RBE 500 — Foundations of Robotics

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Spring 2022

Problem Set 2

Due: Feb 18, 2022 at 11:59 pm

Please show your work. Correct answers not accompanied by sufficient explanations will receive little or no credit.

Problem 1 (4 points)

Consider the cylindrical manipulator shown in Figure 1. Derive the solution to the inverse position kinematics for this robot. That is, given the position of the end-effector as $p_{ef} = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix}^T$, find a closed-form solution for each of the joint variables.

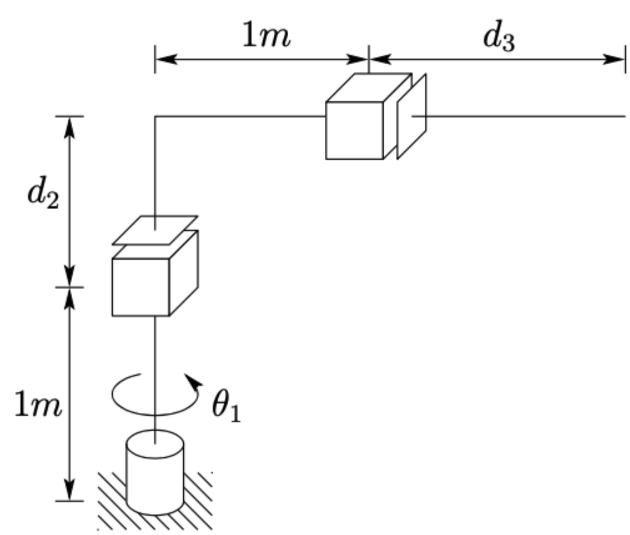

Figure 1: Cylindrical robot manipulator for Problem 1

Problem 2 (4 points)

Consider the RRPR robot manipulator shown in Figure 2. From Forward Kinematics solution of the robot, the end-effector position is calculated as:

$$p_x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$
$$p_y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$
$$p_z = d_3$$

Derive the solution to the inverse kinematics for the robot. That is, given the the end-effector position as $p_{ef} = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix}^T$ and the end-effector orientation as angle ϕ with respect to the zero frame, find a closed-form solution for each of the joint variables.

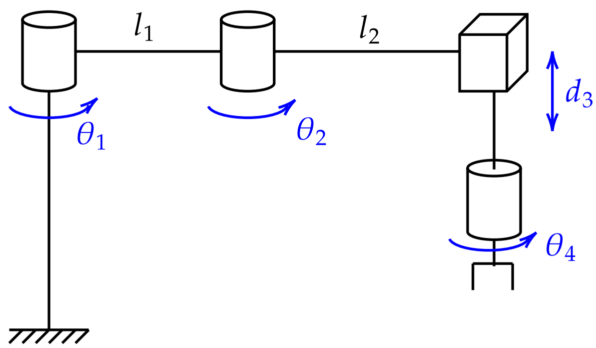

Figure 2: RRPR robot manipulator for Problem 2

Problem 3 (4 points)

Consider the vector $a = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix}$ and the rotation matrix $R = R_{y,\pi/2}$. By direct calculation, show that:

$$R\,S(a)\,R^T = S(R\,a)$$

Hint: R_1^0 is basically a rotation about the z axis by an angle θ. The final answer will be in terms of θ.

Problem 4 (4 points)

Consider a fixed Frame 0 as the base frame. A point $p^1 = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}^T$ is rigidly attached to a moving Frame 1. Frame 1 is rotating with angular velocity $\omega_1^0 = 1\,k$ rad/s and translating along the x_0 axis at a rate of 3 m/s. Find the linear velocity of point p with respect to Frame 0, i.e. $\dot{p}^0$.

Problem 5 (4 points)

Consider the 6-DoF robot manipulator shown in Figure 3. Derive the manipulator Jacobian matrix for this robot.

For the resulting terms such as z_1^0, z_2^0, o_3^1, etc., explain how you could calculate them using homogeneous transformation matrices (for example: "I could calculate z_1^0 using the homogeneous transformation matrix [X] as follow..." and explain. No need to perform matrix multiplications associated to the forward kinematics.).

(Note: use the same coordinate frames shown in Figure 3).

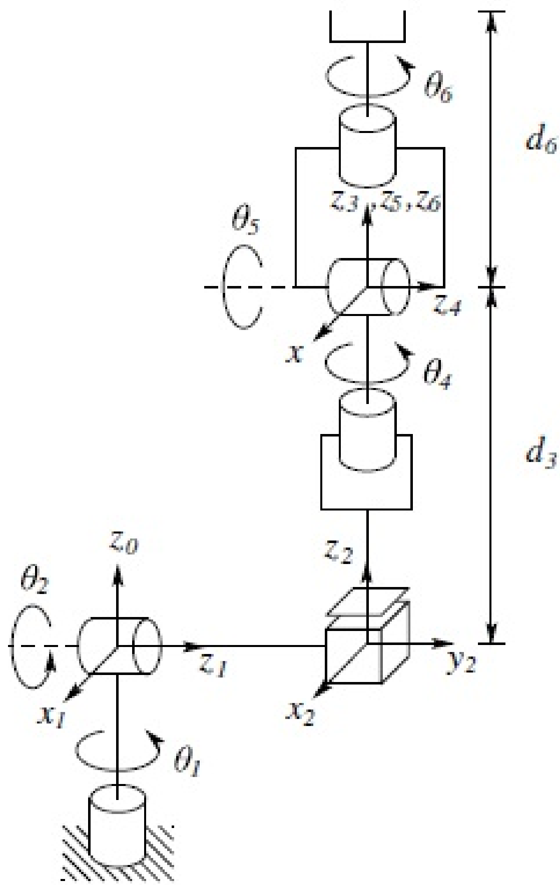

Figure 3: 6-DOF robot manipulator for Problem 4