IBS notes for JSPEC

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We document the model implemented in JSPEC.

The model is based on that of Martini and implemented in Betacool. Let us consider the evolution of the emittances which are defined as

$$
\begin{aligned}
\epsilon_x &= \gamma_x x^2 + 2\alpha_x x x' + \beta_x x^2 && (1)\\
\epsilon_y &= \gamma_y y^2 + 2\alpha_y y y' + \beta_y y^2 && (2)\\
\epsilon_z &= \gamma_z z^2 + 2\alpha_z z\delta + \beta_z \delta^2 && (3)
\end{aligned}
$$

where $\delta = \frac{E-E_0}{E_0}$. The IBS growth rates are defined as

$$
\frac{1}{\tau_a} = \frac{1}{\epsilon_a}\frac{d\epsilon_a}{dt} \tag{4}
$$

with $a = x, y, z$. In the coupled case, we say that $a = 1, 2, 3$. With coupling, the emittance definition is more complicated, coupling horizontal and vertical, for example.

These rates are given by Martini (implemented in Betacool and JSPEC) as

$$
\begin{aligned}
\frac{1}{\tau_x} &= \langle \frac{A}{2}\left[f_2 + (d^2 + \tilde{d}^2) f_1 \right] \rangle && (5)\\
\frac{1}{\tau_y} &= \langle \frac{A}{2} f_3 \rangle && (6)\\
\frac{1}{\tau_z} &= \langle \frac{A}{2} (1 - d^2) f_1 \rangle && (7)
\end{aligned}
$$

(check factor of 2) with

$$
A = \frac{\sqrt{1+\alpha_x^2}\sqrt{1+\alpha_y^2}\, c r_i^2 \lambda}{16\pi\sqrt{\pi}\sigma_{x\beta}\sigma_{x'\beta}\sigma_y\sigma_{y'}\beta^3\gamma^4} \tag{8}
$$

The quantities f_i are given by the 3-D integral

$$
f_i = k_i \int\int\int \sin\mu\, g_i(\mu,\nu) e^{-D(\mu,\nu)z} \ln(1+z^2)\, d\nu\, d\mu\, dz \tag{9}
$$

References