

## MP-1 TUTORIAL-10

1. Demonstrate the Initial Basic Solution in Transportation problem using Vogel method in Linear Programming (U-V method)., Least time Transportation problem.

## QUESTION:

Factory/Warehouse	$W_1$	$W_2$	$W_3$	$W_4$	Factory Capacity
$F_1$	19	30	50	10	7
$F_2$	70	30	40	60	9
$F_3$	40	8	70	20	18
Warehouse Requirement	5	8	7	14	

190031187

Radhakrishna

Row diff

<del>5</del> 19	30	<del>10</del>	<del>2</del> 10	<del>2</del> 0	7	9	40	40
70	30	<del>7</del> 40	<del>2</del> 60	<del>1</del> 9	10	20	20	20
40	<del>8</del>	70	<del>10</del> 20	<del>0</del> 18	12	20	<u>50</u>	-
<del>8</del> 0	<del>8</del> 0	<del>7</del> 0	<del>1</del> 19	<del>10</del> 1				
Col diff								
21	<u>22</u>	10	10					
<u>21</u>	-	10	10					
-	-	10	10					
-	-	10	<u>50</u>					

Transportation Cost :-

$$\begin{aligned}
 &= 5 \times 19 + 8 \times 8 + 7 \times 40 + 2 \times 10 + 10 \times 20 + 2 \times 60 \\
 &= 95 + 64 + 280 + 20 + 200 + 120 \\
 &= 779
 \end{aligned}$$

u-v method

	$W_1$	$W_2$	$W_3$	$W_4$	
	19	30	50	10	7
	70	30	40	60	9
	40	8	70	20	18
	5	8	7	14	

Step 1

$m = \text{no. of origins} = 3$

$n = \text{no. of destinations} = 4$

$m+n-1 = 6 \Rightarrow \text{no. of allocation} = 6$

And the allocations are independent positions.

$\therefore$  The problem is non-degenerate

Step 2

$u_i, v_j$  - allocated cells.

$$u_i + v_j = c_{ij}$$

5	19	30	50	2	10	$u_1 = 0$	
	70	30	7	40	2	60	$u_2 = 50$
	40	8		70	10	20	$u_3 = 10$

$$v_1 = 19 \quad v_2 = -2 \quad v_3 = -10 \quad v_4 = 10$$

cell	eqns	Values
(1,1)	$u_1 + v_1 = 19$ $0 + v_1 = 19$	$v_1 = 19$
(1,4)	$u_1 + v_4 = 10$ $0 + v_4 = 10$	$v_4 = 10$
(2,4)	$u_2 + v_4 = 60$ $u_2 + 10 = 60$	$u_2 = 50$
(2,3)	$u_2 + v_3 = 40$ $50 + v_3 = 40$	$v_3 = -10$
(3,4)	$u_3 + v_4 = 20$ $u_3 + 10 = 20$	$u_3 = 10$
(3,2)	$u_3 + v_2 = 8$ $10 + v_2 = 8$	$v_2 = -2$

Step: 3

<sup>(1)</sup> 19	<sup>(32)</sup> 30	<sup>(60)</sup> 50	<sup>(2)</sup> 10
<sup>(1)</sup> 70	<sup>(18)</sup> 30	<sup>(7)</sup> 40	<sup>(1)</sup> 60
<sup>(10)</sup> 40	<sup>(8)</sup> 8	<sup>(70)</sup> 70	<sup>(10)</sup> 20

 $u_i$ 

0

50

10

 $v_j$  19 -2 -10 10

cell	eqn	cell evaluation
(1,2)	$30 - (u_1 + v_2)$ $30 - (0 - 2)$	32
(1,3)	$50 - (u_1 + v_3)$	60
(2,1)	$70 - (u_2 + v_1)$	1
(2,2)	$30 - (u_2 + v_2)$	-18
(3,1)	$40 - (u_3 + v_1)$	11
(3,3)	$70 - (u_3 + v_3)$	70

If all the cells evaluations are non-negative then the solution is optimal

The cell evaluation -18 is negative so sol is not optimal

∴ Entering cell (2,2)

19	30	50	<sup>(2)</sup> 10
70	30	<sup>(7)</sup> 40	<sup>(1)</sup> 2-0 → 60
40	<sup>(8)</sup> 8 ← 8-0	70	<sup>(10)</sup> 20 ↓ 10-0

Form a loop starting from cell (2,2)

$$\theta = \min \{ 2-0, 8-0 \}$$

$$2-0 = 0$$

$$\theta = 2$$



<sup>5</sup> 19	30	<sup>5</sup> 10	<sup>2</sup> 10
70	<sup>2</sup> 30	<sup>7</sup> 40	<sup>0</sup> 60 ← leaving cell
40	<sup>6</sup> 8	70	<sup>12</sup> 20

updated allocations

<sup>5</sup> 19	<sup>(22)</sup> 30	<sup>(42)</sup> 10	<sup>2</sup> 10	0
<sup>(7)</sup> 70	<sup>2</sup> 30	<sup>7</sup> 40	<sup>(18)</sup> 60	32
40 <sup>(11)</sup>	<sup>6</sup> 8	<sup>(12)</sup> 70	<sup>(12)</sup> 20	10

$V_j \rightarrow 19 \quad -2 \quad 8 \quad 10$

Now all the cell evaluations are non-negative

Transportation cost

$$5 \times 19 + 2 \times 10 + 2 \times 30 + 7 \times 40 + 6 \times 8 + 12 \times 20$$

$$95 + 20 + 60 + 280 + 48 + 240$$

$$743$$