29/08/2020.

Graphical method

Given $z = 3x+2y$

constraints are $2x+y \le 18$

$2x+3y \le 42$

$3x+y \le 24$

$y \ge 0$.

so Now to get optimal solution, do below steps.

$2x+y \le 18$

let $x=0$

$\therefore 2(0)+y \le 18$

$\therefore y = 18$

$(x,y) = (0,18)$

(ii) let $y=0$

$2x+y \le 18$

$2x+0 \le 18$

$x \le 9$

$(x,y) = (9,0)$

$2x+3y \le 42$

let $x=0$

$2(0)+3y \le 42$

$3y \le 42$

$y \le 14$

$\therefore (x,y)\ (0,14)$

(ii) let $y=0$

$2x+3(0) \le 42$

$x = 21$

$\therefore (x,y) = (21,0)$

$3x+y \le 24$

(i) let $x=0$

$3(0)+y \le 24$

$y \le 24$

$(x,y) = (0,24)$

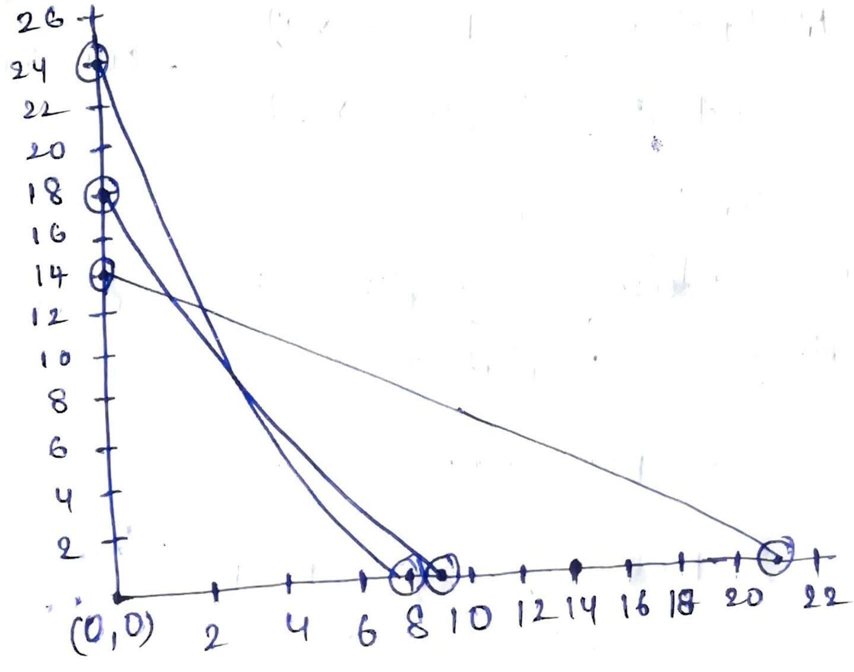
(ii) let $y=0$

$3x+0 \le 24$

$x = 8$

$\therefore (8,0) = (x,y)$

Now draw a Graph



Now we will get intersection of points and points at x and y axes.

so points are (0,14) (3,12) (6,6) (8,0)

Now optimal sol is

$Z = 3x+2y$

(i) $3(0)+2(14)$

$Z = 28$

(ii) (3,12)

$3(3)+2(12)$

$9+24$

$= 33$

(iii) $3(6)+2(6)$

$= 18+12$

$= 30$

(iv) (8,0)

$3(8)+2(0)$

$= 24$

∴ out of 33 is greatest

so $Z = 33$.

Note:- we use Graphical method, only when there are 2 decission variables.

Simplex Process

- This process is also called iterative method

Problem:-

max $Z = 12x_1 + 16x_2$ → objective function.

$$\left.\begin{array}{l} 10x_1 + 20x_2 \leq 120 \rightarrow (1) \\ 8x_1 + 8x_2 \leq 80 \rightarrow (2) \end{array}\right\} \text{- constraints.}$$

x_1 & $x_2 \geq 0$.

we are adding equality to constraints by adding 1 variable.

max $Z = 12x_1 + 16x_2 + 0S_1 + 0S_2$

$10x_1 + 20x_2 + S_1 = 120$ - (1)

$8x_1 + 8x_2 + S_2 = 80$ - (2)

$x_1, x_2, S_1, S_2 \geq 0$.

Simplex table

CBi	c_j / Basic variable	12 x_1	16 x_2	0 S_1	0 S_2	sol	Ratio min
0	S_1	10	20	1	0	120	120/20 = 6
0	S_2	8	8	0	1	80	80/8 = 10
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	12	16	0	0		

key row

key column

* $Z_j = \sum_{i=1}^{2} (CB_i)(a_{ij})$

* for maximisation all $C_j - Z_j \leq 0$
* for minimization all $C_j - Z_j \geq 0$.
* C_j = values of coefficients of objective function.

* Z_j is calculated as $\sum_{i=1}^{n} (C_{bi})(a_{ij})$

$\boxed{S_1, S_2 \text{ are slack variables}}$

* No as we need maximisation we take $C_j - Z_j \leq 0$.

* Now at $C_j - Z_j$ row takes highest column and take the column value as key column.

* Now in key column, the highest valued row is taken as key row. (wrong) ✗

* Intersection of key column & key row produce 1 element, that element is called key element

Note;- Cbi means coefficient of slack variable

CBi	c_j / Basic var	12 / x_1	12 / x_2	0 / S_1	0 / S_2	Sol	Ratio
0	S_1	10	20	1	0	120	$120/20 = 6$
0	S_2	8	8	0	1	80	$80/8 = 10$
	Z_j	0	0	0	0		
	$c_j - Z_j$	12	16	0	0		

→ key column

max

∴ we select key row as follows.
After you find key column, find ratio as sol/ key column elements

Now we get 2 different ratio's out of them take row, which as least ratio. and mark as key row.

Now intersection of key row & column gives element called key element.

$$\text{New value} = \text{old value} - \frac{\text{key col} \times \text{key row}}{\text{key element}}$$

$$= 8 - \frac{8 \times 10}{20}$$

$$= 8 - 80/20$$

$$= 8 - 4$$

New value = 4

key col = 8 becoz 20 is already key element

same is for key row

Iteration - I

we make iteration becoz, in last table we got $x_1 = 12$ & $x_2 = 16$ $C_j - Z_j \leq 0$ but they are > 0 so we need iteration.

CBi	Cj / Basic	12 / x_1	16 / x_2	0 / S_1	0 / S_2	Sol	Ratio
16	x_1	$10/20 = 1/2$	$\frac{20}{20} = 1$	$1/20$	$0/20 = 0$	6	$6/\frac{1}{2} = 12$
0	~~S_2~~	4	0	$-2/5$	1	32	$32/4 = 8$
	Z_j	8	16	$4/5$	0	96	
	$C_j - Z_j$	$\boxed{4}$	0	$-4/5$	0		

1) New value = old val - $\frac{\text{key col} \times \text{key row}}{\text{key element}}$

$$= 8 - \frac{8 \times 20}{20}$$

$$= 8 - 8$$

$$= 0.$$

2) N.V = $0 - \frac{8 \times 1}{20} = -2/5 = -2/5$

3) N.V = $1 - \frac{8 \times 0}{20} = 1 - 0 = 1$

4) N.V = $80 - \frac{8 \times \cancel{120}^{6}}{\cancel{20}} = 80 - 48 = 32$

Note:- we will fill 1 row in iteration table, based on previous table
In that table, key column, one variable is there. For that row, we will start

Big-M Method

we solve this, by adding artificial variables, a_1 & a_2

A_1, A_2 ---> artificial variables
S_1, S_2 ---> slack variable
M ---> High penality value.

minimum $z = x_1 + x_2$ - optimum function.

$x_1 + 2x_2 \geq 2$
$x_1 + 7x_2 \geq 7$ } These are inequality conditions
$x_1, x_2 \geq 0$.

To get equality we do as follows.

$x_1 + 2x_2 - S_1 + A_1 = 2$
$x_1 + 7x_2 - S_2 + A_2 = 7$
$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$.

$$Z = x_1 + x_2 + \underbrace{0S_1 + 0S_2}_{\text{+ve zero}} + \underbrace{MA_1 + MA_2}_{\text{max +ve value}}$$

Initial Table

CBi	cj	1	1	0	0	M	M	sol	Ratio
	Basic	x_1	x_2	S_1	S_2	A_1	A_2		
m	A_1	1	$\boxed{2}$	-1	0	1	0	2	2/2 = 1
m	A_2	1	7	0	-1	0	1	7	7/7 = 1
	zj	2m	9m	-m	-m	m	m	9m	
	cj-zj	1-2m	1-9m	+m	m	0	0		

optimal condition checking

$c_j - z_j \geq 0$.

A_1 - leaving var
x_2 - entering var

$1-2m$

Here 1st 2 terms are negitive.
out of those $1-9m$ is least

Iteration Table - I

CB_i	c_j	1	1	0	0	m	m	sol	Ratio
	Basic	x_1	x_2	S_1	S_2	A_1	A_2		
1	x_2	$1/2$	1	$-1/2$	0	$1/2$	0	1	$1/(-1/2)$
m	A_2	$-5/2$	0	$7/2$	-1	$-7/2$	1	0	$0/(7/2)$ = 0
	z_j	$1-5m/2$	1	$7m-1/2$	$-m$	$\frac{1-7m}{2}$	m	1	
	c_j-z_j	$1+5m/2$	0	$\frac{1-7m}{2}$	m	$\frac{9m-1}{2}$	0		

x_2 coeff. val

① $N.V = 1 - \frac{1 \times 7}{2} = 1 - 7/2 = -5/2$

② $N.V = 7 - \frac{2 \times 7}{2} = 7 - 7 = 0$

③ $N.V = 0 - \frac{(-1) \times 7}{2} = +7/2$

④ $N.V = -1 - \frac{(0 \times 7)}{2} = -1$

⑤ $N.V = 0 - \left(\frac{1 \times 7}{2}\right) = -7/2$

⑥ $N.V = 1 - (0 \times 7/2) = 1$

⑦ $N.V = 7 - (2 \times 7/2) = 0$.

optimal condition checking

all $c_j - z_j$ values ≥ 0.

Here 3 & 5 are negitive values

so Here 3 is highest negitive.

so consider it as key column.

If ratio is negitive we don't consider that row as key row. So consider next row as key row.

S_1 - entering.

Iteration Table - II

CBi	c_j	1	1	0	0	m	m	sol	Ratio
	Basic	x_1	x_2	S_1	S_2	A_1	A_2		
1	x_2	$1/7$	1	0	$-1/7$	0	$1/7$	1	
0	S_1	$-5/7$	0	1	$-2/7$	-1	$2/7$	0	
	z_j	$1/7$	1	0	$-1/7$	0	$1/7$	1	
	$c_j - z_j$	$6/7$	0	0	$1/7$	m	$m-1/7$		

optimal condition checking.

$c_j - z_j$, all values are ≥ 0. (✓)

$\therefore$ solution $z_j = 1$

CO-2

Duality in Linear programming.

- If problem is rewritten by exchanging rows & columns of a matrix problem, then it is called dual of the problem.
- original problem is also called primal problem.
- The optimal solution of dual problem gives complete information about .....
- Constraints means rows of a problem.
- variables means columns of a problem.
- By computing dual of the primal problem the solution is become easy.

Rules for Primal and Dual:

* If primal contains m variables and n constraints, then dual contains n variables and m constraints.

* If primal is a maximisation, dual will be minimisation and vice-versa.

In Primal problems.

* Maximisation problems must have ≤ constraints
  minimisation " " " ≥ constraints

Coefficient matrix of constraints of the dual is the transpose of the primal.

Constants of primal becomes the coefficients of dual

Constraints of primal becomes the coefficients of dual.

Variables in primal & dual are ≥ 0

1) construct the dual of the primal problems.

Maximise

$$2x_1 + 6x_2 \leq 50$$
$$3x_1 + 2x_2 \leq 35$$
$$5x_1 - 3x_2 \leq 10.$$
$$x_2 \leq 20.$$

where $x_1, x_2 \geq 0.$

a) Let y_1, y_2, y_3 & y_4 be the corresponding dual variables, then the dual problem is given by

minimisation

$$W = 50y_1 + 35y_2 + 10y_3 + 20y_4$$

subject to constraints.

$$2y_1 + 3y_2 + 5y_3 \geq 3$$
$$6y_1 + 2y_2 - 3y_3 + y_4 \geq 5$$

where $y_1, y_2, y_3, y_4 \geq 0$.

As a dual problem has lesser number of constraints, it requires lesser work & effort to solve it

Note:- Computational difficulty in LPP is mainly associated with no of constraints rather than no of variables.

Problem:- Construct the dual of the problem,

Minimise $z = 3x_1 - 2x_2 + 4x_3$ subject to the constraint

$$3x_1 + 5x_2 + 4x_3 \geq 7 \quad -(1)$$

$$6x_1 + x_2 + 3x_3 \geq 4 \quad -(2)$$

$$7x_1 - 2x_2 - x_3 \leq 10 \quad -(3)$$

$$x_1 - 2x_2 + 5x_3 \geq 3 \quad -(4)$$

$$4x_1 + 7x_2 - 2x_3 \geq 2 \quad -(5)$$

& $x_1, x_2, x_3 \geq 0$.

Sol:- The given problem is of minimisation.

* All constraints should be of type >=

* By multiplying 3rd constraint (equation) with -1 on both sides, we get

$$-7x_1 + 2x_2 + x_3 \geq -10 \quad -(6)$$

$\therefore$ dual of given problem will be

maximise $w = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$

subject to the constraints

$$3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3$$

$$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$$

$4y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4$

where y_1, y_2, y_3, y_4 & $y_5 \geq 0$.
and are called as dual variables.

Dual problem, when primal is in standard form.

problem:-

max $z = 3x_1 + 10x_2 + 2x_3$. subject to

$2x_1 + 3x_2 + 2x_3 \leq 7$

$3x_1 - 2x_2 + 4x_3 = 3$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

a) Since a given problem is of maximisation, all constraints should be of type $<=$

The equation $3x_1 - 2x_2 + 4x_3 = 3$ can be expressed as a pair of inequalities

$3x_1 - 2x_2 + 4x_3 \leq 3$. $\quad 3x_1 - 2x_2 + 4x_3 \geq 3$.

(or)

$3x_1 - 2x_2 + 4x_3 \leq 3$

$-3x_1 + 2x_2 - 4x_3 \leq -3$.

The primal problem becomes maximised

$Z = 3x_1 + 10x_2 + 2x_3$ subject to constraints

$2x_1 + 3x_2 + 2x_3 \leq 7$

$3x_1 - 2x_2 + 4x_3 \leq 3$.

$-3x_1 + 2x_2 - 4x_3 \leq -3$

$x_1, x_2, x_3 \geq 0$.

let y_1, y_2, y_2'', y be associative non-negitive dual variables. Then dual of primal problem is given by

minimise $w = 7y_1 + 3y_2' - 3y_2''$ subject to constraints.

$$2y_1 + 3y_2' - 3y_2'' \geq 3$$

$$3y_1 - 2y_2' + 2y_2'' \geq 10$$

$$2y_1 + 4y_2' - 4y_2'' \geq 2.$$

where $y_1, y_2', y_2'' \geq 0.$

substituting $y_2' - y_2'' = y_2$ where y_2 is unrestricted in sign, then dual problem becomes min $w = 7y_1 + 3y$ subject to constraints.

$$2y_1 + 3y_2 \geq 3$$

Assignment :-

construct dual of problem $\min z = x_2 + 3x_3$
subject to $2x_1 + x_2 \leq 3$.

$$x_1 + 2x_2 + 6x_3 \geq 5$$
$$-x_1 + x_2 + 2x_3 = 2$$
where $x_1, x_2, x_3 \geq 0$.

Solving LPP by using it's dual.

1) Solve the following LPP by using it's dual,

$\max \; z = 5x_1 - 2x_2 + 3x_2$ subject to

$$2x_1 + 2x_2 - x_3 \geq 2$$
$$3x_1 - 4x_2 \leq 3$$
$$x_2 + 3x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0.$$

a) The given problem can be written as

$\max \; z = 5x_1 - 2x_2 + 3x_3$ subject to constraints

$$-2x_1 - 2x_2 + x_3 \leq -2$$
$$3x_1 - 4x_2 \leq 3$$
$$x_2 + 3x_3 \leq 5,$$
$$x_1, x_2, x_3 \geq 0.$$

The associated dual of the primal problem above is given by

minimise $w = -2y_1 + 3y_2 + 5y_3$ subject to the constraint

$$-2y_1 + 3y_2 + \cancel{y} \geq 5$$
$$-2y_1 - 4y_2 + y_3 \geq -2$$
$$y_1 + 3y_3 \geq 3.$$
where $y_1, y_2, y_3 \geq 0$.

The solution of the dual by simplex method consists of the following steps.

Step 1:- Express the problem in standard form

note:- whenever you try to compute dual, the right hand side value should not be negitive.

So multiplying 2nd constraints with -1, then it can be written as.

$$2y_1 + 4y_2 - y_3 \leq 2$$

Introducing the slack and surplus variables, we get an artificial system given by.
minimise $w = -2y_1 + 3y_2 + 5y_3$ subject to constraints.

$$-2y_1 + 3y_2 - s_1 + A_1 = 5$$
$$2y_1 + 4y_2 - y_3 + s_2 = 2$$
$$y_1 + 3y_3 - s_3 + A_2 = 3$$

Note:- If you have $\geq$ in equation, then add $-s_1 + A_1$ If you have $\leq$ in equation, then we add s_2 (slack variable

Step-2:- Find the inital basic feasible solution.

put $y_1 = y_2 = y_3 = s_1 = s_3 = 0$.

$A_1 = 5$ $s_2 = 2$ $A_2 = 3$

which is not BFS

C_B	Basis	y_1	y_2	y_3	s_1	s_2	s_3	A_1	A_2
	c_j	-2	3	5	0	0	0	M	M
m	A_1	-2	3	0	-1	0	0	1	0
0	S_2								
m	A_2								

C_B	Basis	y_1	y_2	y_3	S_1	s_2	S_3	A_1	A_2	b	θ
	c_j	-2	3	5	0	0	0	m	m		
m	A_1	-2	3	0	-1	0	0	1	0	5	
0	s_2	2	4	-1	0	1	0	0	0	2	
m	A_2	1	0	3	0	0	-1	0	1	3	
	$z_j = \Sigma C_B{}_{ij}$	-m	3m	3m	-m	0	-m	m	m		
	$\bar{c}_j = c_j - z_j$	-2+m	3-3m	5-3m	m	0	m	0	0		

$y_2 \nearrow$

y_2 is incoming vector

Final simplex table

C_B	Basis	y_1	y_2	y_3	s_1	s_2	s_3	A_1	A_2	b
	c_j	-2	3	5	0	0	0	m	m	
0	S_3	-15	0	0	-4	-3	1	4	-1	11
3	y_2	-2/3	1	0	-1/3	0	0	1/3	0	5/3
5	y_3	-14/3	0	1	-4/3	-1	0	4/3	0	14/3
	z_j	-76/3	3	5	-23/3	-5	0	23/3	0	
	$\bar{c}_j = c_j - z_j$	70/3	0	0	23/3	5	0	-23/3+m	m	

optimal solution is given by $y_1 = 0$

$y_2 = 5/3$, $y_3 = 14/3$