MP-1 TUTORIAL-7

1. Demonstrate the Initial Basic Solution in Transportation problem using NW method in Linear Programming (U-V method).

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Tutorial-7

NWCR

5 19	2 30	50	10	7 2 0
70	6 30	3 40	60	9 3 0
40	8	4 70	14 20	18 14 0
5 0	8 6 0	7 4 0	14 0	

Transportation cost =

$5*19 + 2*30 + 6*30 + 3*40 + 4*70 + 14*20$

$95 + 60 + 180 + 120 + 280 + 280$

1015

U-V Method

5 19	2 30	50	10	7
70	6 30	3 40	60	9
40	8	4 70	14 20	18
5	8	7	14	

Step : 1 m = no. of origins = 3

n = no. of destinations = 4

$m+n-1 = 6 \Rightarrow$ No. of allocations = 6

And the allocations are in independent positions ∴ The problem is non-degenerate

Step - 2 u_i, v_j - allocated cells

$u_i + v_j = c_{ij}$

[5] 19	[2] 30	50	10	$u_1 = 0$
70	[6] 30	[3] 40	60	$u_2 = 0$
40	8	[4] 70	[14] 20	$u_3 = 30$
$v_1 = 19$	$v_2 = 30$	$v_3 = 40$	$v_4 = -10$	

cells	eqns	values
(1,1)	$u_1 + v_1 = 19$ $0 + v_1 = 19$	$v_1 = 19$
(1,2)	$u_1 + v_2 = 30$ $0 + v_2 = 30$	$v_2 = 30$
(2,2)	$u_2 + v_2 = 30$ $u_2 = 0$	$u_2 = 0$
(2,3)	$u_2 + v_3 = 40$ $v_3 = 40$	$v_3 = 40$
(3,3)	$u_3 + v_3 = 70$ $u_3 = 30$	$u_3 = 30$
(3,4)	$u_3 + v_4 = 20$ $30 + v_4 = 20$	$v_4 = -10$

Step :- 3

				u_i
[5] 19	[2] 30	(40) 50	(20) 10	0
(51) 70	[6] 30	[3] 40	(70) 60	0
(-9) 40	(-51) 8	[4] 70	[14] 20	30
v_j 19	30	40	-10	

cell	eqn	cell evaluation:
(1,3)	$50-(u_1+v_3)$	40
(1,4)	$10-(u_1+v_4)$	20
(2,1)	$70-(u_2+v_1)$	51
(2,4)	$60-(u_2+v_4)$	70
(3,1)	$40-(u_3+v_1)$	-9
(3,2)	$8-(u_3+v_2)$	-52

If all the cells evaluations are non-negative then the solution is optimal.

Here cell evaluation -9, -12 are negative

So sol is not optimal

∴ Entering cell (3,2)

5 19	2 30	50	10
70	6 30 (6-θ)	3 40 (3+θ)	60
40	θ 8 (4)	4 70 (4-θ)	14 20

Form a loop starting from (3,2)

$\theta \Rightarrow \min\{4-\theta, 6-\theta\} = 0$

$\because 4-\theta = 0$

$\theta = 4$

5 19	2 30	50	10
70	2 30	7 40	60
40	4 8	0 70	14 20

↑ leaving cell

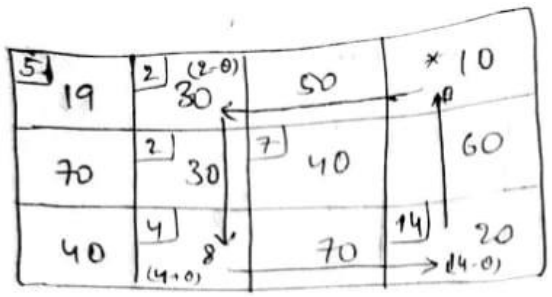
[5] 19	[2] 30	50	10	$u_1 = 0$
70	[2] 30	[7] 40	60	$u_2 = 0$
40	[4] 8	70	[14] 20	$u_3 = -22$
$v_1 = 19$	$v_2 = 30$	$v_3 = 40$	$v_4 = 42$	

cells	eqn	values
(1,1)	$u_1 + v_1 = 19$	$v_1 = 19$
	$0 + v_1 = 19$	
(1,2)	$u_1 + v_2 = 30$	$v_2 = 30$
	$0 + v_2 = 30$	
(2,2)	$u_2 + v_2 = 30$	$u_2 = 0$
	$u_2 = 0$	
(3,2)	$u_3 + v_2 = 8$	$u_3 = -22$
	$u_3 = -22$	
(3,4)	$u_3 + v_4 = 20$	$v_4 = 42$
	$-22 + v_4 = 20$	
(2,3)	$u_2 + v_3 = 40$	$v_3 = 40$
	$v_3 = 40$	

[5] 19	[2] 30	(10) 50	(32) 10
(51) 70	[2] 30	[7] 40	(18) 60
(43) 40	[4] 8	(62) 70	[14] 20

Here (1,4) cell evaluation is -32
i.e negative

∴ Entering cell (1,4)



Form a loop starting from the cell (1,4)

$\theta = \min\{2-\theta, 14-\theta\} = 0$

$2-\theta = 0$

$\boxed{\theta = 2}$

leaving cells

5 19	0 30	50	2 10
70	2 30	7 40	60
40	6 8	70	12 20

5 19	30	50	2 10	$u_1 = 0$
70	2 30	7 40	60	$u_2 = 32$
40	6 8	70	12 20	$u_3 = 10$
$v_1 = 19$	$v_2 = -2$	$v_3 = 8$	$v_4 = 10$	

cell	eqn	value
(1,1)	$u_1 + v_1 = 19$ $0 + v_1 = 19$	$v_1 = 19$
(1,4)	$u_1 + v_4 = 10$ $0 + v_4 = 10$	$v_4 = 10$
(2,2)	$u_2 + v_2 = 30$ $u_2 = 32$	$u_2 = 32$
(3,4)	$u_3 + v_4 = 20$ $u_3 = 10$	$u_3 = 10$

(3,2) $u_3 + v_2 = 8$ $v_2 = -2$

$v_2 = -2$

(2,3) $u_2 + v_3 = 40$ $v_3 = 8$

$v_3 = 8$

~~(3,4)~~ ~~$u_3 + v_4$~~

5 19	(22) 30	(42) 50	2 10
(19) 70	2 30	7 40	(18) 60
(11) 40	6 8	(52) 70	12 20

Now all the cell evaluations are non-negative

Transportation cost

$5*19 + 2*10 + 2*30 + 7*40 + 6*8 + 12*20$

$95 + 20 + 60 + 280 + 48 + 240$

$= 743$