Relativity

Reference Frame :- It is a space in which we are making observation and measuring physical quantities. There are two types of reference frames.

Inertial Refrence Frame :-

It is a reference frame in which Newton's first law of motion holds good. That is an object at rest remains at rest and ~~and~~ an object in motion remains in ~~~~ motion, unless acted upon by a net force. A inertial reference frame is either at rest or moves with constant velocity.

Non-~~inter~~inertial reference frame :-

It is a reference frame that is accerlating, either in a linear fashion or rotating around some axis.

Label the types of refrence frame

1) Train moving with Constant velocity (Inertial)

2) A rotating merry-go-round (Non Intertial)

3) A turning car moving with Constant speed (Non Inertial)

4) The rotating Earth. (Non intertial)

Principle of Relativity :-

states that the basic laws of Physics are same in all inertial reference frames. That is as you go from one reference frame to another, things like forces, mass, length and time does not change. These quantities are said to be absolute.

Since the laws of mechanics do not change for different inertial refrence frames, no one inertial refronce frame is special in any sence.
Therefore we conclude that
" All inertial refrence frames are equivalent".

Einstein Postulates

1) All laws of Physics including maxwell's theory or electromagnetism, hold in all inertial reference frames. This is an extension of Principle of relativity.

2) Light can move through empty space and without the presence of a medium. Infact, light travells through empty space with a constant velocity 'c' that is independent of the speed of observer or source of light.

Considerations   Galileo Transformation Equation

S Frame of refrn → stationary (Rest)

S' '' '' → Non station (moving along X axis)

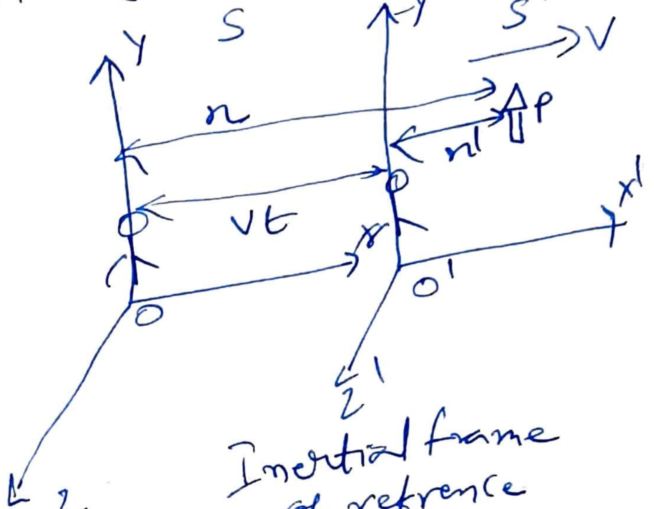
V → Constant velocity

O & O' → observer

(P) event obs by O and O'

x → distance from obser to P

Same clock



Inertial frame of refrence

from fig for S' frame for O' observer

$x' = x - vt$

$y' = y$

$z' = z$

$\left.\begin{matrix} y' = y \\ z' = z \end{matrix}\right\}$ No motion along y and z axis

$\boxed{t' = t}$

Galileam Transformation equation

from fig for S ~~frame for O observer~~

$x = x' + vt'$

$y = y'$

$z = z'$

$\left.\begin{matrix} y = y' \\ z = z' \end{matrix}\right\}$ No ~~variation~~ along y and z axis

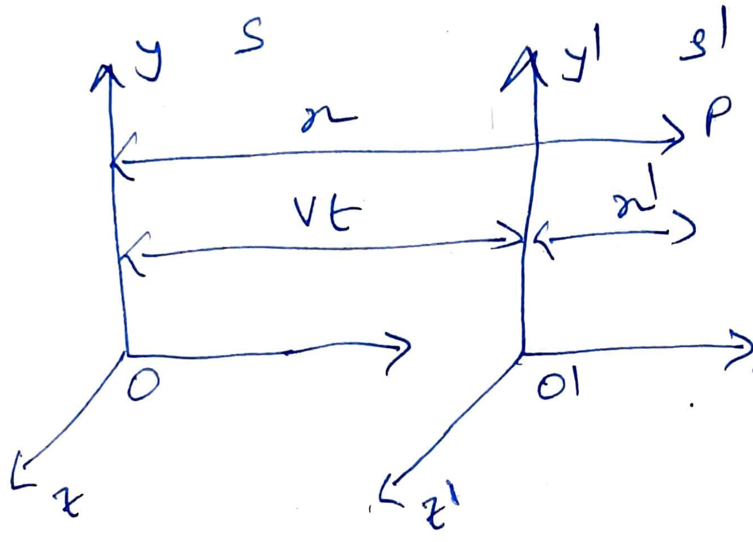
$t = t'$

~~Gatt~~ Galileam inverse Transformation eqatn

(1)

Lorentz Transformation

According to Lorentz Transformation equation the measurement in x direction made in frame 'S' must be linearly proportional to that made in S' frame. Hence a constant K should be there so,



Considerations

$S \to$ rest frame

$S' \to$ moving frame with velocity v along x axis

O & $O' \to$ Two observers S & S' fram of ref

~~$x' = x - vt$~~

$$x' = K(x - vt) \quad \text{— (1)}$$

$$x = K(x' + vt') \quad \text{— (2)}$$

Using Einstein I Postulate

Principle of equivalence: All Physic laws are valid and same for all inertial frame of refrence

So put value of x' in eq (2)

$$x = K[K(x - vt) + vt']$$

$$\frac{x}{K} = Kx - Kvt + vt'$$

$$vt' = \frac{x}{K} - Kx + Kvt$$

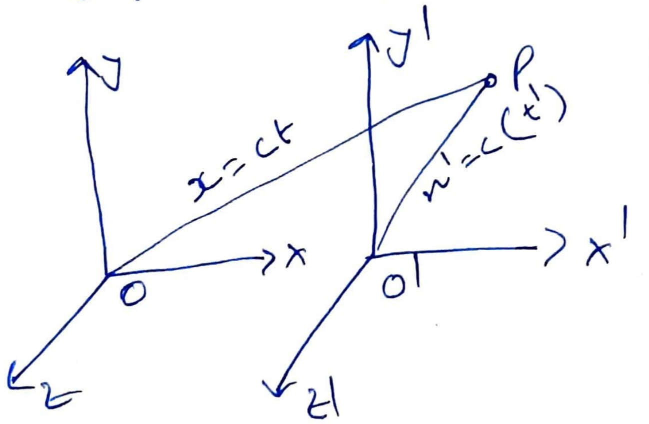
$$t' = \frac{x}{Kv} - \frac{Kx}{v} + \frac{Kt\cancel{v}}{\cancel{v}}$$

$$t' = \frac{x}{Kv} - \frac{Kx}{v} + Kt$$

$$\boxed{t' = Kt - \frac{Kx}{v}\left(1 - \frac{1}{K^2}\right)} \quad -(3)$$

<u>Using Einstein II Postulate</u> (for finding K)

<u>Constancy of speed of light:</u>



laser ligt from O to P

When two high light beam flashes according to fig.

$x = ct$ & $x' = ct'$

Put these values in eq (1) and (2)

from eq (1)

$$ct' = K(ct - vt) \quad -(5)$$

from eq (2)

$$ct = K(ct' + vt') \quad -(6)$$

(3) To find the value of constant K multiply eq (4) and (5)

$$c^2 tt' = K(ct - vt) \cdot K(ct' + vt')$$

$$= (Kct - Kvt)(Kct' + Kvt')$$

$$c^{\cancel{2}}\cancel{tt'} = K^2 \cancel{t}(c-v)\,\cancel{t'}(c+v)$$

$$c^2 = K^2 (c-v)(c+v)$$

$$K^2 = \frac{c^2}{(c-v)(c+v)}$$

$$= \frac{c^2}{c^2 - v^2} = \frac{\cancel{c^2}}{\cancel{c^2}(1 - v^2/c^2)}$$

$$K^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\frac{1}{K^2} = 1 - \frac{v^2}{c^2} \quad \text{--- (7)}$$

$$\boxed{K = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}}} \quad \text{--- (6)}$$

Put values from (6) and (7) into (1) and (3)

$$\boxed{x' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}(x - vt) = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad (8)$$

$$t' = K\left[t - \frac{x}{v}\left(1 - \frac{1}{K^2}\right)\right]$$

$$t' = \frac{t - \frac{x}{v}\left(\cancel{\gamma} - \cancel{\gamma} + \frac{v^2}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{t - \frac{x}{\cancel{v}}\left(\frac{v^{\cancel{2}}}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}, \quad x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

As there is no motion along y and z axis

hence
$$y' = y$$
$$z' = z$$

Inverse Lorentz Transformation

$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{x'v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Time Dilation

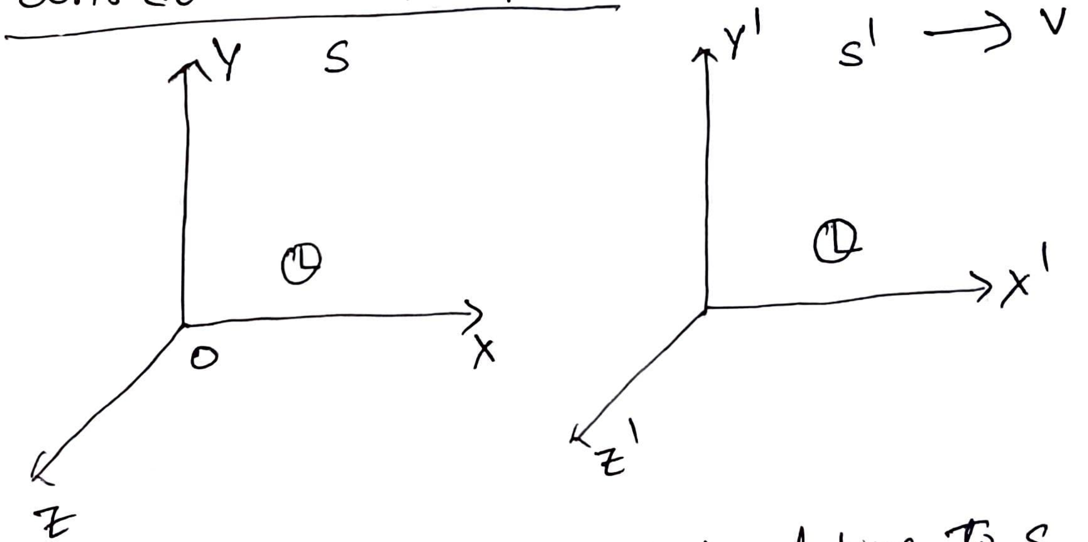
Time Dilation :- A clock in a moving frame of refrence measures a longer time interval between two events while for the same event the clock in the stationary frames measures short time interval.

$$\boxed{t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{t_0}{\sqrt{1-\beta^2}} = \gamma t_0$$

$$\beta = \frac{v^2}{c^2} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\boxed{t = \gamma t_0}$$

Consider an example



S' is moving with velocity 'v' relative to S.

The Person in S' observes that the time difference between two events is 10 sec. The Same two events when observed by a Person from S frame feels that the time difference between two events is 15 sec. The difference in time is called Time dilation.

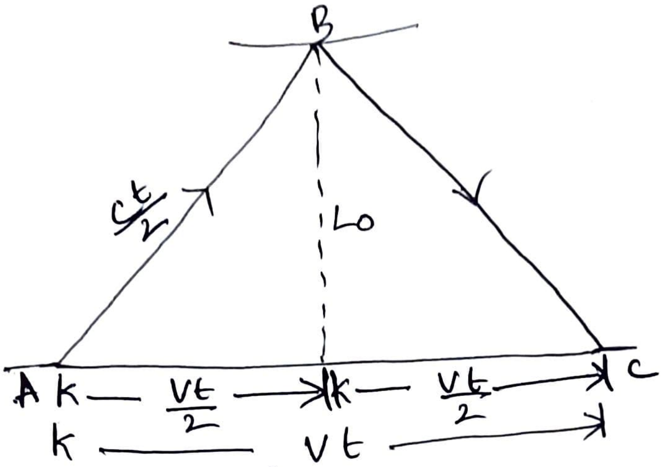


fig 2

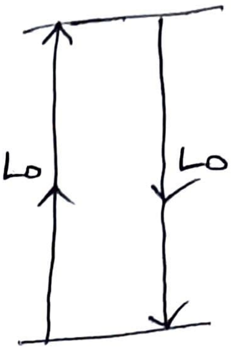


fig 1

Derivation :- Consider that a person in frame s' throws ball up to a distance L_0 then the time taken to reach the point of throw is as shown in fig 1

total distance travalled is $L_0 + L_0 = 2L_0$

velocity $= c$

$\therefore$ for s' frame

$$t_0 = \frac{2L_0}{c} \quad \text{(proper time)}$$

Event observed from S frame

Now the same event when observed from the frame S the person feels that the distance travelled by the ball to go up and down is time 't' as shown in fig 2

The person feels that the ball has travelled from A to B and from B to C. The total time taken as 't'.

so the ~~time~~ distance for ~~going from~~ for A to B is $\frac{ct}{2}$

Apply Pythagoras theorem in ~~eq~~ fig ②

$$\left(\frac{ct}{2}\right)^2 = \left(\frac{vt}{2}\right)^2 + L_0^2$$

$$\frac{c^2t^2}{4} = \frac{v^2t^2}{4} + L_0^2$$

$$\frac{c^2t^2}{4} - \frac{v^2t^2}{4} = L_0^2$$

$$\frac{t^2}{4}(c^2 - v^2) = L_0^2$$

$$t^2 = \frac{4L_0^2}{c^2 - v^2} = \frac{4L_0^2}{c^2\left(1 - \frac{v^2}{c^2}\right)}$$

$$t^2 = \frac{4L_0^2}{c^2\left(1 - \frac{v^2}{c^2}\right)}$$

$$t = \frac{2L_0}{c\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t_0}{\sqrt{1 - \beta^2}} = \gamma t_0$$

$$\beta = \frac{v^2}{c^2} \quad ; \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\boxed{t = \gamma t_0}$$

$V \ll c$ so $\beta < 1$ then so $\gamma > 1$

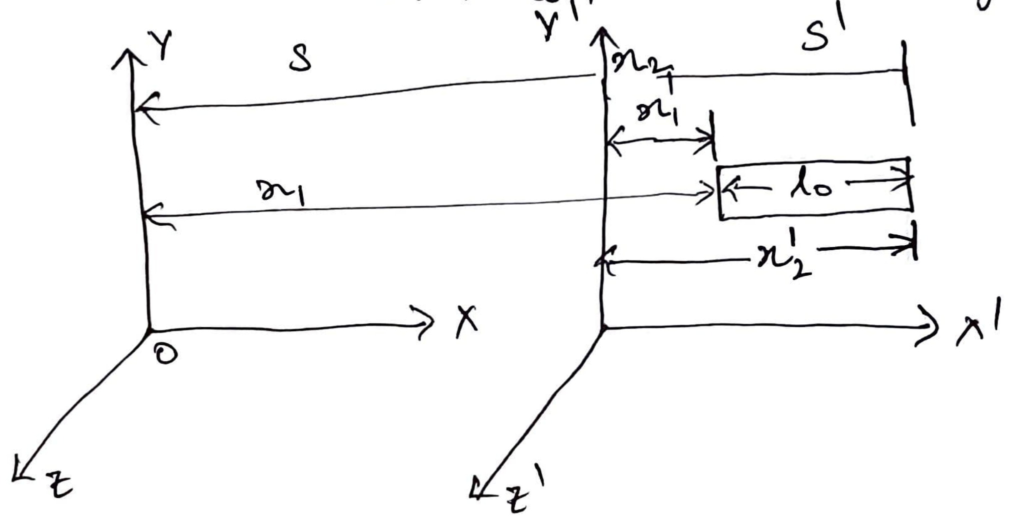
then $t > t_0$

"Speed of the light is same"

Length contraction

Proper Length :- The length of any object measured by an observer at rest wrt object is called proper length.

Length contraction :- The length of any object moving with high velocity (approachable to 'c') ~~relative~~ relative to an observer is measured contracted in the direction of motion while no change in other directions ⊥ or to motion. This phenomenon is called length contraction



S → rest ref. frame

S' → moving ref. frame with velocity 'v' along x

L_0 → Proper length

L → length contraction

In the frame S'

$$L_0 = x_2' - x_1' \quad \text{(Proper length)}$$

From the frame S

$$L = x_2 - x_1 \quad \text{(contracted length)}$$

From Lorentz Transformation

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_0 = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - v^2/c^2}} (x_2 - \cancel{vt} - x_1 + \cancel{vt})$$

$$L_0 = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{L = L_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

Condition 1) $v <<< c \Rightarrow \frac{v}{c} <<< 1$

$$L = L_0 \sqrt{1 - 0}$$

$\boxed{L = L_0}$ No Contraction

Condition 2 : $V = c$

$$L = L_0\sqrt{1-\frac{c^2}{c^2}}$$

$\boxed{L = 0}$ Not valid

Condition 3: v is near c

$$v \approx c$$

$$L = L_0\sqrt{1-0.\ldots}$$

$$L < L_0$$

Rest Motion