Mathematical Programming

- Management science is characterized by a scientific approach to managerial decision making.
- Management science has been known by a variety of other names
  - operations research in US
  - operational research in Great Britan

To identify the scientific approach to managerial problem solving under such other names as systems analysis, cost–benefit analysis, and cost-effectiveness analysis

Management science

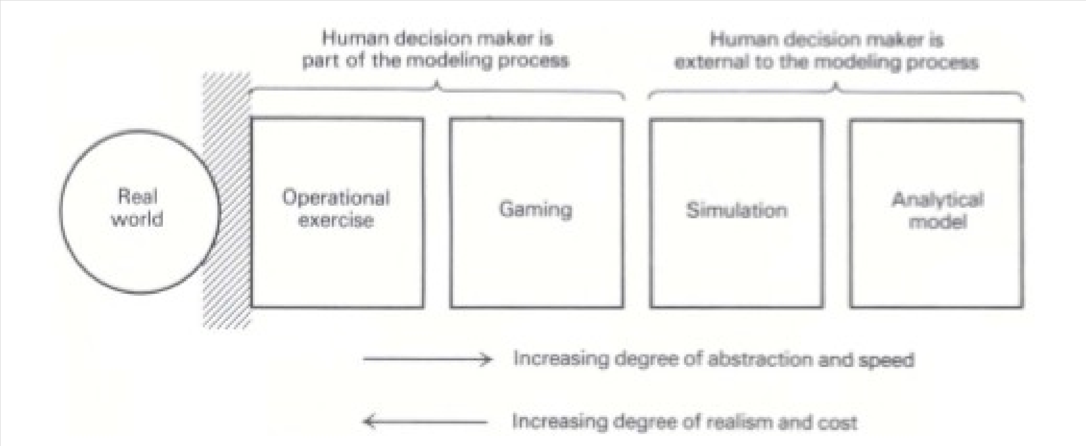
- Management science is characterized by the use of mathematical models in providing guidelines to managers for making effective decisions within the state of the current information, or in seeking further information if current knowledge is insufficient to reach a proper decision

- There are several elements of this statement that are deserving of emphasis
- First, the essence of management science is the model-building approach— that is, an attempt to capture the most significant features of the decision under consideration by means of a mathematical abstraction.
- Second, through this model-design effort, management science tries to provide guidelines to managers or, in other words, to increase managers' understanding of the consequences of their actions. The aim is to support the managers.

- Finally, it is the complexity of the decision under study, and not the tool being used to investigate the decision-making process, that should determine the amount of information needed to handle that decision effectively.

MODEL CLASSIFICATION

- The management-science literature includes several approaches to classifying models.



Types of model representation

The first model type is an operational exercise

This approach to operate successfully, it is mandatory to

design experiments to be conducted carefully

To evaluate the experimental results in light of errors that can be introduced by measurement inaccuracies,

To draw inferences about the decisions reached, based upon the limited number of observations performed

Gaming

- The second type of model in this classification is gaming
- In this case, a model is constructed that is an abstract and simplified representation of the real environment.
- This model provides a responsive mechanism to evaluate the effectiveness of proposed alternatives, which the decision-maker must supply in an organized and sequential fashion.

Gaming

- The model is simply a device that allows the decision-maker to test the performance of the various alternatives that seem worthwhile to pursue.
- In addition, in a gaming situation, all the human interactions that affect the decision environment are allowed to participate actively by providing the inputs they usually are responsible for in the actual realization of their activities.
- The model should reflect, with an acceptable degree of accuracy, the relationships between the inputs and outputs of the refinery process.

- Subsequently, all the personnel who participate in structuring the decision process in the management of the refinery would be allowed to interact with the model.
- The production manager would establish production plans
- The marketing manager would secure contracts and develop marketing strategies
- The purchasing manager would identify prices and sources of crude oil and develop acquisition programs, and so forth.

- The cost of processing each alternative has been reduced, and the speed of measuring the performance of each alternative has been increased.
- Gaming is used mostly as a learning device for developing some appreciation for those complexities inherent in a decision-making process.
- Several management games have been designed to illustrate how marketing, production, and financial decisions interact in a competitive economy.

Simulation

- Simulation models are similar to gaming models except that all human decision-makers are removed from the modeling process.
- The model provides the means to evaluate the performance of a number of alternatives, supplied externally to the model by the decision-maker, without allowing for human interactions at intermediate stages of the model computation.

- Like operational exercises and gaming, simulation models neither generate alternatives nor produce an optimum answer to the decision under study.
- These types of models are inductive and empirical in nature; they are useful only to assess the performance of alternatives identified previously by the decision-maker.
- Many simulation models take the form of computer programs, where logical arithmetic operations are performed in a prearranged sequence.

Analytical Model

- Finally, the fourth model category proposed in this framework is the analytical model
- In this type of model, the problem is represented completely in mathematical terms
- Normally by means of a criterion or objective, which we seek to maximize or minimize
- Subject to a set of mathematical constraints that portray the conditions under which the decisions have to be made.

- The model computes an optimal solution, that is, one that satisfies all the constraints and gives the best possible value of the objective function.
- Most of the work undertaken by management scientists has been oriented toward the development and implementation of analytical models.

Classification of Analytical and Simulation Models

	Strategy evaluation	*Strategy generation*
Certainty	Deterministic simulation Econometric models Systems of simultaneous equations Input-output models	Linear programming Network models Integer and mixed-integer programming Nonlinear programming Control theory
Uncertainty	Monte Carlo simulation Econometric models Stochastic processes Queueing theory Reliability theory	Decision theory Dynamic programming Inventory theory Stochastic programming Stochastic control theory

Statistics and subjective assessment are used in all models to determine values for parameters of the models and limits on the alternatives.

The classification presented in Table is not rigid, since strategy evaluation models are used for improving decisions by trying different alternatives until one is determined that appears‘‘best.’

Session-2
Mathematical Modeling of Linear Programming Problem

Example Problem

- A small cooperative craft workshop makes two types of table: a standard rectangular table and a de luxe circular table. The market can absorb as many of either type of table as the workshop can produce, and so we can assume unlimited demand.
- Each type of table is made from the same wood and, once the wood has been cut, each table has to go through three processes: joinery, pre finishing and final finishing (in that order). Sufficient cut wood is always available. Each rectangular table takes 2 hours for joinery, 40 minutes for pre finishing and 5 hours 20 minutes for final finishing.

Cont…

- Each circular table requires 3 hours for joinery, 2 hours for pre-finishing and 4 hours for final finishing. The workshop employs five joiners, two sanders and eight polishers.
- The joiners each work a fixed six-hour day, while the sanders and polishers each work a fixed eight-hour day on the pre finishing and final finishing respectively.
- No overtime is worked, and full six-hour or eight-hour days are worked by each employee irrespective of whether there is work for that employee to do. All running costs, including wages, are fixed.
- The cooperative sells each rectangular table for £120 and each circular table for £150. How many of each type of table should it produce each day in order to maximize its profit?

Formulating the problem

- This example is obviously simplified, but it includes features common to many mathematical programming problems:
- There is a quantity (the profit) to be maximized and there are constraints (the available person-hours) on each process involved in making the tables.
- We want to formulate a mathematical model of the problem, where the purpose of the model is to decide how many of each type of table should be produced each day in order to maximize profit.

- Having specified the purpose of the model, the mathematical modelling process now suggests that we should state the assumptions.
- For the example, we can identify the following assumptions from the given description.

- There is unlimited demand for both types of table.
- Each table is made from the same type of wood.
- Sufficient cut wood is always available.
- No overtime is worked.
- Full days are worked irrespective of whether there is work to do.
- All running costs, including wages, are fixed.

Objective Function

- The quantity to be optimized in a mathematical programming problem is known as the objective function, and the objective is to optimize this function.

- The first step in formulating a linear programming model is to identify the objective and it's objective function.
- In this example, we want to maximize the daily profit, so we could take maximizing the daily profit as the objective and the daily profit as the objective function.
- However, the daily profit is equal to the daily income from selling tables less the fixed daily running costs (including the cost of wood and wages). So, since the running costs are fixed, maximizing daily profit is equivalent to maximizing daily income.

- As omitting the fixed running costs will simplify the model, we shall therefore take the objective to be to maximize daily income and the objective function to the daily income.
- The second step is to identify the variables, other than the objective function, and to specify their units of measurement.
- In this example, the only variables are the numbers of tables of each type made per day

- The third step is to identify the constraints and parameters. In this example the only constraints are the numbers of person-hours available for joinery, pre finishing and final finishing each day. The parameters are the time it takes for each process for each type of table, the hours available for each process each day, and the selling prices of the tables

- The fourth step is to assign algebraic symbols to the objective function, the variables and, if necessary, the parameters. It is usual in linear programming to use z for the objective function and x_1, x_2, … for the variables. So, in this case, we have:
- z the daily income, in pounds
- x_1 the number of rectangular tables made per day
- x_2 the number of circular tables made per day.

- We shall therefore adopt this approach here, so that, usually, instead of assigning symbols to the parameters, we shall state their numerical values in a table that relates them to the variables, the objective function and the constraints, as in Table

	Rectangle Table	Circular Table	upper limit(Per Day)
income (£)	120	150	-
joinery (hours)	2	3	30
Pre finishing (hours)	2/3	2	16
final finishing (hours)	5 1/3	4	64

- The next step is to derive algebraic relationships for the objective function and the constraints.
- Before deriving these relationships, it is important to remember that they must be linear.
- In this case, there is a clear linear relationship between the daily income and the numbers of tables sold, given by

$$z = 120x_1 + 150x_2$$

- This is the objective function and need to maximize it

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$$maximize\ z = 120x_1 + 150x_2$$

- The numbers of hours spent daily on each of joinery, pre finishing and final finishing are given, using Table , by the simple linear expressions

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- $$2x_1 + 3x_2 (Joinery)$$
- $$\frac{2}{3}x_1 + 2x_2 (Pre\ finishing)$$
- $$5\frac{1}{3}x_1 + 4x_2 (final\ finishing)$$

- The upper limits on the numbers of hours available for each of these processes each day can then be combined with these expressions to give the following linear constraints

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- $2x_1 + 3x_2 \leq 30$
- $\frac{2}{3}x_1 + 2x_2 \leq 16$
- $5\frac{1}{3}x_1 + 4x_2 \leq 64$

- Finally we must, as so often in mathematics, state the obvious: the cooperative cannot make a negative number of tables. So we must also include the constraints:

- We can write the objective and constraints succinctly as follows.

- Subject to

- Finally we must, as so often in mathematics, state the obvious: the cooperative cannot make a negative number of tables. So we must also include the constraints:

- $x_1 \geq 0$ and $x_2 \geq 0$

- We can write the objective and constraints succinctly as follows.

- $maximize\ z = 120x_1 + 150x_2$

- Subject to

$$2x_1 + 3x_2 \leq 30$$

$$\frac{2}{3}x_1 + 2x_2 \leq 16$$

$$5\frac{1}{3}x_1 + 4x_2 \leq 64$$

- $x_1 \geq 0$ and $x_2 \geq 0$

Formulating linear programming models

- Given a precise statement of a linear programming problem, including a specified purpose and any appropriate data and assumptions, the problem may be formulated as a linear programming model as follows.
- (a) Identify the objective of the model (for example maximizing profit or minimizing cost) and decide on the units in which the objective function is to be measured (for example units of currency).
- (b) Identify the variables and decide on the units in which each is to be measured.
- (c) Identify the constraints and parameters for the problem.
- (d) Assign algebraic symbols to the objective function (usually z) and to the variables (usually $x_1, x_2, \dots \dots, x_n$), and write down precise definitions, including units of measurement, for all of these. If necessary, assign algebraic symbols to the parameters as well.
- (e) Using a table of parameter values or otherwise, identify the linear relationships between the objective function and the variables and between the constraints and the variables, being careful to use consistent units when identifying these relationships

Formulating linear programming models

- (f) Write down the objective of the problem in the form optimize $z = c_1x_1 + c_2x_2 + \ldots \ldots . + c_n x_n$
- (g) Write down the non-trivial constraints where the ith non-trivial constraint is a linear relationship of one of the following forms:

$$
\begin{aligned}
a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n &\leqslant b_i;\\
a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n &\geqslant b_i;\\
a_{i1}x_i + a_{i2}x_2 + \cdots + a_{in}x_n &= b_i.
\end{aligned}
$$

- (h) Write down the non-negativity or trivial constraints xj > 0 (j = 1,2,…,n).

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- (h) Write down the non-negativity or trivial constraints xj> 0 (j = 1,2,…,n).

Example

- A food manufacturer buys edible oils in their raw state, and refines and blends them to produce margarine. The raw oils can be vegetable or non-vegetable. The vegetable oils are ground-nut, soya-bean and palm. The non-vegetable oils are lard and fish. The margarine sells at £2400 a tonne, and the manufacturer can sell as much margarine as can be made. The prices of the raw oils are shown in Table . Sufficient quantities of all five oils can be assumed to be readily available.

oil	ground-nut	soya-bean	palm	lard	fish
cost (£ per tonne)	960	1600	1600	2200	1900
hardness	1.2	3.4	8.0	10.8	8.3

- To avoid contamination, vegetable and non-vegetable oils are refined separately, using different equipment. It is possible to refine up to 1000 tones of vegetable oil and up to 800 tones of non-vegetable oil in a month. When refined, some or all of the oils are blended to produce the margarine.
- The quantities of oil lost during the refining and blending processes are negligible.

- The hardnesses of the refined oils, given in Table , are assumed to combine linearly to give the hardness of the margarine. To ensure that the margarine spreads easily but is not runny, the hardness of the margarine must be between 5.6 and 7.4, measured in the same units as the hardnesses of the oils in Table . All running costs can be taken to be fixed. The manufacturer wants to know how much of each type of oil to use in order to maximize profit while maintaining the quality of the margarine.

Formulation

We follow the steps of Procedure

(a) The purpose of the model is to maximize profit, subject to constraints on resources, while maintaining quality. Since the refining capacities are given per month, it is sensible to work on a monthly basis. We could therefore take maximizing monthly profit as the objective. However, since the running costs are fixed, maximizing monthly profit is equivalent to maximizing monthly income. Therefore, as omitting the fixed running cost will simplify the model, we shall take the objective as maximizing monthly income. This monthly income could be measured in pounds. However, looking at the quantities involved, it would seem more sensible to work in units of £10 000.

(b) The variables are the quantities of each oil used per month in blending the margarine. Also, as it will be useful to know the total quantity of margarine manufactured per month, we shall regard this as a variable.

(c)identify the parameters

(d) We now assign algebraic symbols to the objective function and the variables, giving precise definitions, including units of measurement:

- z the monthly profit, in tens of thousands of pounds;
- x1 the quantity, in hundreds of tones, of ground-nut oil used per month;
- x2 the quantity, in hundreds of tones, of soya-bean oil used per month;
- x3 the quantity, in hundreds of tones, of palm oil used per month;
- x4 the quantity, in hundreds of tones, of lard used per month;
- x5 the quantity, in hundreds of tones, of fish oil used per month;
- x6 the quantity, in hundreds of tones, of margarine manufactured per month.

(e) We next identify the linear relationships for the objective function and the constraints. We can do this by extending Table , and making

suitable adjustments to the numerical values to take into account the

- units of measurement decided on in (a) and (b), and listed in (d). This
- results in the following Table

	ground-nut oil	soya-bean oil	palm oil	lard	fish oil	upper limit	lower limit
cost (£10 000s per 100 tonnes)	9.6	16	16	22	19	—	—
refining of vegetable oils (100s of tonnes)	✓	✓	✓	—	—	10	—
refining of non-vegetable oils (100s of tonnes)	—	—	—	✓	✓	8	—
hardness (hardness units)	1.2	3.4	8.0	10.8	8.3	7.4	5.6

- (f) Determine the objective function and objective of the problem in terms of x1, . . . , x6
- (g) We now want to express the constraints algebraically in terms of the variables.
- The hardness constraints require some thought.
- Remember that the hardnesses of the components combine linearly to give the hardness of the margarine, which must be at least 5.6.

- Using Table this means that we must have

$$1.2x_1 + 3.4x_2 + 8.0x_3 + 10.8x_4 + 8.3x_5 \geqslant 5.6x_6.$$

- If, for consistency, we keep all variables to the left-hand side of inequality or equality signs, this becomes

$$1.2x_1 + 3.4x_2 + 8.0x_3 + 10.8x_4 + 8.3x_5 - 5.6x_6 \geqslant 0.$$

- We must also remember the quantity constraint, which can be written as

$$x_1 + x_2 + x_3 + x_4 + x_5 = x_6$$

- or, in the format of the other constraints, as

$$x_1 + x_2 + x_3 + x_4 + x_5 - x_6 = 0.$$

(h) Finally, as the manufacturer cannot use negative quantities of oil, we have the trivial constraints

$$x_1, x_2, x_3, x_4, x_5, x_6 \geqslant 0.$$

The linear programming model for Example is thus:

$$\text{maximize } z = -9.6x_1 - 16x_2 - 16x_3 - 22x_4 - 19x_5 + 24x_6$$

subject to

$$\begin{array}{rcll}
x_1 + x_2 + x_3 & \leqslant & 10 & \text{(vegetable refining capacity)} \\
x_4 + x_5 & \leqslant & 8 & \text{(non-vegetable refining capacity)} \\
1.2x_1 + 3.4x_2 + 8.0x_3 + 10.8x_4 + 8.3x_5 - 5.6x_6 & \geqslant & 0 & \text{(lower hardness)} \\
1.2x_1 + 3.4x_2 + 8.0x_3 + 10.8x_4 + 8.3x_5 - 7.4x_6 & \leqslant & 0 & \text{(upper hardness)} \\
x_1 + x_2 + x_3 + x_4 + x_5 - x_6 & = & 0 & \text{(quantity)} \\
x_1, x_2, x_3, x_4, x_5, x_6 & \geqslant & 0 & \blacksquare
\end{array}$$

Session-3
Finding the optimal solution using Graphical Method for a given Linear Programming Problem(LPP)

Topic delivery on Graphical Method

- Any optimization problem with two variables can be solved
- It is visual in nature
- For solving the problems graphically, we need the following
- Inequality Constraints
- Objective Functions

Outlines of Graphical Method

- The linear programming problems of two decision variables can be easily solved by graphical method.
- The outlines of the graphical procedure as follows:
- **Step: 1** Identify the problem-the decision variables, the objective function and the restrictions.
- **Step: 2** set up the mathematical formulation of the problem.
- **Step: 3** consider each inequality –constraint as an equation

- **Step: 4** Plot each equation on the graph, as each one will geometrically represent a straight line.
- **Step: 5** Shade the feasible region. Every point on the line will satisfy the equation of the line. If the inequality constraint corresponding to that line is '≤', then the region below the line lying in the first quadrant (due to non-negativity of variables) is shaded. For the inequality-constraint with '≥' sign, the region above the line in the first quadrant is shaded. The points lying in the common region will satisfy all the constraints simultaneously. The common region thus obtained is called the **feasible region**.

- **Step: 6**: Choose the convenient value of z(say-0) and plot the objective function line.
- **Step: 7** pull the objective function line until the extreme points of the feasible region. In the maximization case, this line will stop farthest from the origin and passing through at least one corner of the feasible region. In the minimization case, this line will stop nearest to the origin and passing through at least one corner of the feasible region.
- **Step: 8** Read the coordinates of the extreme point(s) selected in step6, and find the maximum or minimum value of z.

Students doubts with poll/pop questions.

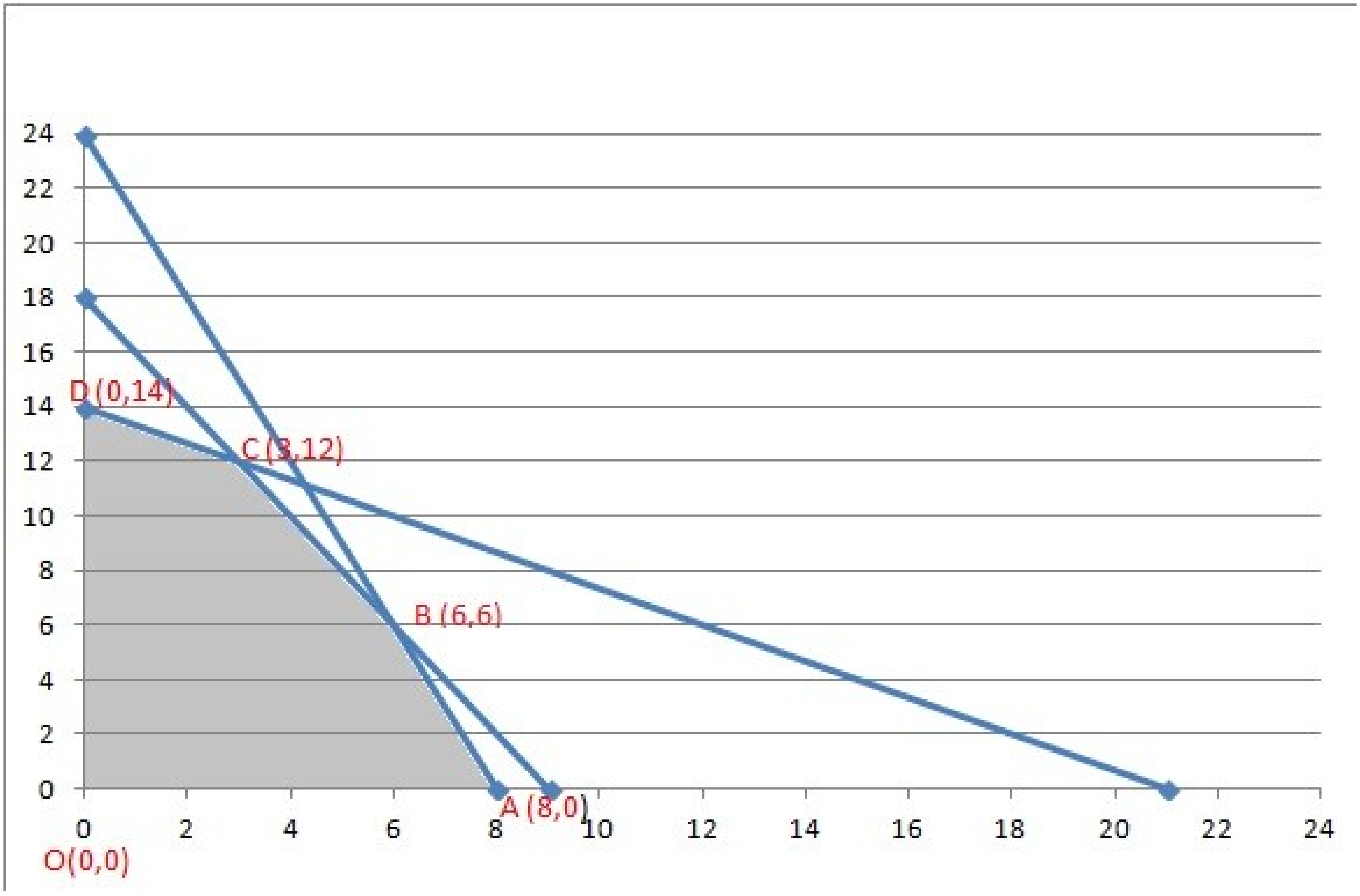
Questions

1. why do we use the graphical method?
2. why graphical method is so popular?
3. what is the constraint in graphical method?

Problem

- Using the graphical method, solve the following problem
- Max $Z=3x+2y$
- subject to the constraints
- $2x+y \le 18$, $2x+3y \le 42$, $3x+y \le 24$ and $y \ge 0$

- Solution:
- First, we draw the lines of all the given equations and shade the common region according to the signs.



24
22
20
18
16
14
12
10
8
6
4
2
0
0 2 4 6 8 10 12 14 16 18 20 22 24
D (0,14)
C (3,12)
B (6,6)
A (8,0)
O(0,0)

- The feasible region is the intersection of the regions defined by the set of constraints and the coordinate axis (conditions of non-negativity of variables). This feasible region is represented by the O-A-B-C-D-O
- As a feasible region exists, extreme values (or polygon vertices) are calculated. These vertices are the point's candidate as optimal solutions. In the example, these points are O, A, B, C, and D, as shown in the figure.

- Finally, the objective function is evaluated in each of these points (results are shown in the tableau below). Since C-point provides the greatest value to the Z-function and the objective is to maximize, this point is the optimal solution: $Z = 33$ with $x = 3$ and $y = 12$

Points	Coordinates	Value of Objective function
O	(0,0)	0
A	(8,0)	24
B	(6,6)	30
C	(3,12)	33
D	(0,14)	28

Find the maximum value of

$$Z = 2x_1 + 3x_2,$$

$$\text{subject to } x_1 + x_2 \le 30,$$

$$x_2 \ge 3,$$

$$x_2 \le 12,$$

$$x_1 - x_2 \ge 0,$$

$$0 \le x_1 \le 20.$$

Find the minimum value of

$$Z = 5x_1 - 2x_2,$$

$$\text{subject to} \qquad 2x_1 + 3x_2 \geq 1,$$

$$x_1, x_2 \geq 0.$$

Formulation of an LPP and finding optimal solution using Graphical method

Let x_1 and x_2 denote the number of old hens and young hens that must be bought.

Objective is to maximize the profit per week. Since old hens lay 3 eggs/week and the young ones lay 5 eggs/week, the total number of eggs laid per week is $3x_1 + 5x_2$ which yields earnings/week of ₹ $[0.3(3x_1 + 5x_2)]$. As a hen costs ₹ 1 to be fed, the total cost of feeding the hens per week is ₹ $[1(x_1 + x_2)]$.

Thus the profit earned per week is ₹ $[0.3\ (3x_1 + 5x_2) - 1(x_1 + x_2)]$ = ₹ $[-\ 0.1x_1 + 0.5\ x_2]$.

Thus the objective function is

$$\text{maximize } Z = ₹\ [-\ 0.1x_1 + 0.5x_2],$$

where $x_1, x_2 \geq 0$.

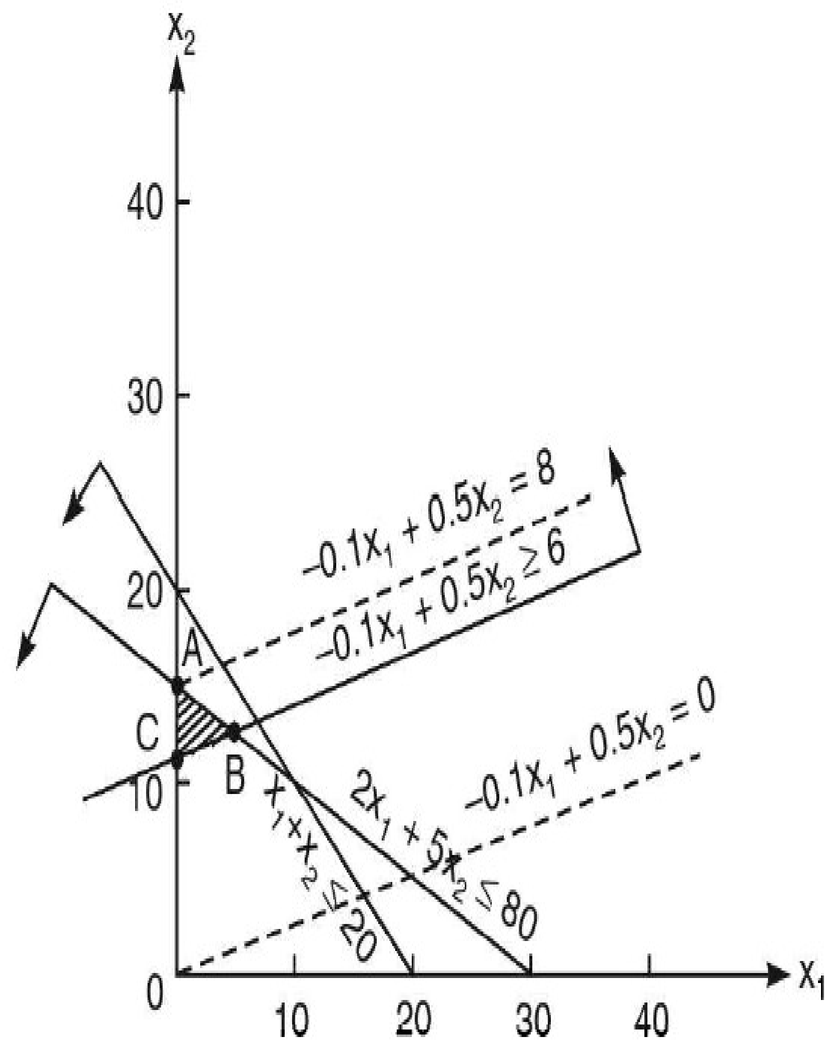
Constraints are

on the amount of money, $2x_1 + 5x_2 \leq 80$,

on the number of hens that can be housed, $x_1 + x_2 \leq 20$,

on the minimum profit to be achieved, $-\ 0.1x_1 + 0.5x_2 \geq 6$.

The solution space satisfying the constraints $2x_1 + 5x_2 \le 80$, $x_1 + x_2 \le 20$, $-0.1x_1 + 0.5x_2 \ge 6$ and meeting the non-negativity restrictions $x_1 \ge 0$, $x_2 \ge 0$ is shown shaded in Fig.



The coordinates of the vertices of the convex polygon ABC are A(0, 16), B(20/3, 40/3) and C(0, 12).

Values of the objective function $Z = -0.1x_1 + 0.5x_2$ at these vertices are Z(A) = ₹ 8, Z(B) = ₹ [– 2/3 + 20/3] = ₹ 6, Z(C) = ₹ 6.

Since the maximum value of Z is ₹ 8, which occurs at the vertex A(0, 16), the solution to the given problem is

$$x_1 = 0,\ x_2 = 16,\ Z_{max} = ₹\ 8.$$

Session-4
Formulating a given problem as a Linear Programming Problem(LPP)

Topic delivery

- Formulating the given problem as a Linear Programming Problem (LPP)
- Graphical Method
- Solving the formulated LPP using Graphical Method to find the optimal solution

Formulating the given problem as a Linear Programming Problem (LPP)

Problem

A person wants to decide the constituent of a diet which will fulfill his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the following table.

Food Type	Yeild per unit			Cost per Unit
	Protiens	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Formulate linear programming problem for the given problem.

Solution:

- Let x_1, x_2, x_3 and x_4 denote the number of units of food type 1,2,3 and 4 respectively. The objective is to minimize the cost.

$$\text{Minimize } Z=45x_1+40x_2+85x_3+65x_4$$

subject to the constraints

$$3x_1+4x_2+8x_3+6x_4>=800$$

$$2x_1+2x_2+7x_3+5x_4>=200$$

$$6x_1+4x_2+7x_3+4x_4>=700$$

where x_1, x_2, x_3 and $x_4>=0$.

This is called an Linear Programming Problem(LPP).

Students doubts with poll/pop questions.

Questions

1. what are x1, x2, x3 and x4 in this problem.
2. what are the constituents of an LP problem?

Problems as Assignments/Quiz
Doubts can be asked in public chat.

Problem

A Firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.

Machines	Time per unit (minutes)			Machine capacity
	Product-1	Product-2	Product-3	
M1	2	3	2	440
M2	4	--	3	470
M3	2	5	--	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product-1, product-2 and product-3 is 4Rs,3Rs and 6Rs respectively. It is assumed that, all the amounts produced are consumed in the market. Formulate the mathematical model LPP that will maximize the daily profit.

Solution:

Let the amounts of products 1,2 and 3 manufactured daily be x1,x2 and x3 units respectively. Clearly x1,x2 and x3 are clearly >=0

In this problem, The objective function is

Maximize $Z=4x_1+3x_2+6x_3$

Since, the constraints are on machine capacities and can be mathematically expressed as

$$2x_1+3x_2+2x_3<=440$$

$$4x_1+0x_2+3x_3<=470$$

$$2x_1+5x_2+0x_3<=430$$

where x_1,x_2,x_3 are >=0.

Hence, The given problem can be formulated as an LP problem by

Maximize $Z=4x_1+3x_2+6x_3$

Subject to the constraints

$$2x_1+3x_2+2x_3<=440$$

$$4x_1+0x_2+3x_3<=470$$

$$2x_1+5x_2+0x_3<=430$$

where x_1,x_2,x_3 are $>=0$

Problem Discussion Duration:5minutes

Session-5
Simplex Method

Simplex Method

Steps to follow:

Step:1 Check whether the objective function of the given L.P.P is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximizing it by using the result Minimum Z = - Maximum(-z).

Step: 2 Check whether all b_i (i=1,2, ..., m) are non-negative. If any one of b_i is negative then multiply the corresponding in equation of the constraints by -1, so as to get all b_i(i=1,2,..., m) non-negative.

Step: 3 convert all the in equations of the constraints into equation by introducing slack/surplus and artificial variables in the constraints. Put the costs of slack/surplus variables equal to zero and the cost of artificial variable -1 or M (depending on the method) in modified objective function.

Step: 4 Obtain an initial basic feasible solution by setting $x_1=x_2=\ldots=x_n=0$ in equations obtained in step 3.

Step: 5 Determine which variable to enter into the solution next. Identify the column— hence the variable—with the largest positive number in the *Cj_ Zj* row of the previous tableau. This step means that we will now be producing some of the product contributing the greatest additional profit per unit.

Step: 6 Determine which variable to replace. Because we have just chosen a new variable to enter into the solution, we must decide which variable currently in the solution to remove to make room for it. To do so, we divide each amount in the quantity column by the corresponding number in the column selected in the step 5. The row with the *smallest nonnegative number* calculated in this fashion will be replaced in the next tableau. This row is often referred to as the **pivot row**, and the column identified in step 5 is called the **pivot column**. The number at the intersection of the pivot row and pivot column is the **pivot number**

Step 7: Compute new values for the pivot row. To find them, we simply divide every number in the row by the *pivot number*.

The new coefficients of the tableau are calculated as follows:

In the pivot row each new value is calculated as: *New value = Previous value / Pivot*

In the other rows each new value is calculated as:

*New value = Previous value - (Previous value in pivot column * New value in pivot row)*

Step 8: Compute the Z_j and $C_j - Z_j$ rows, as demonstrated in the initial tableau. If all numbers in the $C_j - Z_j$ row are zero or negative, we have found an optimal solution. If this is not the case, we must return to step 5.

Session-6

Big-M Method

Step: 1

- Write the given LPP into its standard form and check whether there exists a starting basic feasible solution.
- If there is ready starting basic feasible solution, move on to step 3.
- If there does not exist a ready starting basic feasible solution, move on to step 2.

Step: 2

- Add artificial variable to the left side of each equation that has no obvious starting basic variables.

- Assign a very high penalty (say M) to these variables in the objective function.

Step:3

- Apply simplex method to the modified LPP following cases may arise at the last iteration:

- At least one artificial variable is present in the basis with zero value.

- In such a case the current optimum basic feasible solution is degenerate.

- At least one artificial variable is present in the basis with a positive value, in such a case, the given LPP does not possess an optimum basic feasible solution.

- The given problem is said to have a pseudo-optimum basic feasible solution.

- No artificial variable in the basis and at least one 0, in such a case apply usual simplex algorithm to the modified simplex table to get the optimum solution of the original problem.