RBE 500 — Foundations of Robotics

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Spring 2022

Problem Set 1

Due: Feb 4, 2022 at 11:59 pm

Please show your work. Correct answers not accompanied by sufficient explanations will receive little or no credit.

Problem 1 (5 points)

Consider an initial coordinate frame $o_0x_0y_0z_0$. The frame is subjected to the following transformations:

i. rotation of π about the fixed x-axis (denoted as Frame 1).

ii. translation of 5 m along the current z-axis (denoted as Frame 2)

iii. translation of -3 m along the fixed y-axis (denoted as Frame 3).

a) Find the overall homogeneous transformation matrix associated to this transformation.

b) What are the coordinates of the origin of Frame 3 with respect to the initial Frame 0?

c) Consider a point p with coordinates in Frame 2 as $p^2 = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$. Using the transformations calculated in part (a), determine the coordinates of p w.r.t. the initial Frame 0 (p^0), and with respect to Frame 3 (p^3).

Problem 2 (5 points)

Consider the robot setup shown in Figure 1. The table is placed 1 meter from the robot. The table top is 1 meter high with a surface area of 1 meter square. Frame 1 is attached to the corner of the table, as shown in the figure. A cube of $0.2 \times 0.2 \times 0.2$ meter is placed in the center of the table, with Frame 2 attached to the center of the cube as shown (thus 10 cm above the table top). A camera is installed directly above the center of the block, 2 meters above the table top. Frame 3 is attached to the camera as shown in the figure.

Find the homogeneous transformations relating each of these frames to the base frame $o_0x_0y_0z_0$ (i.e. H_1^0, H_2^0, H_3^0). Moreover, find the homogeneous transformation relating Frame 2 to the camera frame (Frame 3) (i.e. H_3^2).

Hint: calculating H_3^2 first can be helpful for finding H_3^0.

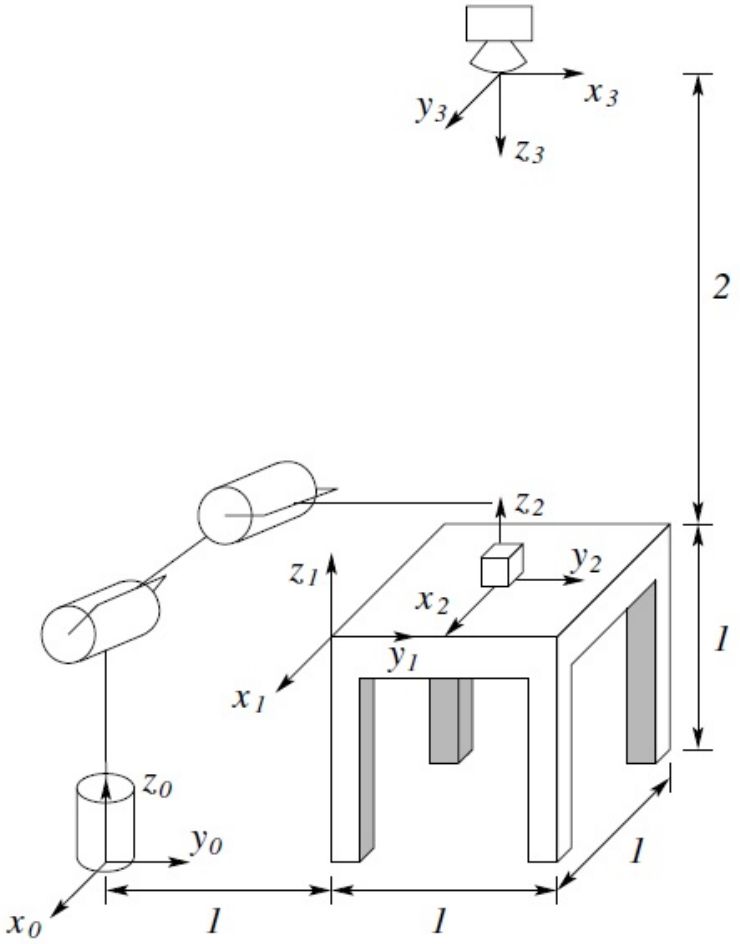

Figure 1: Robot Setup for Problem 2

Problem 3 (5 points)

Consider the three-link Cartesian manipulator in Figure 2. Derive the forward kinematic equations using the DH convention, that is:

- Assign the coordinate frames using the DH convention.
- Form a DH parameters table.
- Write all the A matrices.
- Compute the forward kinematics transformation matrix T_3^0 (no need to multiply the matrices).

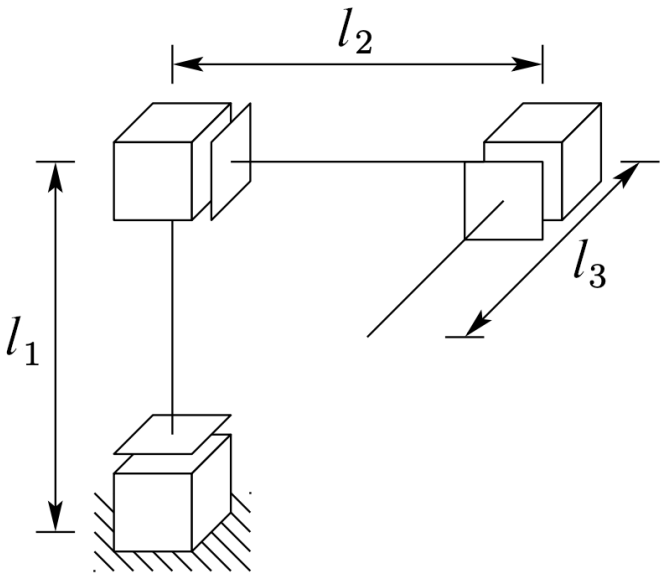

Figure 2: Three-link Cartesian robot manipulator for Problem 3

Problem 4 (5 points)

Consider the 6-DoF robot manipulator in Figure 3. Derive the forward kinematic equations using the DH convention.

- Assign the coordinate frames using the DH convention.
- Form a DH parameters table.
- Write all the A matrices.
- Compute the forward kinematics transformation matrix T_6^0 (no need to multiply the matrices).

Hint: draw the robot in its upward position and consider that configuration as the home position for the robot.

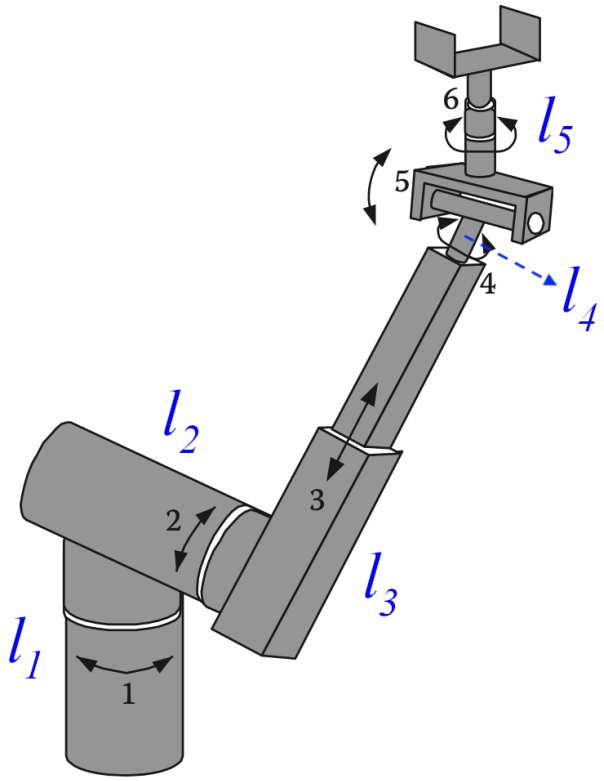

Figure 3: 6-DoF RRPRRR robot manipulator for Problem 4