CO1 HOME ASSIGNMENT

1. Maximize $z = x_1 + 2x_2$, subject to: $x_1 - 2x_2 \leq 3$, $x_1 + x_2 \leq 3$, $x_1, x_2 \geq 0$.

MP1
Home Assignment

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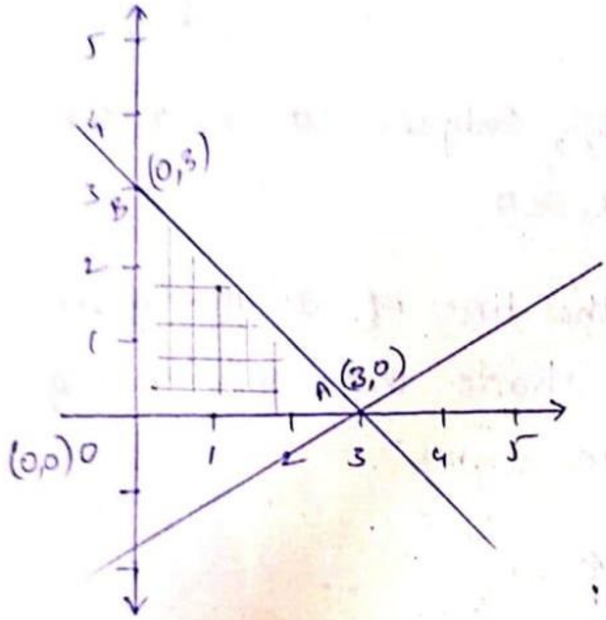
i) First, we draw the lines of all the given equations and shade the common region according to the signs.



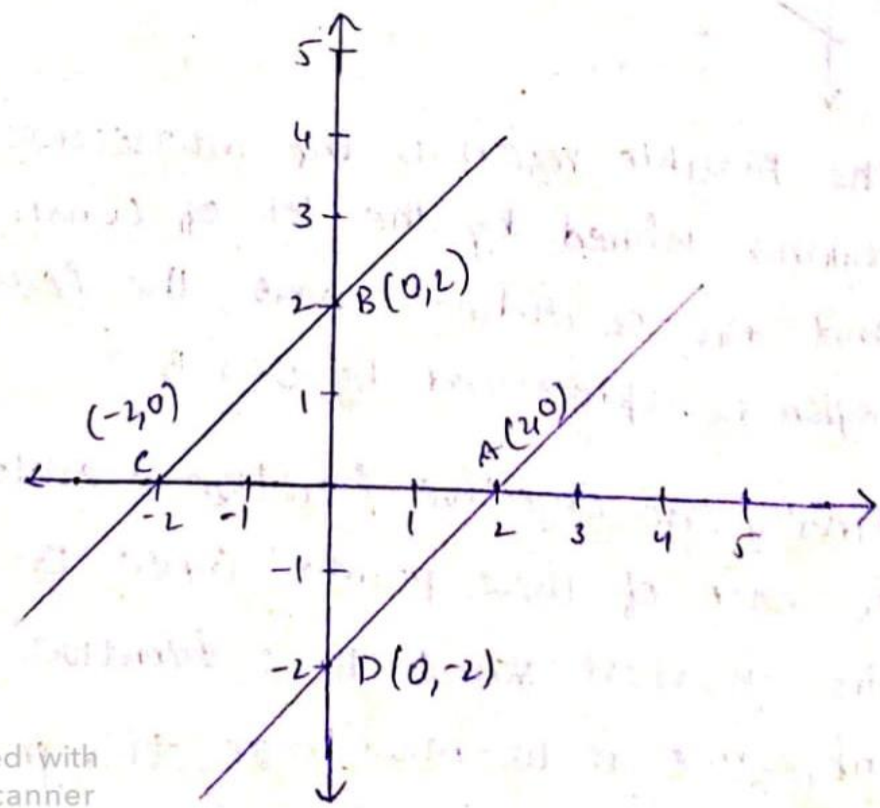
The feasible region is the intersection of the regions defined by the set of constraints and the co-ordinate axis, the feasible region is represented by O-A-B-O

Finally the objective function is evaluated in each of these points. since B- provides the greatest value to z function and objective is to maximize, this point is the optimal solution: z = 6 with x = 0, y = 3

points	coordinates	value
O	(0,0)	0
A	(3,0)	3
B	(0,3)	6

2. Minimize $z = x_1 + x_2$, subject to: $x_1 - x_2 \leq 2$, $x_1 - x_2 \geq -2$, $x_1, x_2 \geq 0$.

Sol) First we draw the lines of all the given equations and shade the common region according to the signs



Here we can say that more than one solution exists. so, there is no optimal solution

3. Consider the following linear program: Maximize $z = 2x_1 + x_2$ subject to: $12x_1 + 3x_2 \leq 6$, $-3x_1 + x_2 \leq 7$, $x_2 \leq 10$, $x_1, x_2 \geq 0$. Draw a graph of the constraints and shade in the feasible region. Label the vertices of this region with their coordinates.

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3. Maximize $z = 2x_1 + x_2$

subject to $12x_1 + 3x_2 \leq 6$

$-3x_1 + x_2 \leq 7$

$x_2 \leq 10$, $x_1, x_2 \geq 0$

$12x_1 + 3x_2 = 6$

x_1	0	1/2
x_2	2	0

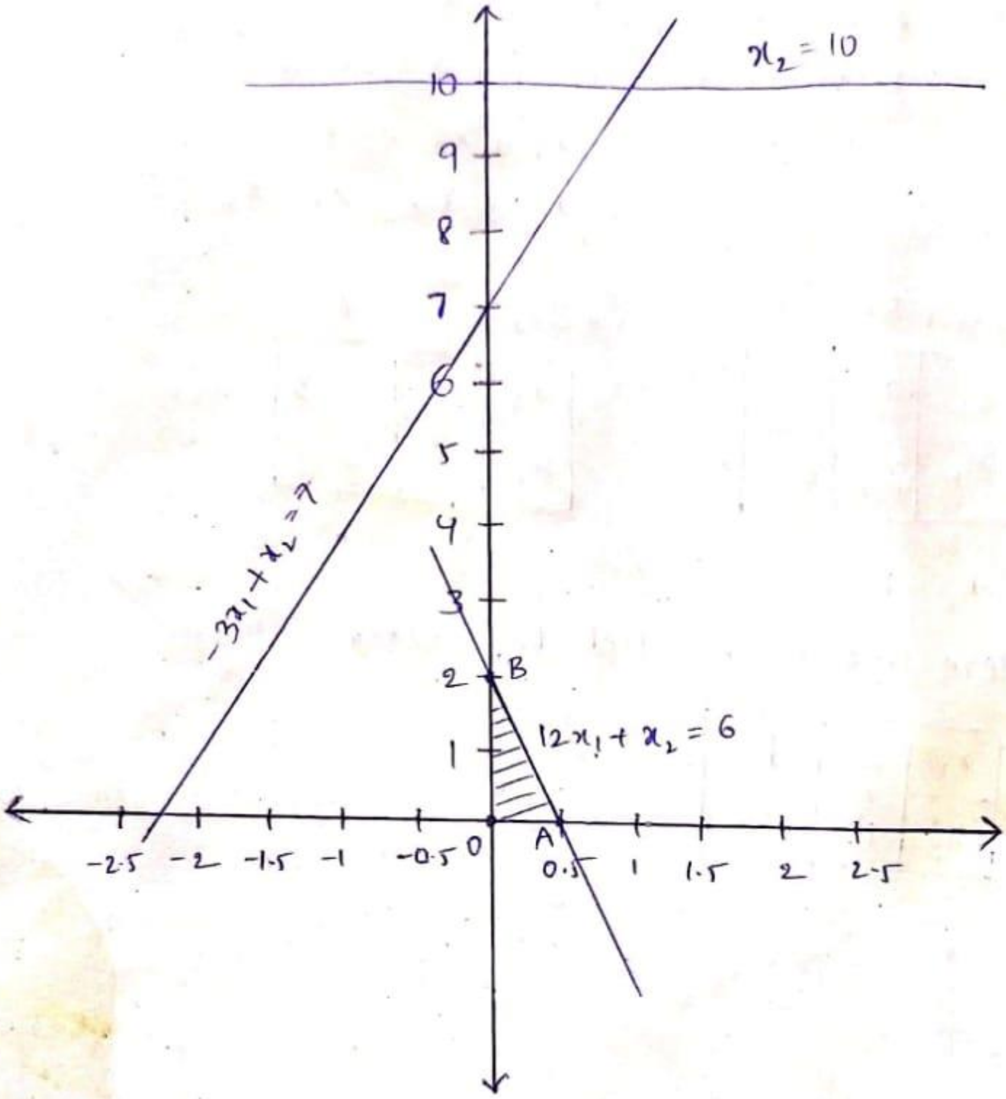
$-3x_1 + x_2 = 7$

x_1	0	-7/3
x_2	7	0

$x_2 = 10$

Here line is parallel to x-axis

x_1	0	1
x_2	10	10



The value of objective function at each of these extreme points is as follows

Extreme point	value
O (0,0)	0
A(0.5,0)	1
B(0,2)	2

The maximum value of $z = 2$ occurs at $(0,2)$

$\therefore$ optimal solution is $x_1 = 0$

$x_2 = 2$

max $z = 2$

4. A company produces 2 types of cowboy hats. Each hat of the first type requires twice as much labour time as the second type. The company can produce a total of 500 hats a day. The market limits the daily sales of first and second types to 150 and 250 hats. Assuming that the profits per hat are \$8 per type A and \$5 per type B, formulate the problem as Linear Programming model in order to determine the number of hats to be produced of each type so as to maximize the profit.

Let the company produces x type of hat A and y type of hat B each day

So, the profit p after selling these two products is $P = 8x + 5y$

Since the company can produce at the most 500 hats in a day and A type of hats requires twice of type B.

$$2x + y \leq 500$$

But there are limitations of the sale of hats

further restrictions, $x \leq 150$
$y \leq 250$

As the company cannot produce negative values $x \geq 0$
$y \geq 0$

So, the final formulation

$$P = 8x + 5y$$

$$2x + y < 500$$

$$x < 150$$

$$y < 250$$

$$x > 0$$

$$y > 0$$

5. A cooperative society of farmers has 50 hectares of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide must be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used to protect fish and wildlife using a pond which collects drainage from this land. How much land should be allocated to each crop to maximise the total profit of the society? (formulating Mathematical modelling of LPP)

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Let the land allocated to crop x be x hectare
Let the land allocated to crop y be y hectare

According to question

Herbicide used for crop x = 20 lt per hect
Herbicide used for crop y = 10 lt per hect

Maximum quantity of herbicide = 800lt

$$20x + 10y \leq 800$$
$$2x + y \leq 80 \quad —(1)$$

Also,
Total land available to grow crops = 50 hect

$$\therefore x + y \leq 50 \quad —(2)$$

Also,
we want to maximize the profit

Hence, the function used here is Maximize z

profit from crop x = ₹ 10500 per hect
profit from crop y = ₹ 9000 per hect

$$\text{Maximize } z = 10600x + 9000y$$

Subject to constraints

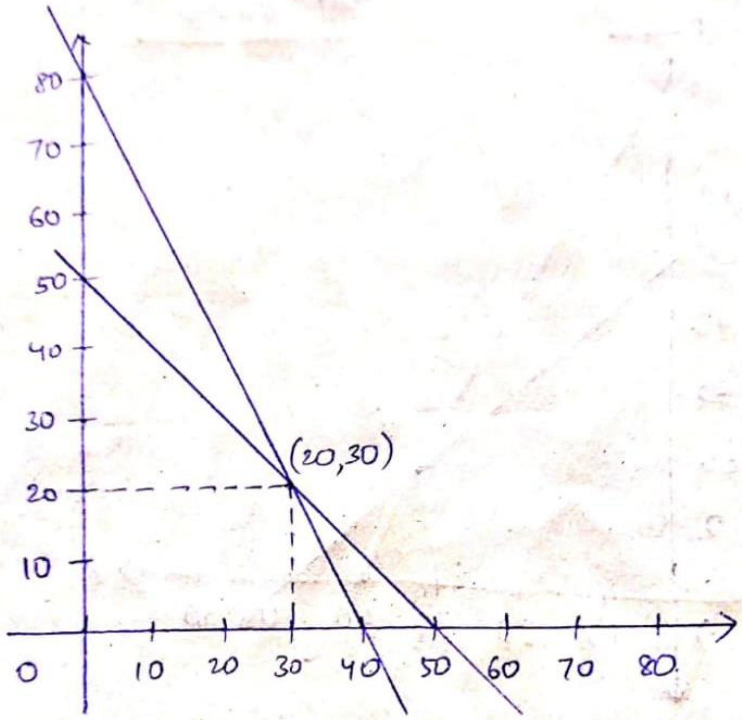
$$2x + y \leq 80$$
$$x + y \leq 50$$
$$x, y \geq 0$$

$x + y \leq 50$

x	0	50
y	50	0

$2x + y \leq 80$

x	0	40
y	80	0



Corner points	Value of z
(0, 50)	450000
(30, 20)	495000 → maximum
(40, 0)	420000
(0, 0)	0

Hence the profit will be maximum if

land allocated to crop $x = 30$ hectare

land allocated to crop $y = 20$ hectare