

MP-1 TUTORIAL-6

1. Demonstrate the Initial Basic Solution in Transportation problem using Vogel method in Linear Programming (U-V method), Least time Transportation problem.

QUESTION:

Factory/Warehouse	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	

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Tutorial - 6

Row diff

19	30	50	10	2	7	9	40	40
70	30	40	60	1	10	20	20	20
40	8	70	20	0	12	20	50	-

Col diff

21	22	10	10
21	-	10	10
-	-	10	10
-	-	10	50

Transportation Cost :-

$$\begin{aligned}
 &= 5 \times 19 + 8 \times 8 + 7 \times 40 + 2 \times 10 + 10 \times 20 + 2 \times 60 \\
 &= 95 + 64 + 280 + 20 + 200 + 120 \\
 &= 779
 \end{aligned}$$

u-v method

	W_1	W_2	W_3	W_4	
	19	30	50	10	7
	70	30	40	60	9
	40	8	70	20	18
	5	8	7	14	

Step 1

$m = \text{no. of origins} = 3$

$n = \text{no. of destinations} = 4$

$m+n-1 = 6 \Rightarrow \text{no. of allocation} = 6$

And the allocations are independent positions.

\therefore The problem is non-degenerate

Step 2

u_i, v_j - allocated cells.

$$u_i + v_j = c_{ij}$$

5	19	30	50	2	10	$u_1 = 0$	
	70	30	7	40	2	60	$u_2 = 50$
	40	8		70	10	20	$u_3 = 10$

$$v_1 = 19 \quad v_2 = -2 \quad v_3 = -10 \quad v_4 = 10$$

cell	eqns	Values
(1,1)	$u_1 + v_1 = 19$ $0 + v_1 = 19$	$v_1 = 19$
(1,4)	$u_1 + v_4 = 10$ $0 + v_4 = 10$	$v_4 = 10$
(2,4)	$u_2 + v_4 = 60$ $u_2 + 10 = 60$	$u_2 = 50$
(2,3)	$u_2 + v_3 = 40$ $50 + v_3 = 40$	$v_3 = -10$
(3,4)	$u_3 + v_4 = 20$ $u_3 + 10 = 20$	$u_3 = 10$
(3,2)	$u_3 + v_2 = 8$ $10 + v_2 = 8$	$v_2 = -2$

Step: 3

⁽¹⁾ 19	⁽³²⁾ 30	⁽⁶⁰⁾ 50	⁽²⁾ 10
⁽¹⁾ 70	⁽¹⁸⁾ 30	⁽⁷⁾ 40	⁽¹⁾ 60
⁽¹⁰⁾ 40	⁽⁸⁾ 8	⁽⁷⁰⁾ 70	⁽¹⁰⁾ 20

 u_i

0

50

10

 v_j 19 -2 -10 10

cell	eqn	cell evaluation
(1,2)	$30 - (u_1 + v_2)$ $30 - (0 - 2)$	32
(1,3)	$50 - (u_1 + v_3)$	60
(2,1)	$70 - (u_2 + v_1)$	1
(2,2)	$30 - (u_2 + v_2)$	-18
(3,1)	$40 - (u_3 + v_1)$	11
(3,3)	$70 - (u_3 + v_3)$	70

If all the cells evaluations are non-negative then the solution is optimal

The cell evaluation -18 is negative so sol is not optimal

\therefore Entering cell (2,2)

19	30	50	⁽²⁾ 10
70	30	⁽⁷⁾ 40	⁽¹⁾ 2-0 → 60
40	⁽⁸⁾ 8 ← 8-0	70	⁽¹⁰⁾ 20 ↓ 10-0

Form a loop starting from cell (2,2)

$$\theta = \min \{ 2-0, 8-0 \}$$

$$2-0 = 0$$

$$\theta = 2$$



⁵ 19	30	⁵ 10	² 10
70	² 30	⁷ 40	⁰ 60 ← leaving cell
40	⁶ 8	70	¹² 20

updated allocations

⁵ 19	⁽²²⁾ 30	⁽⁴²⁾ 10	² 10	0
⁽⁷⁹⁾ 70	² 30	⁷ 40	⁽¹⁸⁾ 60	32
40 ⁽¹¹⁾	⁶ 8	⁽¹²⁾ 70	⁽¹²⁾ 20	10

$$V_j \rightarrow 19 \quad -2 \quad 8 \quad 10$$

Now all the cell evaluations are non-negative

Transportation cost

$$5 \times 19 + 2 \times 10 + 2 \times 30 + 7 \times 40 + 6 \times 8 + 12 \times 20$$

$$95 + 20 + 60 + 280 + 48 + 240$$

$$743$$