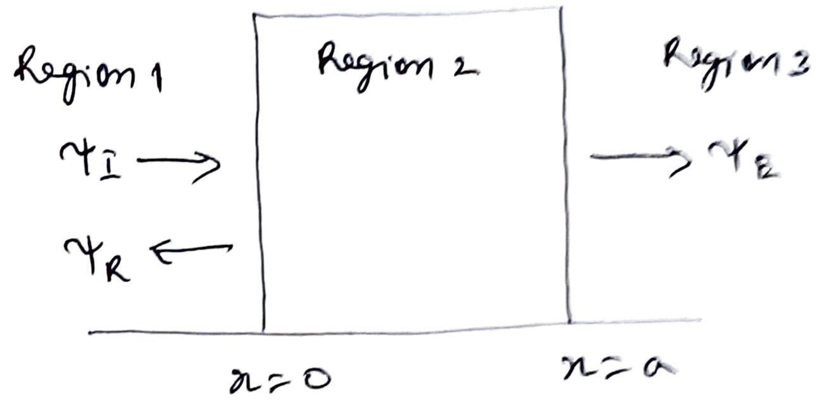
Potential Energy Barrier

The sharpest increase of Potential Energy (PE) to a certain value at a point and sharpest fall of the PE to zero at another point while remaining constant continuously over the interval ~~~~ constitute Potential Barrier.

Thus the Potential Barrier sharply increases to a certain value V_0, remains constant over a ~~~~ certain ~~~~ interval and then decreases again to its original value.



The Potential barrier is defined as

$$V(x) = 0 \quad \text{for } x < 0 \quad (\text{Region } 1)$$

$$V(x) = V_0 \quad \text{for } 0 \leq x \leq a \quad (\text{Region } 2)$$

$$V(x) = 0 \quad \text{for } x > 0 \quad (\text{Region } 3)$$

According to classical physics, if a particle is incident on potential barrier with energy less than the height of the barrier, it will be reflected back by the barrier.

But according to quantum physics, due to the wave nature of matter, there is some probability that the particle incident on the barrier with less energy than the barrier height, penetrates out ~~of~~ to the other side of the barrier. This effect is called tunnel effect.

It a particle is incident on the potential barrier with energy less than the height of the barrier ($E < V_0$), it will not necessarily be reflected by the ~~a~~ barrier but there is always the probability ~~of crossing the barrier~~ that it may cross the barrier and continue its forward motion. This probability of crossing the barrier is called tunnel effect. The tunnel effect is purely quantum mechanicall effect.

For region 1 $x < 0$

Schrodinger time independent wave equation

$$\frac{d^2\psi_1}{dx^2} + \frac{2mE}{\hbar^2}\psi_1 = 0 \quad \text{for } x < 0$$

$$\frac{d^2\psi_1}{dx^2} + k_1^2\psi_1 = 0$$

$$k_1 = \frac{\sqrt{2mE}}{\hbar} = \frac{2\pi}{\lambda}$$

The solution for the above second order differential equation is

$$\psi_1 = A e^{+ik_1x} + B e^{-ik_1x} \quad \text{for } x<0$$

A and B are integration constants. In this solution When $A e^{ik_1x}$ when multiplied by $e^{-i\omega t}$ represents a plane wave going towards the right and the function $B e^{-ik_1x}$ when multiplied by $e^{-i\omega t}$ represents a plane wave going towards the left. so, ψ_1 is a combination of plane waves

For Region 2 $\quad 0 \leq x \leq a$

$V(x) = V_0$ and $\psi(x) = \psi_2$

Schrodinger time independent wave equation

$$\frac{d^2\psi_2}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi_2 = 0 \quad 0 \leq x \leq a$$

$$\frac{d^2\psi_2}{dx^2} - \frac{2m(V_0-E)}{\hbar^2}\psi_2 = 0$$

$$\frac{d^2\psi_2}{dx^2} - k_2^2\psi_2 = 0$$

$$k_2 = \frac{\sqrt{2m(V_0-E)}}{\hbar}$$

The solution of the above second order diff eq is

$$\psi_2 = C e^{k_2x} + D e^{-k_2x} \quad \text{for } 0 \leq x \leq a$$

where C and D are constants. In this solution, the function $Ce^{K_2 x}$ when multplied by the factor $e^{-i\omega t}$ represents a plane wave going towards the right and the function $De^{-K_2 x}$ when multplied by the factor $e^{-i\omega t}$ represents a plane wave going towards left.

In region 3 $V = 0$ and $\psi(x) = \psi_3$

Schrodinger time independent wave equation

$$\frac{d^2\psi_3}{dx^2} + \frac{2mE}{\hbar^2}\psi_3 = 0 \qquad x > 0$$

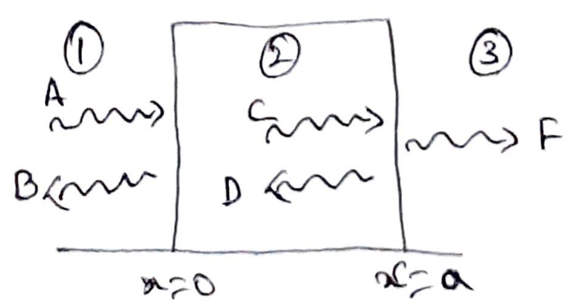
$$\frac{d^2\psi_3}{dx^2} + K_1^2\psi_3 = 0$$

$$K_1 = \frac{\sqrt{2mE}}{\hbar}$$

$$\psi_3 = Fe^{iK_1 x} + Ge^{-iK_1 x} \qquad x > 0$$

where F and G are constants. In this function $Fe^{iK_1 x}$ when multplied by the factor $e^{-i\omega t}$ represents a plane wave going towards the right and the function $Ge^{-iK_1 x}$ when multplied by the factor $e^{-i\omega t}$ represents a plane wave going towards left. However in region 3, there cannot be a reflected wave to move towards the left. Hence $G = 0$

$$\therefore \quad \psi_3 = Fe^{iK_1 x} \qquad x > 0$$



The Probability of the Particles being Reflected

$$R = \frac{|\psi_1 (\text{reflected})|^2}{|\psi_1 (\text{incident})|^2} = \frac{B^* B}{A^* A}$$

The Probability of the Particles being Transmitted

$$T = \frac{|\psi_3 (\text{transmitted})|^2}{|\psi_1 (\text{incident})|^2} = \frac{F^* F}{A^* A}$$