

MP-1 TUTORIAL-13

1. Demonstrate the Discrete Optimization using Cutting Plane method.

QUESTION:

$$\text{Max } z = x_1 + x_2$$

$$\text{Subject To: } 3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Tutorial -13

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$$\text{Max } z = x_1 + x_2$$

subject to :

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$\text{and } x_1, x_2 \geq 0$$

The problem is converted to canonical form by adding slack variables, surplus and artificial variables as appropriate.

$$\text{Max } z = x_1 + x_2 + 0s_1 + 0s_2$$

subject to

$$3x_1 + 2x_2 + s_1 = 5$$

$$x_2 + s_2 = 2 \quad x_1, x_2, s_1, s_2 \geq 0$$

C_B	C_j	1	1	0	0	
	B.V	x_1	x_2	s_1	s_2	sol
0	s_1	3	2	1	0	5
0	s_2	0	1	0	1	2
	Z_j	0	0	0	0	
	$C_j - Z_j$	1	1	0	0	

$$\text{Entering} = 1$$

$$\text{leaving} = s_1$$

$$\text{key element} = 3$$

C_B	C_j	1	1	0	0	
	B.V	x_1	x_2	s_1	s_2	sol
1	x_1	1	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{5}{3}$
0	s_2	0	①	0	1	2
	Z_j	1	$\frac{2}{3}$	$\frac{1}{3}$	0	
	$C_j - Z_j$	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	

Leaving = s_2

Entering = x_2

key element = 1

C_B	C_j	1	1	0	0	
	B.V	x_1	x_2	s_1	s_2	sol
1	x_1	1	0	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$
1	x_2	0	1	0	1	2
	Z_j	1	1	$\frac{1}{3}$	$\frac{1}{3}$	
	$C_j - Z_j$	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	

Since all $C_j - Z_j \leq 0$

Hence non-integer optimal solution is arrived with value of variables as-

$$x_1 = \frac{1}{3} \quad x_2 = 2$$

$$\max Z = \frac{5}{3}$$

To obtain integer valued solution we proceed to construct Gomory's fractional cut, with the help of x_1 -row as follows.

$$0.3333 = 1x_1 + 0.3333s_1 - 0.6667s_2$$

$$0 + 0.3333 = (1+0)x_1 + (0-0.3333)s_1 + (-1+0.3333)s_2$$

The fractional cut will become

$$-0.3333 = s_1 - 0.3333s_1 - 0.3333s_2$$

cut - 1

Adding this constraint at the bottom of optimal simplex table

C_B	C_j	1	1	0	0	0	0
	B.V	x_1	x_2	s_1	s_2	s_3	s_4
1	x_1	1	0	$\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{1}{2}$
1	x_2	0	1	0	1	0	2
0	s_3	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	1	$-\frac{1}{3}$
	Z_j	1	1	$\frac{1}{3}$	$\frac{1}{3}$	0	0
	$C_j - Z_j$	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0
	Ratio	-	-	1	1	-	-

\therefore Entering = s_1

Leaving = s_3

key element = $-\frac{1}{3}$



C_{B_i}	C_j	1	1	0	0	0	
	B-V	x_1	x_2	s_1	s_2	s_3	sol
1	x_1	1	0	0	-1	1	0
1	x_2	0	1	0	1	0	2
0	s_1	0	0	1	1	-3	1
	z_j	1	1	0	0	1	
	$g_j - z_j$	0	0	0	0	-1	

since all $g_j - z_j \leq 0$

Hence integer optimal solution is arrived with value of variables as

$$x_1 = 0 \quad x_2 = 2$$

$$\max z = 2$$

The integer optimal solution found after
1- cuts