# MP-1 TUTORIAL-1
## PRE-LAB

1. Write the algorithm for Linear Programming Graphical Method. Define the following:
a. Objective Function.
b. Decision Variables.
c. Constraints.
d. Non negativity Restrictions.
e. Feasible Solution.

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MP-1 Tutorial-1

Prelab

1. Algorithm for programming graphical method :-

The linear programming problems of two decision variables can be easily solved by graphical method.

step-1 :- Identify the problem - the decision variables, the objective function & the restrictions.

step-2 :- set up the mathematical formulation of the problem.

step-3 :- consider each inequality - constraint as an equation.

step-4 :- plot each equation on the graph as each one will geometrically represent a straight line.

step-5 :- shade the feasible region. Every point on the line will satisfy the equation of line. If the inequality constraint corresponding to that line is '$\leq$' then the region below the line lying in the first quadrant is shaded. For the inequality constraint with '$\geq$' quadrant is shaded. The points lying in the common region will satisfy all the constraints simultaneously

The common region thus formed is called feasible region.

Step-6 :- choose the convenient value of z (say-0) and plot the objective function line.

Step-7 :- pull the objective function line until the extreme points of the feasible region. In the maximization case, this line will stop farthest from the origin and passing through atleast one corner of the feasible region. In the minimization case, this line will stop nearest to the origin and passing through at atleast one corner of the feasible region.

Step-8 :- Read the coordinates of the extreme point(s) selected in step-6 and find the maximum or minimum value of z.

## Objective Function :-

The quantity to be optimized in a mathematical programming problem is known as the objective function, and the objective is to optimize this function.

## Decision variables :-

The variables in a linear program are a set of quantities that need to be determined in order to solve the problem i.e the problem is solved when the best values of the variables have been identified.

## Constraints :-

The constraints are the restrictions or limitations on the decision variables. They usually limit the value of the decision variables.

## Non-negativity Restrictions :-

For all linear programs, the decision variables should always take non-negative values. This means the values for decision variables should be greater than or equal to 0.

## Feasible solution :-

A feasible solution is a set of values for decision variables that satisfies all of the constraints in an optimization problem.

2. Solve the following problem graphically:
Maximize C=3a+2b Subject to the constraints a+2b≤10 3a+b≤15 a,b≥0

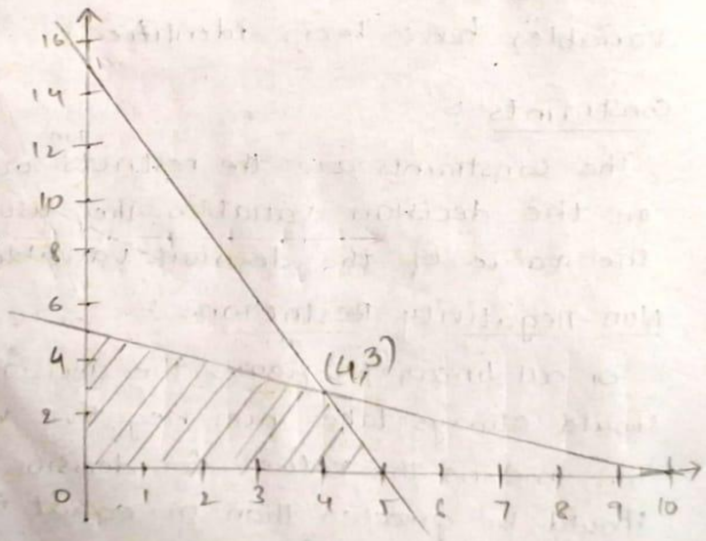
2. Maximize $C = 3a + 2b$
Subject to
$a + 2b \le 10$
$3a + b \le 15$
$a, b \ge 0$

$a + 2b \le 10$

| | | |
|---|---|---|
| a | 0 | 10 |
| b | 5 | 0 |

$3a + b \le 15$

| | | |
|---|---|---|
| a | 0 | 5 |
| b | 15 | 0 |




| corner points | value of c |
|---|---|
| (0,0) | $C = 0$ |
| (5,0) | $C = 3 \cdot 5 + 0 = 15$ |
| (4,3) | $C = 3 \cdot 4 + 2 \cdot 3 = 18$ |
| (0,5) | $C = 3 \cdot 0 + 2 \cdot 5 = 10$ |

The maximum value of c is 18 occurs at extreme point (4,3)
Hence optimal solution to the given LP problem is $a = 4$ $b = 3$ and max $c = 18$

3. Solve LPP using Graphical Method
Minimize
4p + 5q + 6r
Subject to:
p + q >= 11
p - q <= 5
r - p - q = 0
7p >= 35 – 12q
p >= 0 q >= 0 r >= 0

3. Minimize $z =$

$4p + 5q + 6r$

subject to

$p + q >= 11$

$p - q \leq 5$

$r - p - q = 0 \implies r = p + q$

$7p \geq 35 - 12q$

Minimize $z = 4p + 5q + 6(p+q)$

$z = 10p + 11q$

subject to

$p + q \geq 11$

$p - q \leq 5$

$7p + 12q \geq 35$

$p + q \geq 11$ Treat as $p + q = 11$

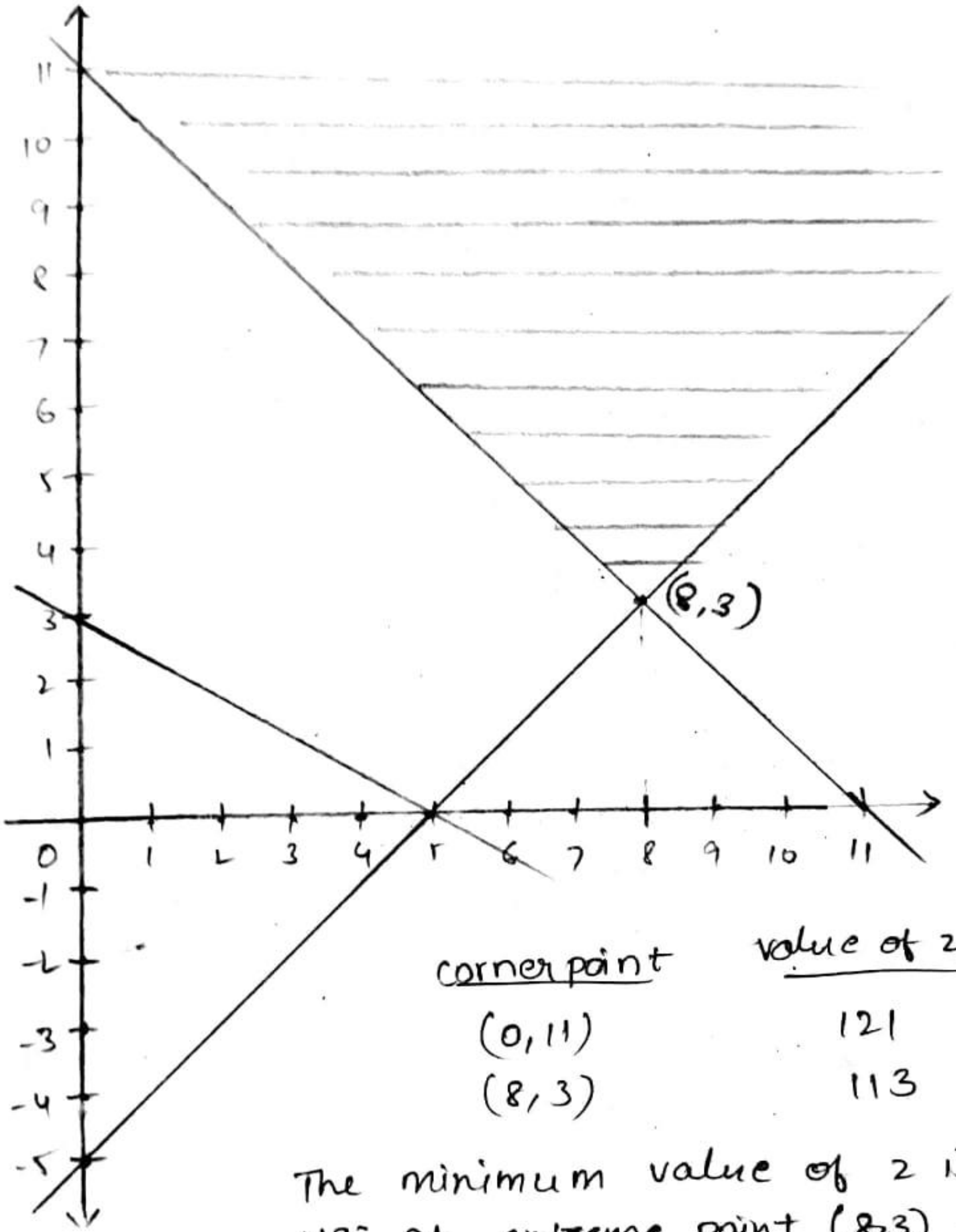
| | | |
|---|---|---|
| p | 0 | 11 |
| q | 11 | 0 |

$p - q \leq 5$ Treat as $p - q = 5$

| | | |
|---|---|---|
| p | 0 | 5 |
| q | -5 | 0 |

$7p + 12q \geq 35$ Treat as $7p + 12q = 35$

| | | |
|---|---|---|
| p | 0 | 5 |
| q | 2.92 | 0 |




| corner point | value of z |
|---|---|
| (0,11) | 121 |
| (8,3) | 113 |

The minimum value of z is 113 at extreme point (8,3)

Hence the optimal solution to given LP problem is $x_1 = 8$, $x_2 = 3$ and min $z = 113$.

## INLAB

1. An Industry started in producing A and B, two products. These two products need two resources, machine power and man power. For product A, require 13 machine power and 20 man power. For the product B, require 19 machine power and 29 man power. Industry has 40 hrs of machine power which is available in the coming working week but only 35hrs of man power. The cost of machine power is $10 per hour worked and man power is $2 per hour worked. Both machine and manpower idle times incur no costs. The revenue received for each product produced (all production is sold) is $20 for A and $30 for B. For a customer, Industry has a specific contract to produce 10 products of A per week.

· Formulate the problem of deciding how much to produce per week as a linear program and represent the linear program graphically.

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1. Let x be the no.of items of A
y be the no.of items of B.

Then the LP is

Maximize

$$20x + 30y - 10\left(\underset{\text{worked}}{\text{machine power}}\right) - 2\left(\underset{\text{worked}}{\text{man power}}\right)$$

subject to :-

$13x + 19y \leq 40(60)$ Machine power time

$20x + 29y \leq 35(60)$ man power time

$x \geq 10$

$x, y \geq 0$

So the objective function becomes

Maximize

$$20x + 30y - 10(13x + 19y)/60 - 2(20x + 29y)/60$$

i.e Maximize

$$17.1667x + 25.8667y$$

subject to

$13x + 19y \leq 2400$

$20x + 29y \leq 2100$

$x \geq 10$

$x, y \geq 0$

$x = 10$

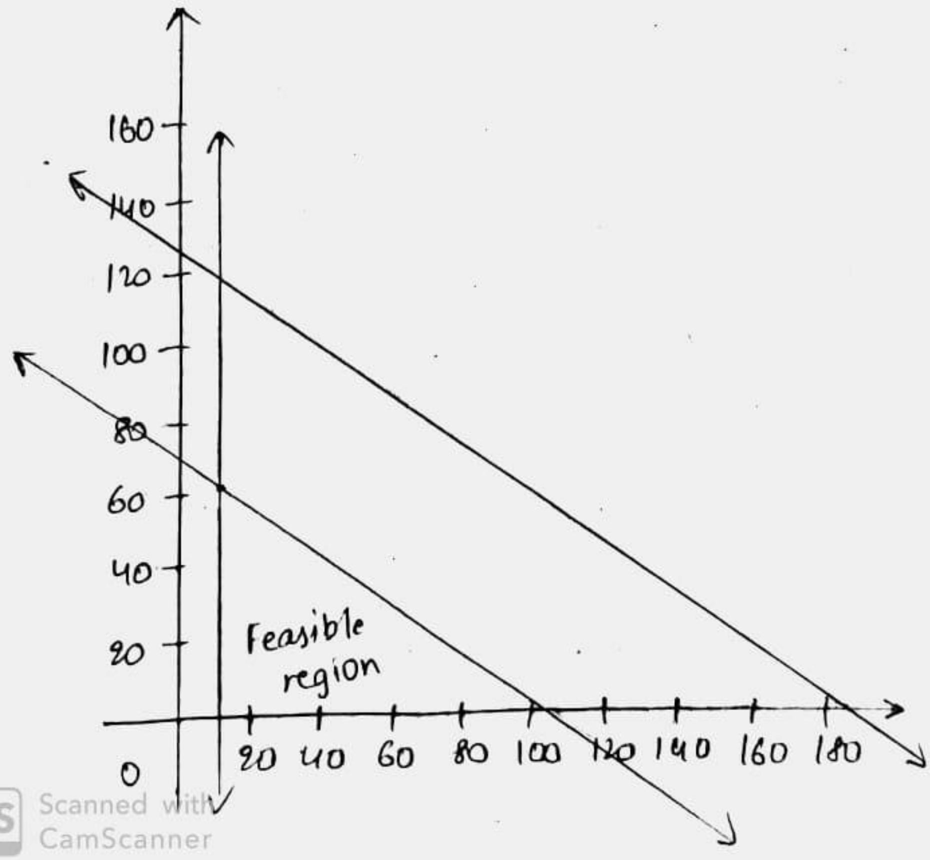
$20x + 29y \leq 2100$ $13x + 19y \leq 2400$

| | | |
|---|---|---|
| x | 105 | 0 |
| y | 0 | 72.4 |

| | | |
|---|---|---|
| x | 184.6 | 0 |
| y | 0 | 126.3 |

Solving simultaneously rather than by reading values off the graph we have that

$x = 10$, $y = 65.52$ with the value of the objective function being 1866.5



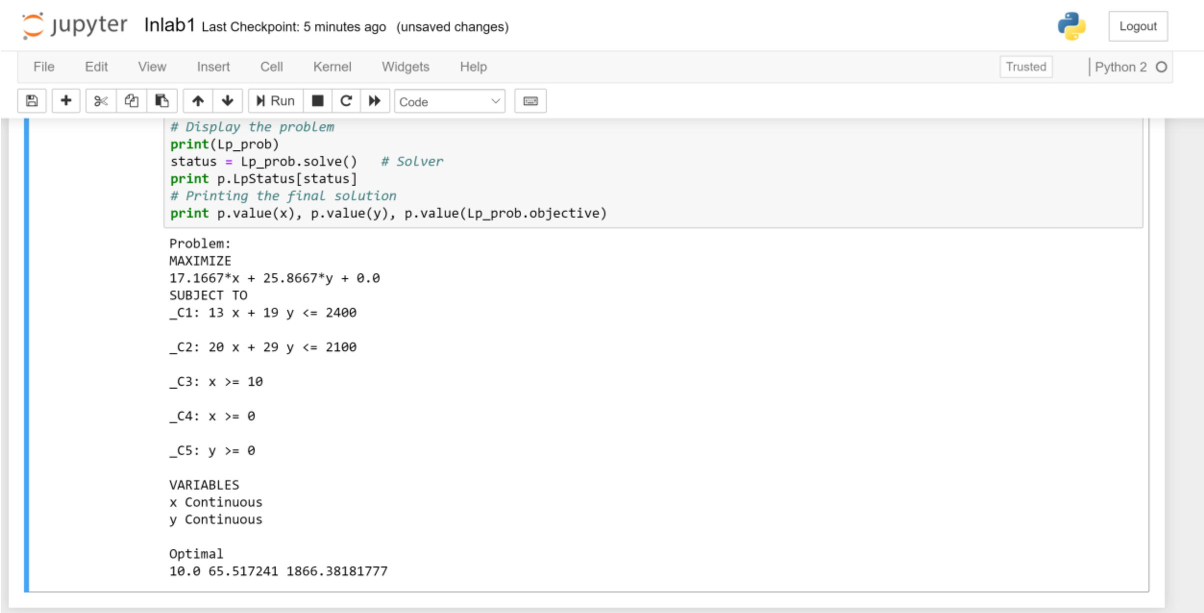



· Solve the problem using python

**Code:**

```
# import the library pulp as p
import pulp as p
# Create a LP Minimization problem
Lp_prob = p.LpProblem('Problem', p.LpMaximize)
# Create problem Variables
x = p.LpVariable("x", lowBound = 0)   # Create a variable x >= 0
y = p.LpVariable("y", lowBound = 0)   # Create a variable y >= 0
# Objective Function
Lp_prob += 17.1667 * x + 25.8667 * y
# Constraints:
Lp_prob +=  13 * x + 19 * y <= 2400
Lp_prob +=  20 * x + 29 * y <= 2100
Lp_prob += x >= 10
Lp_prob += x >= 0
Lp_prob += y >= 0
# Display the problem
print(Lp_prob)
status = Lp_prob.solve()   # Solver
print p.LpStatus[status]
# Printing the final solution
print p.value(x), p.value(y), p.value(Lp_prob.objective)
```

## OUTPUT




2. An Industry makes two items of P and Q by using two devices X and Y. Processing time requires 50hrs for item P on device X and 30hrs requires on device Y. Processing time requires 24hrs for item Q on device X and 33hrs requires on device Y. At starting of the current week, 30 pieces of A and 90 pieces of B are available. Processing time that is available on device X is predict to be 40hrs and on device Y is predict to be 35hrs. Demand for P in the current week is predict to be 75 pieces and for Q is predict to be 95 pieces. Industry policy is to maximize the combined sum of the pieces of P and the

pieces of Q in stock at the end of the week. Formulate the problem of deciding how much of each item to make in the current week as a linear program. Solve this linear program graphically.

2. Let $x$ be no.of pieces of P produced in the current week

let $y$ be no.of pieces of Q produced in the current week

then the constraints are :

$50x + 24y \leq 40(60)$. Machine A time

$30x + 33y \leq 35(60)$ Machine B time

$x >= 75 - 30$

i.e $x \geq 45$, so production of $x >=$ demand (75) − Initial stock (30) which ensures we meet demand

$y \geq 95 - 90$

i.e $y \geq 5$ so production of $y >=$ demand (95) − Initial stock (90) which ensures we meet demand.

The objective function is

Maximize

$$(x + 30 - 75) + (y + 90 - 95) = (x + y - 50)$$

i.e to maximize the no.of units left in stock at the end of the week.

$x = 45 \quad y = 5$

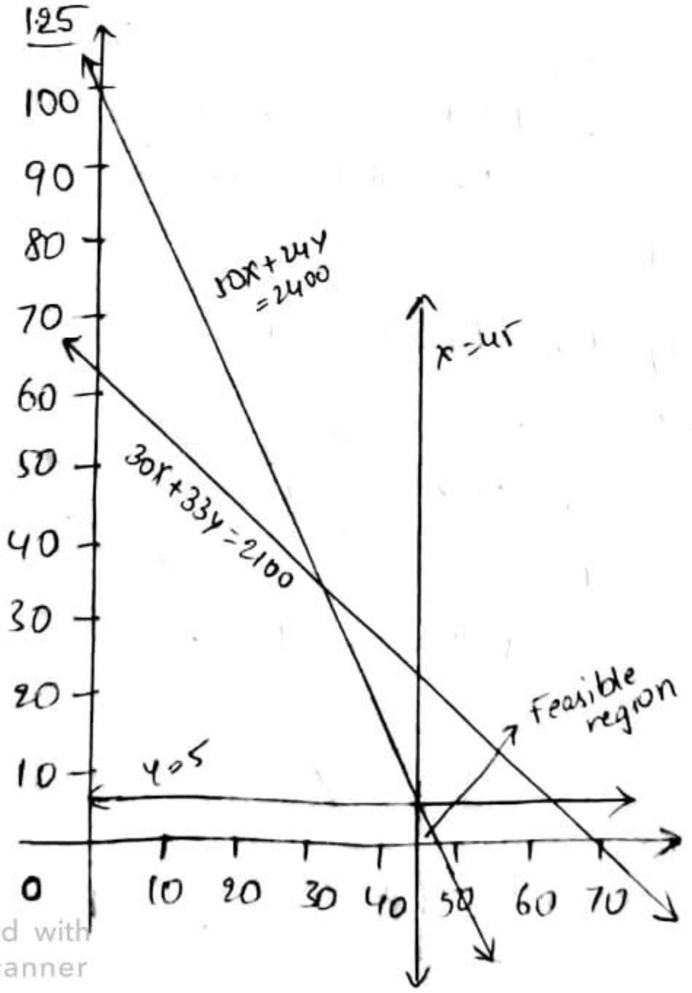
$50x + 24y \leq 2400 \qquad 30x + 33y \leq 2100$

| $x$ | 48 | 0 |
|---|---|---|
| $y$ | 0 | 100 |

| $x$ | 70 | 0 |
|---|---|---|
| $y$ | 0 | 63.6 |

It is plain from the diagram below that the maximum occurs at the intersection of $x = 45$ and $50x + 24y = 2400$

solving simultaneously rather than by reading values off the graph we have $x = 45$ $y = 6.25$ with the value of Objective function being 125



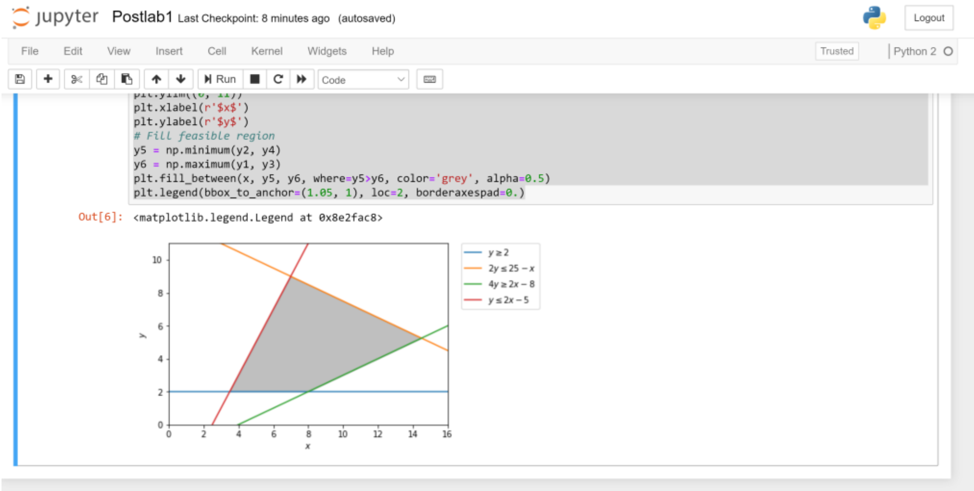
## POSTLAB

1. Maximize
   Z=4x+3yZ=4x+3y
   Subject TO:
   x≥0
   y≥2
   2y≤25–x
   4y≥2x–8
   y≤2x−5
   Solve LP graphically using python

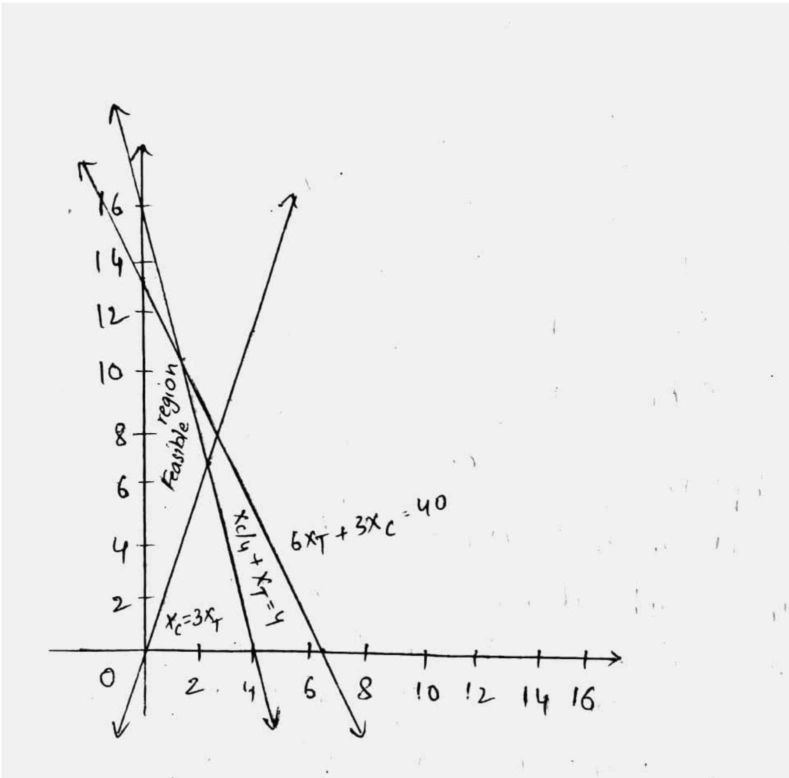
**Code:**

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
# Construct lines
# x > 0
x = np.linspace(0, 20, 2000)
# y >= 2
y1 = (x*0) + 2
# 2y <= 25 - x
y2 = (25-x)/2.0
# 4y >= 2x - 8
y3 = (2*x-8)/4.0
# y <= 2x - 5
y4 = 2 * x -5
# Make plot
plt.plot(x, y1, label=r'$y\geq2$')
plt.plot(x, y2, label=r'$2y\leq25-x$')
plt.plot(x, y3, label=r'$4y\geq 2x - 8$')
plt.plot(x, y4, label=r'$y\leq 2x-5$')
plt.xlim((0, 16))
plt.ylim((0, 11))
plt.xlabel(r'$x$')
plt.ylabel(r'$y$')
# Fill feasible region
y5 = np.minimum(y2, y4)
y6 = np.maximum(y1, y3)
plt.fill_between(x, y5, y6, where=y5>y6, color='grey', alpha=0.5)
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
```

## OUTPUT




2. A cabinetmaker makes benches and desks. Each bench can be sold for a profit of $30 and each desk for a profit of $10. The cabinetmaker can afford to spend up to 40 hrs per week working and takes 6 hrs to make a bench and 3 hrs to make a desk. Customer demand requires that he makes at least 3 times as many desks as benches. Benches take up 4 times as much storage space as desks and there is room for at most four benches each week. Formulate this problem as a linear programming problem and solve it graphically.



## post lab

2. Let $x_T$ = no. of benches made per week

$x_C$ = no. of desks made per week

$6x_T + 3x_C <= 40$

$x_C \geq 3x_T$

$x_C/4 + x_T <= 4$

**objective**

Maximize $30x_T + 10x_C$

The graphical representation of the problem is given below and from that we have that the solution lies at the intersection of $(x_C/4) + x_T = 4$ and $6x_T + 3x_C = 40$

solving simultaneously we get $x_C = 10.667$, $x_T = 1.333$ and the profit = 146.667

$6x_T + 3x_C = 40$

| | | |
|---|---|---|
| $x_T$ | 6.66 | 0 |
| $x_C$ | 0 | 13.33 |

$(x_C/4) + x_T = 4$

| | | |
|---|---|---|
| $x_T$ | 4 | 0 |
| $x_C$ | 0 | 16 |