Conversion Primal into dual Problem

Problem

1) Construct the dual of the primal problem

Maximize

$$Z = 3x_1 + 5x_2$$

$$2x_1 + 6x_2 \le 50$$
$$3x_1 + 2x_2 \le 35$$
$$5x_1 - 3x_2 \le 10$$
$$x_2 \le 20$$

- primal problem

where $x_1, x_2 \ge 0$

Sol:- Let y_1, y_2, y_3, y_4 be the corresponding dual variables then the dual problem is given by

Minimize

$$W = 50y_1 + 35y_2 + 10y_3 + 20y_4$$

Subject to the constraints,

$$2y_1 + 3y_2 + 5y_3 \ge 3$$
$$6y_1 + 2y_2 \mp 3y_3 + y_4 \ge 5$$

- dual problem

Where $y_1, y_2, y_3, y_4 \ge 0$

As the dual problem has lesser no. of constraints it req., lesser work and effort to solve it

Note:- The computational difficulty in LPP is mainly assisted with the no. of contraints rather than no of variables

2) Construct the dual of the problem

Minimize $Z = 3x_1 - 2x_2 + 4x_3$

Subject to the constraints

$$3x_1 + 5x_2 + 4x_3 \ge 7 \quad \text{--- (1)}$$
$$6x_1 + x_2 + 3x_3 \ge 4 \quad \text{--- (2)}$$
$$7x_1 - 2x_2 - x_3 \le 10 \quad \text{--- (3)}$$
$$x_1 - 2x_2 + 5x_3 \ge 3 \quad \text{--- (4)}$$
$$4x_1 + 7x_2 - 2x_3 \ge 2 \quad \text{--- (5)}$$

primal problem

and x_1, x_2 and x_3

Sol: Let y_1, y_2, y_3, y_4, y_5 be the corresponding dual variables

The given problem is of minimization, all constraints should be of type $\geq$.

Multiplying the ③ constraints by -1 on both sides we get $-7x_1 + 2x_2 + x_3 \geq -10$ — ⑥

~~Let~~ Maximize $W = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$

Subject to the constraints

$$3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3$$
$$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$$
$$4y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4$$

where y_1, y_2, y_3 and $y_4, y_5 \geq 0$ and are called as dual variables.

Dual problem when the primal is in standard form:

Problem:

Maximize $Z = 3x_1 + 10x_2 + 2x_3$

subject to $2x_1 + 3x_2 + 2x_3 \leq 7$ — ①

$3x_1 - 2x_2 + 4x_3 = 3$ — ②

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

This problem in standard form.

Sol:- Since, the given problem is of maximization, all constraints should be of type $\leq$. The eq. $3x_1 - 2x_2 + 4x_3 = 3$ can be expressed as a pair of inequalities

$$3x_1 - 2x_2 + 4x_3 \leq 3, \quad 3x_1 - 2x_2 + 4x_3 \geq 3$$
(or)
$$\Rightarrow -3x_1 + 2x_2 - 4x_3 \leq -3$$

The primal problem becomes

maximize $Z = 3x_1 + 10x_2 + 2x_3$

subject to constraints

$$2x_1 + 3x_2 + 2x_3 \leq 7$$
$$3x_1 - 2x_2 + 4x_3 \leq 3$$

$-3x_1 + 2x_2 - 4x_3 \leq -3$

$x_1, x_2, x_3 \geq 0$

Let y_1, y_2, y_3 are the corresponding dual variables

The given problem is ~~of~~ maximization

Minimize $W: 7y_1 + 3y_2 - 3y_3$

Subject to the constraints

$2y_1 + 3y_2 - 3y_3 \geq 3$

$3y_1 - 2y_2 + 2y_3 \geq 10$

$2y_1 + 4y_2 - 4y_3 \geq 2$

where y_1, y_2, y_3 ~~and y_0, y_4~~ ≥ 0 are called as dual variables

substituting $y_2' - y_3 = y_2$ Let y_2 is inrestricted in sign

then the dual problem becomes

Min $W = 7y_1 + 3y_2$

Subject to the Constraints

$2y_1 + 3y_2 \geq 3$, $3y_1 - 2y_2 \geq 10$, $2y_1 + 4y_2 \geq 2$,

$y_1, y_2 \geq 0$ is inrestricted in sign

Assignment Problem

construct the dual of the problem.

Min $Z = x_2 + 3x_3$

Subject to $2x_1 + x_2 \leq 3$

$x_1 + 2x_2 + 6x_3 \geq 5$

$-x_1 + x_2 + 2x_3 = 2$

where $x_1, x_2, x_3 \geq 0$

Solving the LPP by using dual

Solve the following LLP by using it dual

$$\left.\begin{aligned} &\text{Max } Z = 5x_1 - 2x_2 + 3x_3 \\ &\text{Subject to } 2x_1 + 2x_2 - x_3 \geq 2 \\ &3x_1 - 4x_2 \leq 3 \\ &x_2 + 3x_3 \leq 5 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}\right\} \text{Primal Problem}$$

Sol:- The given problem can be written as

Max $z = 5x_1 - 2x_2 + 3x_3$

subject to constraints $-2x_1 - 2x_2 + x_3 \le -2$ — ①

$3x_1 - 4x_2 \le 3$ — ②

$x_2 + 3x_3 \le 5$ — ③

$x_1, x_2, x_3 \ge 0$

The associated dual of the primal problem also is given by

Min $W = -2y_1 + 3y_2 + 5y_3$

subject to constraints

$-2y_1 + 3y_2 \ge 5$

$-2y_1 - 4y_2 + y_3 \ge -2$

$y_1 + 3y_3 \ge 3$

dual of the primal problem

where $y_1, y_2, y_3 \ge 0$

The solution of the dual by simplex method consists of the following steps.

Step1:- Express the problem in standard form.

Multiplying the 2nd constraint by -1, it can be written as

$2y_1 + 4y_2 - y_3 \le 2$

Introducing the slack and simple variables we get an artificial system given by

Minimize

$W = -2y_1 + 3y_2 + 5y_3 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2$

s.t

$-2y_1 + 3y_2 - S_1 + A_1 = 5$

$2y_1 + 4y_2 - y_3 + S_2 = 2$

$y_1 + 3y_3 - S_3 + A_2 = 3$

$\ge$:- add $A_1, -S_1$ to constraints

$\le$ = add S_1 into +ve constraints

where $y_1, y_2, y_3, S_1, S_2, S_3, A_1, A_2 \ge 0$

Step-2: Find the ibfs

Put $y_1 = y_2 = y_3 = s_1 = s_3 = 0$, $A_1 = 5$, $S_2 = 2$, $A_2 = 3$, which is not as bfs

C_j		-2	3	5	0	0	0	M	M		
C_B	Basis	y_1	y_2	y_3	s_1	s_2	s_3	A_1	A_2	b	θ
M	A_1	-2	$\boxed{3}$	0	-1	0	0	1	0	5	5/3
0	S_2	2	4	-1	0	1	0	0	0	2	2/4 1/2
M	A_2	7	0	3	0	0	-1	0	1	3	∞
$Z_j = \Sigma C_B a_j$		-M	3M	3M	-M	0	-M	M	M		
$\bar{C_j} = C_j - Z_j$		-2+M	3-3M	5-3M	M	0	M	0	0		

zg

$\begin{bmatrix} \bar{C_j} \leq 0 \text{ max} \\ \bar{C_j} \geq 0 \text{ min} \end{bmatrix}$

Final simplex table:-

C_j		-2	3	5	0	0	0	M	M	b	θ
C_B	Basis	y_1	y_2	y_3	S_1	S_2	S_3	A_1	A_2		
					-4	-3	1	4	-1	11	
0	S_3	-15	0	0	-11/3	0	0	1/3	0	5/3	
3	y_2	-2/3	1	0	-4/3	-1	0	4/3	0	14/3	
5	y_3	-14/3	0	1	-23/3	-5	0	23/3	0		
C_j		-76/3	3	5							
$\bar{C_j}$		70/3	0	0	23/3	5	0	$-\frac{23}{3}+M$	M		

$W_{min} = -2y_1 + 3y_2 + 5 \cdot y_3 = -2 \times 0 + 3 \times \frac{5}{3} + 5 \times \frac{14}{3} = \frac{85}{3}$

The opt sol given by

$y_1 = 0$, $y_2 = 5/3$, $y_3 = 14/3$