**Space charge effect in Garnett-Wangler approach**

Method to calculate the electrical field inside the bunch of the beam based on the representation of the beam with an ellipsoidal charge distribution is presented in [1].

Electric field  of the ellipsoidal charge distribution equals

 (1)

where

 (2)

Here  is a charge density and  describes an ellipsoidal charge density expressed as a function of a single generalized coordinate  as



and  are the dimensions of the distribution.

Variable substitution

 (3)

allows find the following expressions for  instead formulas (1):

 (4)

Where  is described with expression (2).

For Gaussian charge distribution expressions (4) give:

 (5)

Results shown above, formulas (5), is taken from [2],[3] and were received in the following way. To find the potential and electric field of the charged bunch the following non proved suggestions were used [4] (this reference is taken from [2],[3]):

1. Given a three-dimensional charge distribution such that the density is constant over ellipsoids:



Then distribution depend only on , i.e. Gaussian distribution  is



1. If distribution extends out to infinity and if  is a primitive of , then the potential at the field point  is



The three components of the corresponding electrical field can be derived by differentiating under the integration sign and result is as follows:

 (6)

If we now abandon the particular (Gaussian) type of distribution, i.e. make the replacement , where  is defined by expression (2), then we receive the formulas (1).

Let’s try to reproduce these results as they are received in [2],[3].

The charge distribution inside the bunch is assumed to be a three dimensional Gaussian distribution such as



Where constant  is determined by the total charge  of the bunch:



Then potential of such charge distribution in the point  equals



But



Then





where single integral  equals



To calculate this integral let’s take into account that



Then



Since,



Changing of variable  gives



and then



This expression for potential means, that electrical field  of the bunch equals



**So, we have been received the results (6) with accuracy of definitions of the widths of charge distribution and normalization factor before integrals**; this factor is defined by total charge (total current ) of the beam.

References.

1. R.W. Garnett, T.P. Wangler. *Space-Charge Calculation for Bunched Beams with 3-D Ellipsoidal Symmetry.* Proceeding 0f the IEEE Particle Accelerator Conference, San Francisco. **1** (1991) 330.
2. M. Martini, M. Prome. *Computer Studies if Beam Dynamics in a Proton Linear Accelerator with Space Charge*. Particle Accelerators, **2** (1971) 289.
3. M. Prome. Effets de la Charge D’Espace Dans les Accelerateurs Lineares a Protons. PhD Thesis, Saclay,1971.

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