2.

	X	Y	Availabilty
Eggs	1	1	5
Nutella	3	2	12
profit	6	5	

Maximize $Z = 6x + 5y$

Subject to $1x + 1y \le 5$

$3x + 2y \le 12$

Maximize $z = 6x + 5y + 0S_1 + 0S_2$

subject to $1x + 1y + S_1 = 5$

$3x + 2y + S_2 = 12$

	C_j	6	5	0	0		
C_B	Basic variables	x	y	S_1	S_2	sol	Ratio(min)
0	S_1	1	1	1	0	5	$5/1 = 5$
0	S_2	3	2	0	1	12	$12/3 = 4$
	Z_j	0	0	0	0		
	$C_j - Z_j$	6	5	0	0		

Here all $c_j - z_j$ must be ≤ 0 since it is failed we need to go for next iteration

Entering $= x_1$ Leaving $= S_2$ key element $= 3$

	C_j	6	5	0	0		
C_B	Basic variables	x	y	S_1	S_2	sol	Ratio (min)
0	S_1	0	$1/3$	1	$-1/3$	1	$1/\frac{1}{3} = 3$
6	x_1	1	$2/3$	0	$1/3$	4	$4/\frac{2}{3} = 6$
	Z_j	6	4	0	2		
	$C_j - Z_j$	0	1	0	-2		

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Entering = y leaving = s_1 key-element = 1/3

C_B	Basic variables	c_j: 6 x	5 y	0 s_1	0 s_2	sol	Ratio(min)
5	y	0	1	3	-1	3	
6	x	1	0	-2	1	2	
	z_j	6	5	3	1		
	$c_j - z_j$	0	0	-3	-1		

$\frac{1}{3} - \frac{-1}{3} \times \frac{2}{3} \times \frac{3}{1}$

$4 - 1 \times \frac{2}{3} \times \frac{3}{1}$

$\because c_j - z_j \leq 0$

optimal solution is

$x = 2 \quad y = 3$

$$\text{Max} = 6x + 5y = 6(2) + 5(3) = 27$$

4.

	S1	S2	Availability
Chemical A	12	24	480
Chemical B	[redacted] 9	5	180
Chemical C	30	30	720
profit	100	85	

Maximize $z = 100x + 85y$

subject to $12x + 24y \leq 480$

$\Rightarrow 3x + 6y \leq 120$

$\Rightarrow x + 2y \leq 40$ — (1)

$9x + 5y \leq 180$ — (2)

$3\cancel{0}x + 3\cancel{0}y \leq 72\cancel{0}$ — (3)

$x + y \le 24$ — (3)

$z = 100x + 85y$

$x + 2y \le 40$

$9x + 5y \le 180$

$x + y \le 24$

becomes $x + 2y + s_1 = 40$

$9x + 5y + s_2 = 180$

$x + y + s_3 = 24$

$z = 100x + 85y + 0s_1 + 0s_2 + 0s_3$

C_B	Basic variables	x	y	s_1	s_2	s_3	Sol	Ratio
	c_j	100	85	0	0	0		
0	s_1	1	2	1	0	0	40	40
0	s_2	9	5	0	1	0	180	20
0	s_3	1	1	0	0	1	24	24
	z_j	0	0	0	0	0		
	$c_j - z_j$	100	85	0	0	0		

Entering = x   leaving = s_2 key-element = 9

C_B	Basic variables	x	y	s_1	s_2	s_3	Sol	Ratio
	c_j	100	85	0	0	0		
0	s_1	0	13/9	1	-1/9	0	20	$20 \times \frac{9}{13} = 13.8$
100	x	1	5/9	0	1/9	0	20	$20 \times \frac{9}{5} = 36$
0	s_3	0	4/9	0	-1/9	1	4	$4 \times \frac{9}{4} = 9$
	z_j	100	$\frac{500}{9}$	0	$\frac{100}{9}$	0		
	$c_j - z_j$	0	$\frac{265}{9}$	0	$\frac{-100}{9}$	0		

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Entering = y   leaving = s_3 key-element = 4/9

		c_j	100	85	0	0	0		
C_B	Basic variables		x	y	s_1	s_2	s_3	Sol	Ratio
0	s_1		0	0	1	1/4	-13/4	7	
100	x		1	0	0	1/4	-5/4	15	
85	y		0	1	0	-1/4	9/4	9	
		z_j	100	85	0	$\frac{15}{4}$	66.25		
		$c_j - z_j$	0	0	0	-15/4	-66.25		

$\because$ All $c_j - z_j \le 0$

$x = 15 \Rightarrow s_1 = 15$ (solvent 1)

$y = 9 \Rightarrow s_2 = 9$ (solvent 2)

Max $z = 100(15) + 85(9)$

$= 1500 + 765$

Max $z = 2265$

Maximum profit = \$2265

$0 - \frac{1 \times 5/9}{4/9}$

$-5/4$

$0 - \frac{1 \times 13/9}{4/9}$

$\frac{1}{9} - \frac{(-\frac{1}{9})(\frac{5}{9})}{4/9}$

$\frac{4+5}{36}$

$-\frac{1}{9} - \frac{(\frac{-1}{9})(\frac{13}{9})}{4/9}$

$\frac{-4+13}{36}$

$20 - \frac{4 \times 5/9}{4/9}$

$20 - \frac{4 \times 13/9}{4/9}$

6.

Minimize $z = 4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$

As the constraint - 1 is of type '≥' we should subtract surplus variable S_1 and add Aritifical variable A_1

As the constraint - 2 is of type '≤' we should add slack variable S_2

As the constraint - 3 is of type '≥' we should subtract surplus variable S_3 and add Aritificial variable A_2

Minimize

$$z = 4x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2$$

subject to

$$2x_1 + x_2 - S_1 + A_1 = 10$$
$$-3x_1 + 2x_2 + S_2 = 6$$
$$x_1 + x_2 - S_3 + A_2 = 6$$

	c_j	4	3	0	0	0	M	M		
C_B	Basic Variables	x_1	x_2	S_1	S_2	S_3	A_1	A_2	Sol	Ratio
M	A_1	2	1	-1	0	0	1	0	10	$10/2 = 5$
0	S_2	-3	2	0	1	0	0	0	6	$6/-3 = -2$
M	A_2	1	1	0	0	-1	0	1	6	$6/1 = 6$
	z_j	3M	2M	-M	0	-M	M	M		
	$c_j - z_j$	4-3M	3-2M	M	0	M	0	0		

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	c_j	4	3	0	0	0	M		
C_B	Basic variables	x_1	x_2	s_1	s_2	s_3	A_2	sol	Ratio
4	x_1	1	$1/2$	$-1/2$	0	0	0	5	10
0	s_2	0	$7/2$	$-3/2$	1	0	0	21	6
M	A_2	0	$1/2$	$1/2$	0	-1	1	1	2
	z_j	4	$\frac{M}{2}+2$	$\frac{M}{2}-2$	0	$-M$	M		
	$c_j - z_j$	0	$1-\frac{M}{2}$	$2-\frac{M}{2}$	0	M	0		

	c_j	4	3	0	0	0	
C_B	Basic variable	x_1	x_2	s_1	s_2	s_3	sol
4	x_1	1	0	-1	0	1	4
0	s_2	0	0	-5	1	7	14
3	x_2	0	1	1	0	-2	2
	z_j	4	3	-1	0	-2	
	$c_j - z_j$	0	0	1	0	2	

$\because$ All $c_j - z_j \geq 0$

$21 - \frac{1 \times 7/2}{1/2}$

optimal solution

$x_1 = 4, \; x_2 = 2$

Min $z = 22$