MP-1 TUTORIAL-7
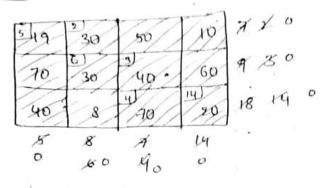
1. Demonstrate the Initial Basic Solution in Transportation problem using NW method in Linear Programming (U-V method).

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Tutorial-7

NWCR

Transportation cost =

$5 * 19 + 2 * 30 + 6 * 30 + 3 * 40 + 4 * 70 + 14 * 20$

$95 + 60 + 180 + 120 + 280 + 280$

1015

U-V method

step: 1   m = no. of origins = 3
           n = no. of destinations = 4
           m + n - 1 = 6 ⇒ No. of allocations = 6

And the allocations are in independent positions ∴ The problem is non-degenerate

step - 2 u_i, v_j - allocated cells
            $u_i + v_j = c_{ij}$

$\boxed{5}$ 19	$\boxed{2}$ 30	50	10	$u_1 = 0$
70	$\boxed{6}$ 30	$\boxed{3}$ 40	60	$u_2 = 0$
40	8	$\boxed{4}$ 70	$\boxed{14}$ 20	$u_3 = 30$

$v_1 = 19 \quad v_2 = 30 \quad v_3 = 40 \quad v_4 = -10$

cells	eqns	values
$(1,1)$	$u_1 + v_1 = 19$ $0 + v_1 = 19$	$v_1 = 19$
$(1,2)$	$u_1 + v_2 = 30$ $0 + v_2 = 30$	$v_2 = 30$
$(2,2)$	$u_2 + v_2 = 30$ $u_2 = 0$	$u_2 = 0$
$(2,3)$	$u_2 + v_3 = 40$ $v_3 = 40$	$v_3 = 40$
$(3,3)$	$u_3 + v_3 = 70$ $u_3 = 30$	$u_3 = 30$
$(3,4)$	$u_3 + v_4 = 20$ $3C + v_4 = 20$	$v_4 = -10$

step :- 3

u_i

$\boxed{5}$ 19	$\boxed{2}$ 30	(40) 50	(20) 10	0
(51) 70	6 30	$\boxed{3}$ 40	(70) 60	0
(-9) 40	(-52) 8	$\boxed{4}$ 70	$\boxed{14}$ 20	30

v_j　　19　　　30　　　40　　　−10

cell	eqn	cell evaluation:
$(1,3)$	$50 - (u_1 + v_3)$	40
$(1,4)$	$10 - (u_1 + v_4)$	20
$(2,1)$	$70 - (u_2 + v_1)$	51
$(2,4)$	$60 - (u_2 + v_4)$	70
$(3,1)$	$40 - (u_3 + v_1)$	-9
$(3,2)$	$8 - (u_3 + v_2)$	-52

If all the cells evaluations are non-negative then the solution is optimal.

Here cell evaluation -9, -52 are negative

So sol is not optimal

∴ Entering cell $(3,2)$

$\boxed{5}$ 19	$\boxed{2}$ 30	50	10
70	$\boxed{6}$ $30_\uparrow$ $6-\theta$ →	$\boxed{3}$ 40 $3+\theta$ ↓	60
40	θ $8_{(+)}$ ←	$\boxed{4}$ -70 $4-\theta$	$\boxed{14}$ 20

Form a loop starting from $(3,2)$

$$\theta \Rightarrow \min \{4-\theta, 6-\theta\} = 0$$

$$\therefore 4-\theta = 0$$

$$\theta = 4$$

$\boxed{5}$ 19	$\boxed{2}$ 30	50	10
70	$\boxed{2}$ 30	$\boxed{7}$ 40	60
40	$\boxed{4}$ 8	$\boxed{0}$ 70	$\boxed{14}$ 20

leaving cell

⑤ 19	② 30	50	10	$u_1 \quad 0$
70	② 30	⑦ 40	60	$u_2 \quad 0$
40	④ 8	70	⒁ 20	$u_3 \quad -22$

$V_1 = 19 \quad V_2 = 30 \quad V_3 = 40 \quad V_4 = 42$

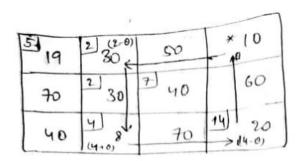| cells	eqn	values
$(1,1)$	$u_1 + V_1 = 19$ $0 + V_1 = 19$	$V_1 = 19$
$(1,2)$	$u_1 + V_2 = 30$ $0 + V_2 = 30$	$V_2 = 30$
$(2,2)$	$u_2 + V_2 = 30$ $u_2 = 0$	$u_2 = 0$
$(3,2)$	$u_3 + V_2 = 8$ $u_3 = -22$	$u_3 = -22$
$(3,4)$	$V_3 + V_4 = 20$ $-22 + V_4 = 20$	$V_4 = 42$
$(2,3)$	$u_2 + V_3 = 40$ $V_3 = 40$	$V_3 = 40$

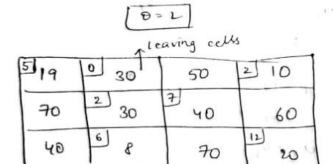| ⑤ 19	② 30	(10) 50	(-32) 10
(51) 70	② 30	⑦ 40	(18) 60
(43) 40	④ 8	(62) 70	⒁ 20

Here $(1,4)$ cell evaluation is -32

i.e negative

∴ Entering cell $(1,4)$

$\boxed{5}$ 19	$\boxed{2}$ 30 $^{(2-\theta)}$	50	× 10
70	$\boxed{2}$ 30	$\boxed{7}$ 40	60
40	$\boxed{4}$ 8 $_{(4+\theta)}$	70	$\boxed{14}$ 20 $_{(14-\theta)}$

Form a loop starting from the cell $(1,4)$

$\theta = \min \{ 2-0, 14-0 \} = 0$

$2-\theta = 0$

$$\boxed{\theta = 2}$$

↑ leaving cells

$\boxed{5}$ 19	$\boxed{0}$ 30	50	$\boxed{2}$ 10
70	$\boxed{2}$ 30	$\boxed{7}$ 40	60
40	$\boxed{6}$ 8	70	$\boxed{12}$ 20

$\boxed{5}$ 19	30	50	$\boxed{2}$ 10	$u_1 = 0$
70	$\boxed{2}$ 30	$\boxed{7}$ 40	60	$u_2 = 32$
40	$\boxed{6}$ 8	70	$\boxed{12}$ 20	$u_3 = 10$

$v_1 = 19$　$v_2 = -2$　$v_3 = 8$　$v_4 = 10$

cell	eqn	value
$(1,1)$	$u_1 + v_1 = 19$ $0 + v_1 = 19$	$v_1 = 19$
$(1,4)$	$u_1 + v_4 = 10$ $0 + v_4 = 10$	$v_4 = 10$
$(2,2)$	$u_2 + v_2 = 30$ $u_2 = 32$	$u_2 = 32$
$(3,4)$	$u_3 + v_4 = 20$ $u_3 = 10$	$u_3 = 10$

$(3, 2)$ $u_3 + v_2 = 8$ $v_2 = -2$

                        $v_2 = -2$

$(2, 3)$ $u_2 + v_3 = 40$ $v_3 = 8$

                        $v_3 = 8$

$(3, 4)$ $u_3 + v_4$

[5] 19	(32) 30	(42) 50	[2] 10
(19) 70	[2] 30	[7] 40	(18) 60
(11) 40	[6] 8	(52) 70	[12] 20

Now all the cell evaluations are non-negative

Transportation cost

$5*19 + 2*10 + 2*30 + 7*40 + 6*8$

$\qquad + 12*20$

$95 + 20 + 60 + 280 + 48 + 240$

$\qquad\qquad = 743$