IBS module in AT

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Reminder of theory

IBS is multiple scattering that causes emittance evolution

Radiation damp/diff IBS

$$\frac{d\varepsilon_a}{dt} = -2\frac{\varepsilon_a - \varepsilon_{a0}}{\tau_{a,rad}} + \frac{\varepsilon_a}{T_a} \qquad a = 1,2,3$$

IBS growth rates: $\frac{1}{T_{a,IBS}} = \frac{1}{\varepsilon_a}\frac{d\varepsilon_a}{dt}\Big|_{IBS}$

How to compute?

Reminder on eigen-emittance

Given one turn map matrix M, we find 3 invariant matrices

$$M^T G_a M = G_a$$

Eigenemittance defined as average value of invariants

$$\varepsilon_a = \left\langle G_{a,ij} z_i z_j \right\rangle = Tr(G_a \Sigma) \qquad \Sigma_{ij} = \left\langle z_i z_j \right\rangle$$

Uncoupled 1-D example:

$$G_x = \begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix} \Rightarrow \varepsilon_x = \gamma_x \sigma_x^2 + 2\alpha \sigma_{xx'} + \beta \sigma_{x'}^2$$

Implementation in Matlab

[G1,G2,G3]=find_inv_G(m66);

A=amat(m66)

Normalization matrix A transforms
Map into rotations

$$A^{-1}MA = R$$

$$G_a = A^{-1} I_a A$$

Where, e.g.

$$I_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Beam matrices

Matrices defined via distribution matrix in beam rest frame

$$f(\vec{z}) = \frac{1}{(2\pi)^{3/2}\varepsilon_1\varepsilon_2\varepsilon_3} e^{-z_i z_j M_{ij}}$$

$$M = M_1 + M_2 + M_3 = \frac{G_1}{\varepsilon_1} + \frac{G_2}{\varepsilon_2} + \frac{G_3}{\varepsilon_3} = \begin{pmatrix} A & B \\ B & C \end{pmatrix}$$

$$\overline{M} = L^T M L$$

Transforms to beam frame

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$$

Lorentz transformation

Effect of IBS

Invariant from one turn map

Growth of moments From IBS

$$\frac{d\varepsilon_a}{dt} = Tr[G_a \frac{d\Sigma}{dt}]$$

$$\frac{d\Sigma}{dt} = \mathrm{A}\pi L_c Tr(C_a K)$$

Coulomb log

$$A = \frac{N_b r_0^2 c}{8\pi^3 \beta^3 \gamma^4 \varepsilon_1 \varepsilon_2 \varepsilon_3}$$

IBS functions

Given emittances, find one turn map at each position needed around ring.
From one turn maps and emittances, compute local growth rates

[tau1_loc,tau2_loc,tau3_loc]=IBS_GR_local(m66,eps1,eps2,eps3,Nb,gamma)

Can implement different algs here

Now average the growth rates around the ring to get global IBS growth rates.

[tau1_glob,tau2_glob,tau3_glob]=IBS_GR_global(ring,eps1,eps2,eps3,Nb,gamma)

Now use growth rates and radiation damping to evolve emittances by dt.

Iterate until equlibrium is reached.

Growth rates

$$K_{ab} = \int d\Omega \frac{h_{ab}}{h_3}$$

$$h_{ab} = 1 - 3\hat{e}_a \hat{e}_b$$

$$h_3 = \hat{e}_a \overline{C} \hat{e}_b$$

$$\hat{e} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

$$\int d\Omega = \int_0^{\pi} \int_0^{\pi/2} \sin\theta d\theta d\phi$$

Given the C_a matrices and these computed K matrices, we get the IBS emittance growth rates:

$$\frac{1}{T_a} = \frac{1}{2} Tr\left[\overline{C}_a K\right]$$

This agrees with the equations of Bjorken and Mtingwa in the uncoupled Case, but generalizes to coupling also.

Coulomb log

Coulomb log comes from integration over beam
Assuming uniform distribution. It is a logarithmic
Divergence that would be cut off at large distances
By the beam size.

$$L_c = \log \frac{b_{\max}}{b_{\min}}$$

Different definitions of minimum impact
Parameter.

Min/max impact parameters

Max impact parameter choices…

Minimum beam size:

$$\sigma_y = \sqrt{\varepsilon_y \beta_y}$$

Average interparticle spacing

$$\bar{\rho} = \frac{N_b}{(2\pi)^{3/2}\sigma_x\sigma_y\gamma\sigma_z}$$

$$\bar{r}_{IPS} = \bar{\rho}^{-1/3}$$

Min impact parameter choices…

Distance of minimum approach

$$\frac{r_0}{\gamma\sigma_{x'}}$$

“Tail cut” procedure

$$\sqrt{\frac{1}{\pi\rho\sigma_v\tau_{\max}}}$$

$$\sigma_v = c\gamma\sqrt{\frac{\varepsilon_x}{\beta_x}}$$

Maximum damping rate

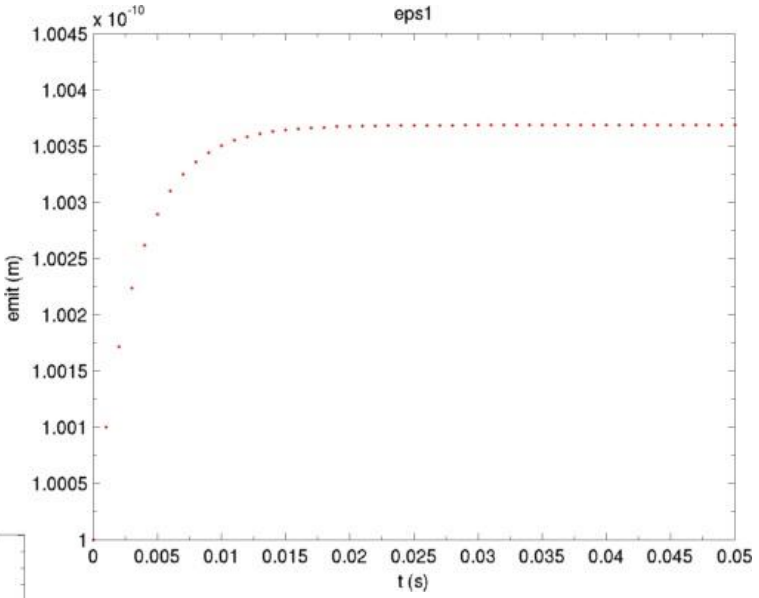
Find C matrices from m66

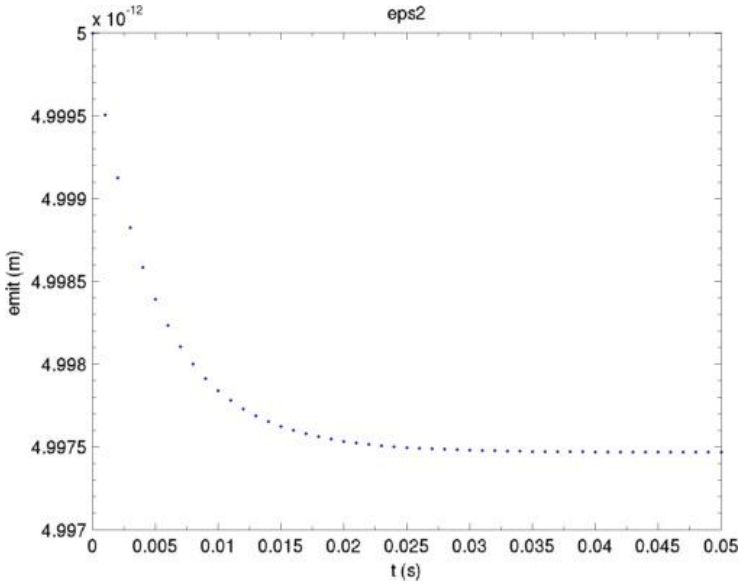
```
[cm,cm1,cm2,cm3] = find_Cmat_IBS(m66, eps1,eps2,eps3, gamma)
%compute the matrix of momentum associated with the beam matrix,
in the
%beam frame.  Used for the IBS computation.
[G1,G2,G3]=find_inv_G(m66);
Mmat1 = G1/eps1;
Mmat2 = G2/eps2;
Mmat3 = G3/eps3;

L=[1 0 0 0 0 0;...
  0 1 0 0 0 0;...
  0 0 1 0 0 0;...
  0 0 0 1 0 0;...
  0 0 0 0 gamma 0;...
  0 0 0 0 0 1/gamma];

Mbar1 = L'*Mmat1*L;
Mbar2 = L'*Mmat2*L;
Mbar3 = L'*Mmat3*L;

%pick out momentum components of Mbar (px:2,py:4,delta:5)
cm1 = [Mbar1(2,2) Mbar1(2,4) Mbar1(2,5);...
      Mbar1(4,2) Mbar1(4,4) Mbar1(4,5);...
      Mbar1(5,2) Mbar1(4,5) Mbar1(5,5)];

cm2 = [Mbar2(2,2) Mbar2(2,4) Mbar2(2,5);...
      Mbar2(4,2) Mbar2(4,4) Mbar2(4,5);...
      Mbar2(5,2) Mbar2(4,5) Mbar2(5,5)];

cm3 = [Mbar3(2,2) Mbar3(2,4) Mbar3(2,5);...
      Mbar3(4,2) Mbar3(4,4) Mbar3(4,5);...
      Mbar3(5,2) Mbar3(4,5) Mbar3(5,5)];

cm=cm1+cm2+cm3;
```

Example Iteration

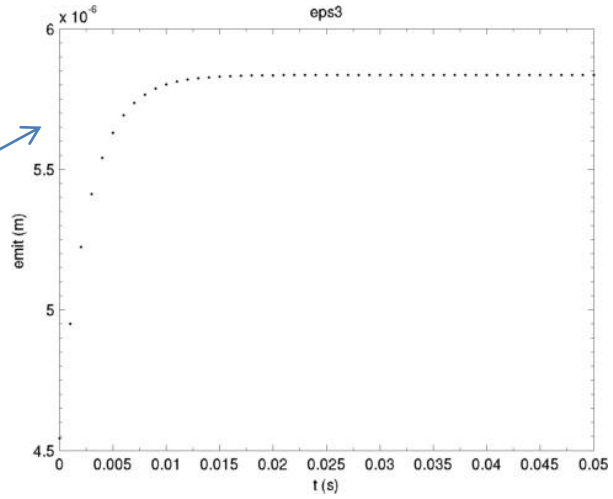
Nb=1e11 electrons
Dt = 1ms
50 iterations

Computed at beginning
Of S28C lattice.
Small dispersion, so
little transverse growth.





Longitudinal
growth



Average growth rates around ring

$$\frac{1}{\tau_{a,global}} = \frac{1}{C}\oint \frac{1}{\tau_l} ds$$

This takes around 30-35 minutes for one growth rate calc when whole ring is used.
With the algorithm given here.
So 50 iterations takes 25 hours.

With just one cell, 32 times faster, so under an hour for the equilibrium calculation.

To do

- Benchmarking results: what to compare to?

--Elegant uses code from ZAP. Coulomb log computed with horizontal beamsize for bmax.

MADX module?

- Implement fast algorithms for when they are valid
- Examine code to improve speed.