

**MP-1 HOME ASSIGNMENT-4**

1. A manufacturer of baby-dolls makes two types of dolls. Doll X and Doll Y. Processing of these two dolls is done on two machines, A and B. Doll X requires two hours on machine A and six hours on machine B. Doll Y requires five hours on machine A and also five hours on machine B. there are 16 hours of time available on machine A and thirty hours on machine B. The profit gained on both the dolls is same, i.e. one rupee per doll. What should be the daily production of each of the two dolls? Formulate but not solve the mathematical programming problem. Suggest the suitable algorithm to solve it.

Solve the following L.P.P by Gomory technique :

*Maximize  $z=3x_2$*

*Subject to  $3x_1+2x_2\leq 7$*

*$x_1-x_2\leq -2$*

*$x_1, x_2 \geq 0$  are integers.*

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# Home Assignment - 4

$$\text{Max } Z = 3x_2$$

$$\text{Subject to } 3x_1 + 2x_2 \leq 7$$

$$x_1 - x_2 \leq -2$$

$$\text{Here } b_2 = -2 < 0 \text{ so}$$

multiply 2<sup>nd</sup> constraint by -1 to make  $b_2 > 0$

$$-x_1 + x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

After Adding slack, surplus, Artificial variables

$$\text{Max } Z = 0x_1 + 3x_2 + 0s_1 + 0s_2 - MA_1$$

Subject to

$$3x_1 + 2x_2 + s_1 = 7$$

$$-x_1 + x_2 - s_2 + A_1 = 2$$

$$x_1, x_2, s_1, s_2, A_1 \geq 0$$

|       |             |       |       |       |       |       |     |
|-------|-------------|-------|-------|-------|-------|-------|-----|
| $C_B$ | $C_j$       | 0     | 3     | 0     | 0     | -M    |     |
|       | B.V         | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | for |
| 0     | $s_1$       | 3     | 2     | 1     | 0     | 0     | 7   |
| -M    | $A_1$       | -1    | 1     | 0     | -1    | 1     | 2   |
|       | $Z_j$       | M     | -M    | 0     | M     | -M    |     |
|       | $G_j - Z_j$ | -M    | 3+M   | 0     | -M    | 0     |     |

Entering =  $x_2$

Departing =  $A_1$

key element = 1



Iteration - 2

| $C_{B_i}$ | $C_j$   | 0     | 3     | 0     | 0     | -M    |      |
|-----------|---------|-------|-------|-------|-------|-------|------|
|           | B.V     | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | do 1 |
| 0         | $s_1$   | 5     | 0     | 1     | 2     | -2    | 3    |
| 3         | $x_2$   | -1    | 1     | 0     | -1    | 1     | 2    |
|           | $Z_j$   | -3    | 3     | 0     | -3    | 3     |      |
|           | $G-Z_j$ | 3     | 0     | 0     | 3     | -M-3  |      |

pivot element is 5

Iteration - 3

| $C_{B_i}$ | $C_j$   | 0     | 3     | 0     | 0     | -M     |      |
|-----------|---------|-------|-------|-------|-------|--------|------|
|           | B.V     | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$  | do 1 |
| 0         | $x_1$   | 1     | 0     | 0.2   | 0.4   | -0.4   | 0.6  |
| 3         | $x_2$   | 0     | 1     | 0.2   | -0.6  | 0.6    | 2.6  |
|           | $Z_j$   | 0     | 3     | 0.6   | -1.8  | 1.8    |      |
|           | $G-Z_j$ | 0     | 0     | -0.6  | 1.8   | -M-1.8 |      |

pivot element - 0.4

Entering -  $s_2$

Departing -  $x_1$

key element = 0.4



Iteration - 4

|       |             |       |       |       |       |       |     |
|-------|-------------|-------|-------|-------|-------|-------|-----|
| $C_B$ | $C_j$       | 0     | 3     | 0     | 0     | -M    |     |
|       | B.V         | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | Sol |
| 0     | $s_2$       | 2.5   | 0     | 0.5   | 1     | -1    | 1.5 |
| 3     | $x_2$       | 1.5   | 1     | 0.5   | 0     | 0     | 3.5 |
|       | $Z_j$       | 4.5   | 3     | 1.5   | 0     | 0     |     |
|       | $C_j - Z_j$ | -4.5  | 0     | -1.5  | 0     | M     |     |

Since all  $C_j - Z_j \leq 0$

Hence non-integer optimal solution is arrived with value of variables as

$$x_1 = 0, x_2 = 3.5$$

$$\text{Max } Z = 10.5$$

To obtain the integer valued solution, we proceed to construct Gomory's fractional cut with the help of  $x_2$  row as follows.

$$3.5 = 1.5x_1 + 1x_2 + 0.5s_1$$

$$(3 + 0.5) = (1 + 0.5)x_1 + (1 + 0)x_2 + (0 + 0.5)s_1$$

The fraction cut will become

$$-0.5 = s_1 - 0.5x_1 - 0.5s_1 \rightarrow \text{cut 1}$$

Adding this additional constraint bottom of optimal simplex table. The new <sup>at</sup> table so obtained is



Iteration 1

| $C_B$ | $C_j$       | 0     | 3     | 0     | 0     | 0        |      |
|-------|-------------|-------|-------|-------|-------|----------|------|
|       | B.V         | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_{g1}$ | sol  |
| 0     | $s_2$       | 2.5   | 0     | 0.5   | 1     | 0        | 1.5  |
| 3     | $x_2$       | 1.5   | 1     | 0.5   | 0     | 0        | 3.5  |
| 0     | $s_{g1}$    | -0.5  | 0     | -0.5  | 0     | 1        | -0.5 |
|       | $Z_j$       | 4.5   | 3     | 1.5   | 0     | 0        |      |
|       | $C_j - Z_j$ | -4.5  | 0     | -1.5  | 0     | 0        |      |

Entering =  $s_1$ Leaving =  $s_{g1}$ 

key element = -0.5

Iteration-2

| $C_B$ | $C_j$       | 0     | 3     | 0     | 0     | 0        |     |
|-------|-------------|-------|-------|-------|-------|----------|-----|
|       | B.V         | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_{g1}$ | sol |
| 0     | $s_2$       | 2     | 0     | 0     | 1     | 1        | 1   |
| 3     | $x_2$       | 1     | 1     | 0     | 0     | 1        | 3   |
| 0     | $s_1$       | 1     | 0     | 1     | 0     | -2       | 1   |
|       | $Z_j$       | 3     | 3     | 0     | 0     | 3        |     |
|       | $C_j - Z_j$ | -3    | 0     | 0     | 0     | -3       |     |

Since  $C_j - Z_j \leq 0$ Hence integer optimal solution is arrived  
with value of variables as:

$$x_1 = 0, x_2 = 3$$

$$\text{Max } Z = 9$$

The integer optimal solution found after

1- cuts