Density-Based Spatial Clustering of Applications with Noise

Comprehensive Notes with Equations, Diagrams, and Implementation

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🎯 Introduction to DBSCAN

DBSCAN (Density-Based Spatial Clustering of Applications with Noise) is a powerful density-based clustering algorithm introduced by Martin Ester, Hans-Peter Kriegel, Jörg Sander, and Xiaowei Xu in 1996. Unlike centroid-based algorithms like K-means, DBSCAN can discover clusters of arbitrary shapes and automatically determine the number of clusters.

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Key Features of DBSCAN:

- **Density-Based:** Groups points that are closely packed
- **Noise Handling:** Identifies and handles outliers effectively
- **Arbitrary Shapes:** Can find clusters of any shape
- **Automatic:** No need to specify the number of clusters
- **Robust:** Relatively insensitive to parameter choices

🎨 Visual Comparison: K-means vs DBSCAN

📊 Diagram

```
K-means (Spherical Clusters) DBSCAN (Arbitrary
Shapes) ●●● ●●● ●●●●●●● ●● ● ● ●● ●● ●● ●● ● ● ●● ●●
●●●●●● ●● ● ● ●● ●● ●● ● ●●● ●●● ●●●●●● ●● ●●● Fails
with non-spherical Successfully identifies clusters
and noise complex shapes and noise
```

🔍 Core Concepts and Definitions

Point Classifications

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Core Point

Has at least **MinPts** neighbors within **ε** distance

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Border Point

Within ε distance of a core point but has fewer than MinPts neighbors

Noise Point

Neither a core point nor a border point (outlier)

Key Definitions

ε (Epsilon)

The maximum distance between two points for them to be considered neighbors

MinPts

The minimum number of points required to form a dense region (including the point itself)

ε-Neighborhood

N_ε(p) = {q ∈ D | dist(p,q) ≤ ε}

Density Reachable

Point q is density reachable from p if there exists a chain of core points from p to q

Visual Representation of Core Concepts

📊 Diagram

```
DBSCAN Point Classification Example (ε = 2, MinPts = 4) A ●
── ● B ● F (Noise) | \ / | | ● | | / \ | C ● ── ● D | | ●
● E Legend: ● A, B, C, D = Core Points (≥4 neighbors within
ε=2) ● E = Border Point (<4 neighbors but within ε of core
point) ● F = Noise Point (isolated, <4 neighbors and not
within ε of core) Cluster Formation: - Cluster 1: {A, B, C,
D, E} (all density-connected) - Noise: {F}
```

📐 Mathematical Formulation

Distance Metrics

DBSCAN can work with various distance metrics. The most common is Euclidean distance:

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Euclidean Distance: $d(p,q) = \sqrt{(\Sigma_i(p_i - q_i)^2)}$

📐

Manhattan Distance: $d(p,q) = \Sigma_i|p_i - q_i|$

📐

Minkowski Distance: $d(p,q) = (\Sigma_i|p_i - q_i|^p)^\wedge(1/p)$

Core Mathematical Definitions

📐

ε-Neighborhood: $N_\varepsilon(p) = \{q \in D \mid dist(p,q) \leq \varepsilon\}$

📐

Core Point Condition: $|N_\varepsilon(p)| \geq MinPts$

📐

Directly Density-Reachable: $q \in N_\varepsilon(p) \wedge |N_\varepsilon(p)| \geq MinPts$

📐

Density-Connected: $\exists o \in D : p\ \rho^*\ o \wedge q\ \rho^*\ o$

Where ρ* denotes density-reachability relation.

Cluster Definition

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A cluster C in DBSCAN is defined as a maximal set of density-connected points:

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$$C = \{p \in D \mid \exists q \in C : p \text{ is density-reachable from } q\}$$

⚙️ The DBSCAN Algorithm

🔧 **Algorithm**

DBSCAN Pseudocode

PSEUDOCODE

```
Algorithm DBSCAN(D, ε, MinPts) Input: Dataset D,
radius ε, minimum points MinPts Output: Set of
clusters C 1. Initialize: C = 0 (cluster counter),
Mark all points as UNVISITED 2. For each point p in
D: a) If p is VISITED, continue to next point b)
Mark p as VISITED c) NeighborPts = regionQuery(p, ε)
d) If |NeighborPts| < MinPts: - Mark p as NOISE e)
Else: - C = C + 1 (increment cluster counter) -
expandCluster(p, NeighborPts, C, ε, MinPts) Function
expandCluster(p, NeighborPts, C, ε, MinPts) 1. Add p
to cluster C 2. For each point p' in NeighborPts: a)
```

```
If p' is UNVISITED: - Mark p' as VISITED -
NeighborPts' = regionQuery(p', ε) - If
|NeighborPts'| ≥ MinPts: * NeighborPts = NeighborPts
∪ NeighborPts' b) If p' is not member of any
cluster: - Add p' to cluster C Function
regionQuery(p, ε) 1. Return all points q where
dist(p,q) ≤ ε
```

Step-by-Step Algorithm Execution

1

Initialization

Mark all points as unvisited and initialize cluster counter to 0

2

Point Selection

Select an unvisited point and mark it as visited

3

Neighborhood Query

Find all points within ε distance (ε-neighborhood)

4

Core Point Check

If neighborhood has ≥ MinPts points, it's a core point

5

Cluster Expansion