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DAA Assignment-4

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3rd B.Tech CSE-C

UNIT-4

2 Marks

1) Given 'n' objects and a knapsack with capacity m, select some objects to fill the knapsack in such a way that it should not exceed the capacity of the knapsack & maximum profit can be earned.

The 0/1 knapsack problem can be solved using branch and bound strategy by considering it as a minimization problem.

$\Sigma p_i x_i$ is maximum iff minimize $\sum_{i=1}^{n} p_i x_i$

subject to $\sum_{i=1}^{n} w_i x_i \leq m$ and $x_i = 0$ or 1.

→ for a minimum cost answer node, we need to define

$u(x) = -\sum_{i=1}^{n} p_i x_i$ for every node x

$$\hat{c}(x) = u(x) - \left[\frac{m - \text{current total weight}}{\text{Actual wt of remaining wt}}\right] * \left[\begin{array}{c}\text{Actual profit of} \\ \text{remaining wt}\end{array}\right]$$

2) Applications of Branch and Bound:-

1. 0/1 knapsack problem
2. Travelling sales person problem
3. LC Branch and Bound.
4. FIFO Branch and Bound
5. LIFO Branch and Bound

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3A) multiplying Triangular matrices:-

An n×n matrix A whose elements are $a_{ij}=0$ for $i>j$ is known as upper triangular matrix. Ex: $\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$

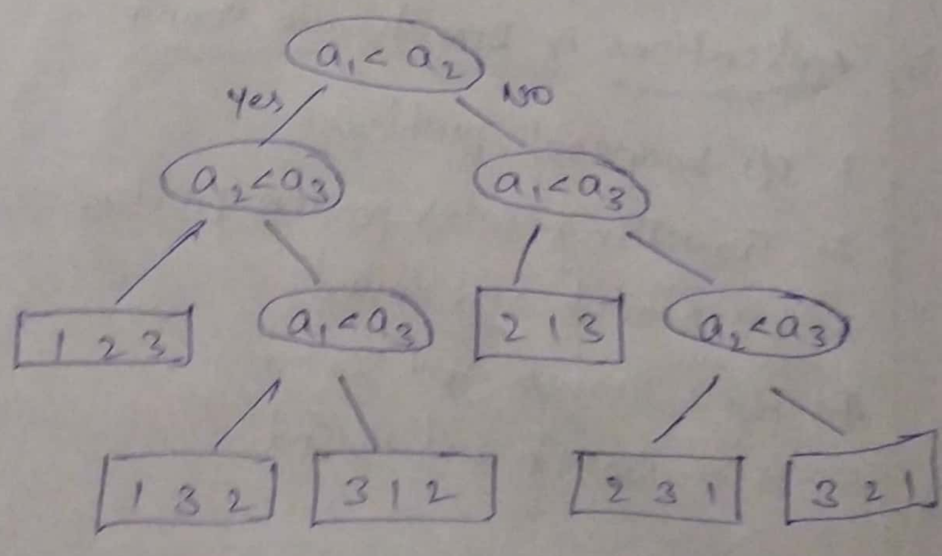
An n×n matrix "A" whose elements are $a_{ij}=0$ for $i<j$ is known as lower triangular matrix. Ex:- $\begin{bmatrix}1&0&0\\2&3&0\\4&5&6\end{bmatrix}$

we derive lower bounds for the problem of multiplying two lower triangular matrices.

4A) reduction in lower bound theory:-

let P_1 & P_2 be any two problems. we say P_1 reduces to P_2 ($P_1 \propto P_2$) in time $T(n)$ if an instance of P_1 can be converted into an instance of P_2 and a solution for P_1 can be obtained from a solution for P_2 in time $\leq T(n)$.

5A) comparison tree for sorting of three items:-



10 Marks

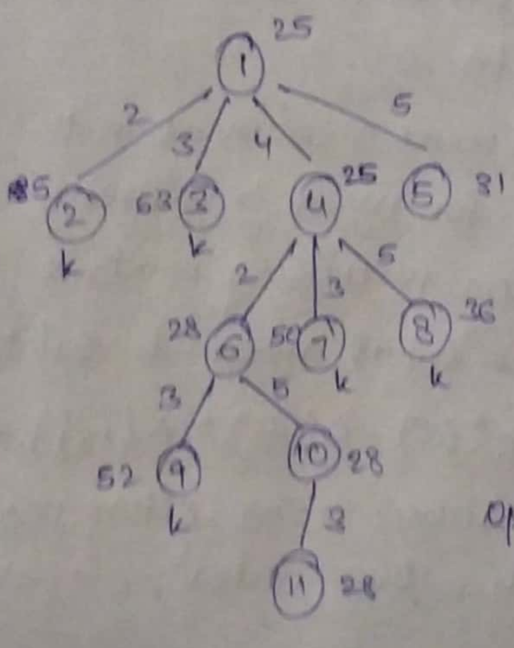
1A) Travelling sales person problem

In order to use LCBB to search the travelling sales person state space tree, we need to define a cost function $c(\cdot)$ and other two functions $\hat{c}(\cdot)$ and $u(\cdot)$ such that $\hat{c}(r) \le c(r) \le u(r)$ $\forall r$. The cost $c(\cdot)$ is such that the solution node with least $c(\cdot)$ corresponds to a shortest tour in G.

Ex: Consider the cost matrix and obtain the optimal tour

$$\begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}$$

Sol: Given $A = \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 14 & 4 & 7 & 16 & \infty \end{bmatrix}$



optimal path is :

1 - 4 - 2 - 5 - 3 - 1

select minimum value from each row & subtract

$$\begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \\ \hline 21 \end{array}$$

select minimum value from each column & subtra

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

$1+0+3+0+0 = 4$

∴ Reduced cost matrix at node ① is 1 $\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 2 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$

∴ Reduced cost $= 21 + 4 = 25$

⇒ Node ① generates it children nodes 2, 3, 4 & 5

Now, the paths are $1 \to 2, 1-3, 1-4, 1-5$

consider the path $1-2$:

Make all entries in row 1 & column 2 to ∞, $A = (2,1) = \infty$

$$\Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

Reduced cost at node 2 is $\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$

cost at node 2 : $\hat{c}(R) + A[1][2] + r$

$25 + 10 + 0 = 35$

consider the path 1-3 : $A[3][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 2 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix}$$

select minimum value from each row & column and subtract it

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

$11 + 0 + 0 + 0 + 0 = 11$

$\therefore$ Reduced cost matrix of node 3 :

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

$\therefore$ cost at node 3 : $\hat{c}(R) + A[1][3] + r$

$25 + 17 + 11 = 53$

consider path 1-4 : $A[4][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

$\therefore$ cost at node 4 :

$\hat{c}(R) + A[1][4] = l$

$25 + 0 + 0 = 25$

consider path 1-5 : $A[5][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

$$\begin{array}{cc} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} & \begin{array}{c} \\ 2 \\ \\ \\ 3 \end{array} \end{array}$$

$\overline{5}$

cost at node 5 : $\hat{c}(R) + A[1][5] + r$

$25 + 1 + 5 = 31$

Now it generates children node 6, 7, 8

consider path 1-4-2:

$A[4][1] = \alpha$; $A[2][1] = \alpha$

$$\begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & 11 & \alpha & 0 \\ 0 & \alpha & \alpha & \alpha & 2 \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ 11 & \alpha & 0 & \alpha & \alpha \end{bmatrix}$$

cost at node 6: $C^{\wedge}(R) + A[4][2] + r$

$= 25 + 3 + 0$

$= 28$

consider path $A[4][1] = \alpha$: $A[3][1] = \alpha$

$$\begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ 12 & \alpha & \alpha & \alpha & 0 \\ \alpha & 3 & \alpha & \alpha & 2 \\ \alpha & \alpha & \alpha & \alpha & 0 \\ 11 & 0 & \alpha & \alpha & \alpha \end{bmatrix} - 2 \Rightarrow \begin{bmatrix} \alpha & \alpha & \alpha & \alpha \\ 1 & \alpha & \alpha & 0 \\ \alpha & 1 & \alpha & 0 \\ \alpha & \alpha & \alpha & \alpha \\ 0 & 0 & \alpha & \alpha \end{bmatrix}$$

11 $= 13 (11+2)$

Reduced cost matrix of node 7:

$$\begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ 1 & \alpha & \alpha & \alpha & 0 \\ \alpha & 1 & \alpha & \alpha & 0 \\ \alpha & \alpha & \alpha & \alpha & 0 \\ 0 & 0 & \alpha & \alpha & \alpha \end{bmatrix}$$

cost at node 7: $C^{\wedge}(R) + A[4][3] + r$

$= 25 + 12 + 13 = 50$

consider in path 1-4-5:

$A[4][1] = \alpha$; $A[5][1] = \alpha$

$$\begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ 12 & \alpha & 11 & \alpha & \alpha \\ 0 & 3 & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & 0 & 0 & \alpha & \alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ 1 & \alpha & 0 & \alpha & \alpha \\ 0 & 3 & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & 0 & 0 & \alpha & \alpha \end{bmatrix} \begin{matrix} 11 \\ \\ \\ \\ \\ \hline 11 \end{matrix}$$

Reduced cost matrix at node 8: $C^{\wedge}(R) + A[4][5] +$

$= 25 + 0 + 11 = 36$

$\therefore$ Now, node 6 generates its children node 9 & 10

consider path 1-4-2-3:

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$A[4][1] = \alpha$; $A[2][1] = \alpha$, $A[3][1] = \alpha$

$$\begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & 2 \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ 11 & \alpha & \alpha & \alpha & \alpha \end{bmatrix} \Rightarrow \begin{array}{c} \begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & 0 \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ 0 & \alpha & \alpha & \alpha & \alpha \end{bmatrix} \begin{matrix} \\ \\ 2 \\ \\ 11 \end{matrix} \\ \begin{matrix} 11 & & & & & \end{matrix} \end{array} \quad \overline{13}$$

cost at node 9 : $c^\wedge(R) + A[2][3] + r = 28 + 11 + 13 = 52$

consider path 1-4-2-5 :

$A[4][1] = \alpha$, $A[2][1] = \alpha$, $A[5][1] = \alpha$

$$\begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ 0 & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & 0 & \alpha & \alpha \end{bmatrix}$$

Cost at node 10 : $c^\wedge(R) + A[2][5] + r$

$= 28 + 0 + 0 = 28$

Now node 10 generates its children node 11

consider the path 1-4-2-3 :

$A[4][1] = \alpha$; $A[2][1] = \alpha$; $A[3][1] = \alpha$

$$\begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & 2 \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ 11 & \alpha & \alpha & \alpha & \alpha \end{bmatrix}$$

$$\Rightarrow \begin{array}{c} \begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & 0 \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ 0 & \alpha & \alpha & \alpha & \alpha \end{bmatrix} \begin{matrix} \\ \\ 2 \\ \\ 11 \end{matrix} \\ \begin{matrix} 11 & & & & & \end{matrix} \end{array} \quad \overline{13}$$

$\therefore$ cost at node 10 :

$c^\wedge(R) + A[2][5] + r$

$28 + 0 + 0 = 28$

Now node 10 generates it children node 11

consider the path 1-4-2-5-3 :

$A[4][1] = \alpha$, $A[2][1] = 8$, $A[5][1] = \alpha$, $A[3][1] = \alpha$

$\therefore$ Cost of node 11 : $c^\wedge(R) + A[5][3] + r$

$28 + 0 + 0 = 28$

$$\begin{bmatrix} \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \\ \alpha & \alpha & \alpha & \alpha & \alpha \end{bmatrix}$$

$\therefore$ The optimal tour is 1-4-2-5-3-1 with an optimal as of 28 procedure:-

1. The given matrix is reduced by reducing rows and columns of matrix. A row (column) is said to be reduced iff it contains atleast one zero and all remaining entries are non-negative.

For this, choose minimum entry in row i (column j) and subtract it from all entries in row i (column j).

2. The total amount subtracted from all the rows and columns is a lower bound on the length of a minimum cost tour and used as c^ value for root node of state space tree.

3. with everynode in the TSP state space tree, we may get a reduced cost matrix.

If A is the reduced cost matrix of node R, and if S is a child of R, such that (R,S) corresponds to including the edge (i,j) in the tower, and if S is not a leaf then the reduced cost matrix of S is computed as,

a) change all entries in row i & column j to ∞

b) set $A(j,1)$ to ∞

c) Reduced all rows & columns in the resulting matrix except for rows & columns containing only ∞. Let the resulting matrix B.

If γ is the total amount subtracted in step c then $c^\wedge(S) = c^\wedge(R) + A(i,j) + \gamma$

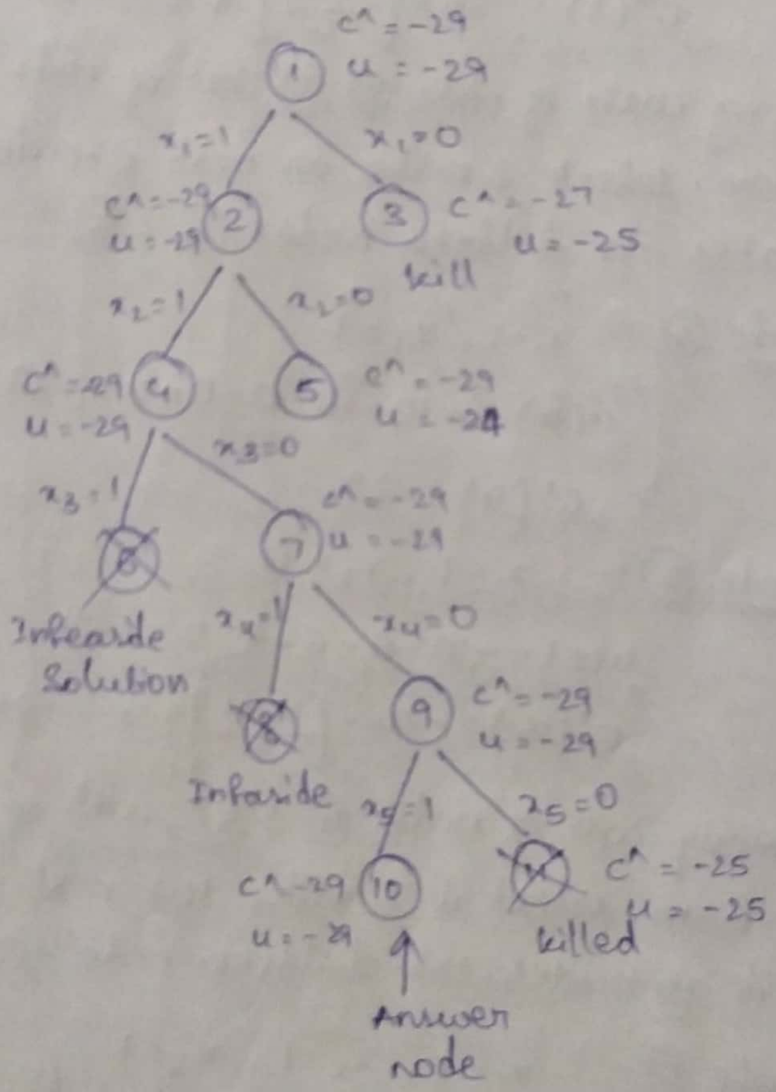
4. Initially assume $u = \infty$, after reaching final leaf node, kill the live nodes, $c^\wedge(x) \geq u$.

5. when the process reached to leaf node in the tour then it produces optimal tour.

3A) Given that

$n=5, (P_1, P_2 \cdots P_5) = (10,15,6,8,4)$ &

$(w_1, w_2, w_5) = (4,6,3,4,2)$ & $m=12$



Solution is $(x_1, x_2, x_3, x_4, x_5) = (1,1,0,0,1)$

At node ①: $u(x) = -10-15-0-0-4$

$= -29$

$\hat{c}(x) = -29$

$\hat{c}(1) = -29$

At node ② $x_1 = 1$

$u(2) = -10-15-0-0-4 = -29$

$c^\wedge(2) = -29$

At node (3) : $x_1 = 0$

$u(3) = -0-15-6-0-4 = -25$

$c^\wedge(3) = -25 - \left[\frac{12-4}{4}\right] * 8 = -27$

Among costs of node ②, ③ cost of node ② is minimum. select & make as next E-node, now it generates its children nodes ④ & ⑤

At node ④ :- $x_1 = 1$, $x_2 = 1$

$u(4) = -10-15-0-0-4 = -29$

$c^\wedge(4) = -29$

At node ⑤ :- $x_1 = 1$, $x_2 = 0$

$u(5) = -10-0-6-8-0 = -24$

$c^\wedge(15) = -24 - \left[\frac{12-11}{2}\right] * 4 = -26$

∴ Among cost of node ④ & ⑤, cost of node ④ is minimum. select it & make the next E-node. now it generates the children node ⑥ & ⑦

At node ⑥ :- $x_1 = 1$, $x_2 = 1$, $x_3 = 1$ (∴ 4+6+3 ≯ 1

$u(6) = -10-15-6-0-0 = -31$

At node ⑦ :- $x_1 = 1$, $x_2 = 1$, $x_3 = 0$

$u(7) = -10-15-0-0-4 = -29$

$c^\wedge(7) = -27 - \left[\frac{12-10}{7}\right] * 14 = -29$

Now, make node ⑦ as E-node. now it gener ~~ates~~ children node ⑧ & ⑨.

At node ⑧ :- $x_1=1, x_2=1, x_3=1, x_4=1$

$(\because 4+6+3+4 \nleq 12$

$u(8) = -10-15-0-8-0 = -33$

(Infeasible solution)

At node ⑨ :- $x_1=1, x_2=1, x_3=0, x_4=0$

$u(9) = -10-15-0-0-4 = -29$ $(\because 4+6+2 \leq 12)$

$c\hat{}(9) = -29$

$\therefore$ Now make node ⑨ as E-node. Now it generate the children node ⑩, ⑪

At node ⑩ :- $x_1=1, x_2=1, x_3=0, x_4=0, x_5=1$

$u(10) = -10-15-0-0-4 = -29$

$c\hat{}(10) = -29 - \left[\frac{12-12}{0}\right] \times 0 = -29$

At node ⑪ :- $x_1=1, x_2=1, x_3=0, x_4=0, x_5=0$

$u(11) = -10-15-0-0-0 = -25$

$c\hat{}(11) = -25 - \left[\frac{12-10}{0}\right] * 0 = -25$

From among costs of node ⑩ & ⑪, cost of node ⑩ is minimum

$\therefore$ optimal solution is $(x_1, x_2, x_3, x_4, x_5) = (1,1,0,0,1)$