

Solution: The given problem can be written as

(20)

$$\text{Maximization } Z = 5x_1 - 2x_2 + 3x_3$$

$$\text{Subject to } -2x_1 - 2x_2 + x_3 \leq -2$$

$$3x_1 - 4x_2 \leq 3$$

$$x_2 + 3x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0.$$

The associated dual is given by

$$\text{Minimize } W = -2y_1 + 3y_2 + 5y_3$$

$$\text{Subject to } -2y_1 + 3y_2 \geq 5$$

$$-2y_1 - 4y_2 + y_3 \geq -2$$

$$y_1 + 3y_3 \geq 3$$

$$y_1, y_2 \text{ and } y_3 \geq 0.$$

The solution of the dual by Simplex method consists of the following steps.

Step 1: Express the problem in standard form.

Multiplying the second constraint by  $-1$ , it can be written as

$$2y_1 + 4y_2 - y_3 \leq 2$$

Introducing the slack and surplus variables, we get an artificial system given by

$$\text{minimize } W = -2y_1 + 3y_2 + 5y_3 + 0s_1 + 0s_2 + 0s_3 + MA_1 + Mx_4$$

$$\text{Subject to } -2y_1 + 3y_2 - s_1 + A_1 = 5 \quad y_1 + 3y_3 - s_3 + A_2 = 3$$