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6. Schrodinger Wave Equation (Time-Independent)

$\psi = \psi_0 \sin(\omega t - kx)$ wave Equation

ψ_0 = Amplitude of wave

ω = angular frequency

k = wave vector

$$\boxed{k = \frac{2\pi}{\lambda}}$$

$$\psi = \psi_0 \sin(\omega t - kx) \quad — ①$$

Diff eqn ① wrt x

$$\frac{\partial \psi}{\partial x} = \psi_0 (-k) \cos(\omega t - kx)$$

$$\frac{\partial \psi}{\partial x} = -k\psi_0 \cos(\omega t - kx) \quad — ②$$

Diff eqn ② wrt x

$$\frac{\partial^2 \psi}{\partial x^2} = -k\psi_0 (-k) [-\sin(\omega t - kx)]$$

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi_0 \sin(\omega t - kx)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0$$

From DeBrogile

$$k = \frac{2\pi}{\lambda} \Rightarrow k^2 = \frac{4\pi^2}{\lambda^2}$$

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

sub k^2 in eqn ③

$$\lambda^2 = \frac{h^2}{p^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad — ④$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 \psi}{\lambda^2} = 0 \Rightarrow \boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 p^2 \psi}{h^2} = 0} \quad — ⑤$$

According to Bohr's theory

$$\hbar = \frac{h}{2\pi} \Rightarrow \hbar^2 = \frac{h^2}{4\pi^2} \Rightarrow h^2 = \hbar^2 4\pi^2$$

sub in eq (5)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\cancel{4\pi^2} p^2 \psi}{\hbar^2 \cancel{4\pi^2}} = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{p^2 \psi}{\hbar} = 0 \quad \text{—— (6)}$$

$$E = KE + PE$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2m}(mv)^2$$

$$E = \frac{p^2}{2m} + V$$

$$p^2 = 2m(E-V)$$

Sub p^2 in eqn (6)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)}{\hbar^2}\psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-V)\,4\pi^2}{h^2}(E-V)\psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8m\pi^2}{h^2}(E-V)\psi = 0$$

This is Time Independent Schrodinger wave equation in 1 Dimension

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

Divide by $\frac{2m}{\hbar^2}$

$$\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + (E-V)\psi = 0$$

$$\boxed{\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + E\psi = V\psi}$$ 1 Dimension

$$\frac{\hbar^2}{2m}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right] + E\psi = V\psi$$

$$\boxed{\frac{\hbar^2}{2m}\nabla^2\psi + E\psi = V\psi}$$ 3 Dimension

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