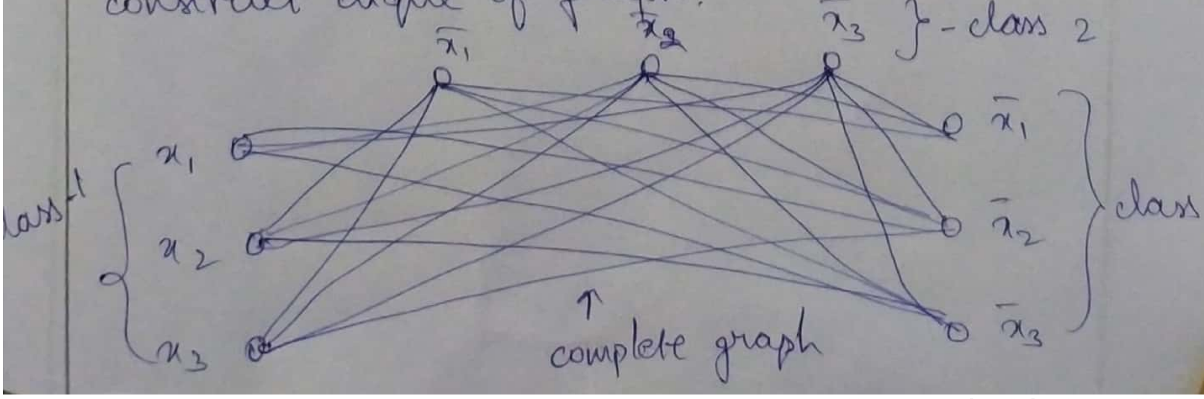
let $x_1 = 0$, $x_2 = 1$, $x_3 = 1$ (Assignments)

$\Rightarrow (0 \vee 1 \vee 1) \wedge (1 \vee 0 \vee 0) \wedge (1 \vee 1 \vee 0)$

$\Rightarrow 1 \wedge 1 \wedge 1$

$\Rightarrow 1$ (True) (The 3CNF formula is satisfiable)

construct clique of graph:



$\therefore$ clique of graph size $k=3$

$\therefore$ clique of graph is "Np-hard"

becoz , 3CNF-SAT $\propto$ clique

$\therefore$ we construct clique is a Np-complete

$\Rightarrow$ p is satisfiable, iff $F(P) = G(V,E)$ has a clique of size k.

$\therefore$ clique is a Np-complete.