Arithmetic

for Ashton, Lyla, Neomi, Ruth, and Malakai

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Factor $23 = 23 \times 1$, factors $23-1$	0
Factor $23 = 1 \times 23$, factors 1-23	1
The Number 24	2
Factor $24 = 24 \times 1$, factors $24-1$	2
Factor $24 = 1 \times 24$, factors 1-24	3
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Arithmetic Preface

Preface

for Ashton, Lyla, Neomi, Ruth, and Malakai

Arithmetic Copying

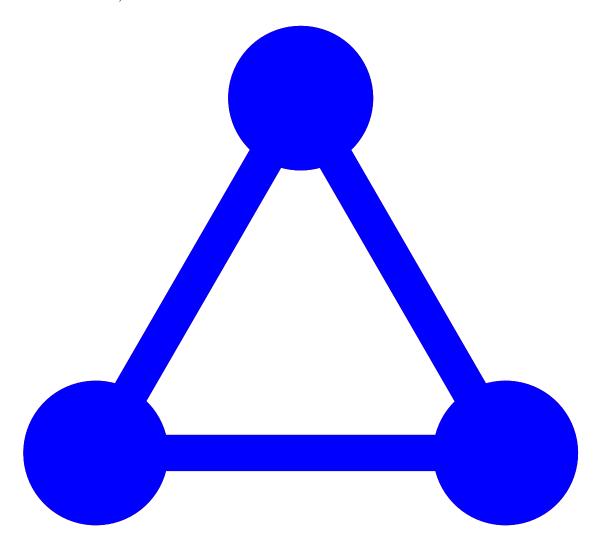
Copying

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Multiplication

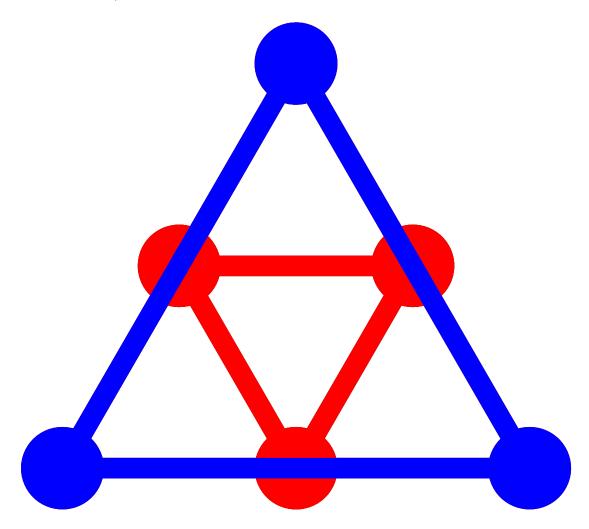
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Factor $1 = 1 \times 1$, factors 1-1

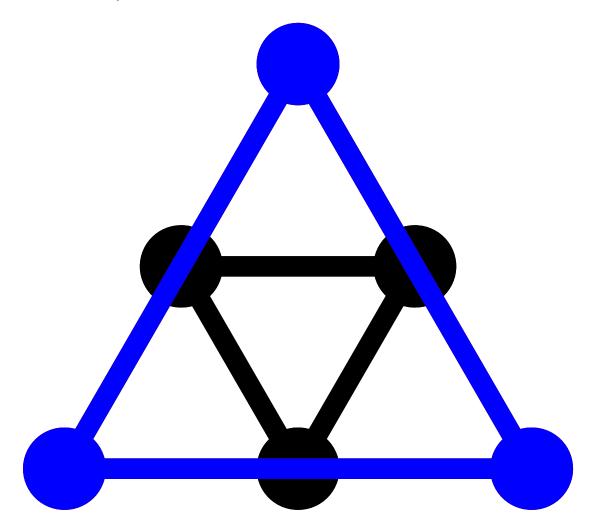


The Number 2

Factor $2 = 1 \times 2$, factors 1-2

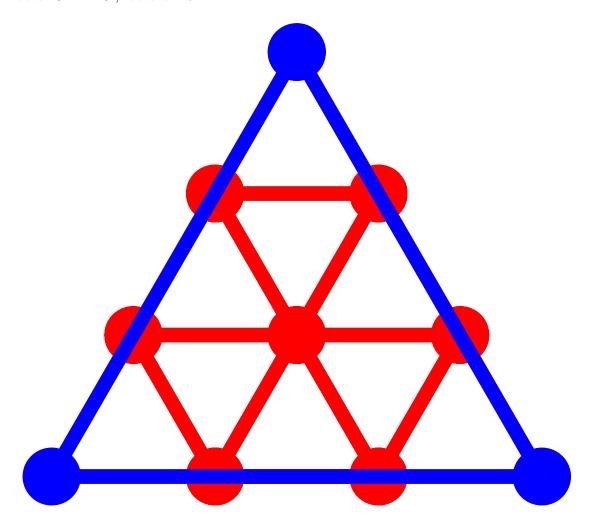


Factor $2 = 2 \times 1$, factors 2-1

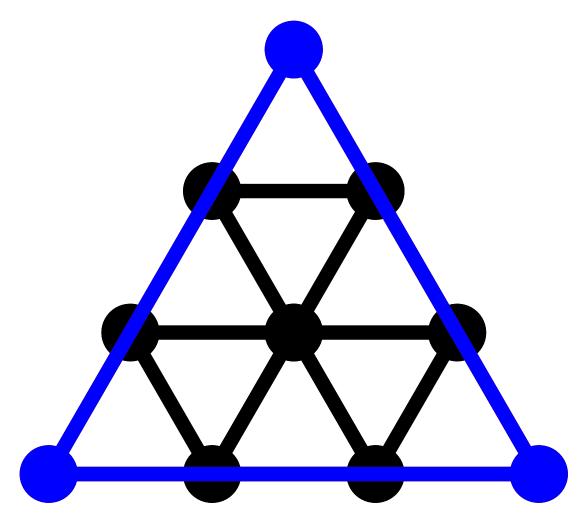


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Factor $3 = 1 \times 3$, factors 1-3

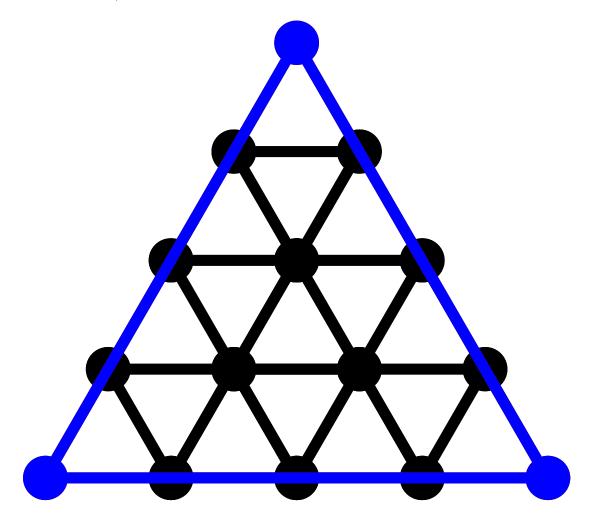


Factor $3 = 3 \times 1$, factors 3-1

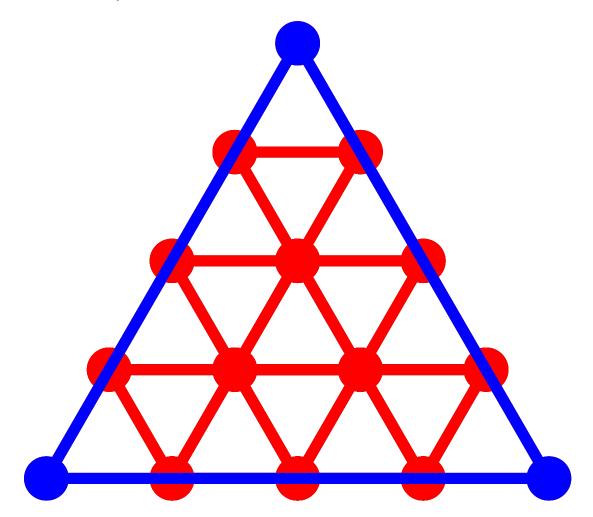


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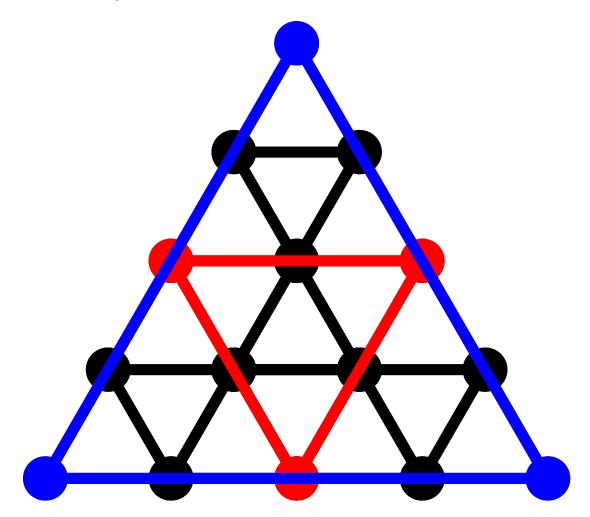
Factor $4 = 4 \times 1$, factors 4-1



Factor $4 = 1 \times 4$, factors 1-4

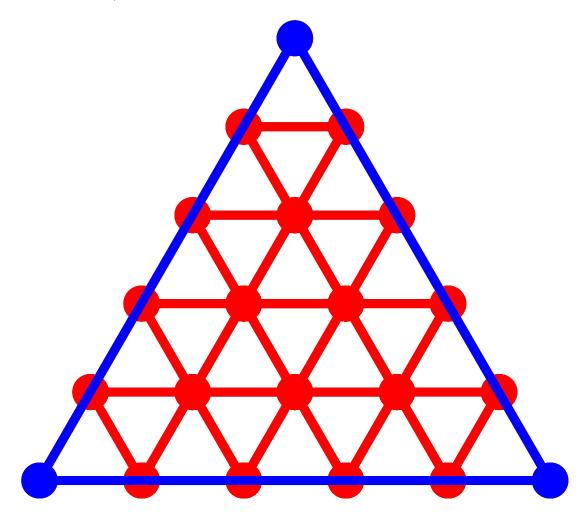


Factor $4 = 2 \times 2$, factors 2-2

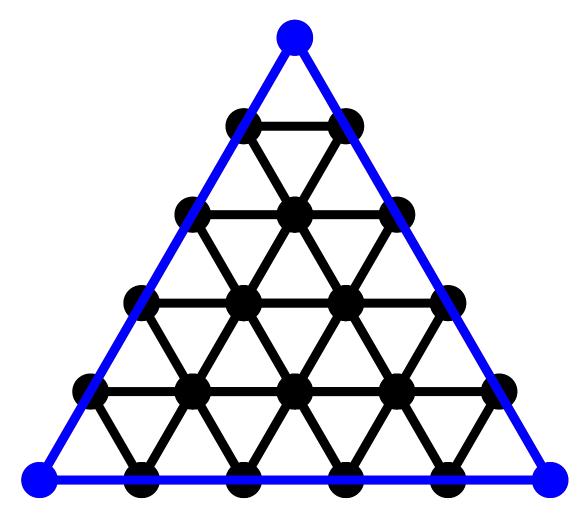


The Number 5

Factor $5 = 1 \times 5$, factors 1-5

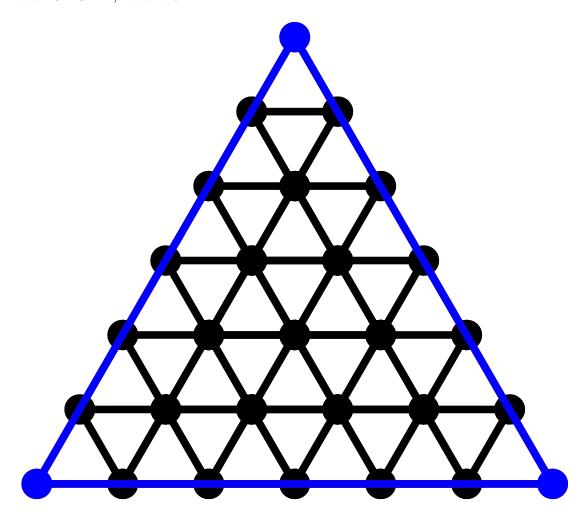


Factor $5 = 5 \times 1$, factors 5-1

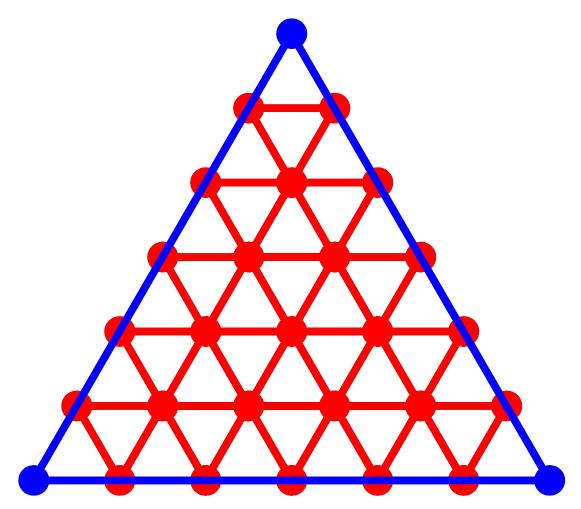


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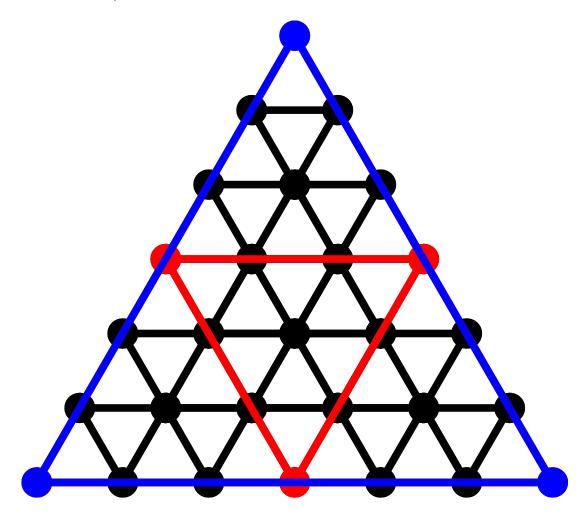
Factor $6 = 6 \times 1$, factors 6-1



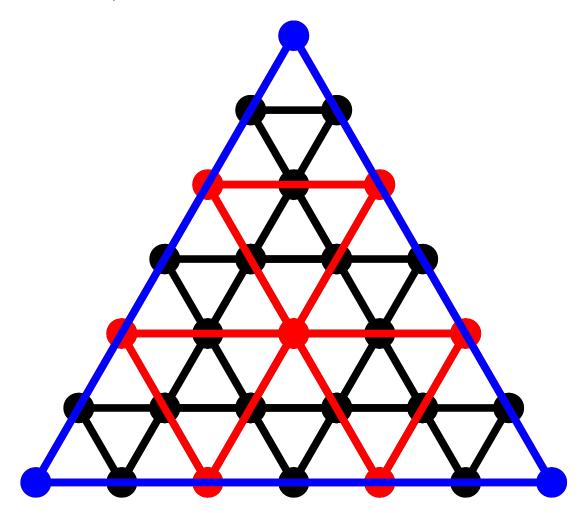
Factor $6 = 1 \times 6$, factors 1-6



Factor $6 = 3 \times 2$, factors 3-2

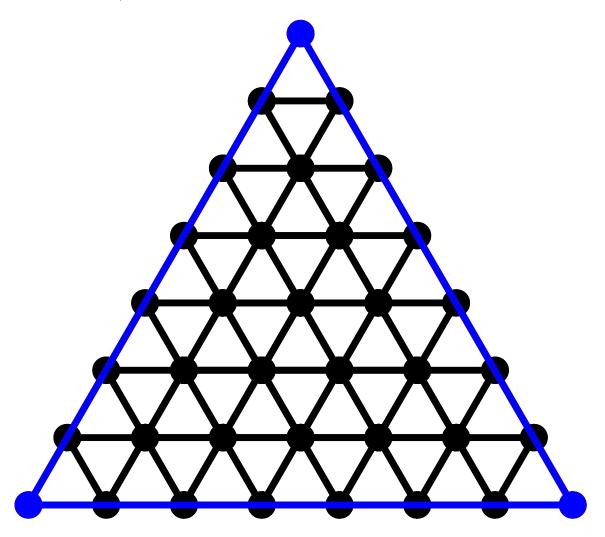


Factor $6 = 2 \times 3$, factors 2-3

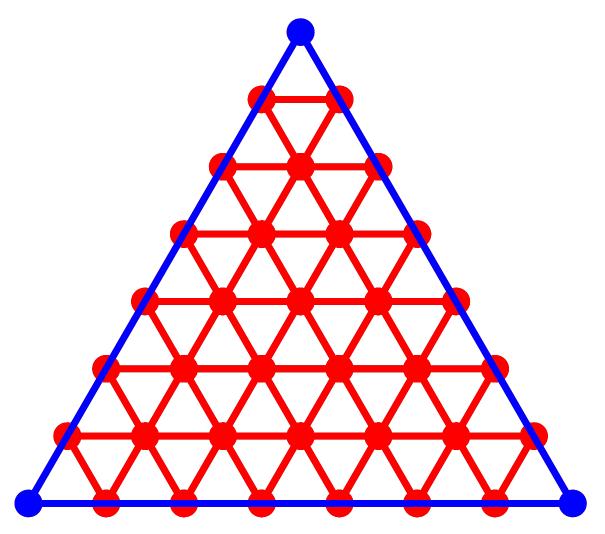


The Number 7

Factor $7 = 7 \times 1$, factors 7-1

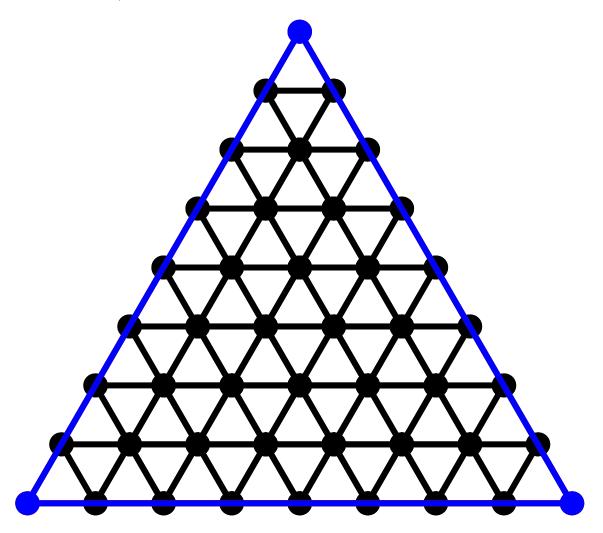


Factor $7 = 1 \times 7$, factors 1-7

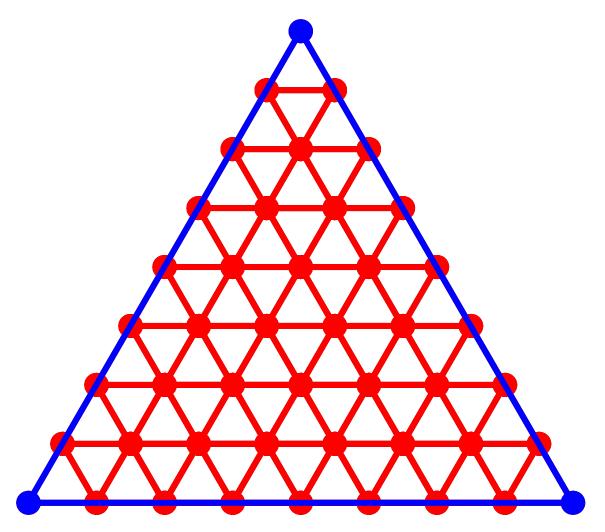


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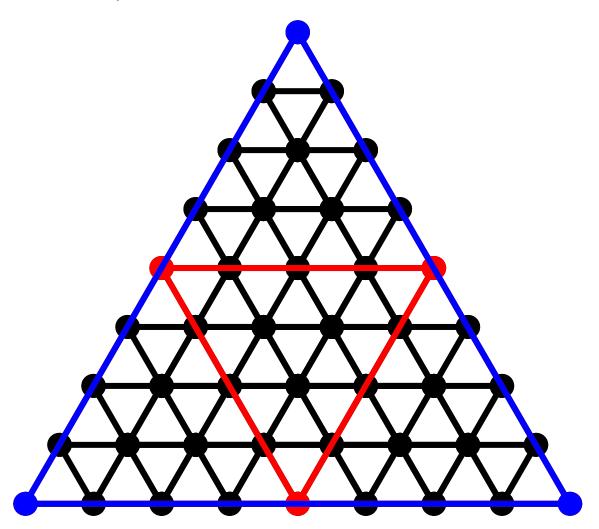
Factor $8 = 8 \times 1$, factors 8-1



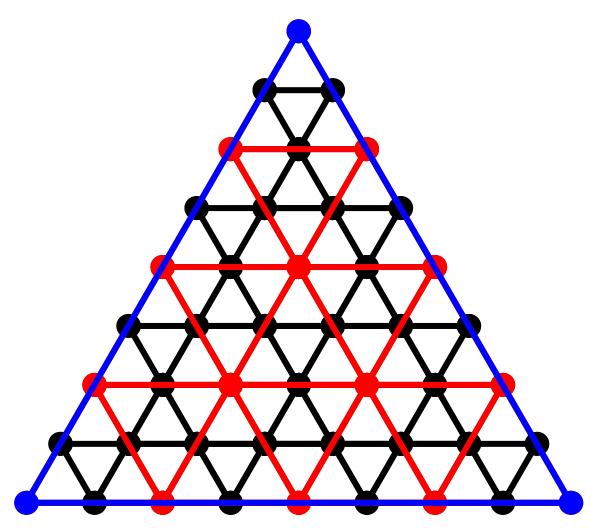
Factor $8 = 1 \times 8$, factors 1-8



Factor $8 = 4 \times 2$, factors 4-2

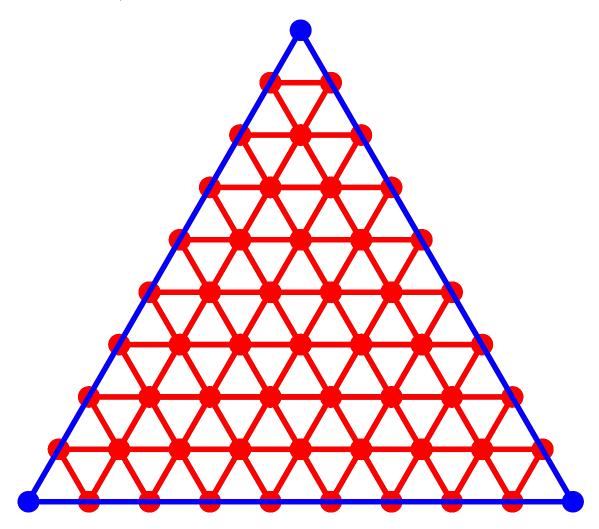


Factor $8 = 2 \times 4$, factors 2-4

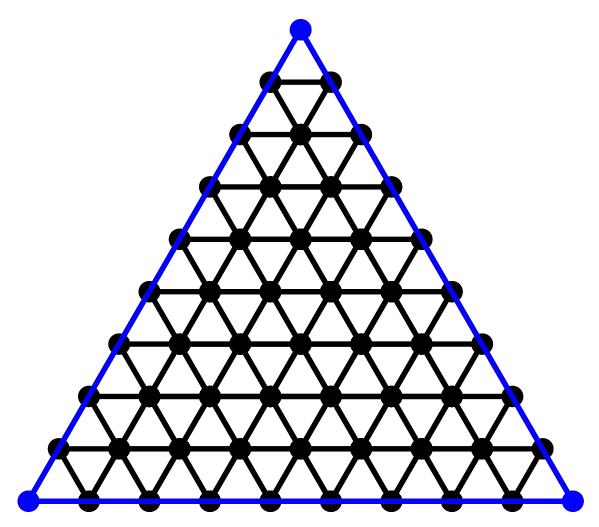


The Number 9

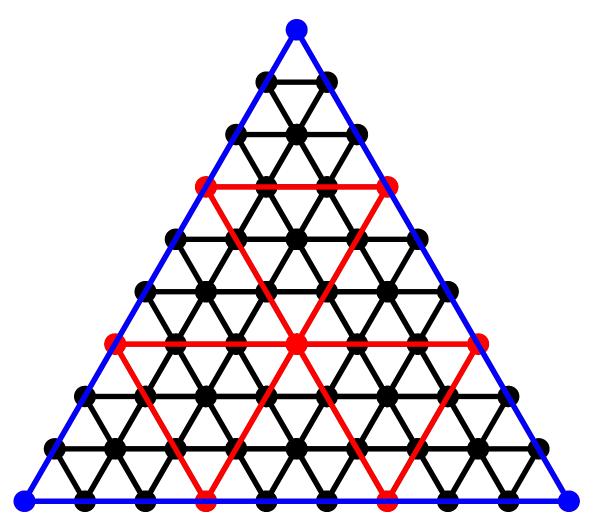
Factor $9 = 1 \times 9$, factors 1-9



Factor $9 = 9 \times 1$, factors 9-1

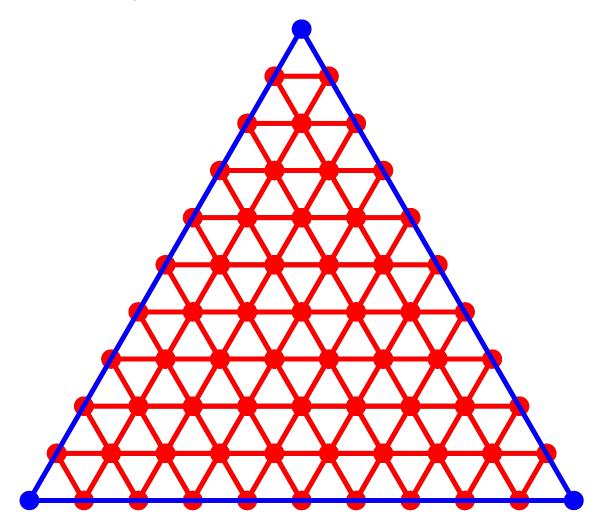


Factor $9 = 3 \times 3$, factors 3-3

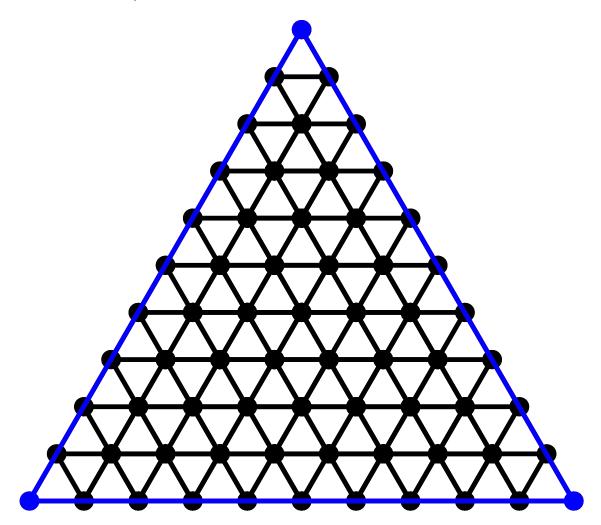


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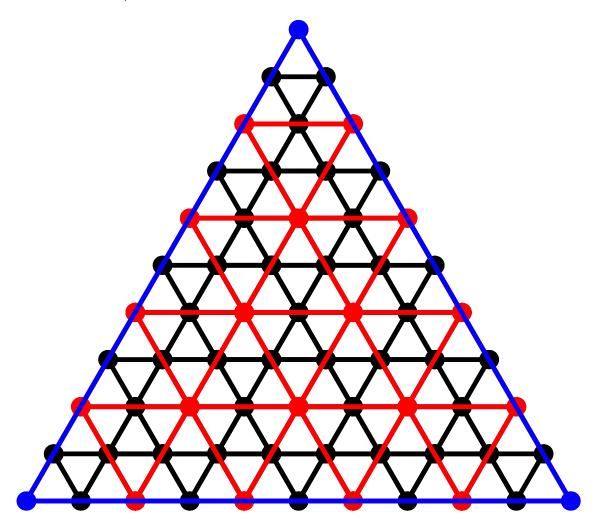
Factor $10 = 1 \times 10$, factors 1-10



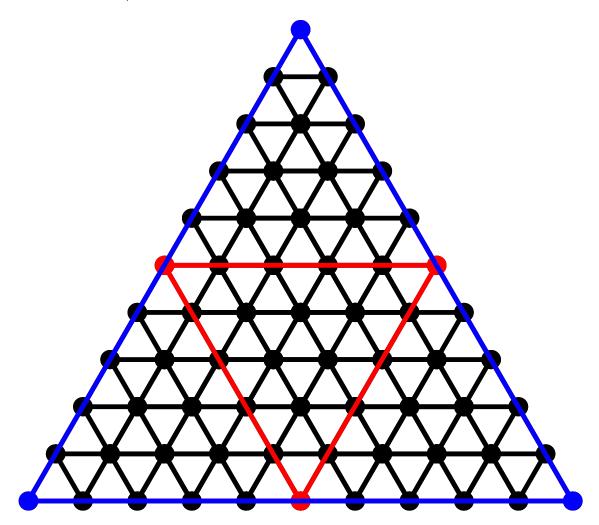
Factor $10 = 10 \times 1$, factors 10-1



Factor $10 = 2 \times 5$, factors 2-5

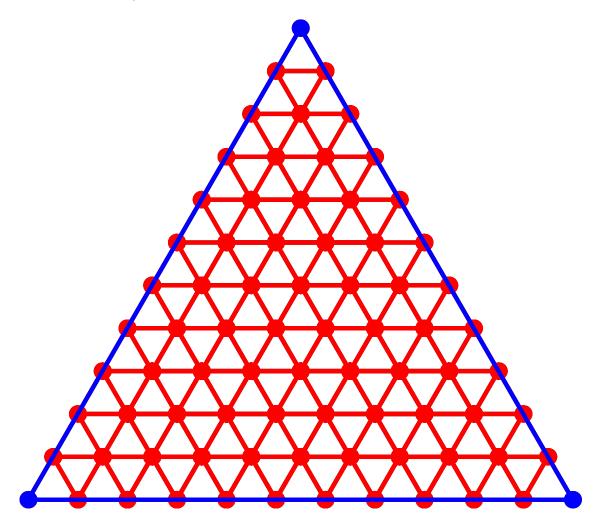


Factor $10 = 5 \times 2$, factors 5-2

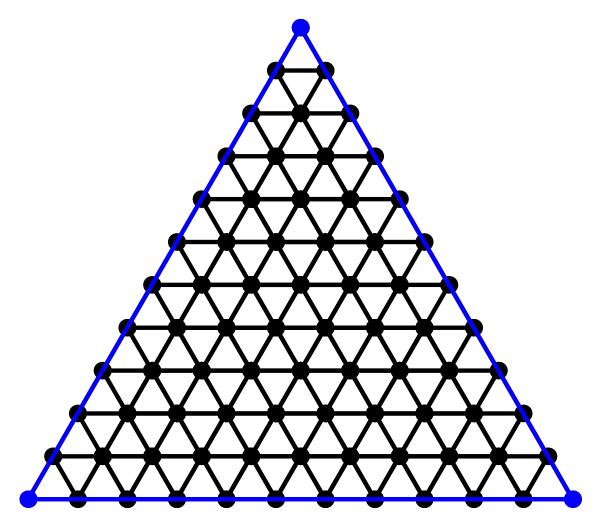


The Number 11

Factor $11 = 1 \times 11$, factors 1-11

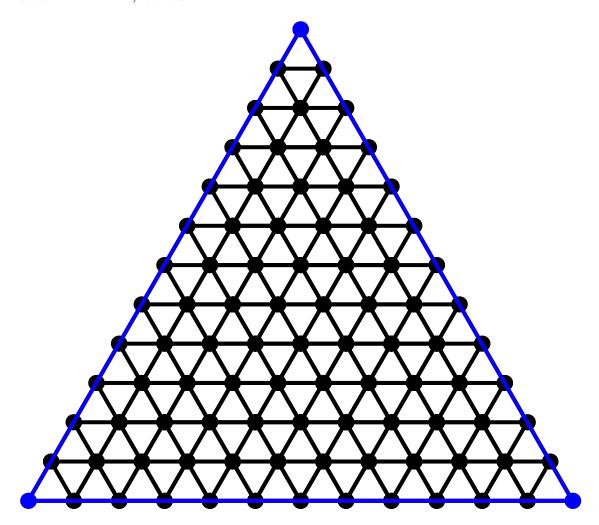


Factor $11 = 11 \times 1$, factors 11-1

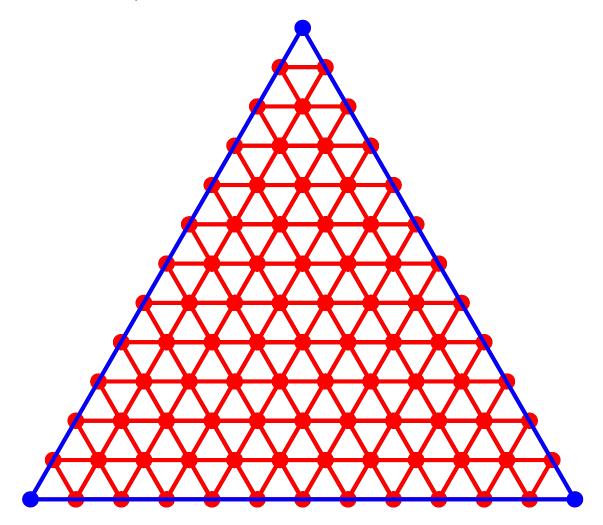


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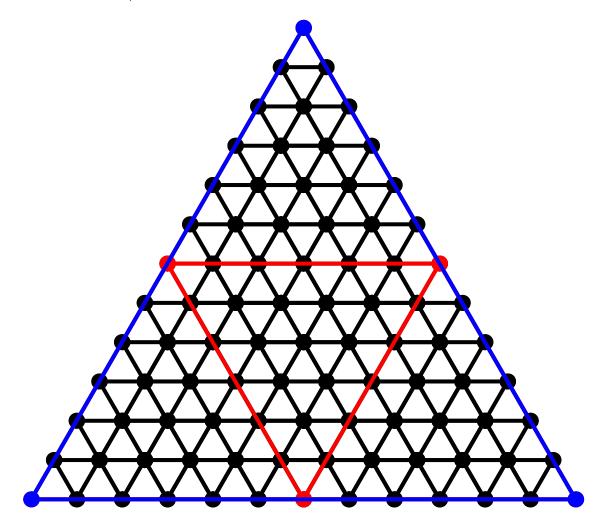
Factor $12 = 12 \times 1$, factors 12-1



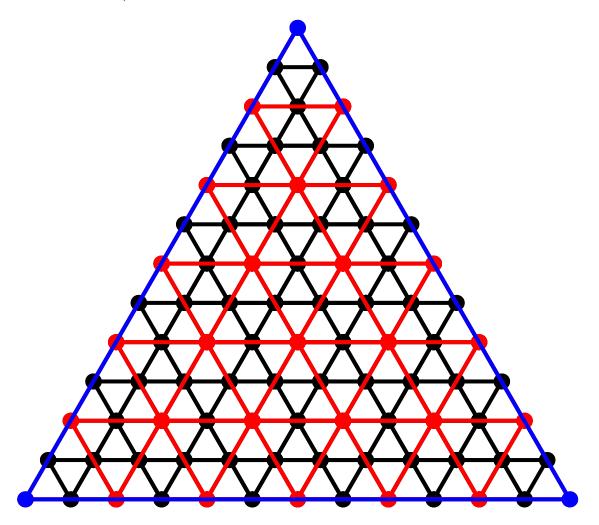
Factor $12 = 1 \times 12$, factors 1-12



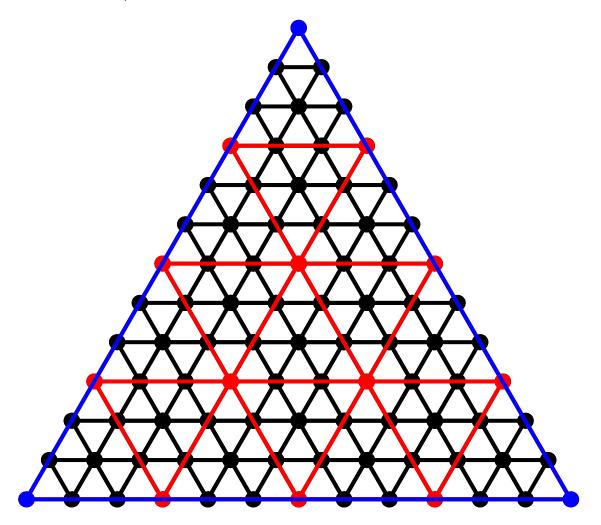
Factor $12 = 6 \times 2$, factors 6-2



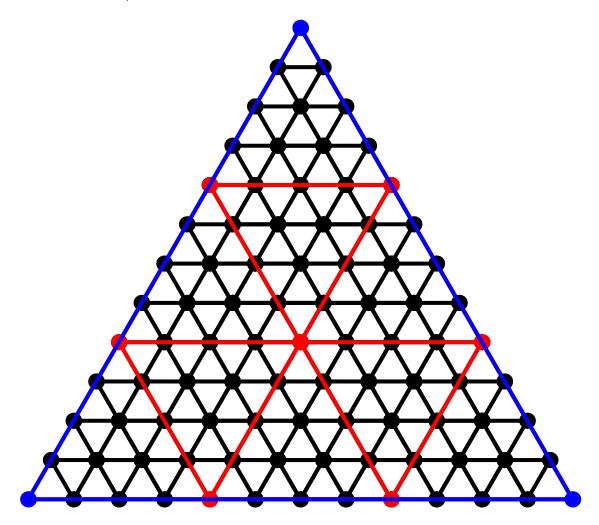
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Factor $12 = 3 \times 4$, factors 3-4

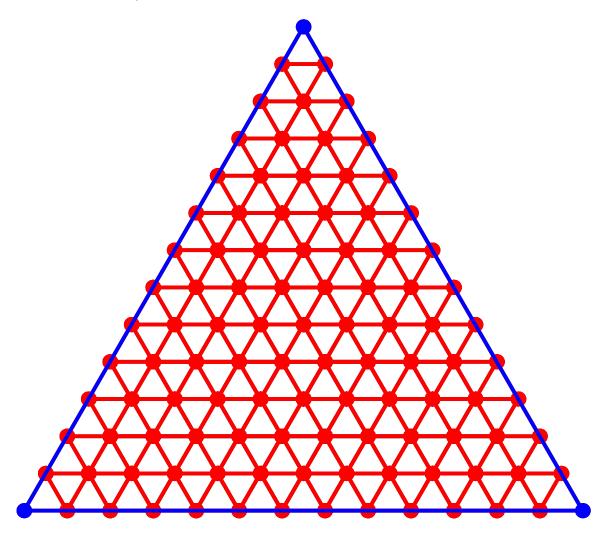


Factor $12 = 4 \times 3$, factors 4-3

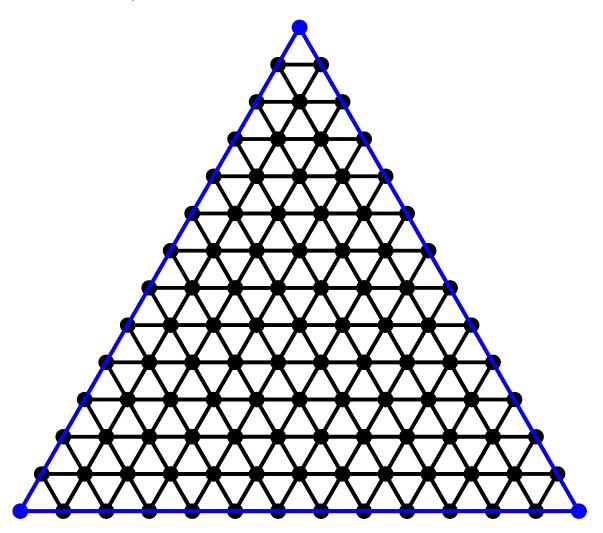


The Number 13

Factor $13 = 1 \times 13$, factors 1-13

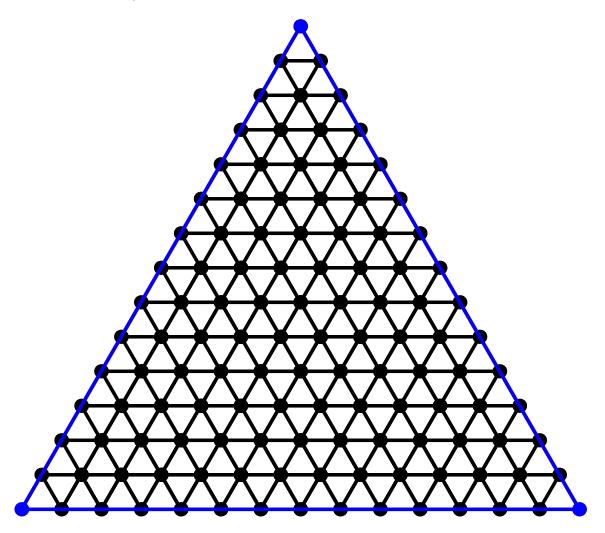


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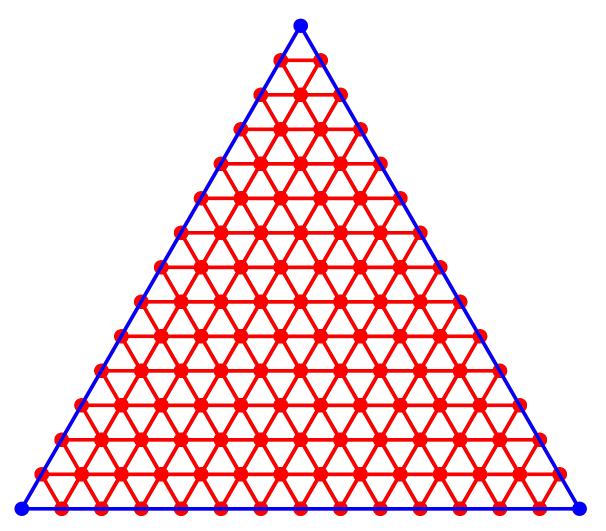


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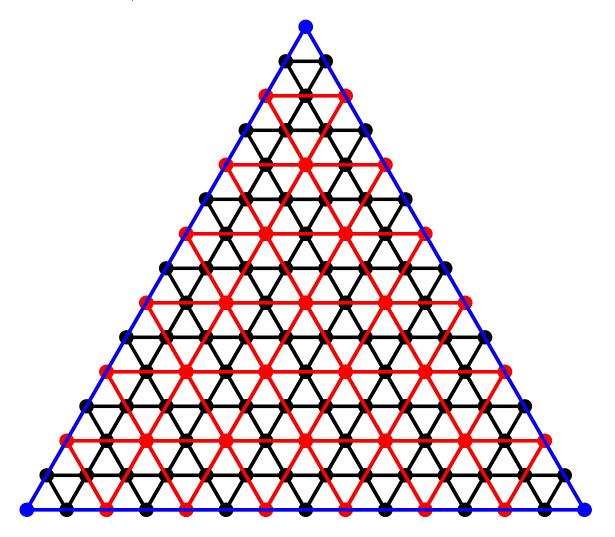
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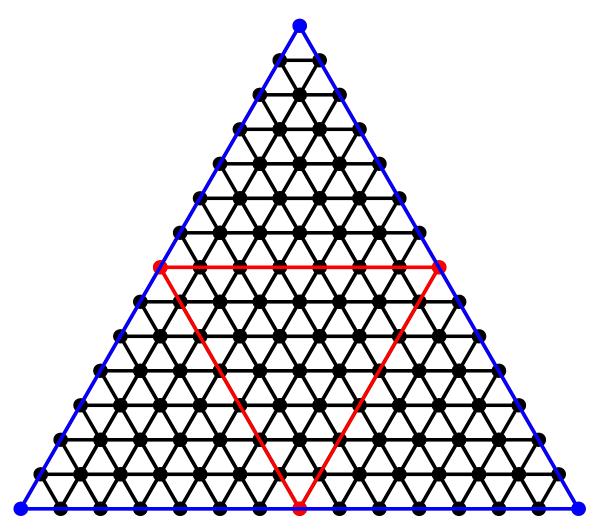
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Factor $14 = 2 \times 7$, factors 2-7

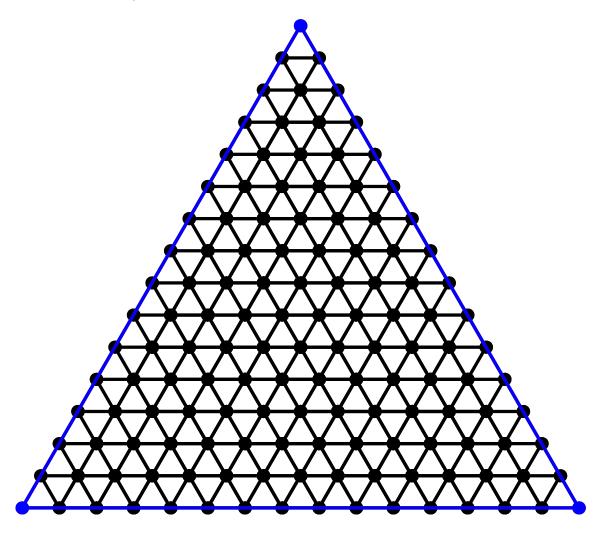


Factor $14 = 7 \times 2$, factors 7-2

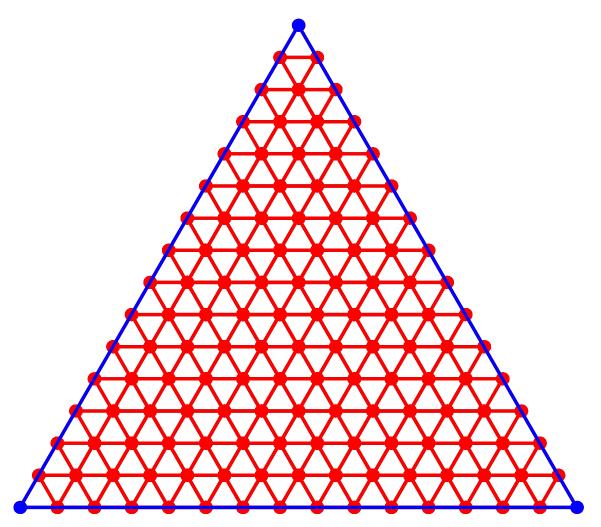


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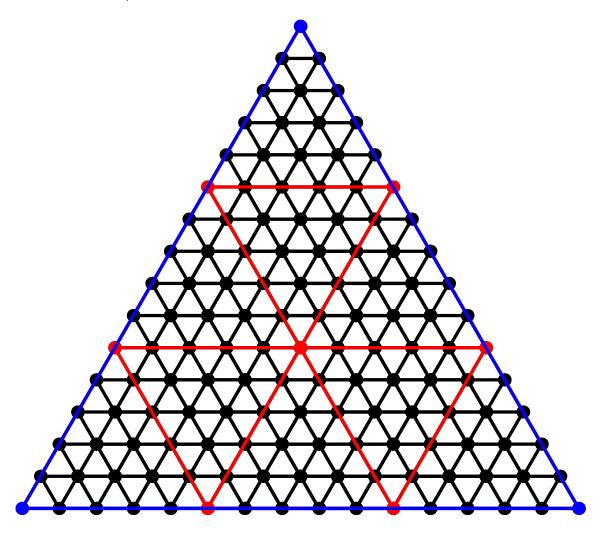
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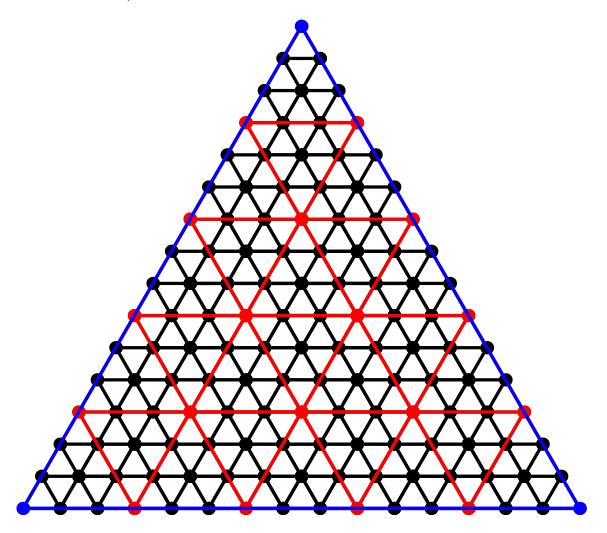
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Factor $15 = 5 \times 3$, factors 5-3

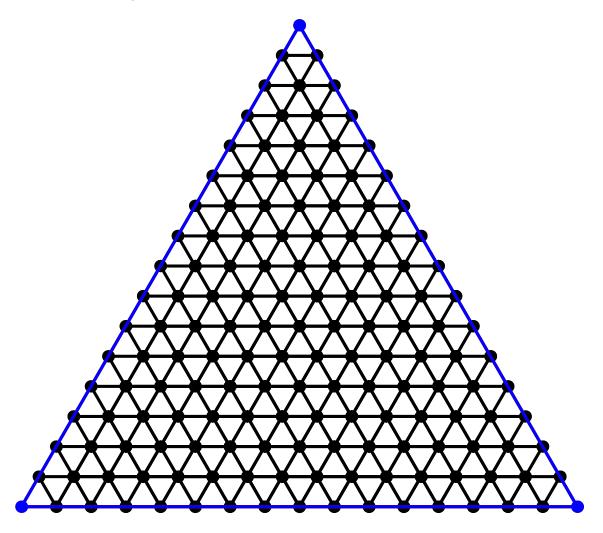


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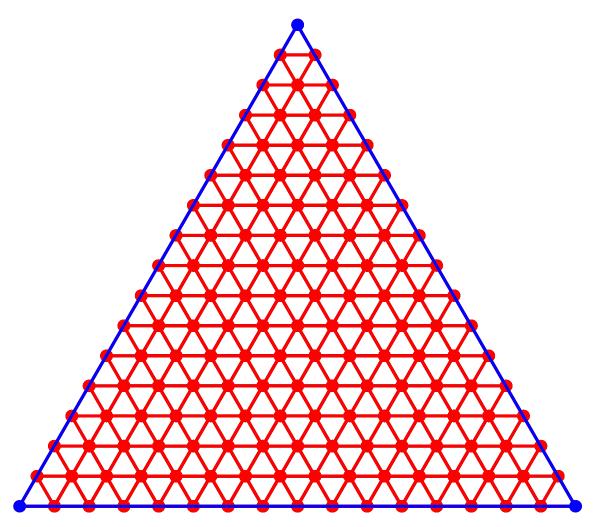


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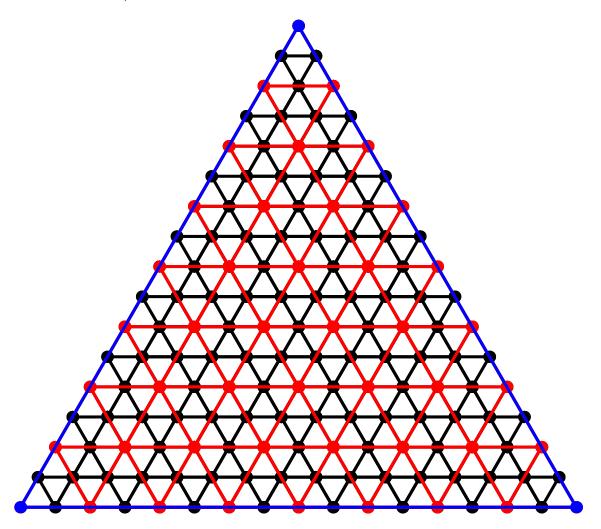
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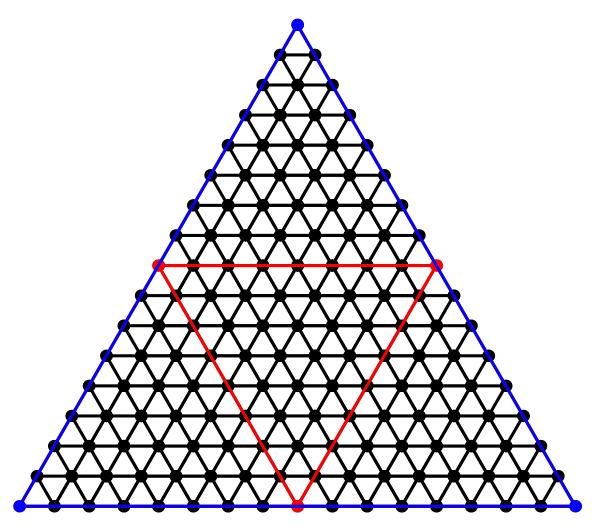
Factor $16 = 1 \times 16$, factors 1-16



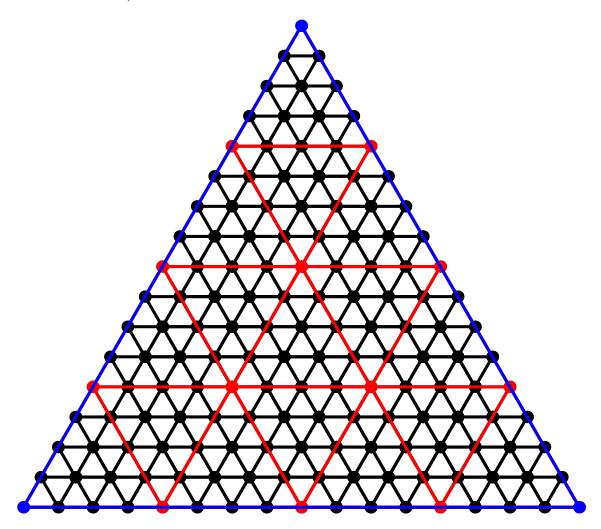
Factor $16 = 2 \times 8$, factors 2-8



Factor $16 = 8 \times 2$, factors 8-2

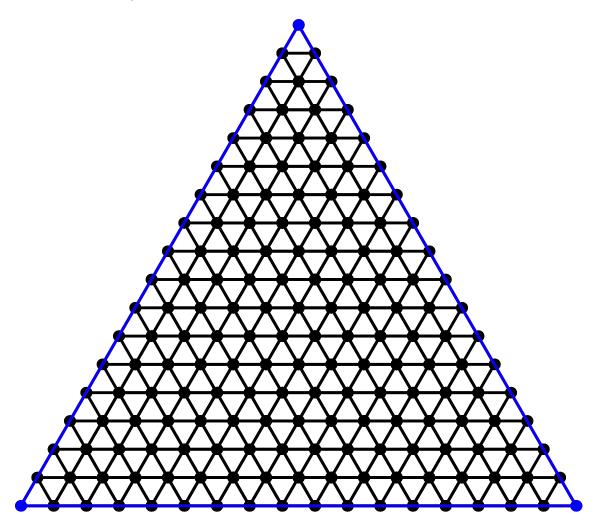


Factor $16 = 4 \times 4$, factors 4-4

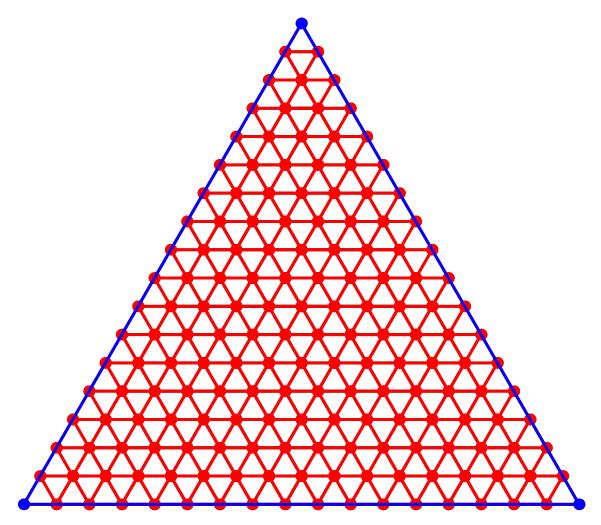


The Number 17

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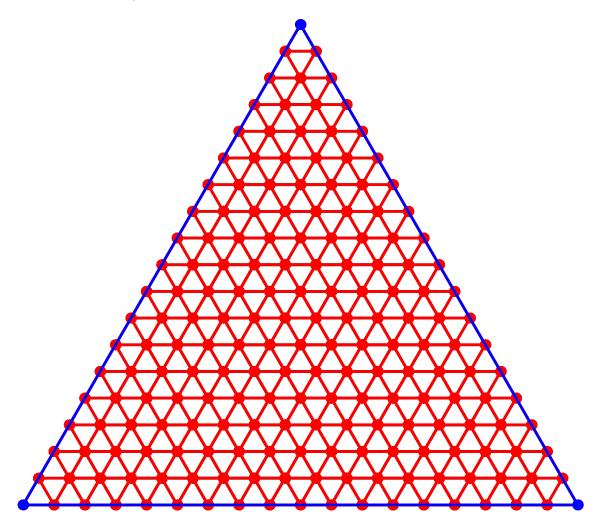


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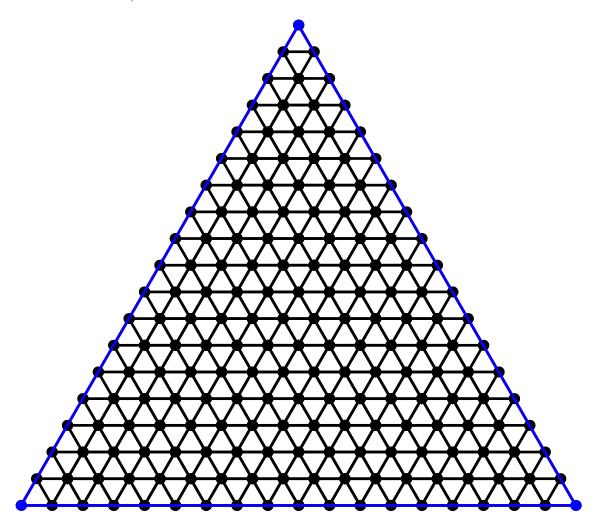


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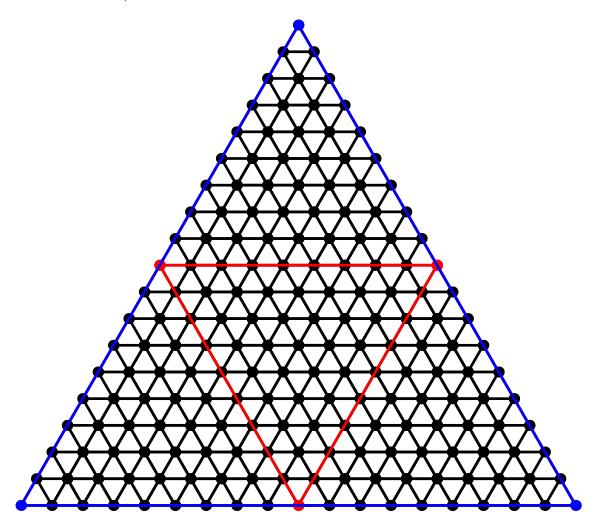
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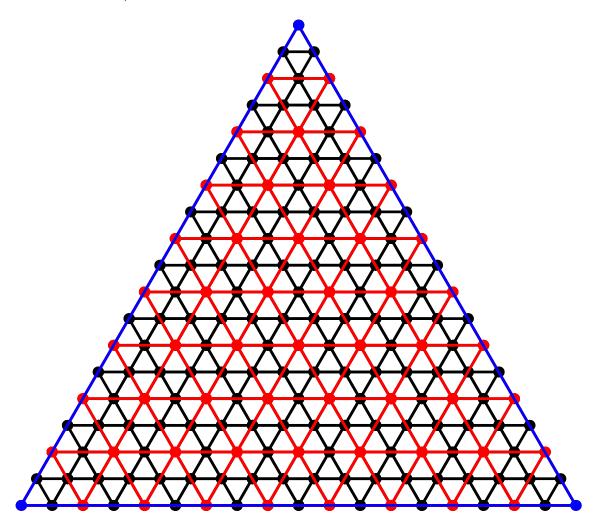
Factor $18 = 18 \times 1$, factors 18-1



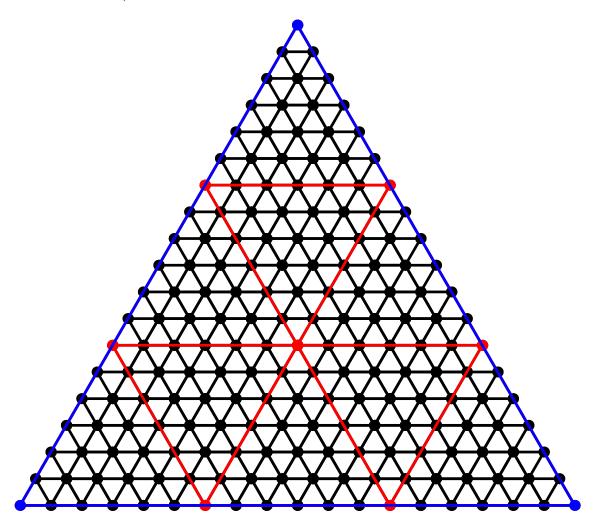
Factor $18 = 9 \times 2$, factors 9-2



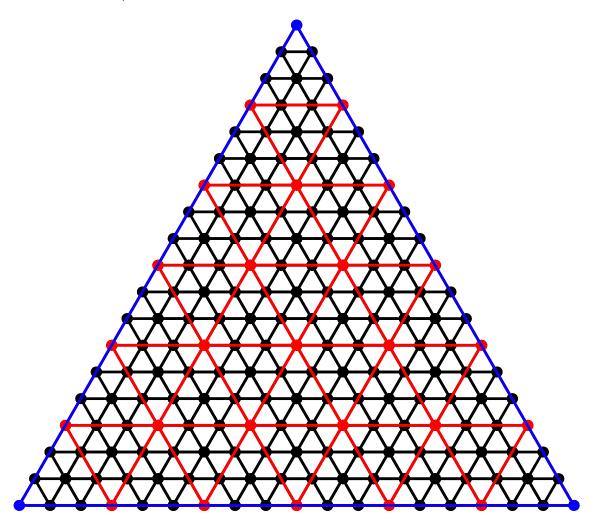
Factor $18 = 2 \times 9$, factors 2-9



Factor $18 = 6 \times 3$, factors 6-3

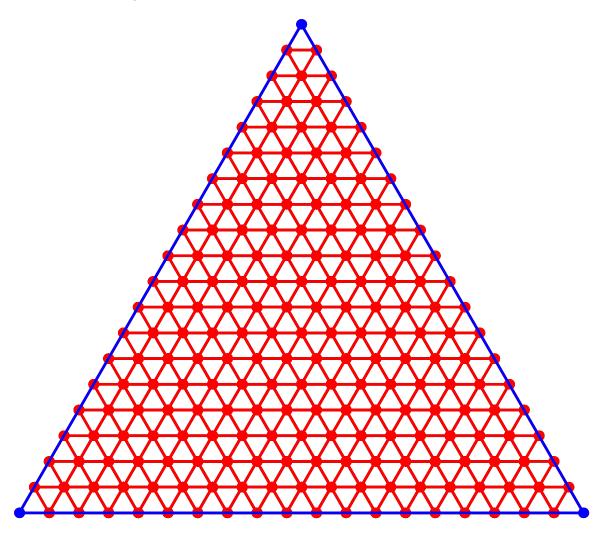


Factor $18 = 3 \times 6$, factors 3-6

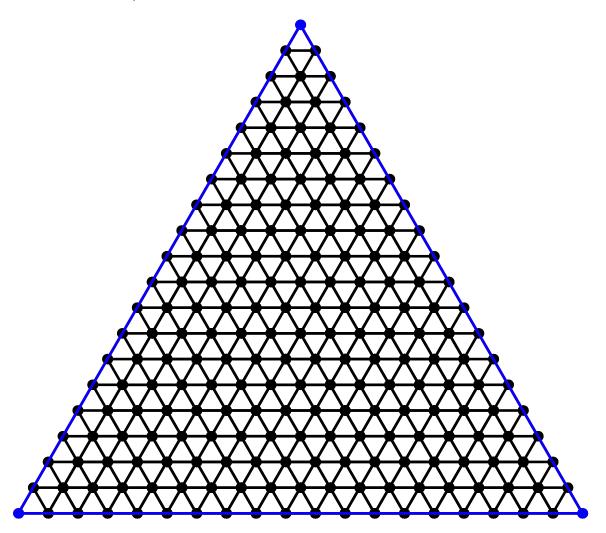


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Factor $19 = 1 \times 19$, factors 1-19

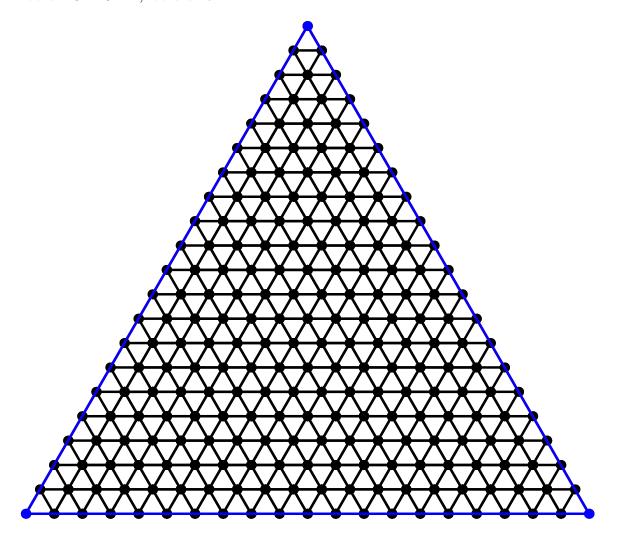


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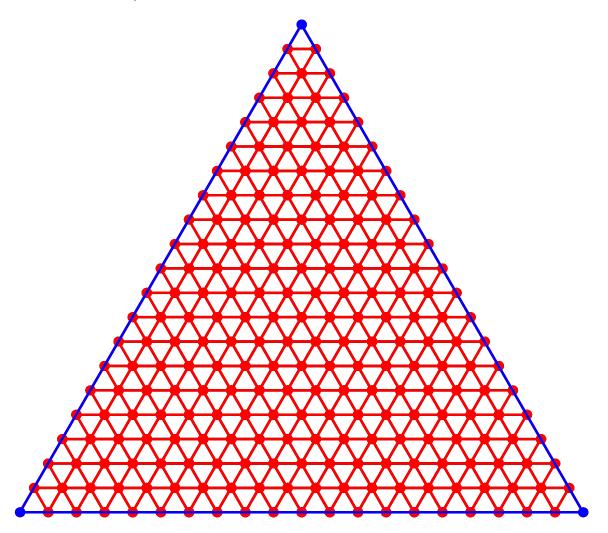


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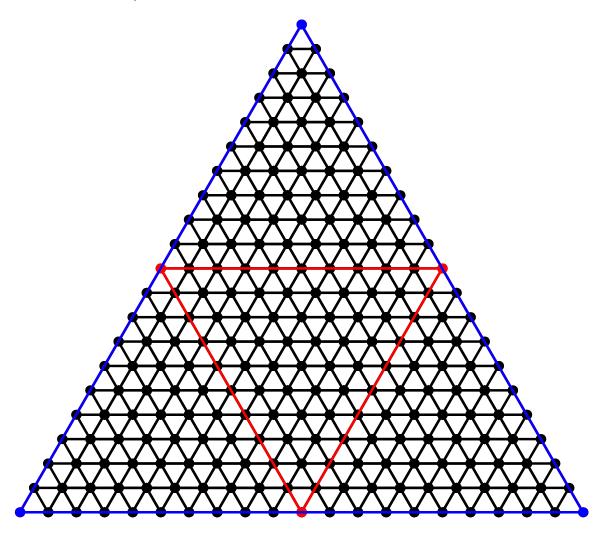
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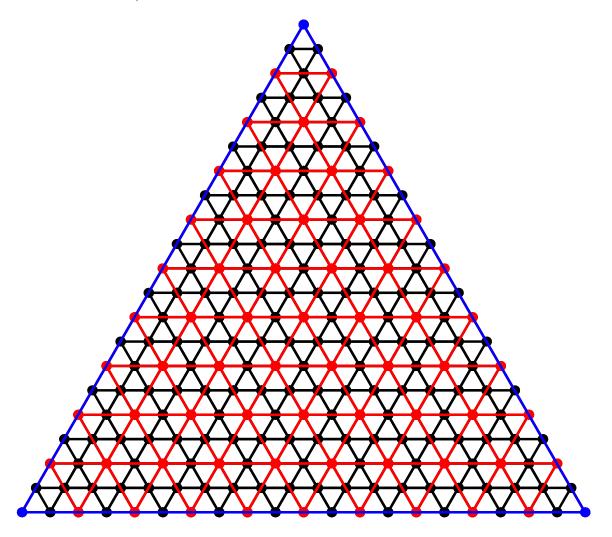
Factor $20 = 1 \times 20$, factors 1-20



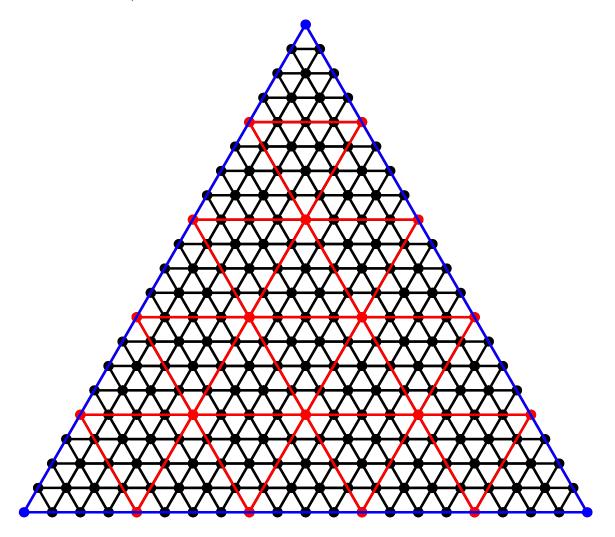
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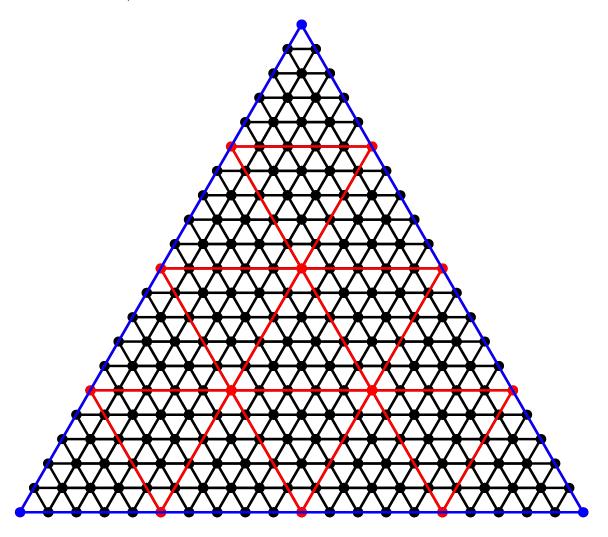
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Factor $20 = 4 \times 5$, factors 4-5

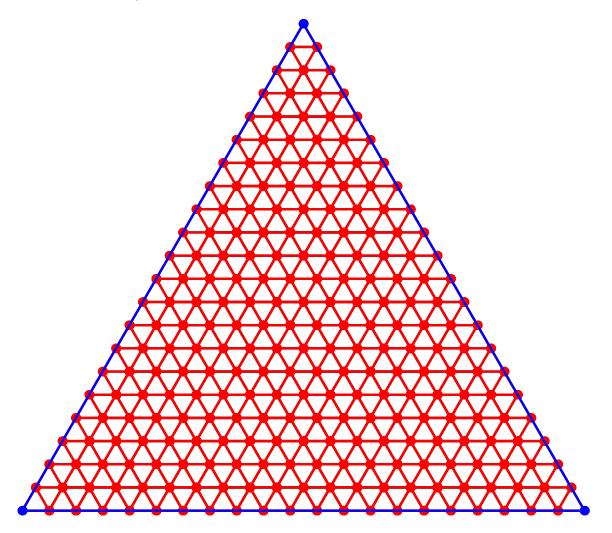


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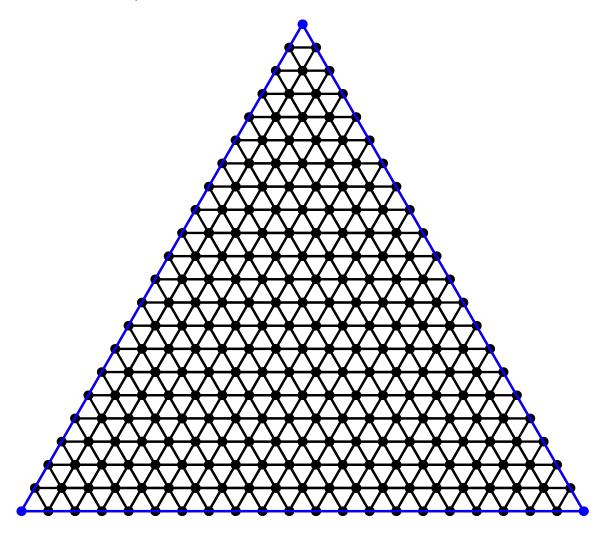


The Number 21

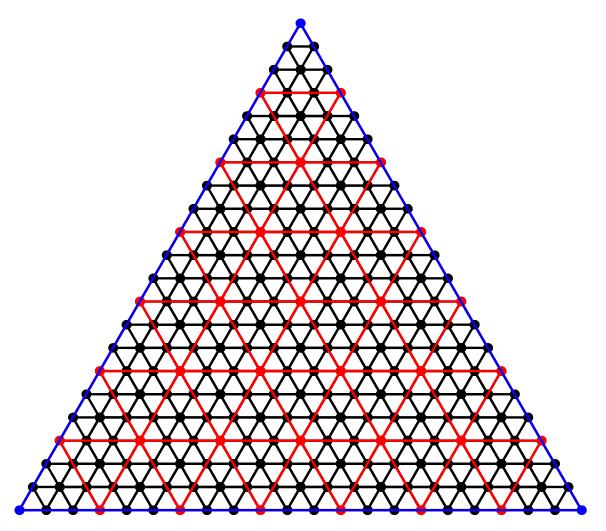
Factor $21 = 1 \times 21$, factors 1-21



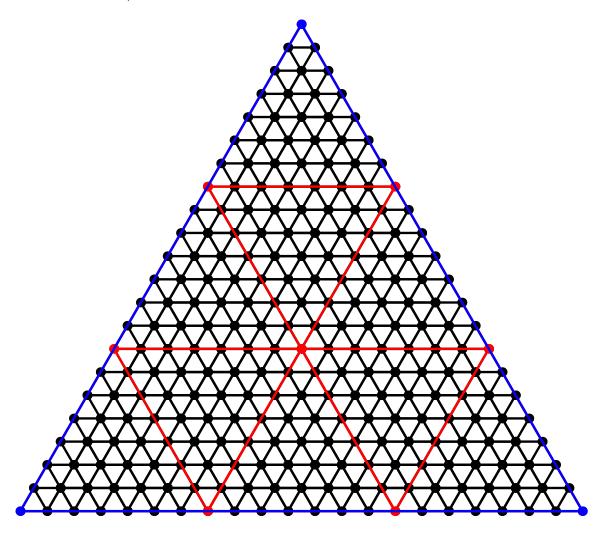
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Factor $21 = 3 \times 7$, factors 3-7

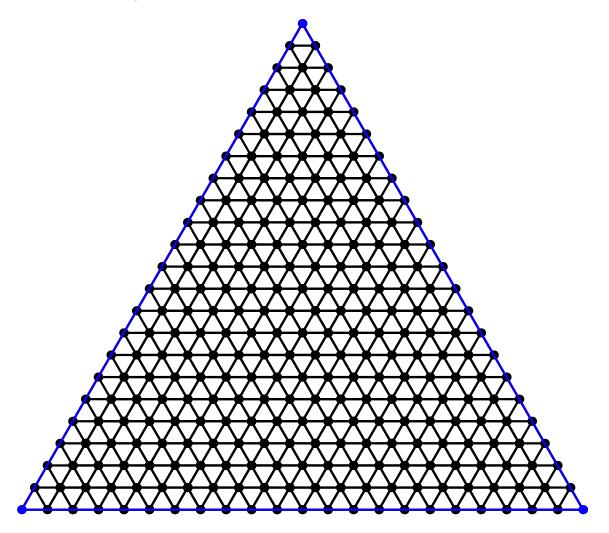


Factor $21 = 7 \times 3$, factors 7-3

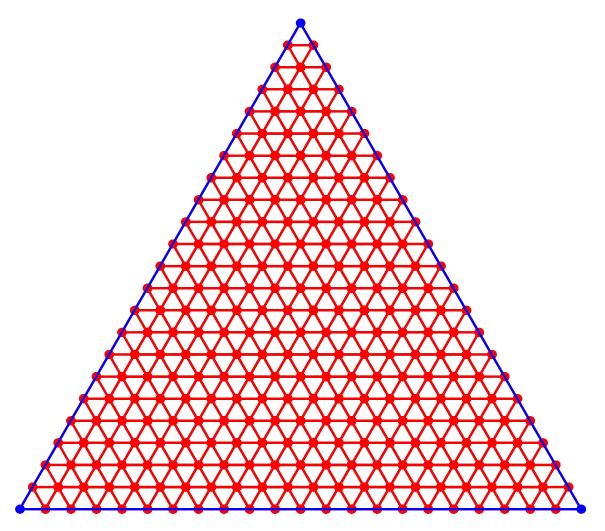


The Number 22

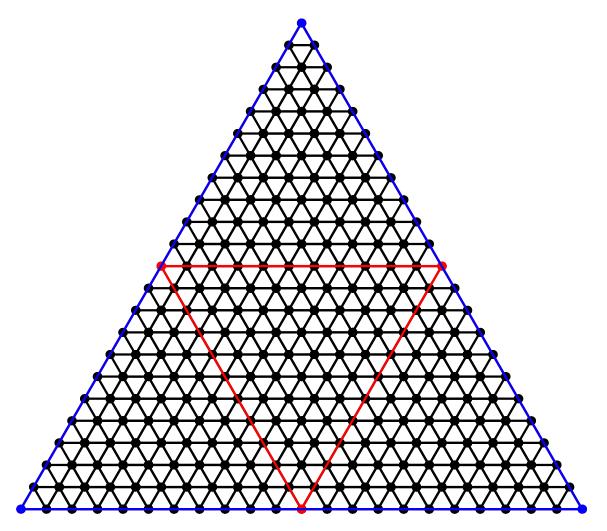
Factor $22 = 22 \times 1$, factors 22-1



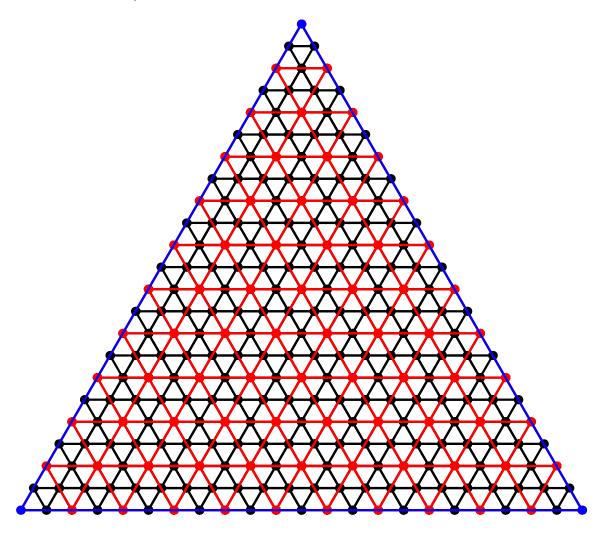
Factor $22 = 1 \times 22$, factors 1-22



Factor $22 = 11 \times 2$, factors 11-2

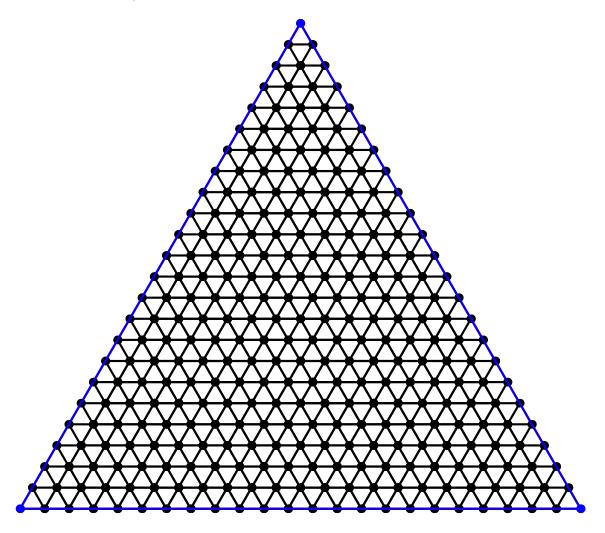


Factor $22 = 2 \times 11$, factors 2-11

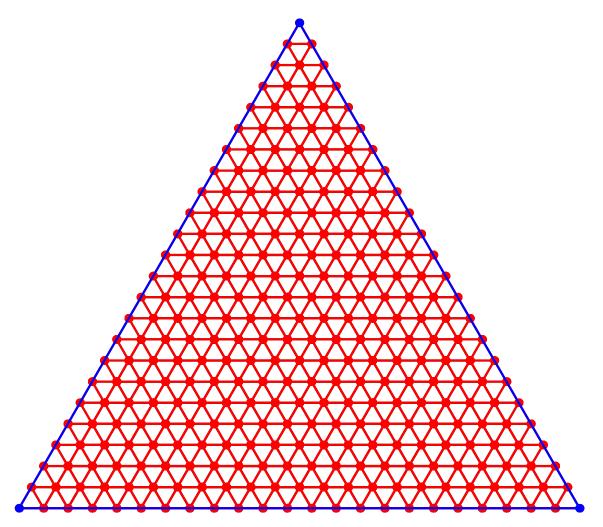


The Number 23

Factor $23 = 23 \times 1$, factors 23-1

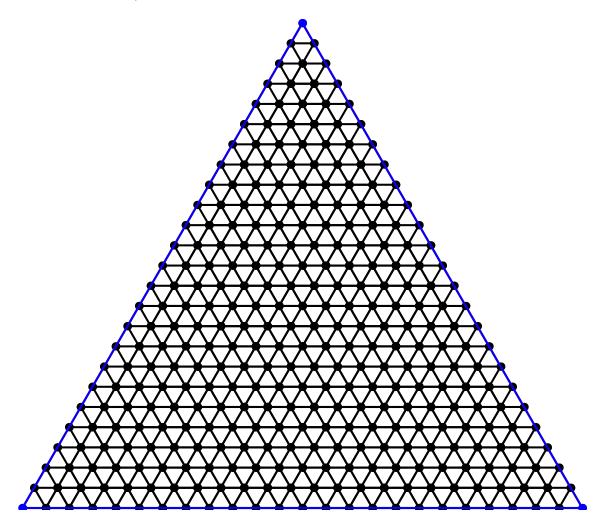


Factor $23 = 1 \times 23$, factors 1-23

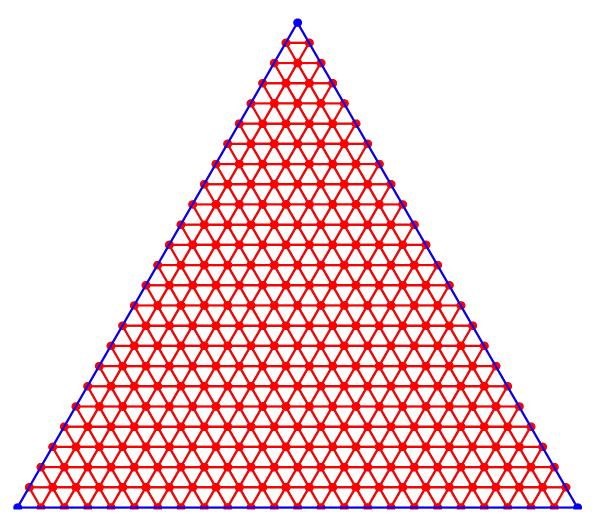


The Number 24

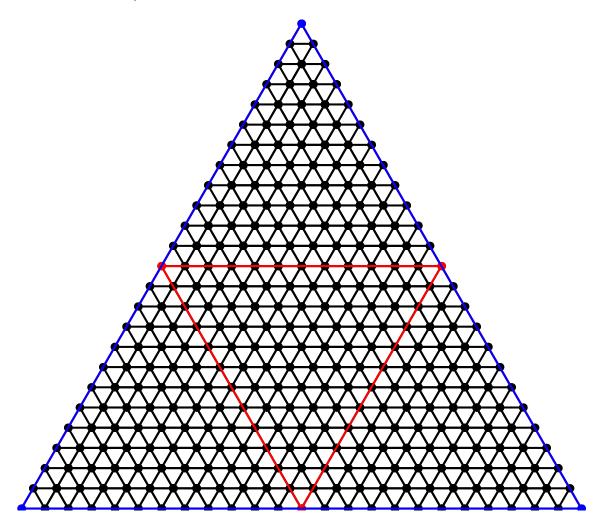
Factor $24 = 24 \times 1$, factors **24-1**



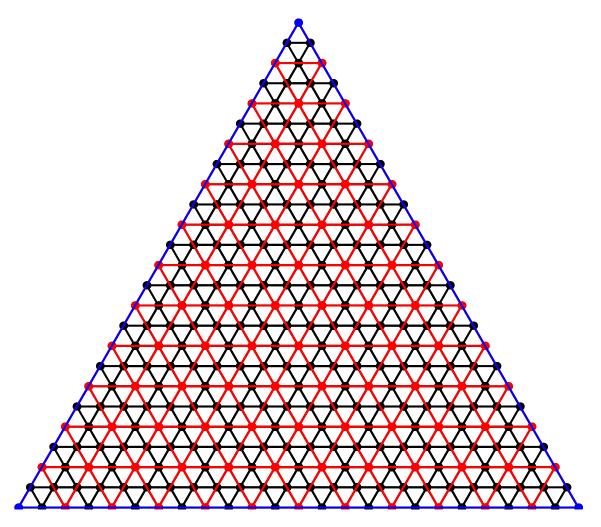
Factor $24 = 1 \times 24$, factors 1-24



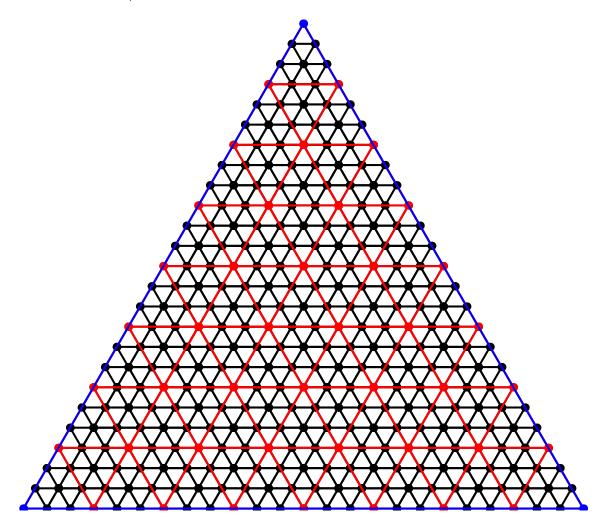
Factor $24 = 12 \times 2$, factors 12-2



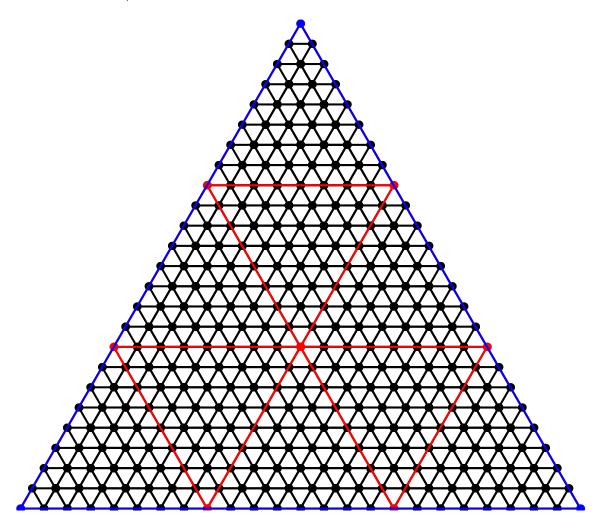
Factor $24 = 2 \times 12$, factors **2-12**



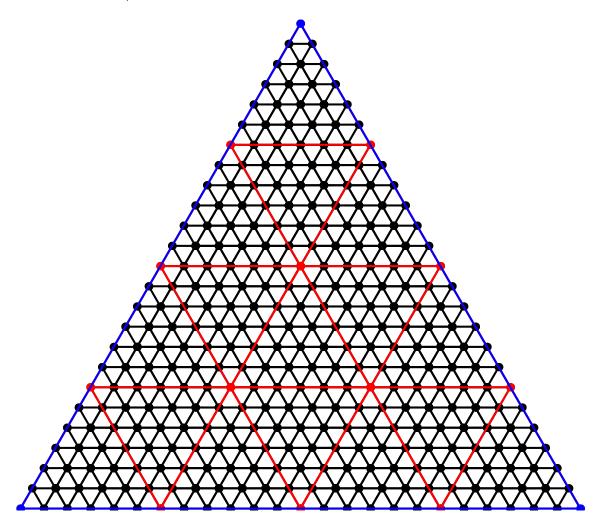
Factor $24 = 3 \times 8$, factors 3-8



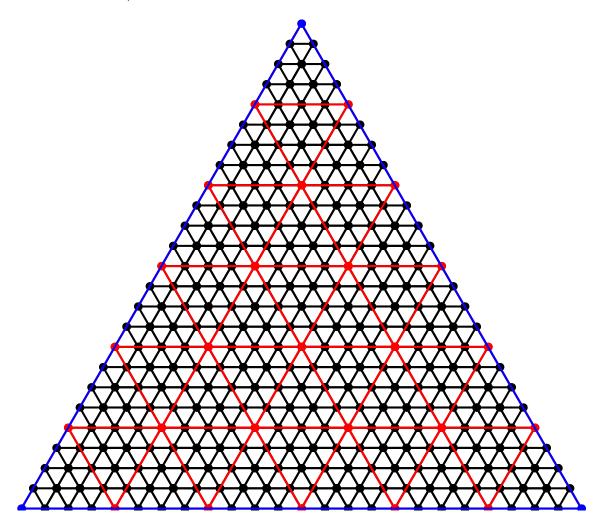
Factor $24 = 8 \times 3$, factors 8-3



Factor $24 = 6 \times 4$, factors 6-4



Factor $24 = 4 \times 6$, factors 4-6



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