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Generation of Periodic Trajectories for Optimal Robot Excitation

This paper describes the parameterization of robot excitation trajectories for optimal robot identification based on finite Fourier series. The coefficients of the Fourier series are optimized for minimal sensitivity of the identification to measurement disturbances, which is measured as the condition number of a regression matrix, taking into account motion constraints in joint and cartesian space. This approach allows obtaining small condition numbers with few coefficients for each joint, which simplifies the optimization problem significantly.

The periodicity of the resulting trajectories and the fact that one has total control over their frequency content, are additional features of the presented parameterization approach. They allow further optimization of the excitation experiments through time domain data averaging and optimal selection of the excitation bandwidth, which both help the reduction of the disturbance level on the measurements, and therefore improve the identification accuracy.

1 Introduction

Accurate robot control and realistic robot simulation require an accurate dynamic robot model. The design of an advanced robot controller, such as a computed torque or a computed velocity controller is based on the robot model, and its performance depends directly on the model accuracy (Spong and Vidyasagar, 1989; Freund, 1982; Torfs and De Schutter, 1992; Markiewicz, 1973). Robot simulation without a dynamic robot model can not give realistic execution time estimates, for example, in the case of spot welding operations where the time required to stop the robot end effector at the different spot welding places depends on the robot dynamics.

Experimental robot identification is the only time and effort efficient way to obtain accurate robot models as well as specifications on their accuracy, confidence and validity. The data provided by the robot manufacturers are insufficient, inaccurate, or often nonexisting if friction and compliance characteristics are concerned. Direct measurement of the physical parameters is unrealistic because of the complexity of most robots.

It is well recognized that reliable, accurate, and efficient robot identification requires specially designed experiments. When designing an identification experiment for a robot manipulator, it is essential to consider whether the excitation is sufficient to provide accurate and fast parameter estimation in the presence of disturbances. The influence of disturbances, such as measurement noise and actuator disturbances, on the parameter estimates depends directly on the condition (number) of the set of equations that generate the parameters (Golub and Van Loan, 1989; Lawson and Hanson, 1974). The condition of this set of equations depends on the excitation during the identification experiment.

The generation of a robot trajectory that optimizes the condition of the parameter estimation involves nonlinear optimization with motion constraints (i.e. constraints on joint positions, velocities, and accelerations). Several approaches have been presented. The main difference between these approaches lies in the parameterization of the excitation trajectory. The parameters that describe the excitation trajectory are the degrees of freedom of the optimization problem. Armstrong (1989) describes an approach in which the degrees of freedom are the elements of a sequence of joint accelerations. This approach is the most general one, but requires a large number of degrees of freedom, such that optimization is cumbersome. Moreover, motion constrains are difficult to satisfy. Gautier (1993) optimizes a linear combination of the condition number and the so-called equilibrium of the set of equations that generate the parameters. The degrees of freedom are a finite set of joint angles and velocities separated in time. The actual trajectory is continuous and smooth, and calculated by interpolating a line between the optimized points, assuming zero initial and final acceleration and using a fifth-order polynomial. Motion constraints can be satisfied. Otani and Kakizaki (1993) use trajectories which are a combination of a cosine and a ramp, such that the joint velocities change sinusoidally between zero and their maximum value. Excitation is optimized through carefully selecting the frequency and amplitude of the sinusoidal movements for each joint, and the initial robot configuration in a rather ad hoc way.

This paper presents a new approach to the parameterization of exciting robot trajectories. It combines the advantages of the method described in (Gauthier, Janin and Presse, 1993) with new interesting features. The excitation trajectory for each joint is a finite sum of harmonic sine and cosine functions, i.e. a finite fourier series. The amplitude of the sine and cosine functions are the degrees of freedom that have to be optimized. This approach guarantees periodic trajectories which is advantageous because it allows time domain data averaging, which improves the signal to noise ratio of the experimental data and allows estimation of the characteristics of the measurement noise. This information is valuable in case of maximum likelihood parameter estimation. The use of finite Fourier series allows

- specifying the bandwidth of the excitation trajectories, such that excitation of the robot flexibility can be either completely avoided or intentionally brought about.
- calculating the joint velocities and accelerations in an analytic way from the measured response. For this purpose, the measured encoder readings are first approximated, in a least squares sense, as a finite sum of sine and cosine functions.

None of the existing methods possesses these features.

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Extending the length of a Fourier series to infinity allows approximation of any arbitrary periodic function, while non-periodic functions can be approximated arbitrarily close if non-harmonic sine and cosine functions are allowed in the Fourier series. In this sense, the method presented in this paper approximates Armstrong's approach (Armstrong, 1989), but has the advantage that motion constraints can be met. Moreover, simulation results show that Fourier series with only a few harmonic sine and cosine functions for each joint are sufficient to reduce the condition number of the set of equations that generate the parameters to values below 10, such that there is no need for more degrees of freedom in the parameter space and to allow arbitrary trajectories.

The following section describes the least squares robot parameter estimation approach, i.e. the influence of the robot excitation experiment on the condition of the estimation. Section 3 describes the parameterization of the excitation trajectories and formulates the constrained optimization problem which generates the optimal Fourier coefficients. Section 4 illustrates the presented approach through its application on a rotational joint 3 dof robot.

2 Least Squares Robot Parameter Estimation

The following set of n differential equations describes the dynamic behavior of an n-degree-of-freedom rigid robot (Spong and Vidyasagar, 1989):

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}. \qquad (1)$$

$\mathbf{q}$ is the n-vector of the joint angles. $\mathbf{M}$ the $n \times n$ mass matrix, which is function of the joint angles. $\mathbf{f}$ the n-vector which specifies the gravitation, Coriolis, viscous and Coulomb friction effects. $\boldsymbol{\tau}$ the n-vector of the actuator torques. The parameters related to the mass distribution and the friction coefficients are the unknown parameters that have to be estimated. The dynamic model is linear in the friction coefficients, and the parameters of the mass distribution if they are combined in the so called barycentric parameters (Fisette, Raucent and Samin, 1993). In that case, the dynamic model (1) can be rewritten as a set of linear equations:

$$\boldsymbol{\Phi}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\theta} = \boldsymbol{\tau}. \qquad (2)$$

$\boldsymbol{\Phi}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ is the $n \times r$ regressor matrix, depending on the joint angles, velocities, and accelerations. r is the number of independent robot parameters. $\boldsymbol{\theta}$ is the r-vector containing the unknown barycentric parameters and friction coefficients.

Robot identification deals with the problem of estimating the model parameters $\boldsymbol{\theta}$ from the response measured during a robot excitation experiment. In most cases, the data obtained from an experiment is a sequence of joint angles and motor currents, from which a sequence of joint velocities, accelerations, and motor torques are calculated. These data are entered in equation (2), yielding a regressor matrix $\boldsymbol{\Phi}(\mathbf{q}_k, \dot{\mathbf{q}}_k, \ddot{\mathbf{q}}_k)$ and vector $\boldsymbol{\tau}_k$ for each time sample k in the data sequence. Combining these matrices and vectors yields the following overdetermined set of equations:

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{b}, \qquad (3)$$

with

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{\Phi}(\mathbf{q}_1, \dot{\mathbf{q}}_1, \ddot{\mathbf{q}}_1) \\ \vdots \\ \boldsymbol{\Phi}(\mathbf{q}_M, \dot{\mathbf{q}}_M, \ddot{\mathbf{q}}_M) \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_M \end{bmatrix}.$$

M represents the length of the data sequence. The choice of M depends on the signal-to-noise ratio of the measured signals and on the required accuracy of the estimated parameters. If the data sequence is long, i.e. if M is large, more data are taken into account during the parameter estimation, yielding more accurate parameter estimates. If the excitation is periodic, data averaging is possible, such that very long data sequences can be taken into account while keeping M equal to the number of samples measured during one period of the excitation. By keeping M, i.e. the number of rows of matrix $\mathbf{A}$, small, one can reduce the number of calculations required for the estimation of $\boldsymbol{\theta}$.

The parameter vector $\boldsymbol{\theta}$ can be estimated as the linear least squares solution $\boldsymbol{\theta}_{ls}$ of this overdetermined set of equations:

$$\boldsymbol{\theta}_{ls} = \arg\min_{\boldsymbol{\theta}} \|\mathbf{A}\boldsymbol{\theta} - \mathbf{b}\|_2^2 = \mathbf{A}^{+}\mathbf{b}, \qquad (4)$$

where $\|\cdot\|_2$ represents the 2-norm of a vector and $\mathbf{A}^+$ the pseudo inverse of the matrix $\mathbf{A}$ (Golub and Van Loan, 1989).

The 2-norm condition number of matrix $\mathbf{A}$**, cond ($\mathbf{A}$), is a measure for the sensitivity of the least squares solution $\boldsymbol{\theta}_{ls}$ to perturbations on the elements of $\mathbf{A}$ and $\mathbf{b}$, provided that the matrix is well equilibrated (Golub and Van Loan, 1989), (Gautier, Janin and Presse, 1993), (Lawson and Hanson, 1973). cond ($\mathbf{A}$) is therefore taken as the criterion to design the excitation experiment: the robot trajectory during the excitation experiment should be such that cond ($\mathbf{A}$) is minimal, without violating the motion constraints. The calculation of such a trajectory is a complex constrained nonlinear optimization problem, which can be solved using sequential quadratic programming methods, which are available in commercially available software packages such as Matlab (Grace, 1992). The complexity of the optimization problem depends on the motion constraints and the parametrization of the trajectory. This paper presents a novel approach to trajectory parameterization, which allows obtaining small condition numbers with few parameters.

Gautier (1993) states that even for small values of cond ($\mathbf{A}$), the accuracy of $\boldsymbol{\theta}_{ls}$ can be poor when $\boldsymbol{\theta}$ is badly scaled. Gautier suggests solving this problem by multiplying the columns of $\mathbf{A}$ with *a priori* available values of their respective parameters. This reduces the identification problem to the estimation of multiplicative corrections to the *a priori* available parameter values. With respect to the calculation of the excitation trajectory, this different approach to the parameter estimation only changes the regressor matrix of which the condition number has to be minimized. The trajectory parameterization presented in the following section can be applied to both approaches without changing the optimization algorithm.

3 Parameterization of the Robot Excitation Trajectory

The angular position q_i, velocity $\dot{q}_i$, and acceleration $\ddot{q}_i$ trajectories for joint i of a n degrees-of-freedom robot are finite Fourier series:

$$q_i(t) = \sum_{l=1}^{N_i} \frac{a_l^i}{2\pi f_f l} \sin(2\pi f_f l t) - \frac{b_l^i}{2\pi f_f l} \cos(2\pi f_f l t) + q_{i0}$$

$$\dot{q}_i(t) = \sum_{l=1}^{N_i} a_l^i \cos(2\pi f_f l t) + b_l^i \sin(2\pi f_f l t)$$

$$\ddot{q}_i(t) = \sum_{l=1}^{N_i} -a_k^i 2\pi f_f l \sin(2\pi f_f l t) + b_l^i 2\pi f_f l \cos(2\pi f_f l t), \qquad (5)$$

with f_f the fundamental frequency of the Fourier series. This Fourier series specifies a periodic function with period $T_f = 1/f_f$. The fundamental frequency is common for all joints, in order to preserve the periodicity of the overall robot excitation. Each Fourier series contains $2 \times N_i + 1$ parameters, that constitute the degrees of freedom for the optimization problem: a_l^i, and b_l^i, for $l = 1$ to N_i, which are the amplitudes of the cosine and sine functions, and q_{i0} which is the offset on the position trajectory.

** The 2-norm condition number of a matrix A equals the ratio of the largest singular value of A to the smallest (Golub and Van Loan, 1989) tory. The offset determines the robot configuration around which the robot excitation will occur. For simplicity of notation, we consider that the vector δ contains the trajectory parameters a_l^i, b_l^i (for $l = 1$ to N_i), and q_{i0} for all joints i.

The Fourier series for each joint consists of a fundamental frequency and $N_i - 1$ harmonic frequencies. The number of harmonic frequencies determines the bandwidth of the trajectory. Certain harmonic frequencies can be omitted from the Fourier series. This allows restricting the energy of the trajectory within a specific frequency band. The choice of the fundamental frequency f_f depends on (1) the desired period of the excitation, and is therefore related to the duration of the experiment, and (2) the minimum frequency resolution of the excitation trajectory, which is f_f. Lower frequency resolution can be obtained by skipping harmonic frequencies.

This trajectory parameterization approach allows controlling the frequency content (frequency resolution and bandwidth) of the excitation, which is extremely important if the excitation of the robot flexibility must be either completely avoided or intentionally brought about.

The motion constraints are limitations on the joint angles, velocities, and accelerations, and on the robot end effector position in the cartesian space in order to avoid collision with the environment and with itself. This last type of constraint involves forward kinematics calculations. All constraints are implemented as continuous functions which are negative if the constraint is satisfied and positive if it is violated. Section 4 gives an example of these constraints.

The following expression gives the mathematical formulation of this constrained optimization problem:

$$\hat{\delta} = \arg\min_{\delta} \operatorname{cond}(\mathbf{A}(\delta, f_f)), \qquad (6)$$

with

$$\begin{cases} \mathbf{q}_{min} \le \mathbf{q}(pT_s, \delta) \le \mathbf{q}_{max} \\ -\dot{\mathbf{q}}_{max} \le \dot{\mathbf{q}}(pT_s, \delta) \le \dot{\mathbf{q}}_{max} \\ -\ddot{\mathbf{q}}_{max} \le \ddot{\mathbf{q}}(pT_s, \delta) \le \ddot{\mathbf{q}}_{max} \\ \{\mathbf{s}(\mathbf{q}(pT_s, \delta))\} \subset \mathbf{S} \end{cases}$$

for

$$0 \le p \le \frac{T_f}{T_s},$$

with T_s the sampling period for the data acquisition during the experiment. The fundamental frequency f_f of the trajectories is fixed. Its value depends on the desired frequency spectrum and the data acquisition capacity. The constraints in (6) are elementwise inequalities. $\mathbf{q}_{min}$, $\mathbf{q}_{max}$, $\dot{\mathbf{q}}_{max}$, and $\ddot{\mathbf{q}}_{max}$ are the vectors containing the minimum *min*, and maximum *max* joint angle, velocity and acceleration values. $\mathbf{S}$ represents the available working space of the robot. $\{s(\mathbf{q})\}$ represents the set of end effector positions resulting from the joint trajectories, using forward kinematics calculation.

4 Verification on an Industrial Robot

This section illustrates the presented trajectory parameterization through its application on a KUKA IR/361 industrial robot (see Fig. 1), which is an elbow type articulated manipulator. Only the first three axes are considered for simplicity of description: the influence of the movement of the wrist axes on the dynamic equations is neglected.

4.1 The Robot Model. The dynamic robot model is simplified: the inertia tensor is diagonal, and the center of gravity of each link lies in the Y-Z plane of its link frame (see Fig. 1). It is the same as the simplified model for a PUMA 562 robot (Fisette, Raucent and Samin, 1993). The robot inertial parameters are:

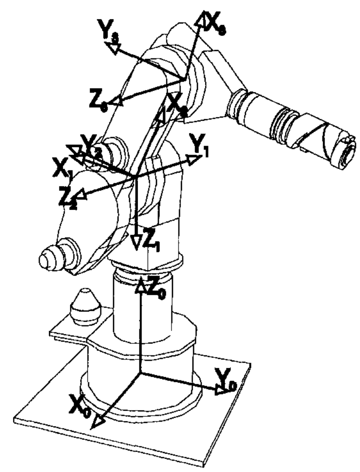

Fig. 1 Schematic representation of a KUKA IR/361 industrial robot

- I_{xx}^i, I_{yy}^i, I_{zz}^i: the inertia parameters of link i, $i = 1, 2, 3$;
- l_y^{ii}, l_z^{ii}: position of center of mass of a link i in the link frame of link i, $i = 1, 2, 3$;
- $l_z^{i(i+1)}$: position of the joint of link $i+1$ in the link-frame of link i, $i = 1, 2$;
- m_z^i: mass of link i, $i = 1, 2, 3$.

These inertial parameters are combined into a nonredundant set of barycentric parameters (Fisette, Raucent and Samin, 1993):

$$\begin{aligned} k_{zz}^1 = {} & I_{zz}^1 + l_y^{11}*l_y^{11}*m^1 + 0.5*(I_{xx}^2 + m^3*l_z^{23}*l_z^{23} \\ & - m^2*(-l_y^{22}*l_y^{22} - l_z^{22}*l_z^{22}) + I_{zz}^2 + l_y^{22}*l_y^{22}*m^2 \\ & + I_{xx}^3 - m^3*(-l_y^{33}*l_y^{33} - l_z^{33}*l_z^{33}) + I_{zz}^3 + l_y^{33}*l_y^{33}*m^3) \end{aligned}$$

$$k_{yz}^2 = -l_y^{22}*l_z^{22}*m^2 - l_y^{33}*m^3$$

$$\begin{aligned} k_d^2 = {} & 0.5*(I_{zz}^2 + l_y^{22}*l_y^{22}*m^2 - I_{xx}^2 + m^3*l_z^{23}*l_z^{23} \\ & - m^2*(-l_y^{22}*l_y^{22} - l_z^{22}*l_z^{22})) \end{aligned}$$

$$k_{yy}^2 = I_{yy}^2 + m^3*l_z^{23}*l_z^{23} + l_z^{22}*l_z^{22}*m^2$$

$$mb_z^2 = l_z^{22}*m^2 + l_z^{23}*m^3$$

$$k_{yz}^3 = -l_y^{33}*l_z^{33}*m^3$$

$$\begin{aligned} k_d^3 = {} & 0.5*(I_{zz}^3 + l_y^{33}*l_y^{33}*m^3 - I_{xx}^3 \\ & - m^3*(-l_y^{33}*l_y^{33} - l_z^{33}*l_z^{33})) \end{aligned}$$

$$k_{yy}^3 = I_{yy}^3 + l_z^{33}*l_z^{33}*m^3$$

$$mb_z^3 = l_z^{33}*m^3$$

These minimal barycentric parameters are combined with the friction parameters into the following nonredundant dynamic robot model:

$$\begin{aligned} \tau_1 = {} & k_{zz}^1*\ddot{q}_1 + k_{yz}^2*(\ddot{q}_2*\cos(q_2) - \dot{q}_2*\dot{q}_2*\sin(q_2)) \\ & + k_d^2*(\ddot{q}_1*\cos(2*q_2) - \dot{q}_1*\dot{q}_2*\sin(2*q_2)) \\ & + k_{yz}^3*(\ddot{q}_2*\cos(q_2+q_3) + \ddot{q}_3*\cos(q_2+q_3) \\ & - \dot{q}_2*\dot{q}_2*\sin(q_2+q_3) - \dot{q}_3*\dot{q}_3*\sin(q_2+q_3) \\ & - 2*\dot{q}_2*\dot{q}_3*\sin(q_2+q_3)) \\ & + k_d^3*(\ddot{q}_1*\cos(2*q_2+2*q_3) \\ & - \dot{q}_1*\dot{q}_2*\sin(2*q_2+2*q_3) \\ & - \dot{q}_1*\dot{q}_3*\sin(2*q_2+2*q_3)) \end{aligned}$$

$$
\begin{aligned}
&+ mb_z^3 * (2 * \ddot{q}_1 * \sin(q_2) * \sin(q_2 + q_3) \\
&+ 2 * \dot{q}_1 * \dot{q}_2 * \sin(2 * q_2 + q_3) \\
&+ 2 * \dot{q}_1 * \dot{q}_3 * \cos(q_2 + q_3) * \sin(q_2)) * l^2 3_z \\
&\qquad + c_1 * (\text{sign}(\dot{q}_1)) + v_1 * (\dot{q}_1) \quad (7)
\end{aligned}
$$

$$
\begin{aligned}
\tau_2 = {} & k_{yz}^2 * (\ddot{q}_1 * \cos(q_2)) + k_d^2 * (2 * \dot{q}_1 * \dot{q}_1 * \cos(q_2) * \sin(q_2)) \\
&+ k_{yy}^2 * (\ddot{q}_2) + mb_z^2 * (-g * \sin(q_2)) \\
&+ k_{yz}^3 * (\ddot{q}_1 * \cos(q_2 + q_3)) \\
&+ k_d^3 * (2 * \dot{q}_1 * \dot{q}_1 * \cos(q_2 + q_3) * \sin(q_2 + q_3)) \\
&+ k_{yy}^3 * (\ddot{q}_2 + \ddot{q}_3) + mb_z^3 * (-g * \sin(q_2 + q_3) \\
&+ (\ddot{q}_2 * \cos(q_3) - \dot{q}_1 * \dot{q}_1 * \cos(q_2 + q_3) * \sin(q_2) \\
&+ \dot{q}_2 * \dot{q}_2 * \sin(q_3) + \ddot{q}_2 * \cos(q_3) + \ddot{q}_3 * \cos(q_3) \\
&- \dot{q}_1 * \dot{q}_1 * \cos(q_2) * \sin(q_2 + q_3) - (\dot{q}_2 * \dot{q}_2 * \sin(q_3) \\
&- \dot{q}_3 * \dot{q}_3 * \sin(q_3) - 2 * \dot{q}_2 * \dot{q}_3 * \sin(q_3)) * l_z^{23} \\
&\qquad + c_2 * (\text{sign}(\dot{q}_2)) + v_2 * (\dot{q}_2) \quad (8)
\end{aligned}
$$

$$
\begin{aligned}
\tau_3 = {} & k_{yz}^3 * (\ddot{q}_1 * \cos(q_2 + q_3) \\
&+ k_d^3 * (2 * \dot{q}_1 * \dot{q}_1 * \cos(q_2 + q_3) * \sin(q_2 + q_3)) \\
&+ k_{yy}^3 * (\ddot{q}_2 + \ddot{q}_3) + mb_z^3 * (-g * \sin(q_2 + q_3) \\
&+ (\ddot{q}_2 * \cos(q_3) - \dot{q}_1 * \dot{q}_1 * \cos(q_2 + q_3) * \sin(q_2) \\
&+ \dot{q}_2 * \dot{q}_2 * \sin(q_3)) * l_z^{23}) \\
&\qquad + c_3 * (\text{sign}(\dot{q}_3)) + v_3 * (\dot{q}_3). \quad (9)
\end{aligned}
$$

τ_i is the torque applied at joint i. g is the gravity constant. c_i and v_i are the Coulomb and viscous friction parameters for joint i. The 15 unknown parameters are: k_{zz}^1, k_{yz}^2, k_d^2, k_{yy}^2, k_{yz}^3, k_d^3, k_{yy}^3, mb_z^2, mb_z^3, c_1, c_2, c_3, v_1, v_2, and v_3. l_z^{23} is the distance between joint 2 and 3, and is assumed to be known (480 mm for the KUKA IR/361).

4.2 Trajectory Optimization: Results. Equations (7)–(9) are linear in the (15) unknown (barycentric and friction) parameters, and can therefore be reformulated as a set of linear equations, like Eq. (2). Entering the data obtained from a robot excitation experiment into Eq. (2) yields an overdetermined set

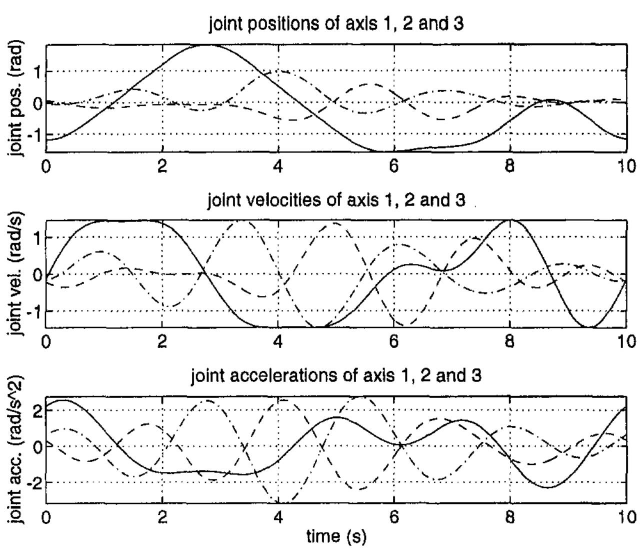

Fig. 2 Optimal excitation profile, axis 1 (solid line), axis 2 (dashed line) and axis 3 (dash-dotted line)

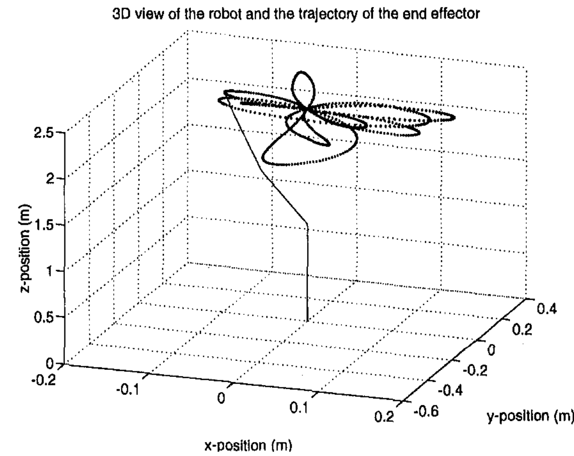

Fig. 3 3D impression of the trajectory of the end effector

of equations like (3). The robot excitation trajectories, parameterized according to Eq. (5), are optimized such that the condition number of matrix **A** in Eq. (3) is minimal, taking into account the motion constraints for the robot.

The motion constraints are limitations on the joint angles, velocities, and accelerations, and on the robot end effector position in the cartesian space in order to avoid collision with the ceiling of the laboratory and with itself. These limitations are specified by the robot control and measurement system (Van De Poel et al., 1993):

- joint angle limits (rad): $-3.2 < q_1 < 3.2$, $-1.95 < q_2 < 1.95$ and $-2.3 < q_3 < 2.3$,
- joint velocity limits (rad/s): $-1.45 < \dot{q}_i < 1.45$.
- joint acceleration limits (rad/s^2): $-3 < \ddot{q}_i < 3$,
- limits on the height of the end effector (mm): $500 < z_{ee} < 2500$,
- the robot touches its base if $r_{ee} < 500$ mm and $z_{ee} < 1600$ mm. r_{ee} is the distance of the end effector from the first robot axis. z_{ee} is the height of the end effector above the ground. r_{ee} and z_{ee} are obtained from forward kinematics calculations.

The excitation trajectories are 5-term Fourier series, yielding 11 parameters for each joint. The fundamental frequency of the trajectories is 0.1 Hz. The sampling rate for the experiment is 75 Hz, yielding data vectors with a length of 750 data samples per period.

The constrained optimization is performed using the "CONSTR" function of the Optimization Toolbox of Matlab (Grace, 1992). This function uses a sequential quadratic programming method. It requires the reformulation of the motion constraints to constraints of the type:

$$g(\delta) \le 0$$

The mentioned motion constraints are therefore reformulated as follows:

$$
\text{for} \quad i = 1{:}3 \begin{cases}
\max_{k=1:750} (q_i(k, \delta) - q_i^{\max}) \le 0 \\
-\min_{k=1:750} (q_i(k, \delta) - q_i^{\min}) \le 0 \\
\max_{k=1:750} (\dot{q}_i(k, \delta) - \dot{q}_i^{\max}) \le 0 \\
-\min_{k=1:750} (\dot{q}_i(k, \delta) - \dot{q}_i^{\max}) \le 0 \\
\max_{k=1:750} (\ddot{q}_i(k, \delta) - \ddot{q}_i^{\max}) \le 0 \\
-\min_{k=1:750} (\ddot{q}_i(k, \delta) - \ddot{q}_i^{\max}) \le 0
\end{cases}
$$

$$\min_{k=1:750} (r_{ee}(k, \delta) - 500) \le 0$$

$$\max_{k=1:750} (2500 - r_{ee}(k, \delta)) \le 0$$

Table 1 Parameters of the optimized trajectory

i/l	a_l^i 1	2	3	4	5	q_{i0}
1	−0.0575	0.8251	0.2563	0.3433	0.2265	−0.2462
2	−0.0294	−0.0954	0.2199	−0.1801	−0.2016	−0.0748
3	0.0741	0.0095	−0.1377	0.4283	−0.1676	0.1385

i/l	b_l^i 1	2	3	4	5
1	0.7284	−0.7267	−0.4001	−0.0079	0.2681
2	−0.0466	0.0514	−0.3723	0.5171	−0.3609
3	0.0752	−0.0954	0.5199	−0.5686	−0.0989

$$\max_{k=1:750}\,(1600 - z_{ee}(k, \delta))(500 - r_{ee}(k, \delta)) \leq 0$$

Figure 2 shows the optimized excitation trajectories. Figure 3 gives a 3D impression of the trajectory of the end effector. The condition number of matrix $\mathbf{A}$ (Eq. (3)) that results from these trajectories is 8.66. Table 1 shows the trajectory parameters for all joints. The condition number of $\mathbf{A}$ can be reduced to 4 if the friction coefficients are omitted from the dynamic model, i.e. if only inertial parameters are considered. This example shows that this parameterization approach allows generating, with few parameters, excitation trajectories which result in a small condition number. Although the obtained optima are not necessarily the global optima, this approach is valuable because the resulting condition numbers are sufficiently small.

5 Conclusion

The presented approach to the parameterization of robot excitation trajectories is based on finite Fourier series. This allows generating periodic robot excitation experiments which are robust with respect to measurement inaccuracies. The robustness is measured as the condition number of a regression matrix containing the robot dynamic equations. The periodicity of the trajectories allows improving the signal to noise ratio of the measurements and accurate frequency domain calculation of the joint velocities and accelerations, which both help obtaining accurate model parameter estimates. The trajectories are band-limited such that the excitation of robot flexibility can be either completely avoided or intentionally brought about.

The application of the trajectory generation approach is illustrated on a KUKA IR 361 industrial robot. The presented approach can be applied in general to any system whose dynamic equations are linear in the parameters, such as other types of industrial robots, machine tools, etc. . . .

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