Dual problem when the primal is in Standard form.

Problem: Construct the dual of the problem

Maximize $Z = 3x_1 + 10x_2 + 2x_3$

Subject to $2x_1 + 3x_2 + 2x_3 \leq 7$

$3x_1 - 2x_2 + 4x_3 = 3$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

Sol: Since the given problem is of Maximization, all Constraints should be of type $\leq$. The equation $3x_1 - 2x_2 + 4x_3 = 3$ can be expressed as a pair of inequalities

$3x_1 - 2x_2 + 4x_3 \leq 3$ and $3x_1 - 2x_2 + 4x_3 \geq 3.$

or $3x_1 - 2x_2 + 4x_3 \leq 3$ and $-3x_1 + 2x_2 - 4x_3 \leq -3.$

∴ The primal problem becomes

Maximize $Z = 3x_1 + 10x_2 + 2x_3$

Subject to $2x_1 + 3x_2 + 2x_3 \leq 7$

$3x_1 - 2x_2 + 4x_3 \leq 3$

$-3x_1 + 2x_2 - 4x_3 \leq -3.$

where $x_1, x_2, x_3 \geq 0.$

Let y_1, y_2', y_2'' be the associated non-negative dual variables. Then the dual of the problem is,

Minimize $W = 7y_1 + 3y_2' - 3y_2''$

Subject to the Constraint

$2y_1 + 3y_2' - 3y_2'' \geq 3$

$3y_1 - 2y_2' + 2y_2'' \geq 10$

$2y_1 + 4y_2' - 4y_2'' \geq 2$, where y_1, y_2' and $y_2'' \geq 0.$