Graphical method

max = $3x+2y$ - objective function

subject to constaints

$2x+y \le 18$ — ①

$2x+3y \le 42$ — ②

$3x+y \le 24$ — ③

} in-equality constraints

$y \ge 0$

x, y these variables are called decision variables

Inequality constrains convert into equality constrains

Sol:- Let $2x+y = 18$ — ①

if $x=0$, $y=18$

$y=0$, $x=9$

x	0	9
y	18	0

$2x+3y=42$

if $x=0$, $3y=42$, $y=14$

$y=0$, $2x=42$, $x=21$

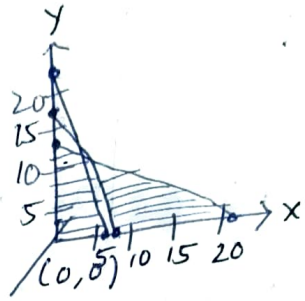
x	0	21
y	14	0

$3x+y=24$

$x=0$, $y=24$

$y=0$, $x=8$

x	0	8
y	24	0



Regional Space

① ② ③ ④ ⑤

O-A-B-C-D-O

At $Z(0) = (0,0)$

A = (8,0), B = (6,6) C = (3,12) D = (0,14)

① $Z(0,0) = 3(0)+2(0) = 0$

② $Z(8,0) = 3(8)+2(0) = 24$

③ $Z(6,6) = 3(6)+2(6) = 30$

④ $Z(3,12) = 3(3)+2(12) = 33$

⑤ $Z(0,14) = 3(0)+2(14) = 28$

$x=3$, $y=12$

Simplex method (Iterative method)

max $Z = 12x_1 + 16x_2$ — objective function.

$10x_1 + 20x_2 \le 120$ — ①

$8x_1 + 8x_2 \le 80$ — ②

} Inequality constraints

$x_1 \& x_2 \ge 0$

slack variables

$S1, S2$

Sol:- $10x_1 + 20x_2 + S1 = 120$ — (1)
$8x_1 + 8x_2 + S2 = 80$ — (2) } equality constraints

$x_1, x_2, S1, S2 \geq 0$

max $Z = 12x1 + 16x2 + 0S1 + 0S2$ — objective function

Simplex table

C_{Bi}	C_j	12	16	0	0	Sol	Ratio
	Basic variables	x_1	x_2	S_1	S_2		
0	S_1	10	20	1	0	120	$\frac{120}{20}=6$ → minimum
0	S_2	8	8	0	1	80	$\frac{80}{8}=10$
	Z_j	0	0	0	0	0	
	$Z_j = \sum_{j=1}^{2} (C_{Bi})(a_{ij})$						
	$C_j - Z_j$	12	16	0	0		

key column, key row, key element = 20

$Z_j = \sum_{i=1}^{2} (C_{Bi})(a_{ij})$

Maximum value

Optimality condition check

for maximization all $C_j - Z_j \leq 0$

for minimization all $C_j - Z_j \geq 0$

N.V = Old value − $\frac{\text{K-col} \times \text{K-row}}{\text{K-elem}}$

Iteration-I

C_{Bi}	C_j	12	16	0	0	Sol	Ratio
	Basic	x_1	x_2	S_1	S_2		
16	x_2	$\frac{10}{20}=\frac{1}{2}$	$\frac{20}{20}=1$	$\frac{1}{20}$	0	6	$\frac{6}{1/2}=12$
0	S_2	4	0	$-\frac{4}{5}$	1	32	$\frac{32}{4}=8$
	Z_j	8	16	$\frac{4}{5}$	0	96	
	$C_j - Z_j$	4	0	$-\frac{4}{5}$	0		

key element = 4

① $8 - \frac{8 \times 10}{20} = 8 - 4 = 4$

② $8 - \frac{8 \times 20}{20} = 8 - 8 = 0$

③ $0 - \frac{8 \times 1}{20} = 0 - \frac{2}{5} = \frac{2}{5}$

④ $1 - \frac{8 \times 0}{20} = 1 - 0 = 1$

⑤ $80 - \frac{8 \times 120}{20} = 80 - 48 = 32$

Iteration-II

C_{Bi}	C_j	12	16	0	0	Sol
	Basic var	x_1	x_2	S_1	S_2	
16	x_2	0	1	$\frac{1}{10}$	0	2
12	x_1	1	0	$-\frac{1}{10}$	$\frac{1}{4}$	8
	Z_j	12/5	16/5	1	3	128
	$C_j - Z_j$	0	0	$-\frac{2}{5}$	1	

① $\frac{1}{2} - \frac{\frac{1}{2} \times 4}{4} = 0$ ② $1 - \frac{\frac{1}{2} \times 0}{4} = 1$ ③ $\frac{1}{20} - \frac{\frac{1}{2} \times 2}{4} =$ ④ $0 - \frac{\frac{1}{2} \times 1}{4} = -\frac{1}{8}$ ⑤ $6 - \frac{\frac{1}{2} \times 32}{4} = 2$

optimality $z = 128$

where $x_2 = 8, x_1 = 8$

Big-M Method:-

A_1, A_2 --- artificial variables

S_1, S_2 --- slack "

M - High penality value.

minimum $z = x_1 + x_2$

$x_1 + 2x_2 \geq 2$ — (1)

$x_1 + 7x_2 \geq 7$ — (2)

$x_1, x_2 \geq 0$

Sol:- $x1 + 2x2 - S1 + A1 = 2$ — (1)

$x_1 + 7x_2 - S_2 + A_2 = 7$ — (2)

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

$z = x1 + x_2 + 0S1, 0S2 + MA_1 + MA_2$

+ve zero maximum +ve value

Initial table:-

	C_j	1	1	0	0	M	M	Solution	Ratio
C_{Bi}	Basic	x_1	x_2	S_1	S_2	A_1	A_2		
M	A_1	1	2	-1	0	1	0	2	2/2=1
M	A_2	1	7	0	-1	0	1	7	7/7=1
	Z_j	2M	9M	-M	-M	M	M	9M	
	$C_j - Z_j$	1-2M	1-9M	M	M	0	0		

optimality condition - checking

max $C_j - Z_j \leq 0$

min $C_j - Z_j \geq 0$

Iteration table-I

C_{Bi}	C_j	1	1	0	0	M	M	Sol	ratio
	Basic	x_1	x_2	S_1	S_2	A_1	A_2		
1	x_2	1/2	1	-1/2	0	1/2	0	1	1/-1/2 = -2
M	A_2	-5/2	0	7/2	-1	-7/2	1	0	0/7/2 = 0
	Z_j	-5M/2	1	7M/2 - 1	-M	1 - 7M/2	M	1	
	$C_j - Z_j$	1+5M/2	0	1-7M/2	M	9M/2 - 1	0		

(1) N.V = old value - $\frac{\text{k-col} \times \text{k-row}}{\text{k-element}}$

(1) $1 - \frac{7 \times 1}{2} = \frac{-5}{2}$ (2) $7 - \frac{7 \times 2}{2} = 0$ (3) $0 - \frac{(-1) \times 7}{2} = \frac{7}{2}$

(4) $-1 - \frac{(7) \times 0}{2} = -1$ (5) $0 - \frac{7 \times 1}{2} = \frac{-7}{2}$ (6) $1 - \frac{7 \times 0}{2} = 1$

(7) $7 - \frac{7 \times 2}{2} = 0$

I. T-II

C_{Bi}	C_j	1	1	0	0	M	M	Sol	ratio
	Basic	x_1	x_2	S_1	S_2	AA_1	A_2		
1	x_2	1/7	1	0	-1/7	0	1/7	1	
0	S_1	-5/7	0	1	-2/7	-1	2/7	0	
	Z_j	1/7	1	0	-1/7	0	1/7	1	
	$C_j - Z_j$	6/7	0	0	1/7	M	M-1/7		

≥ 0

(1) $\frac{7}{2} / -5/2 = \frac{7}{2} \times \frac{-2}{5} = -\frac{7}{5}$

(1) $\frac{1}{2} - \frac{-1/2 \times -5}{7/2} = \frac{1}{2} - \frac{5}{14} \times \frac{2}{7} = \frac{1}{2} - \frac{5}{14} = \frac{7-5}{14} = \frac{2}{14} = \frac{1}{7}$

(2) $1 - \frac{-1/2 \times 0}{7/2} = 1$ (3) $\frac{-1}{2} - \frac{-1}{2} \times \frac{7}{2} = 0$ (4) $0 - \frac{-1/2 \times -1}{7/2} = \frac{-1}{7}$

(5) $\frac{1}{7} = \frac{1}{2} - \frac{-1/2 \times -7/2}{7/2} = \frac{1}{2} 0$ (6) $0 - \frac{-1/2 \times 1}{7/2} = \frac{1}{7}$

$Z = 1$, $x_2 = 1$, $x_1 = 0$

$Z = x_1 + x_2 \Rightarrow Z = 0 + 1 = 1$, $Z = 1$