Flat to Round Transform

Transform matrix for (x,x',y,y')*

$$T = \frac{1}{2}\begin{pmatrix} 1+\alpha & \beta & 1-\alpha & -\beta \\ -\gamma & 1-\alpha & \gamma & 1+\alpha \\ 1-\alpha & -\beta & 1+\alpha & \beta \\ \gamma & 1+\alpha & -\gamma & 1-\alpha \end{pmatrix}$$

*D. Douglas, JLEIC Meeting '16

Parameters of Input Flat Beam

ϵ_{nx}	800 μm
ϵ_{ny}	1 μm
β	5 m
α	0

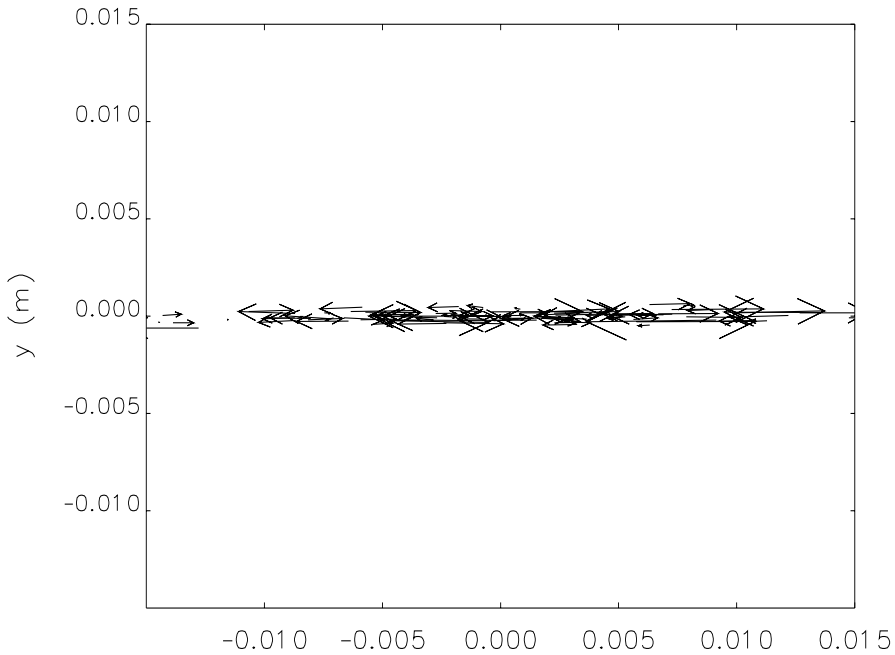
Parameters of Output Magnetized Beam

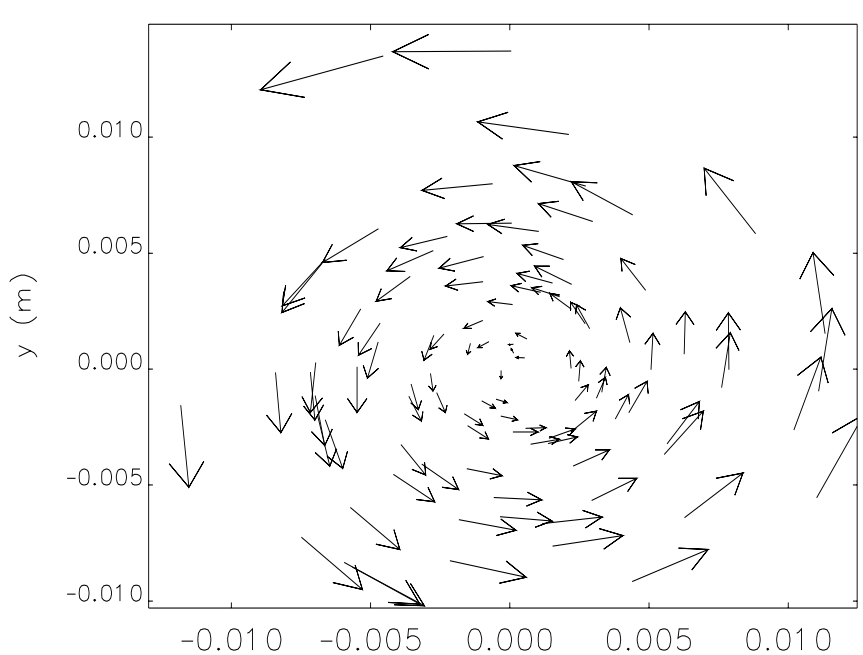
ϵ_{nx}	400 μm
ϵ_{ny}	400 μm
β	5 m
α	0

Beam is coupled so $\epsilon_{x,y}$ are slight misnomers

Flat to Round Transform - Example

Feed a flat bunch through the previously shown transform and you get...

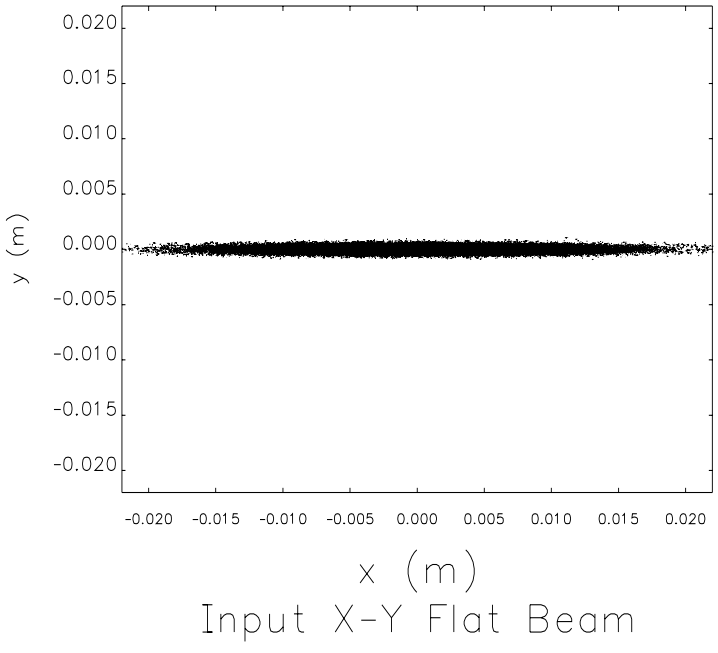




Example of Input and Output X-Y distribution with 100 macroparticles. Arrows show momentum vectors.

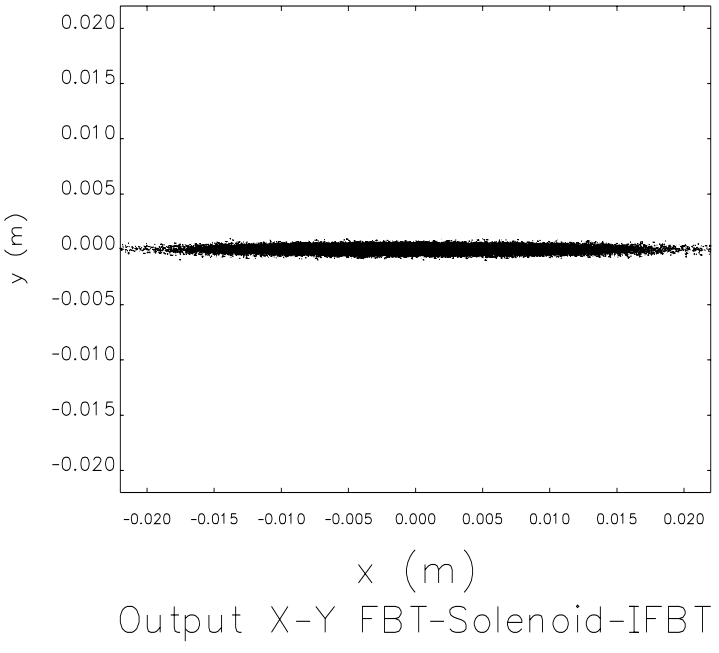
Through a Matched Solenoid

Now start with a flat bunch apply the Flat-to-Round transform / Solenoid / Round-to-Flat transform and you get...



Input X-Y Flat Beam

ϵ_{nx0}	800 μm
ϵ_{ny0}	1 μm
β	5 m
α	0



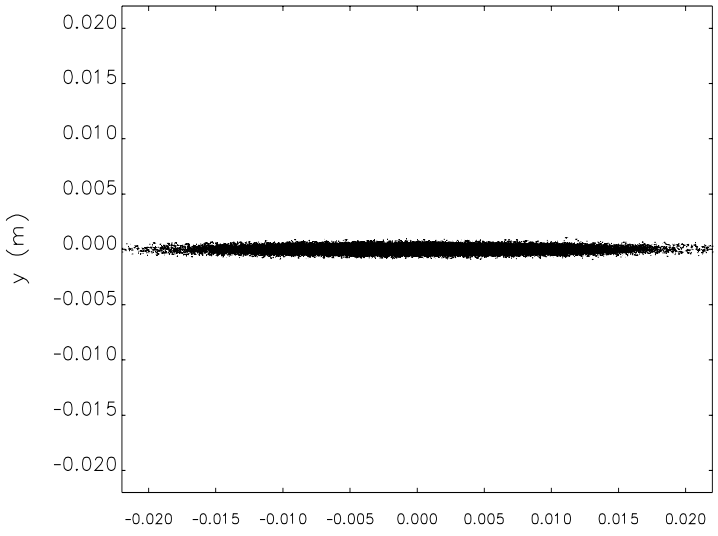
Output X-Y FBT-Solenoid-IFBT

ϵ_{nxf}	800 μm
ϵ_{nyf}	1 μm
β	5 m
α	0

Matched Solenoid with $K_s = 2/\beta$

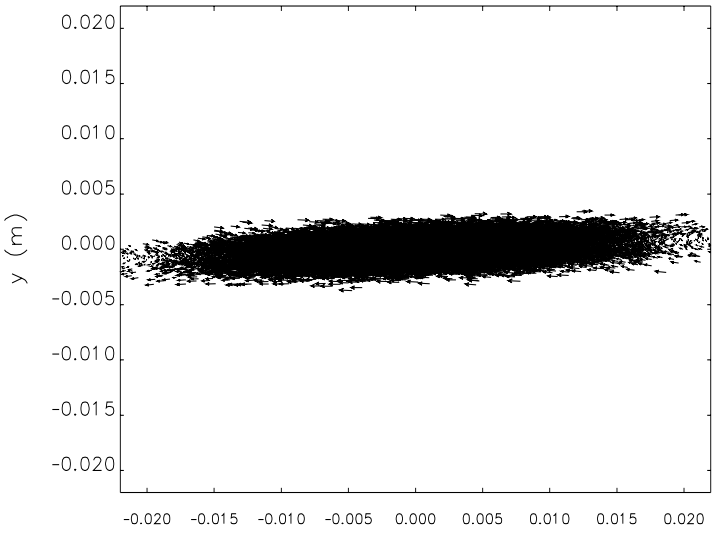
Through a Mismatched Solenoid

Now start with a flat bunch apply the Flat-to-Round transform / Solenoid (**Mismatched**) / Round-to-Flat transform and you get...



Input X-Y Flat Beam

ϵ_{nx0}	800 μm
ϵ_{ny0}	1 μm



Output X-Y FBT-Solenoid-IFBT

ϵ_{nxf}	812 μm
ϵ_{nyf}	13 μm

Mismatched Solenoid with $K_s = 1.5 \cdot 2/\beta$. $\epsilon^f_{nx}/\epsilon^0_{nx} = 1.015$ and $\epsilon^f_{ny}/\epsilon^0_{ny} = 13$