Basic Algebra

Algebra is about using letters in place of numbers. Sometimes it's possible to work out what the letter represents.

- *If you were told that $x + 4 = 10$, you can probably see straight away that $x = 6$.*

- *If you were told that $y - 7 = 5$, you can probably see straight away that $y = 12$.*

These are examples of linear equations and we'll look at them in more detail soon.

Writing Algebraic Expressions:

Writing an algebraic expression is like writing a sentence in maths instead of English. You do this by assigning letters to numbers. An algebraic expression is a set of instructions on how to perform a calculation.

Examples:
- **Write the following as an algebraic expression:**
  Five times a number minus three times another number.

First I need to assign letters to the 'unknown' numbers. I will call the first one 'n' and the second one 'm' so now I have:

Five times n minus three times m. *(Notice I have replaced the 1ˢᵗ and 2ⁿᵈ number with 'n' and 'm').*

Next I replace the words with maths symbols so that I have:

5 x n – 3 x m

Tidy up:

5n – 3m *This is our expression (notice that we don't need the multiplication sign as it is implied).*

- **A number divided by three: can be written as $\frac{x}{3}$**
- **Half of a number plus quarter of another number: can be written $\frac{x}{2} + \frac{y}{4}$**

- **A number plus 5 all multiplied by 3 can be written $(n + 5)3$**

We usually put the number at the front so we could rewrite this as:

$$3(n + 5)$$

When writing algebraic expressions you can choose any letter but make sure that different numbers are assigned different letters.

Simplifying Expressions:

Once you have an algebraic expression it can be simplified by collecting all the 'like terms' together (i.e. combining things that are the same letter or combination of letters). If an expression includes brackets then you may need to multiply out the brackets first to see what will combine.

Examples:

- **Simplify the following expression:** $3a + 7b - 2c - 4b - 6c + a$

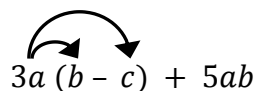Collect together any letters that are the same:

Look at the a's:	$3a + a = 4a$
Look at the b's	$7b - 4b = 3b$
Look at the c's	$-2c - 6c = -8c$

Put everything together and we have: $\underline{4a + 3b - 8c}$

Multiplying out Brackets:

- **Simplify the following:** $3a\,(b - c) + 5ab$

First remove the bracket. To do this you need to multiply the bit (term) outside the bracket by every bit (term) inside the bracket.

$$3a\,(b - c) + 5ab$$
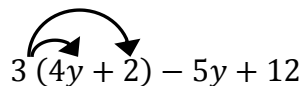
$$3a \times b = 3ab$$
$$3a \times -c = -3ac$$

(These are exactly the same so can add to give 8ab.)

So after multiplying out the brackets we have: $3ab - 3ac + 5ab = \underline{\underline{8ab - 3ac}}$

- **Simplify the following:** $3(4y + 2) - 5y + 12$

First remove the bracket. To do this you need to multiply the bit (term) outside the bracket by every bit (term) inside the bracket.

$$3\,(4y + 2) - 5y + 12$$

$$3 \times 4y = 12y$$
$$3 \times 2 = 6$$

So after multiplying out the brackets we have: $12y + 6 - 5y + 12 = \underline{\underline{7y + 18}}$

Solving Equations with one Unknown:

An equation which doesn't have any squared, cubed, or higher powers is called a linear equation. E.g. $3x + 2 = 14$.

When solving a linear equation you are trying to find the value of the letter in the equation. To do this you need to make the letter the **subject** of the equation (i.e. leave it on its own at one side of the equation).

When rearranging equations we follow the order:

> **Remove Brackets**
> **Add or subtract**
> **Multiply or divide**
> **Indices (powers)**

Notice that, apart from the Brackets still being 1st, this is BIDMAS in reverse because we are 'reversing' or 'undoing' the equation.

Examples:

- **Solve the equation: 5a = 15**

Starting Equation (has a multiplication).	5a = 15
Divide each side by 5 to make 'a' the subject	a = 3

  We have solved this simple equation and found the answer <u>**a = 3**</u>.

- **Solve the equation: 4b – 3 = 17**   *(Hint: collect b's at one side, numbers at the other.)*

Starting Equation.	4b - 3 = 17
Add 3 to each side (to get rid of the -3).	4b – 3 + 3 = 17 + 3
Tidy up.	4b = 20
Divide each side by 4 to leave 'b' on its own.	**b = 5**

- **Solve the equation: 5y – 6 = 2y + 3** *(collect y's at one side, numbers at the other.)*

Starting Equation.	5y – 6 = 2y + 3
Add 6 to each side (to get rid of the -6).	5y – 6 + 6 = 2y + 3 + 6
Tidy up	5y = 2y + 9
Take 2y from each side (so y's only on left).	5y – 2y = 2y – 2y + 9
Tidy up.	3y = 9
Divide each side by 3 to leave 'y' on its own.	**y = 3**

- **Solve the equation: 14 (m + 3) - 5 (2m + 6) = 44**
  First multiply out the brackets so that we can combine like terms.

Starting Equation.	14(m + 3) - 5(2m + 6) = 44
Multiply out brackets.	14m + 42 – 10m - 30 = 44
Collect like terms (m's and numbers)	4m + 12 = 44
Take 12 from each side (to get rid of 12)	4m + 12 - 12 = 44 - 12
Tidy up.	4m = 32
Divide each side by 4 to leave 'm' on its own.	**m = 8**

A quick method:

Look through the above examples again and notice that it looks like:

> **When something moves to the other side of the equals sign it does the "opposite".**
> *E.g. + becomes -, x becomes ÷ and vice versa.*

Notice that in the 2nd example, -3 disappeared from the left and became +3 on the right, in the 3rd example, 2y disappeared from the right and became +2y on the left etc.

 This method can be used to solve equations. When using this method it may help to keep the "=" sign lined up so that you can see when something moves from one side to the other.

Examples:

- **Solve the equation: 5b + 8 = 23**   *(collect b's at one side, numbers at the other.)*

Starting Equation.	5b(+ 8) =	23
+8 crosses the "=" sign and becomes -8.	5b =	23 (- 8)
Tidy up.	5b =	15
X5 crosses the "=" sign and becomes ÷5.	b =	15 ÷ 5
Final answer	**b =**	**3**

- **Solve the equation: 7t + 4 = 3t - 12** *(collect t's at one side, numbers at the other.)*

Starting Equation.	7t + 4 =	3t - 12
+4 crosses the "=" sign and becomes -4.	7t =	3t - 12 - 4
Tidy up.	7t =	3t - 16
+3t crosses the "=" sign and becomes -3t.	7t – 3t =	-16
Tidy up.	4t =	-16
X4 crosses the "=" sign and becomes ÷4.	t =	-16 ÷ 4
Final answer	t **=**	**- 4**

- **Solve: 3(2a – 4) = 2(a + 8)**   *(collect a's at one side, numbers at the other.)*

Starting Equation.	3(2a - 4) =	2(a + 8)
Multiply out brackets (to separate a's and numbers)	6a - 12 =	2a + 16
-12 crosses the "=" sign and becomes +12.	6a =	2a + 16 + 12
Tidy up.	6a =	2a + 28
+2a crosses the "=" sign and becomes -2a.	6a – 2a =	28
Tidy up.	4a =	28
X4 crosses the "=" sign and becomes ÷4.	a =	28 ÷ 4
Final answer	**a =**	**7**