**Physics in Hellweg2D**

Physics in Hellweg2D includes three issues:

1. Equation of the motion;
2. Equations of the electromagnetic fields in the cavity;
3. Equations of the space charge effect.

These equations describe the problem in an axially symmetric geometry (2D) and will be presented and verified. This will allow describe the problem in 3D case in further.

Origin formulas [1-8]

1. Equations of the motion (EoM) together with equations of the fields in the cavity (EoF; formulas (5) from [1], formulas (4) from [2] and formulas (6) – (8) from [3]):

 (1)

where (dot means the differentiation with respect to dimensionless “time” )

 (2)

These formulas can be transformed to the following form. Firstly, let take into account that

 (3)

then (1a) is transformed to



or (with )

 (4a)

Similarly, (1b) is transformed to



or (taking into account that )

 (4b)

At last, (1c) is transformed to

 (4c)

Formulas (4) must be compared with the original formulas of Masunov [5; formulas (4)]:

 (5)

Note: equation (5c) is written for case .

The pares of equations (4a), (5a) and (4b),(5b) are the same (!), but the equations (4c) and (5c) are different. Why? Maybe (4c) is integral of (5c) simply?

Verification of the formulas (1)-(5)

Equations (5) were obtained from formula (3) in [4] as equation for motion of relativistic charged particle in external electrical and magnetic fields:

 (6)

Here are the fields of the RF cavity and is the external magnetic field.

Let come in equation (6) to dimensionless variables:   (again, dot means the differentiation with respect to dimensionless “time” ). Then



Let’s find expression for . Using unit vectors rewrite the vector :



Then

 (7)

But



Then



so that

1) Special case: main symmetrical mode of the field in the RF cavity with cylindrical geometry  and:

 (8)

Note 1: for main mode of the fields in the cylindrical cavity.

Note 2: Because the equation (5c) was written for case .

Returning in these equations to the dimensional variables we have:



then taking into account that and  , we find finally:



So, there are the formulas (5)!

1. Let’s rewrite the equations of the motion (8) in the dimensionless form without any restrictions (it is necessary to remember, that   and ):



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