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Tutorial-12

1. Interior point method:

To apply karmakar's interior point method the LP should be expressed in the following standard form

Minimize $z = c^T x$

Subject To: $Ax = 0$

$Ix = 1$

with: $x \geq 0$

where $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$ $I = [1 \ 1 \ 1 \ \dots \ 1]_{1 \times n}$

$A = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$ and $n \geq 2$

assumed that $x_0 = \begin{bmatrix} 1/n \\ 1/n \\ \vdots \\ 1/n \end{bmatrix}$ is a feasible solution

and $z_{min} = 0$

$r = \frac{1}{\sqrt{n(n-1)}}$ and $\alpha = \frac{(n-1)}{3n}$

In general k^{th} iteration involves

Step - 1

compute $c_p = [I - P^T (PP^T)^{-1} P]$

where $P = \begin{pmatrix} AD_k \\ 1 \end{pmatrix}$, $\bar{c} = c^T D_k$ and

$$D_k = \begin{bmatrix} x_k(1) & 0 & 0 & \cdots & 0 \\ 0 & x_k(2) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & \cdots & & x_k(n) \end{bmatrix}$$

if $c_p = 0$, any feasible solution becomes an optimal solution. Further iteration is not required otherwise compute the following steps

Step - 2 $\quad Y_{New} = X_0 - \alpha r \dfrac{c_p}{\|c_p\|}$

Step - 3 $\quad X_{k+1} = \dfrac{D_k Y_{new}}{I D_k Y_{new}}$ However it can be shown for $k = 0$, $\dfrac{D_k Y_{new}}{I D_k Y_{new}} = Y_{new}$.

Thus $x_1 = Y_{new}$

Step - 4 $\quad Z = c^T x_{k+1}$

Step - 5 $\quad$ Repeat steps 1 through 4 by changing k as $k+1$

Minimize $z = 2x_2 - x_3$

Subject To : $x_1 - 2x_2 + x_3 = 0$

$x_1 + x_2 + x_3 = 1$

$x_1, x_2, x_3 \geq 0$

sol :- Thus , $n = 3$ $c = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$ $A = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$

$$x_0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} \quad r = \frac{1}{\sqrt{(n-1)n}} = \frac{1}{\sqrt{3(2)}} = \frac{1}{\sqrt{6}}$$

$$\alpha = \frac{(n-1)}{3n} = \frac{2}{9}$$

Iteration 0 $(k=0)$:-

$$D_0 = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$\bar{c} = c^T D_0 = \begin{bmatrix} 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2/3 & -1/3 \end{bmatrix}$$

$$AD_0 = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 & -2/3 & 1/3 \end{bmatrix}$$

$$P = \begin{pmatrix} AD_0 \\ 1 \end{pmatrix} = \begin{bmatrix} 1/3 & -2/3 & 1/3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$PP^T = \begin{bmatrix} 1/3 & -2/3 & 1/3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 1 \\ -2/3 & 1 \\ 1/3 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$(PP^T)^{-1} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1/3 \end{bmatrix}$$

$$P^T (PP^T)^{-1} P = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

$$c_p = \left[I - P^T (PP^T)^{-1} P \right] (\bar{c})^T$$

$$= \begin{bmatrix} 1/6 \\ 0 \\ -1/6 \end{bmatrix}$$

$$\|c_p\| = \sqrt{(1/6)^2 + 0 + (-1/6)^2} = \frac{\sqrt{2}}{6}$$

$$Y_{new} = X_0 - \alpha r \frac{c_p}{\|c_p\|} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} - \frac{\frac{2}{9} \times \frac{1}{\sqrt{6}}}{\frac{\sqrt{2}}{6}}$$

$$= \begin{bmatrix} 0.2692 \\ 0.3333 \\ 0.3974 \end{bmatrix} = X_1$$

$$Z = c^T X_1 = \begin{bmatrix} 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 0.2692 \\ 0.3333 \\ 0.3974 \end{bmatrix} = 0.2692$$

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Iteration-1 (k=1)

$$D_1 = \begin{bmatrix} 0.2692 & 0 & 0 \\ 0 & 0.3333 & 0 \\ 0 & 0 & 0.3974 \end{bmatrix}$$

$$\bar{C} = C^T D_1 = \begin{bmatrix} 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 0.2692 & 0 & 0 \\ 0 & 0.3333 & 0 \\ 0 & 0 & 0.3974 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0.6667 & -0.3974 \end{bmatrix}$$

$$AD_1 = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0.2692 & 0 & 0 \\ 0 & 0.3333 & 0 \\ 0 & 0 & 0.3974 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2692 & -0.6666 & 0.3974 \end{bmatrix}$$

$$P = \begin{pmatrix} AD_1 \\ 1 \end{pmatrix} = \begin{bmatrix} 0.2692 & -0.6666 & 0.3974 \\ 1 & 1 & 1 \end{bmatrix}$$

$$PP^T = \begin{bmatrix} 0.2692 & -0.6667 & 0.3974 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.2692 & 1 \\ -0.6667 & 1 \\ 0.3974 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.675 & 0 \\ 0 & 3 \end{bmatrix}$$

$$(PP^T)^{-1} = \begin{bmatrix} 1.482 & 0 \\ 0 & 0.333 \end{bmatrix}$$

$$P^T (PP^T)^{-1} P = \begin{bmatrix} 0.441 & 0.067 & 0.492 \\ 0.067 & 0.92 & -0.059 \\ 0.492 & -0.059 & 0.567 \end{bmatrix}$$

$$x_1 = [0.397483 \quad 0.333333 \quad 0.269183] \quad z = -0.2690$$

$$x_2 = [0.457409 \quad 0.333333 \quad 0.209258] \quad z = -0.2090$$

$$x_3 = [0.509027 \quad 0.333333 \quad 0.157639] \quad z = -0.1580$$

$$x_4 = [0.550656 \quad 0.33333 \quad 0.116011] \quad z = -0.1160$$

$$x_5 = [0.582667 \quad 0.33333 \quad 0.83999] \quad z = -0.0840$$

$\vdots$

$$x_{17} = [0.665417 \quad 0.33333 \quad 0.001249] \quad z = -0.001250$$

$x_{17} \approx (2/3, 1/3, 0)$ and $z \approx 0$ upto 3 decimal places

2 Dynamic programming :

Maximize $z = 2x_1 + 5x_2$

Subject To : $2x_1 + x_2 \leq 43$

$2x_2 \leq 46$

$x_1, x_2 \geq 0$

The given LPP is considered as 2 stage and 2 state dynamic problem because there are two decision variables and two constrain-ts

Assume 2 resources b_1, b_2 because of Two Constraints $F(b_1, b_2)$

Stage 1

$$f_1(b_1, b_2) = \max(2x_1)$$

$$0 \leq x_1 \leq b$$

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To calculate b fro x_1

The feasible of x_1 is non negative that Satisfies given constraints

$$2x_1 \le 43$$

$$x_1 \le 43/2$$

$$b = \min\{43/2, 46/0\}$$

$$= \min\{43/2, \infty\}$$

$$b = 43/2$$

$$x_1 = \min\left\{\frac{43-x_2}{2}, 0\right\}$$

$$x_1 = \frac{43-x_2}{2}$$

$$F_1(43,46) = 2\min\left\{\frac{43-x_2}{2}\right\}$$

<u>stage 2</u> :-

$$F_2(b_1, b_2) = \max_{0 \le x_2 \le b}(2x_1 + 5x_2)$$

$$= \max_{0 \le x_2 \le b}\left(5x_2 + 2\min\left\{\frac{43-x_2}{2}\right\}\right)$$

<u>To calculate b for x_2</u>

$$b = \min\left\{\frac{43}{1}, \frac{46}{2}\right\}$$

$$b = 23$$

$$F_2(43,46) = \max_{0 \le x_2 \le 23}\left(5x_2 + 2\min\left\{\frac{43-x_2}{2}\right\}\right)$$

To calculate $\min\left\{\frac{43-x_2}{2}\right\}$

$$\min\left\{\frac{43-x_2}{2}\right\} \Rightarrow \begin{cases} 43/2, & x_2 = 0 \\ 10, & x_2 = 23 \end{cases}$$

Therefore, $x_2 = 23$

From stage 1,

$$x_1 = \frac{43-x_2}{2}$$

$$\boxed{x_1 = 10}$$

Hence

$$z_{max} = 2x_1 + 5x_2 \quad \text{at } x_1 = 10 \ \& \ x_2 = 23$$

$$= 2(10) + 5(23)$$

$$= 20 + 115$$

$$z_{max} = 135$$

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