**Methods:**

• Venn diagrams can help you visualize the relationships between sets of items, especially when the sets have some items in common.

• The principle of inclusion and exclusion gives a formula for finding the number of items in the union of two or more sets. For two sets, the formula is n(A or B) = n(A) + n(B) − n(A and B).

**\_\_\_\_\_**

• A **combination** is a selection of objects in which order is not important.

• The number of combinations of n distinct objects taken r at a time is denoted as nCr , C(n, r), or 1 n r 2 and is equal to n! / r!(n-r)!

**\_\_\_\_\_**

Use **the formula ( p + 1)(q + 1)(r + 1) … − 1** to find the total number of selections of at least one item that can be made from p items of one kind, q of a second kind, r of a third kind, and so on.

• A set with n distinct elements has **2 n** subsets including the null set.

• For combinations with some identical elements, you often have to **consider all possible cases** individually.

• In a situation where you must choose **at least one particular item**, either consider the **total number of choices available minus the number without the desired item** or add all the cases in which it is possible to have the desired item.

**Questions:**

7. Application: A cookie jar contains three chocolate-chip, two peanut-butter, one lemon, one almond, and five raisin cookies. (p.287)

a) In how many ways can you reach into the jar and select some cookies?

b) In how many ways can you select some cookies, if you must include at least one chocolate-chip cookie?

*Ans: 3C1 X (2+1)(1+1)(1+1)(5+1)*

This is a question from P.287 #8

1. A project team of 6 students is to be selected from a class of 30.

a) How many different teams can be selected?

b) Pierre, Gregory, and Miguel are students in this class. How many of the teams would include these 3 students?

c) How many teams would not include Pierre, Gregory, and Miguel?

**Answers:**

The textbook answers are A) 593775 b) 2925 c) 590850

I solved part a) by doing 30C6.

I was able to solve part b) by doing 27C3=2925 to find all teams when Pierre, Gregory **and** Miguel are part of the team.

*But I don't understand why I couldn't solve using the indirect method of 30C6-27C6 which equals 297765.*

For part C I was able to solve using the indirect method of 30C6-27C3 which equals the given answer of 590850.

*But why could I not solve this using the indirect method of 27C6, which I believe would be choosing 6 students out of the other 27?*

For part (b), the correct way to think about choosing the team is that **Pierre, Gregory, and Miguel automatically occupy three of the positions** in the team and then there are 27 people left, of which you must pick 3. Hence the total number of teams is (27C3) X1 =(27C3). The reason that  (30C6)−(27C6) doesn't give you the same answer is that (27C6) is the number of teams that don't include any of Pierre, Gregory, or Miguel (i.e. all three are not part of the team). **However, you would also need to subtract off teams that just had Pierre, or just had Gregory, or just had Miguel, or teams that have exactly two of them. This is the due to the**[**inclusion-exclusion principle**](https://en.wikipedia.org/wiki/Inclusion%E2%80%93exclusion_principle)**.**

For part (c), I think the confusion is coming from the wording of the question. When they say "how many teams would not include Pierre, Gregory, and Miguel" they mean how many teams would **not include all three members** (so, for example, you could have a team with just Pierre and Gregory and four other members but not Miguel). By using (27C6) you are looking at all teams that don't include any of Pierre, Gregory, or Miguel.

9. The game of euchre uses only the 9s, 10s, jacks, queens, kings, and aces from a standard deck of cards. How many five-card hands have

a) all red cards?

b) at least two red cards?

c) at most two red cards?

**Answer:**

We only use 24 cards, of which 12 are red  
  
number of 5-card hands = C(24,5) = 42504  
  
let's exclude number with no reds and only one red  
number with no reds = C(12,0) x C(12,5) = 792  
number with 1 red = C(12,1) x C(12,4) = 5940  
  
no number with at least two reds  
= 42504 - 792 - 5940  
= 35772

C) Total (42504 – 35772) + 12C2 X 12C3 = 21252

If you are dealing from a standard deck of 52 cards, a) how many different 4-card hands could have at least one card from each suit? b) how many different 5-card hands could have at least one spade? c) how many different 5-card hands could have at least two face cards (jacks, queens, or kings?

b) How many 5-card hands have at least one spade?

There are 13 Spades and 39 Others.  
  
There are: .52C5=2,598,96052C5=2,598,960 possible 5-card hands.  
  
How many have *no* Spades?  
Then the five cards must be chosen from the 39 Other cards.  
There are: .39C5=575,75739C5=575,757 ways.  
  
There are: .2,598,960−575,757=2,023,2032,598,960−575,757=2,023,203 hands with at least one Spade.

c) How many 5-card hands have at least 2 face cards?

There are 12 Face cards and 40 Others.  
  
The opposite of "at least 2 face cards" is "0 or 1 face card."  
  
0 Face cards  
The 5 cards are chosen from the 40 Others.  
There are: .40C5=658,00840C5=658,008 ways to get 0 Face cards.  
  
1 Face card  
There are: .12C1=1212C1=12 ways to get one Face card.  
There are: .40C4=91,39040C4=91,390 to get 4 Others.  
. . There are: .12⋅91,390=1,096,68012⋅91,390=1,096,680 ways to get one Face card.  
  
Hence, there are: .658,008+1,096,680=1,754,688658,008+1,096,680=1,754,688 ways to get 0 or 1 Face card.  
  
  
Therefore, there are: .2,598,960−1,754,688=844,2722,598,960−1,754,688=844,272 hands with at least 2 Face cards.

12. Application A theme park has a variety of rides. There are seven roller coasters, four water rides, and nine story rides. If Stephanie wants to try one of each type of ride, how many different combinations of rides could she choose?

7X4X9 = 252

13. Shuwei finds 11 shirts in his size at a clearance sale. How many different purchases could Shuwei make?

**Ans:** 2^11-1

1. How many different teams of 4 students could be chosen from the 15 students in the grade-12 Mathematics League?

Ans: 15 C 4 =1385

1. How many of the possible teams would include the youngest student in the league?

Ans: 15C4 – 14 C 4=364

c) How many of the possible teams would exclude the youngest student?

**Ans:** 15C4 – 364= 1001

p.288 18

The social convenor has 12 volunteers to work at a school dance. Each dance requires 2 volunteers at the door, 4 volunteers on the floor, and 6 floaters. Joe and Jim have not volunteered before so the social convenor does not want to assign them to work together. In how many ways can the volunteers be assigned?

1. **Break it up into 3 cases**  
   1. Joe at the door, then 10 others at the door, C(10,4) on the floor, C(6,6) as floaters  
   = 10x210x1 = 2100  
   2. Joe on the floor, leaving C(10,3) for the remainder on floor, C(8,2) for the door and C(6,6) as floaters  
   = 120x28x1 = 3360  
   3. Joe as floater leaving C(10,5) for the remaining floaters, C(6,4) for the floor, and C(2,2) for the door  
   = 252x15x1 = 3780  
     
   total = 2100+3360+3780 = 9240

19. Jeffrey is a DJ at a local radio station. For the second hour of his shift, he must choose all his music from the top 100 songs for the week. Jeffery will play exactly 12 songs during this hour. a) How many different stacks of discs could Jeffrey pull from the station’s collection if he chooses at least 10 songs that are in positions 15 to 40 on the charts? b) Jeffrey wants to start his second hour with a hard-rock song and finish with a pop classic. How many different play lists can Jeffrey prepare if he has chosen 4 hard rock songs, 5 soul pieces, and 3 pop classics? c) Jeffrey has 8 favourite songs currently on the top 100 list. How many different subsets of these songs could he choose to play during his shift?

10 songs from 15-40: C(26,10) = 26! / (16! \* 10!) = 5,311,735  
2 songs from 90: C(90,2) = 90! / (88! \* 2!) = 4,005  
  
If order doesn't matter (combinations), the answer is:  
5,311,735 \* 4,005 = 21,273,498,675  
  
If order does matter (permutations), the answer is:  
21,273,498,675 \* 12! = 10,190,039,902,922,880,000

20. There are 52 white keys on a piano. The lowest key is A. The keys are designated A, B, C, D, E, F, and G in succession, and then the sequence of letters repeats, ending with a C for the highest key.

a) If five notes are played simultaneously, in how many ways could the notes all be

i) As? 52 /7 = 7.4 or 8 As possible. 7X7 plus 3 = 52 therefore 8A,B,C 7 of the rest

so 8C5 = 56

ii) Gs? 7C 5 = 21

iii) the same letter?

iv) different letters?

56 possibilites for A, B, C and 21 for D, E, F, G

So 252 possibilities for the same letter

53!/47!5!=2,869,685

So 2,869,685-252=2,869,433 possibities for different letters.

b) If the five keys are played in order, how would your answers in part a) change?

p.288 #22

How many 4 letter word from parallelogram without caring about order?

Parallelogram has 3 l's,3 a's, 2 r's and 1 each of p,e,o,g,m a total of 8 distinct letters.

The possibilities are

3 l's and a single chosen from a,r,p,e,o,g or m: 7

Similarly 3 a's and a single: 7

2 l's and 2 a's, 2 l's and 2 r's or, 2 a's and 2 r's: 3

2 l's and 2 singles chosen from the other 7 letters: 7C2: 21

Similarly 2 a's and 2 singles: 21

Similarly 2 r's and 2 singles: 21

All singles 8C4:70

Total is 7+7+3+21+21+21+70=150

23. Inquiry/Problem Solving Suppose the artist in Example 1 of this section had two apples, two oranges, and two pears in his refrigerator. How many combinations does he have to choose from if he wants to paint a still-life with:

a) two pieces of fruit? 6

b) three pieces of fruit? 7

c) four pieces of fruit? 6

Ans: Cases

2 apples, 2 oranges, 2 pears. Call them a,o,p for short.

a) 2 pieces of fruit

(2a) (ao) (ap) (2o) (op) (2p) which is 6 possibilities.

b) three pieces of fruit

(2ao) (2ap) (a2o) (aop) (a2p) (2op) (o2p) which is 7 choices

c) four pieces of fruit.

(2a2o) (2aop) (2a2p) (a2op) (ao2p) (2o2p) which is 6 choices.

I listed all the possibilities.

36 cards

4 suits (d, sp, h, c)

9 cards in a hand

How many card hands possible?

36 C 9