Wavelet Transform

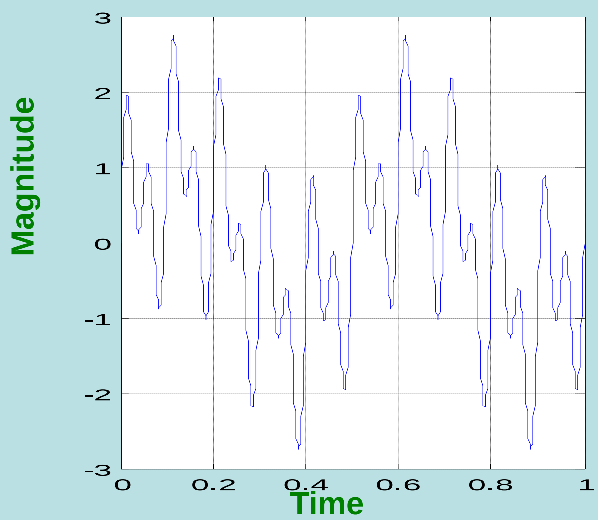
Presented by
Imane Hafnaoui

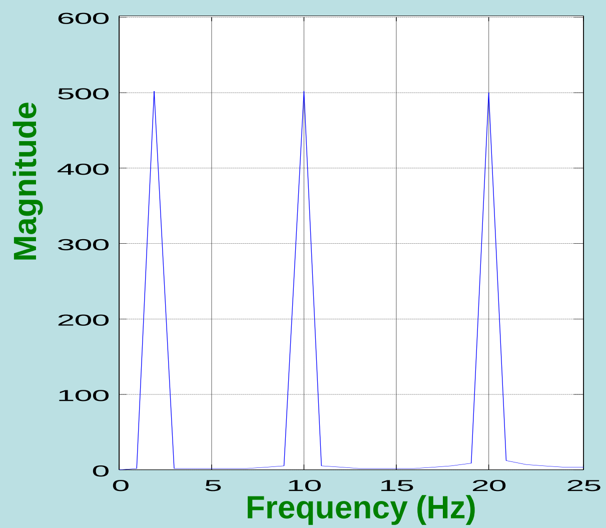
Fourrier Transform Limitations

FT shows what frequencies exist in a signal.

2 Hz + 10 Hz + 20Hz

Stationary

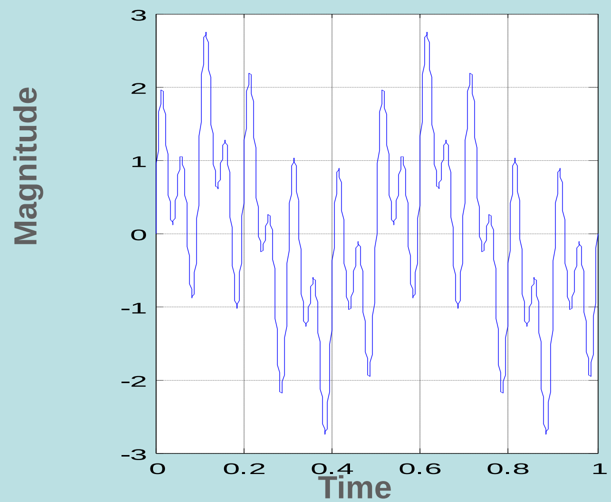


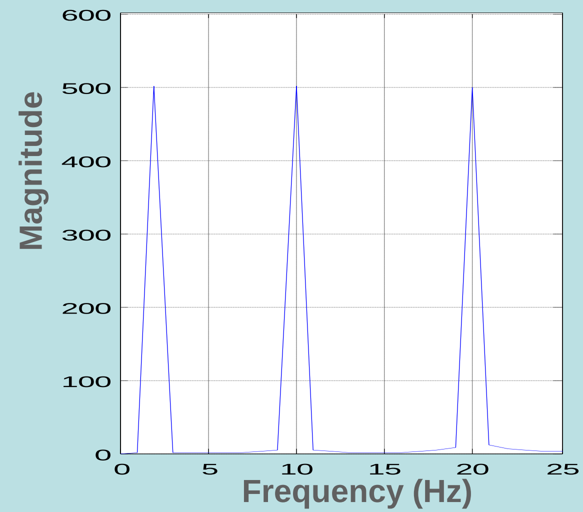


- FT is not good with non-stationary signals

2 Hz + 10 Hz + 20Hz

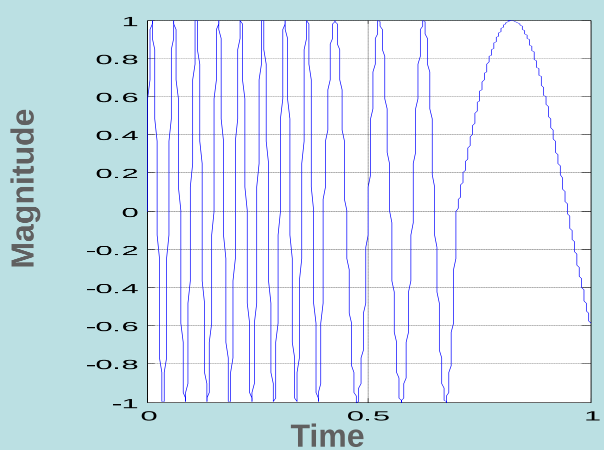
Stationary

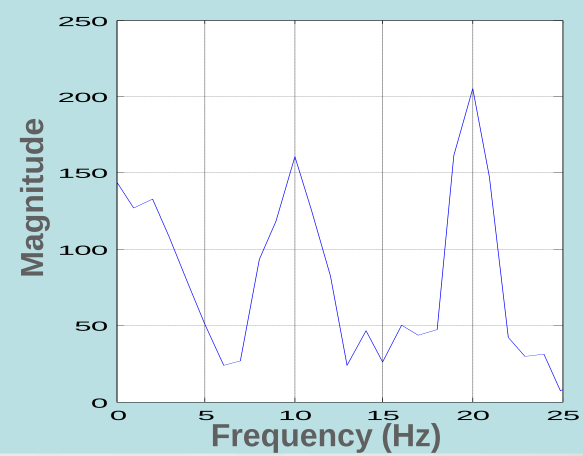




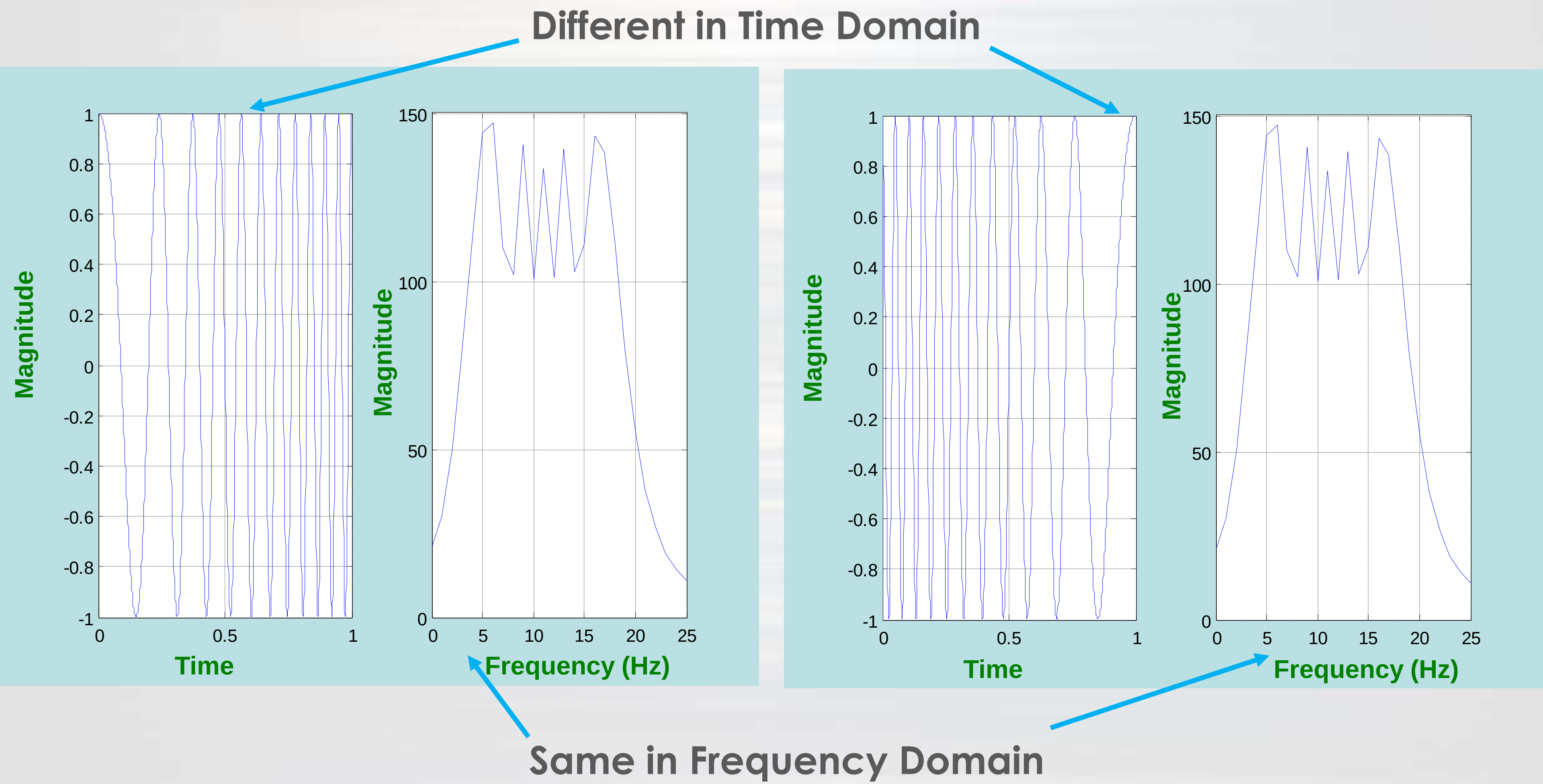
0.0-0.4: 2 Hz +
0.4-0.7: 10 Hz +
0.7-1.0: 20Hz

Non-Stationary





- FT only shows how much of each frequency is present but not at what time it occurs.



Problem

- Time – Frequency representation is needed.

Solution

Wavelet Transform

An introduction to Wavelet Transform

Introduction

- Why Wavelet Transform?

Ans: Analysis signals which is a function of time and frequency

- Examples

  Scores, images, economical data, etc.

Introduction

Conventional Fourier Transform

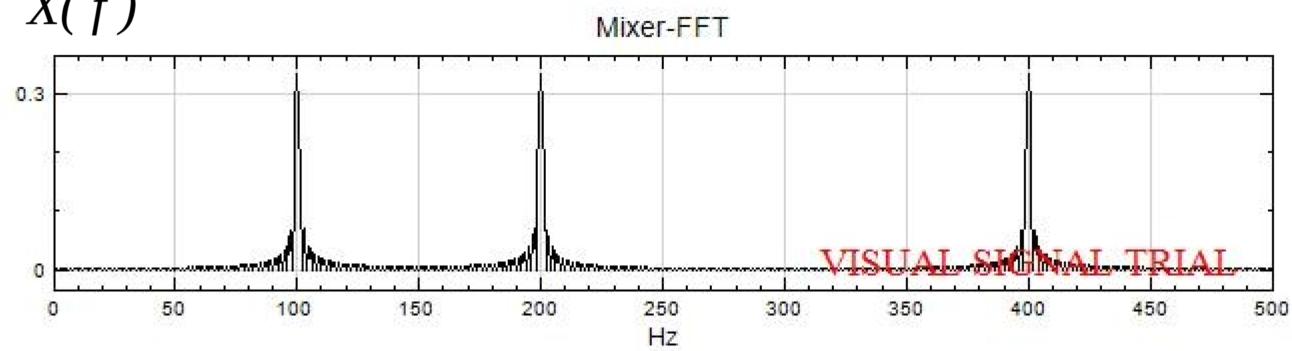
V.S.

Wavelet Transform

Conventional Fourier Transform

$$x(t)=\begin{cases}\sin(2\pi 100t) & 0\le t<0.5\\ \sin(2\pi 200t) & 0.5\le t<1\\ \sin(2\pi 400t) & 1\le t<1.5\end{cases}$$

$X(f)$



Wavelet Transform

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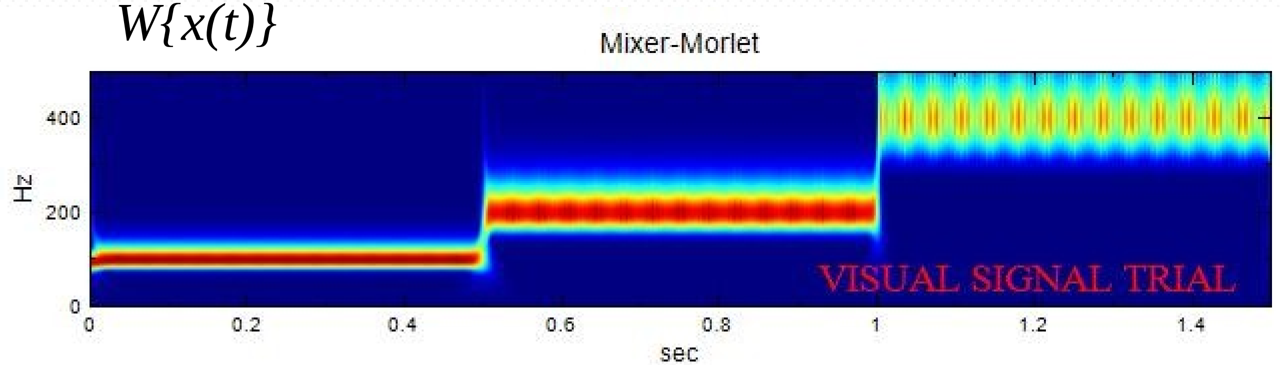
$W\{x(t)\}$



Background

- Image pyramids
- Subband coding

Image pyramids

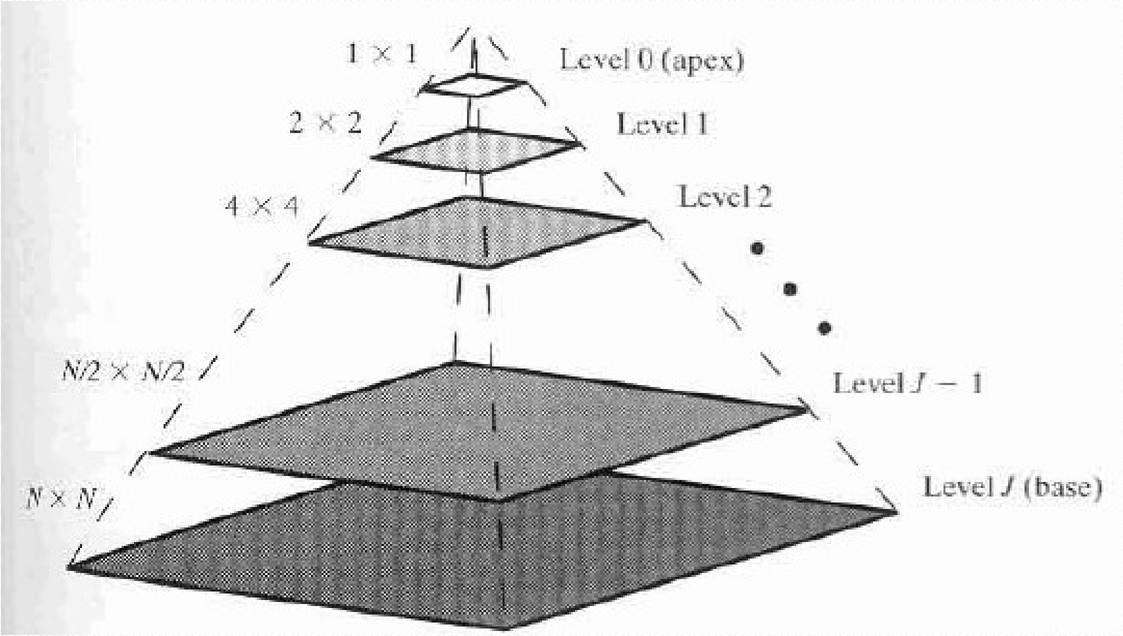


Fig. 1 a J-level image pyramid[1]

Image pyramids

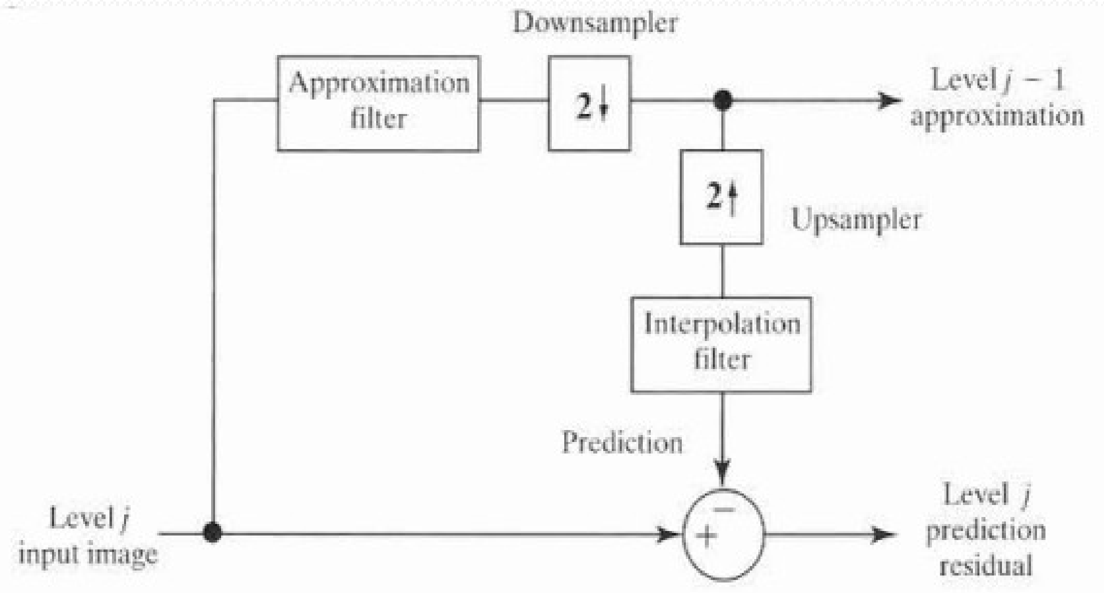


Fig. 2 Block diagram for creating image pyramids[1]

Subband coding

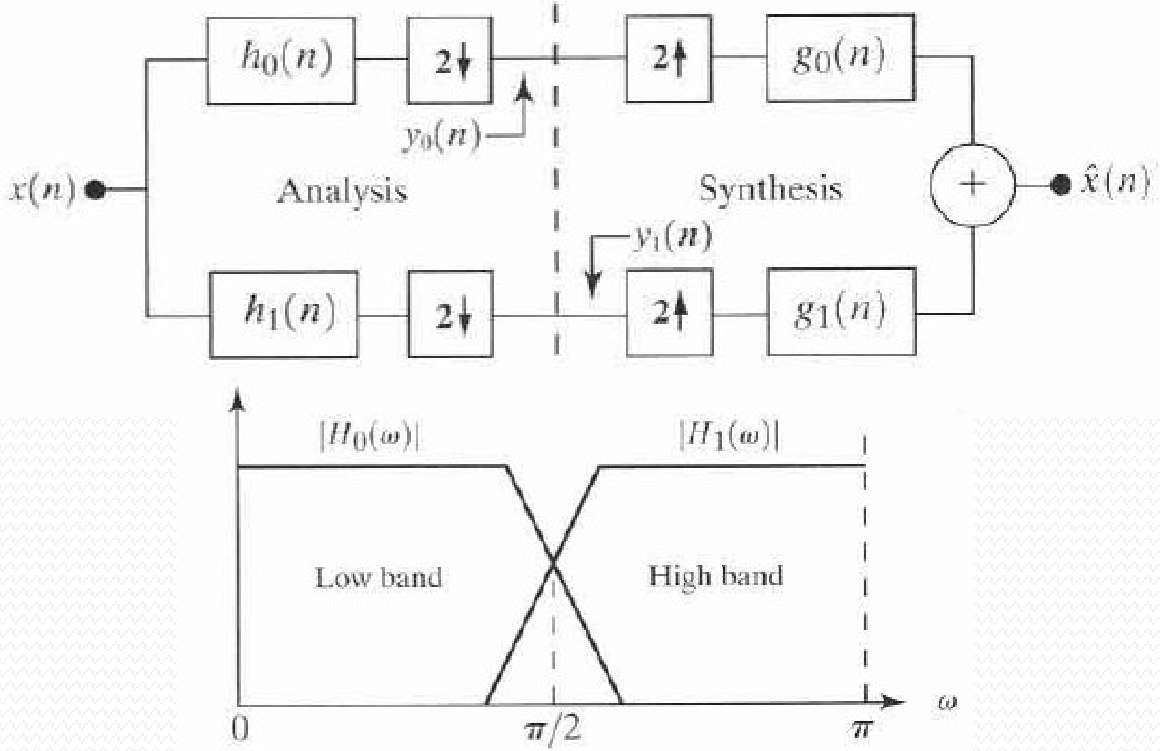


Fig. 3 Two-band filter bank for one-dimensional subband coding and decoding system and the corresponding spectrum of the two bandpass filters[1]

An Animated Introduction to the Discrete Wavelet Transform

Revised Lecture Notes

New Delhi
December 2001

Arne Jensen

Aalborg University

Reference

This is a tutorial introduction to the discrete wavelet transform. It is based on the book

A. Jensen and A. la Cour-Harbo:
Ripples in Mathematics
The Discrete Wavelet Transform
Springer-Verlag 2001.

A first example 1

A signal with 8 samples:

$$56, \quad 40, \quad 8, \quad 24, \quad 48, \quad 48, \quad 40, \quad 16$$

We compute a transform as shown here:

56	40	8	24	48	48	40	16
48	16	48	28	8	-8	0	12
32	38	16	10	8	-8	0	12
35	-3	16	10	8	-8	0	12

To interpretation

A first example 2

First row is the original signal. The second row in the table is generated by taking the mean of the samples pairwise, put them in the first four places, and then the difference between the the first member of the pair and the computed mean. Computations are repeated on the *means*. Differences are kept in each step.

A first example 2

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$$\frac{56+40}{2}$$

56	40	8	24	48	48	40	16
48				8			

$$56 - 48$$

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The transform is invertible. We start from the bottom row. We add and subtract the difference to the mean, and repeat the process up to the first row.

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Lifting 1

We now look at the transform in the first example. The direct transform $(a, b) \to (d, s)$ is given by

$$s = \frac{a+b}{2},$$
$$d = a - s.$$

and the inverse $(d, s) \to (a, b)$ by

$$a = s + d; ,$$
$$b = s - d.$$

DWT 2

A DWT over four scales

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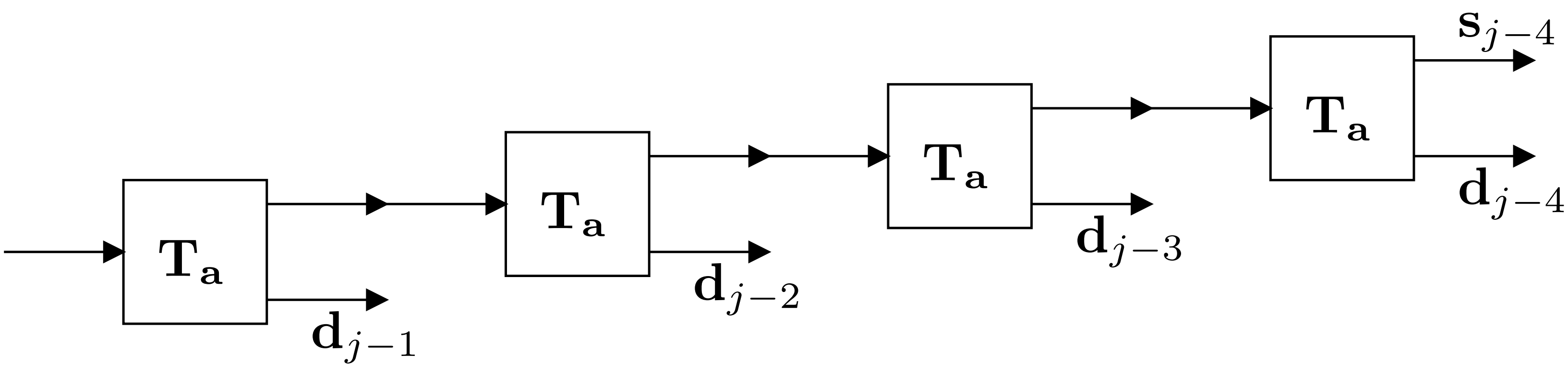
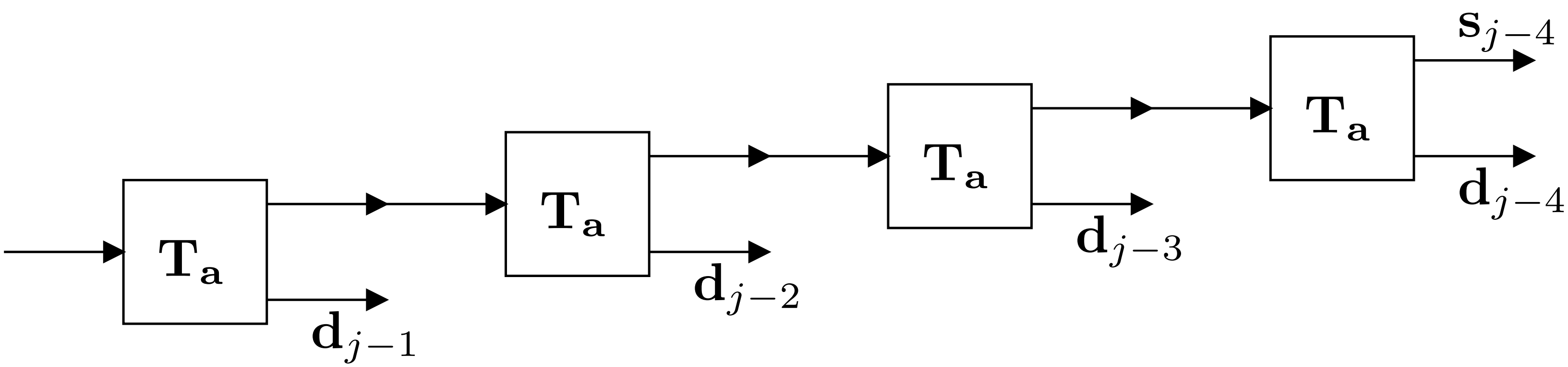


DWT 2

A DWT over four scales



The inverse DWT over four scales

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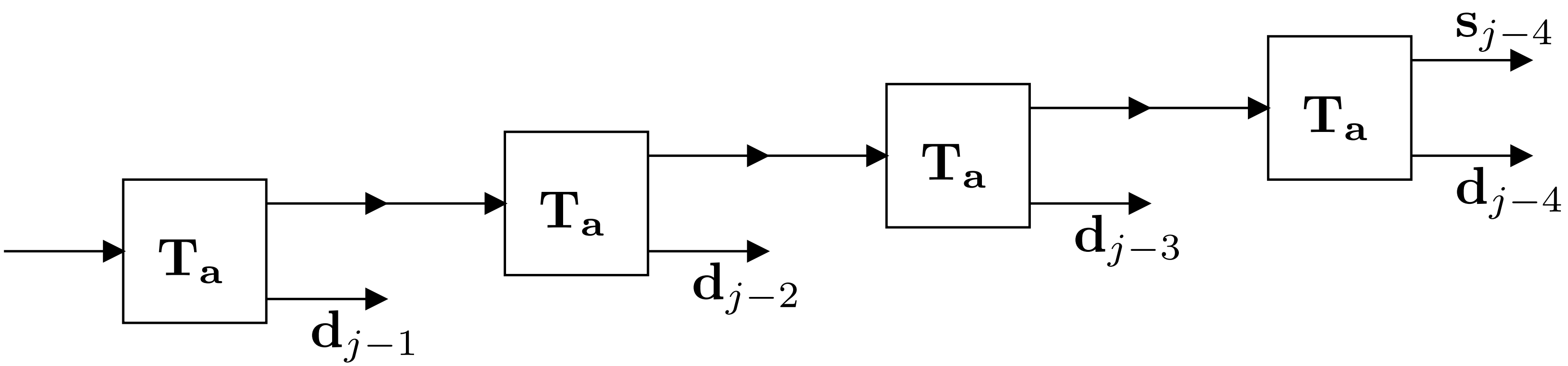

The inverse DWT over four scales