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2.

Schrodinger Wave Equation

Schrodinger wave eqn in its time-dependent form for a particle of energy E moving in a potential V in one dimension is

$$i\hbar \frac{\partial\psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2\psi(x,t)}{\partial x^2} + V(x,t)$$

The general form of the wave fuction is

$$\psi(x,t) = Ae^{i(kx-\omega t)} = A[\cos(kx-\omega t) + i\sin(kx-\omega t)]$$

Time-Independent Schrodinger Wave Equation

The potential in many cases will not depend explicity on time.

The dependence on time and position can then be seperated in the schrodinger wave equation

let $\psi(x,t) = \psi(x) f(t)$

which yields

$$i\hbar\psi(x) \frac{\partial f(t)}{\partial t} = \frac{-\hbar^2}{2m} f(t) \frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) f(t)$$

$$i\hbar \frac{1}{f(t)} \frac{\partial f(t)}{\partial t} = \frac{-\hbar^2}{2m} \frac{1}{\psi(x)} \frac{\partial^2\psi(x)}{\partial x^2} + V(x)$$

The left side of equation depends only on time, and the right side depends only on spatial coordinates. Hence each size must be equal to constant. The time dependent side is

$$i\hbar \frac{1}{f} \frac{\partial f}{\partial t} = B$$

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$$i\hbar \int \frac{df}{f} = \int B\,dt$$

$$i\hbar \ln f = Bt + C$$

where C is integration constant

$$\ln f = \frac{Bt}{i\hbar}$$

$$f = e^{Bt/i\hbar}$$

$$f = e^{-Bti/\hbar}$$

$$i\hbar \frac{1}{f(t)} \frac{df(t)}{dt} = E$$

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

This is known as Time independent Schrondinger wave equation

3. Time Independent Schrodinger Wave Equation

The time independent schrodinger wave eqn for one dimension is of the form

$$\frac{-h^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

where $U(x)$ is potential energy and E represents the system energy. It has a no. of important physical applications in quantum mechanics.

Free particle Wave Function

For a free particle the time dependent Schrodinger equation takes the form

$$\frac{-h^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

and given dependence upon both time & position.

$$\psi = Ae^{ax}e^{bt}$$

presuming that the wave function represents a state of definite Energy E, the equation can be seperated by requirement

$$\frac{-h^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} = E\psi = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

$$\frac{-h^2}{2m} a^2 \psi = E\psi$$

$$E\psi = i\hbar b \psi$$

$$a^2 = \frac{-2mE}{h^2}$$

$$b = \frac{-iE}{h}$$

$$a = i\sqrt{\frac{2mE}{h^2}}$$

$$\boxed{E = h\omega}$$

$$b = -i\omega.$$

Treating the system as a particle where

$$E = \frac{1}{2}mv^2 = \frac{P^2}{2m}$$

$$a = \frac{iP}{h}$$

Using De Broglie and wave relationship

$$a = i\frac{h}{h\lambda} = i\frac{2\pi}{\lambda}$$

This gives a plane wave solution

$$\psi(x,t) = Ae^{i\frac{2\pi}{\lambda} - \omega t}$$

$$= Ae^{i(kx - \omega t)}$$

which as a complex function can be expanded in the form

$$\psi(x,t) = A\left[\cos(kx-\omega t) + i\sin(kx-\omega t)\right]$$

The free particle wave function is associated with a precisely known momentum

$$p = \frac{h}{\lambda} = \frac{hk}{2\pi} = \hbar k$$