

Arithmetic

for Ashton, Lyla, Neomi, Ruth, and Malakai

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Preface

for Ashton, Lyla, Neomi, Ruth, and Malakai

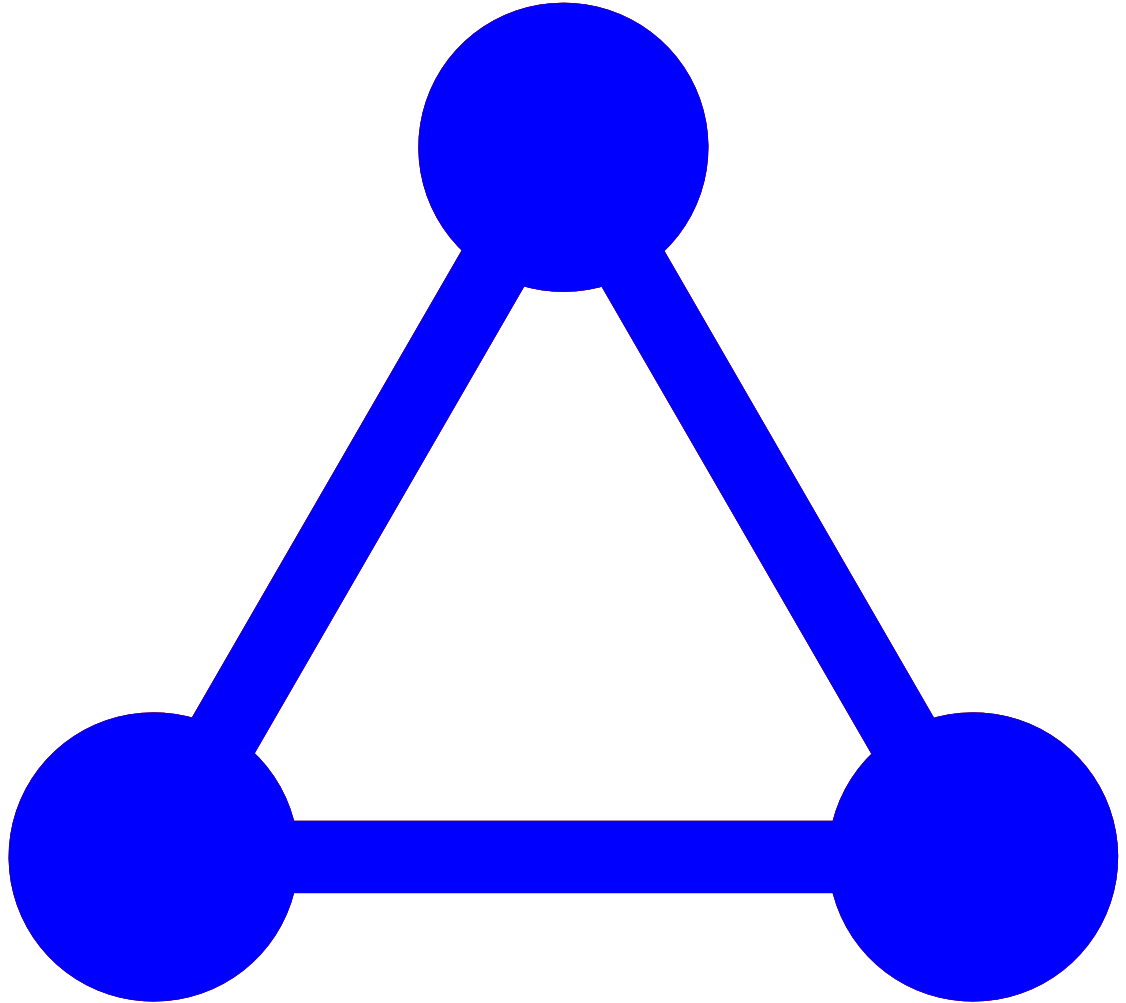
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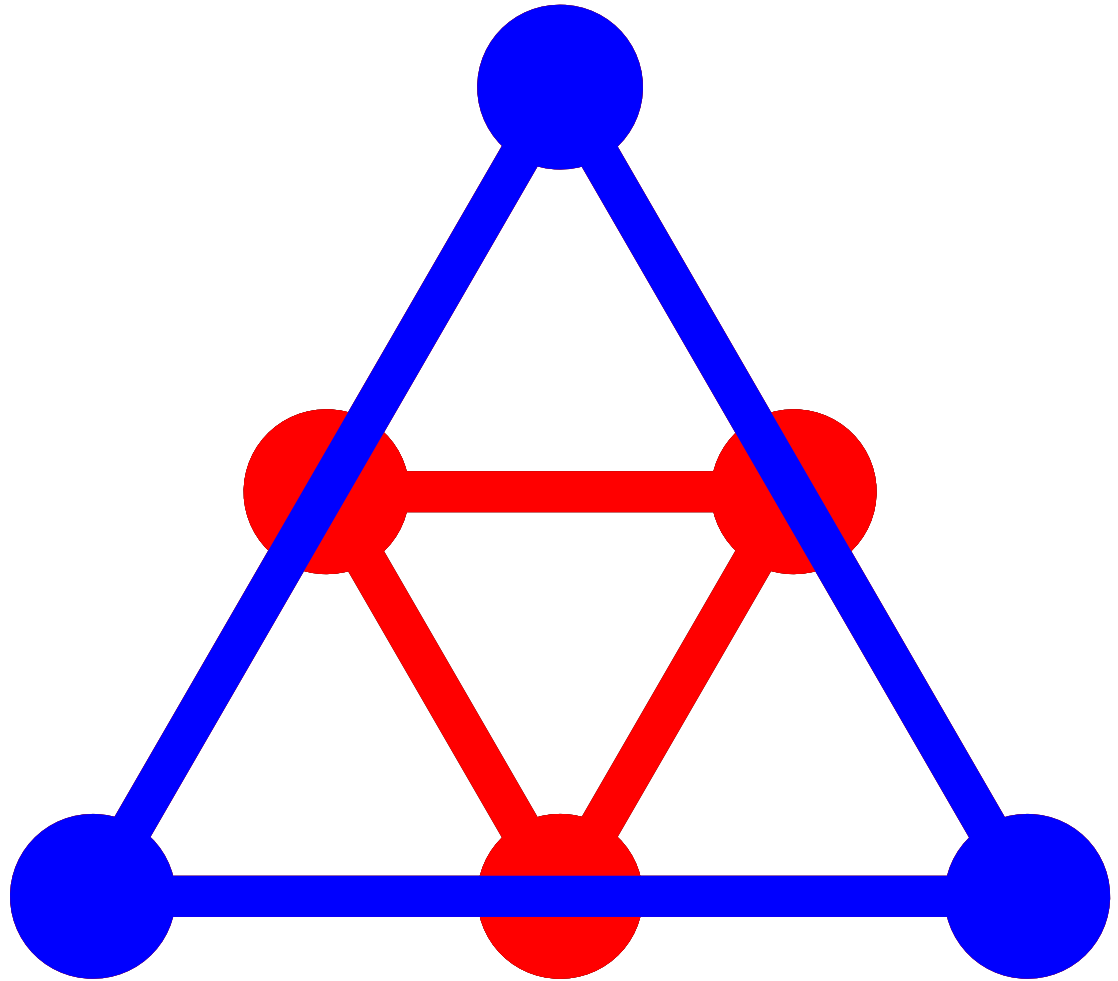
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Factor 1 = 1×1 , factors 1-1



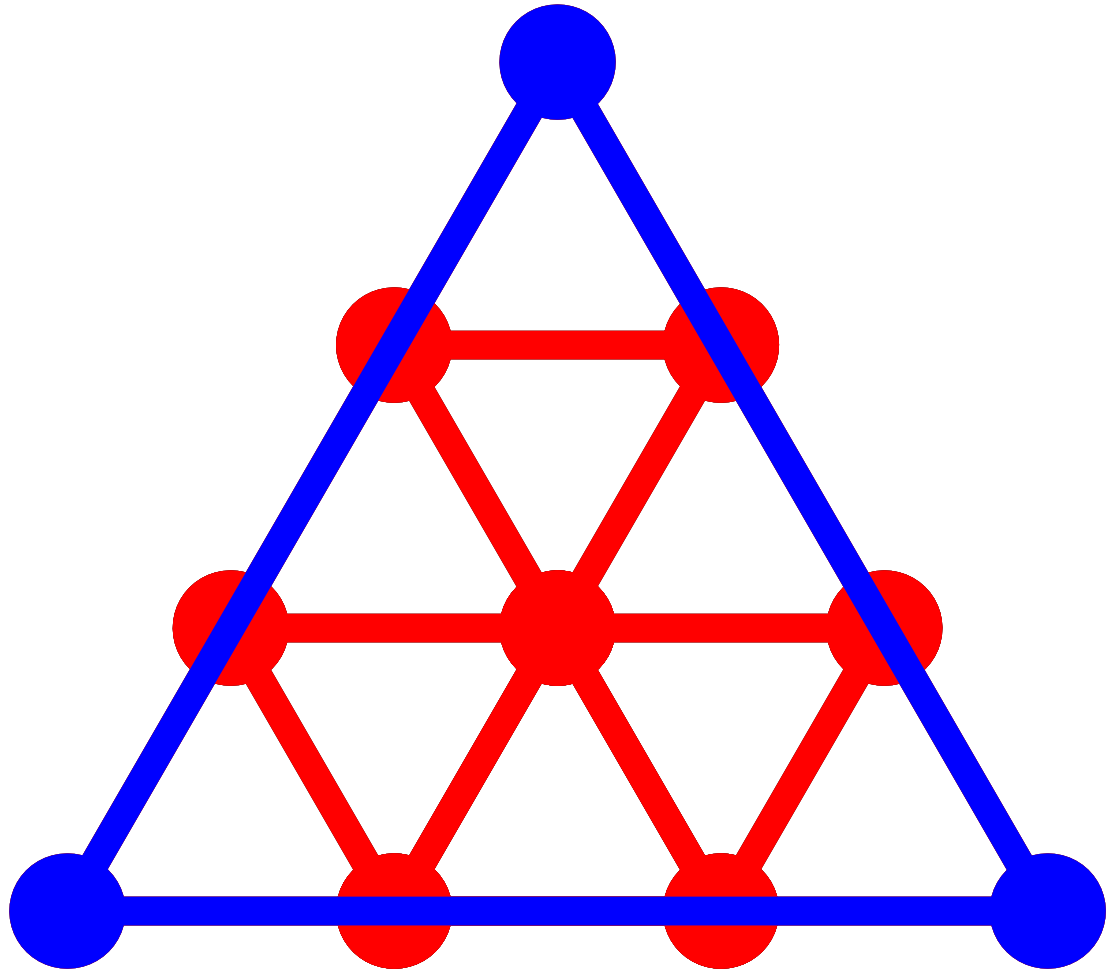
The Number 2

Factor $2 = 1 \times 2$, factors 1-2



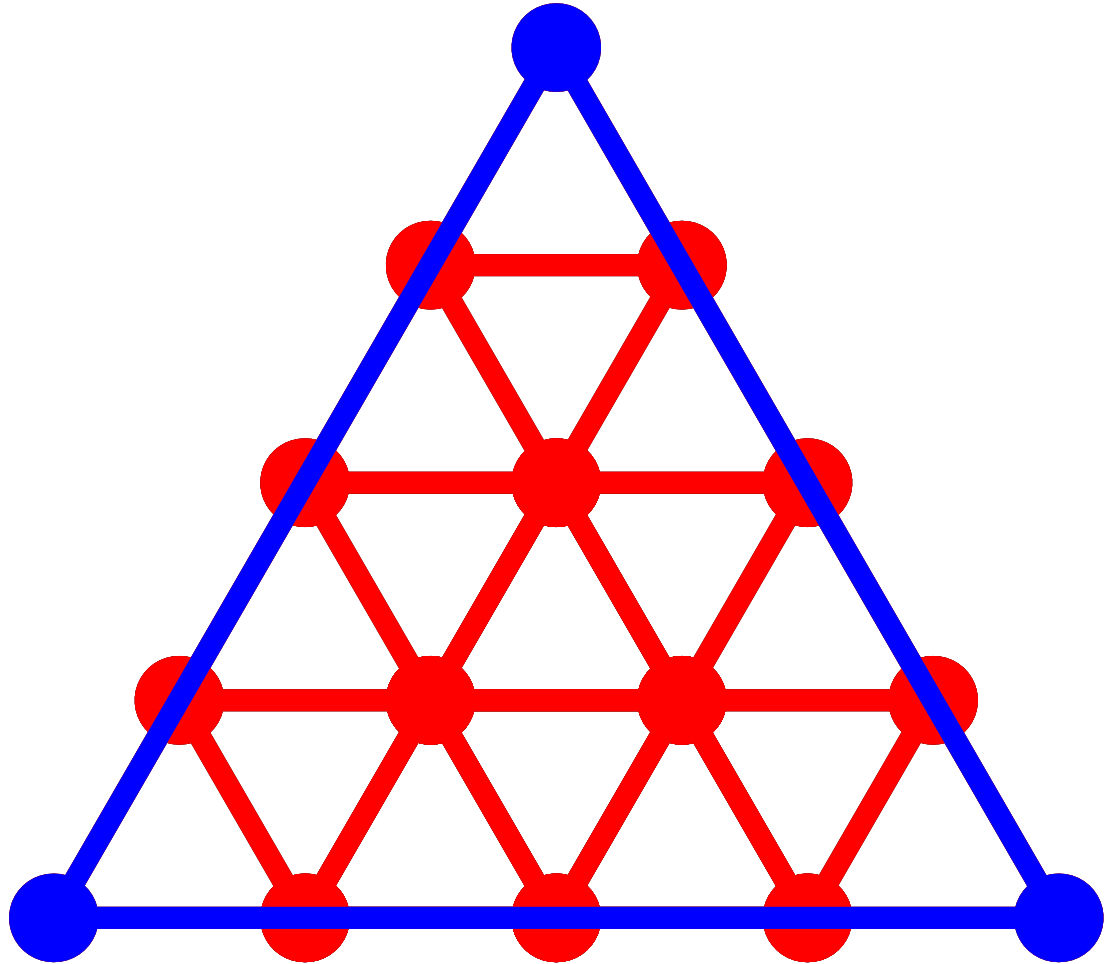
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Factor $3 = 1 \times 3$, factors 1-3

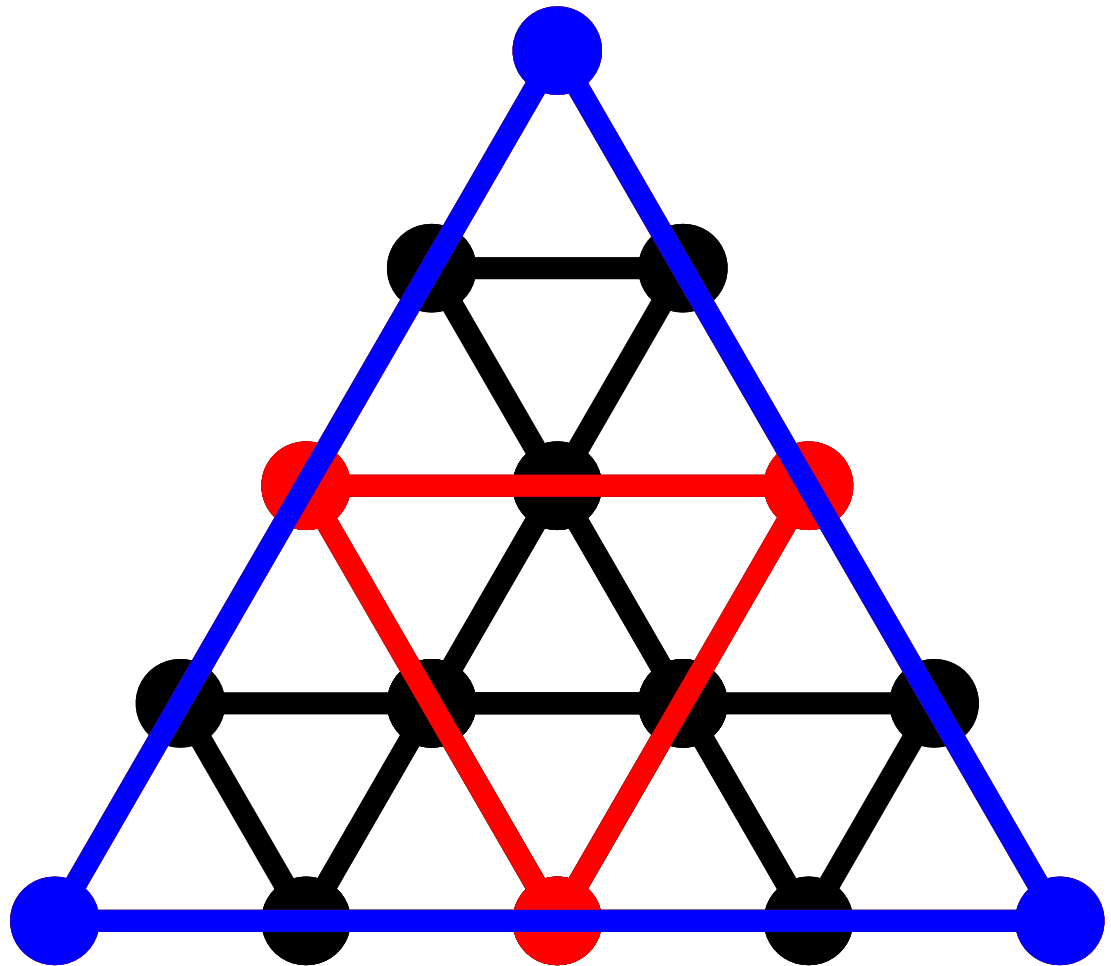


The Number 4

Factor $4 = 1 \times 4$, factors 1-4

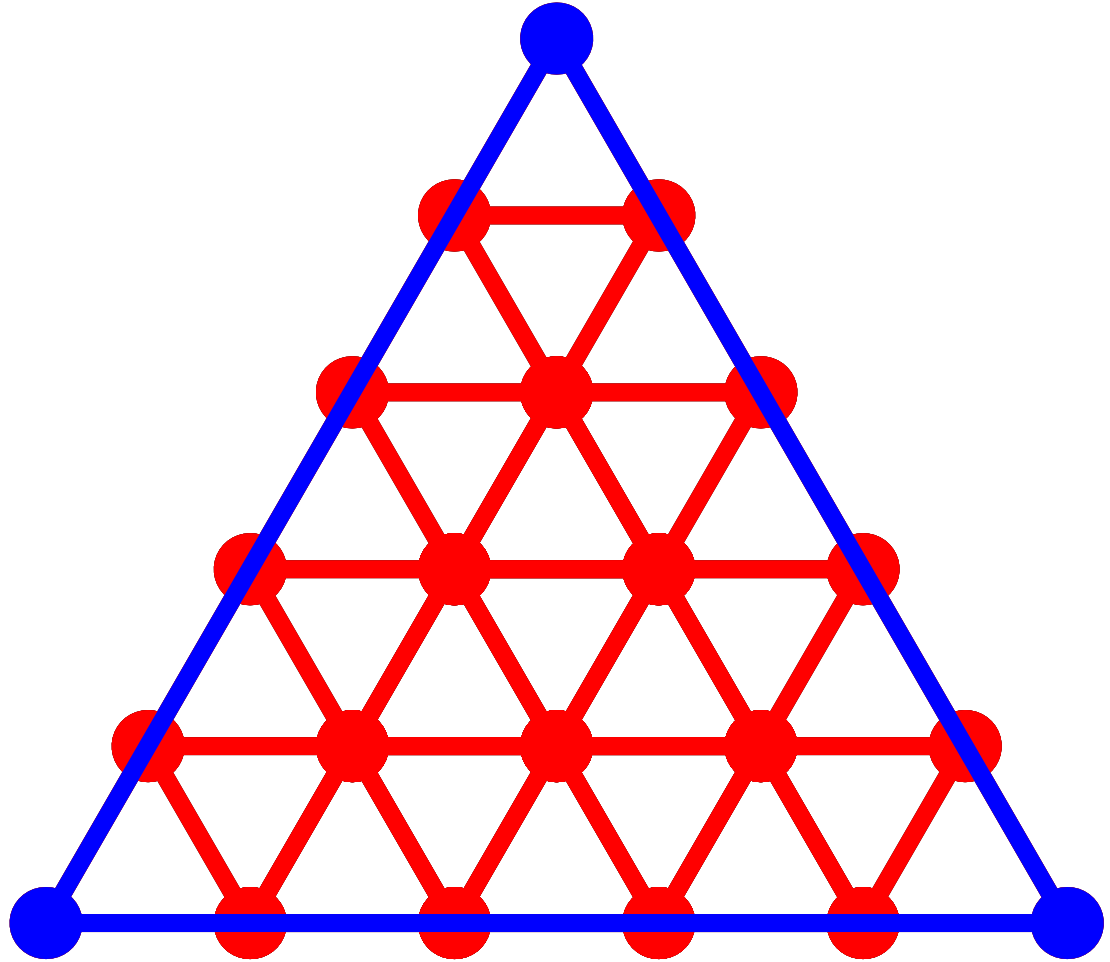


Factor $4 = 2 \times 2$, factors 2-2



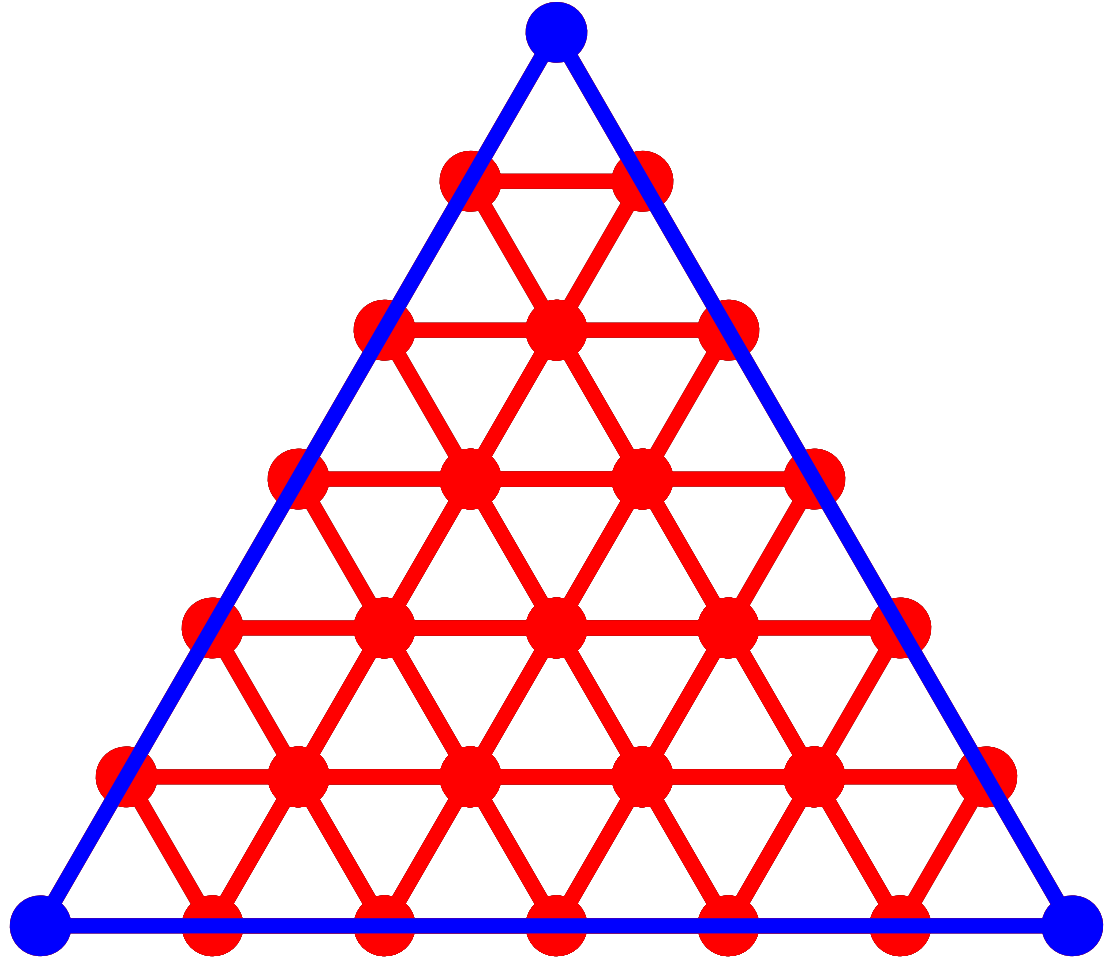
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Factor $5 = 1 \times 5$, factors 1-5

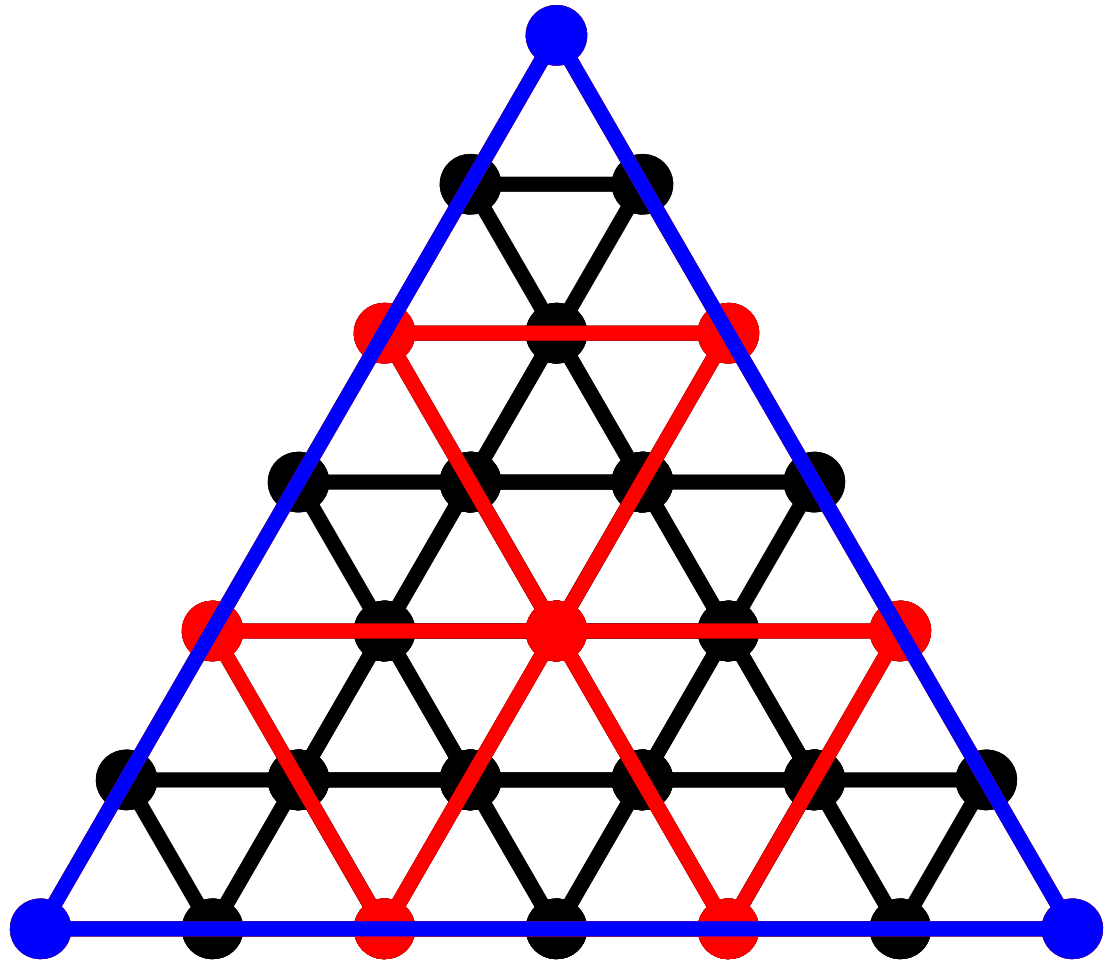


The Number 6

Factor $6 = 1 \times 6$, factors 1-6

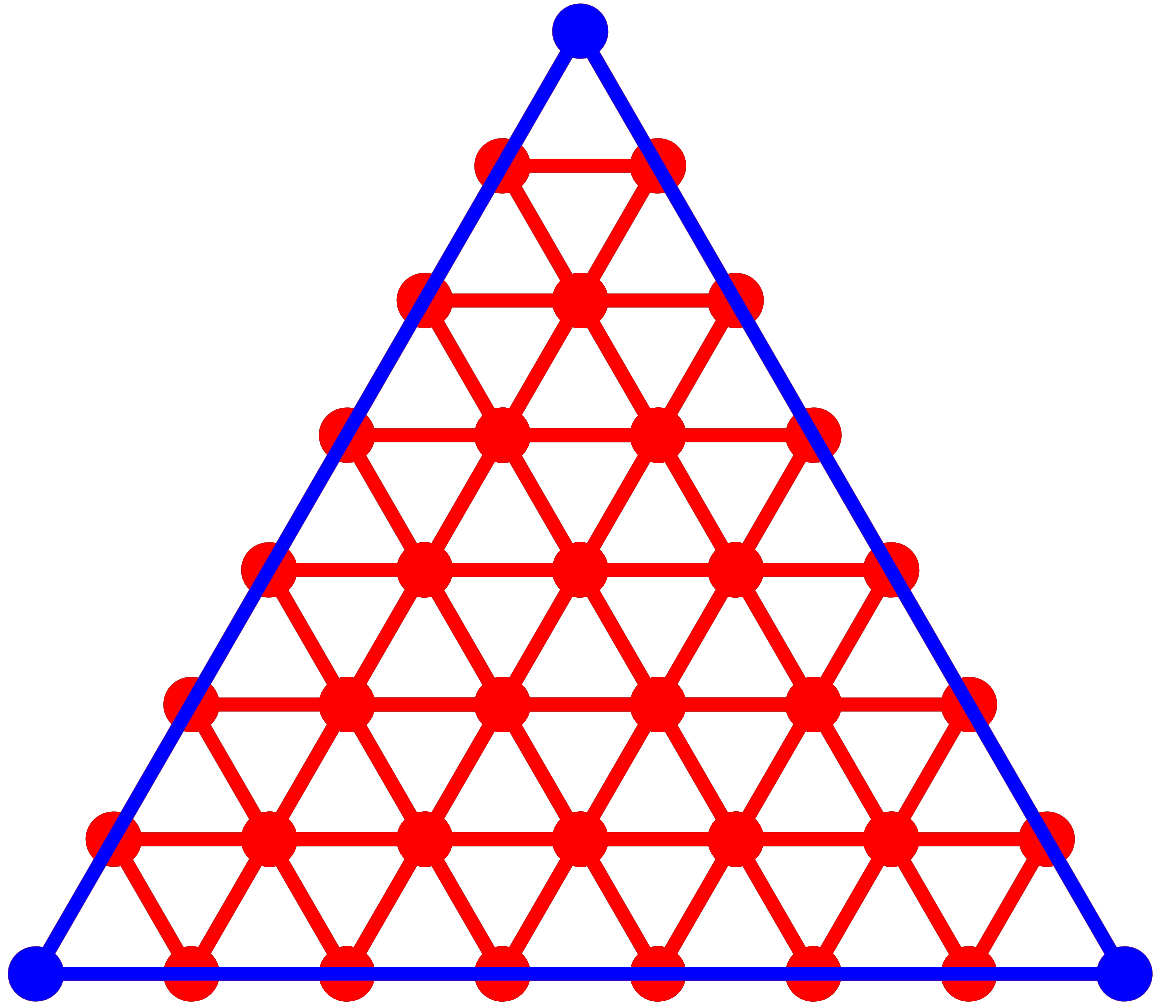


Factor $6 = 2 \times 3$, factors 2-3



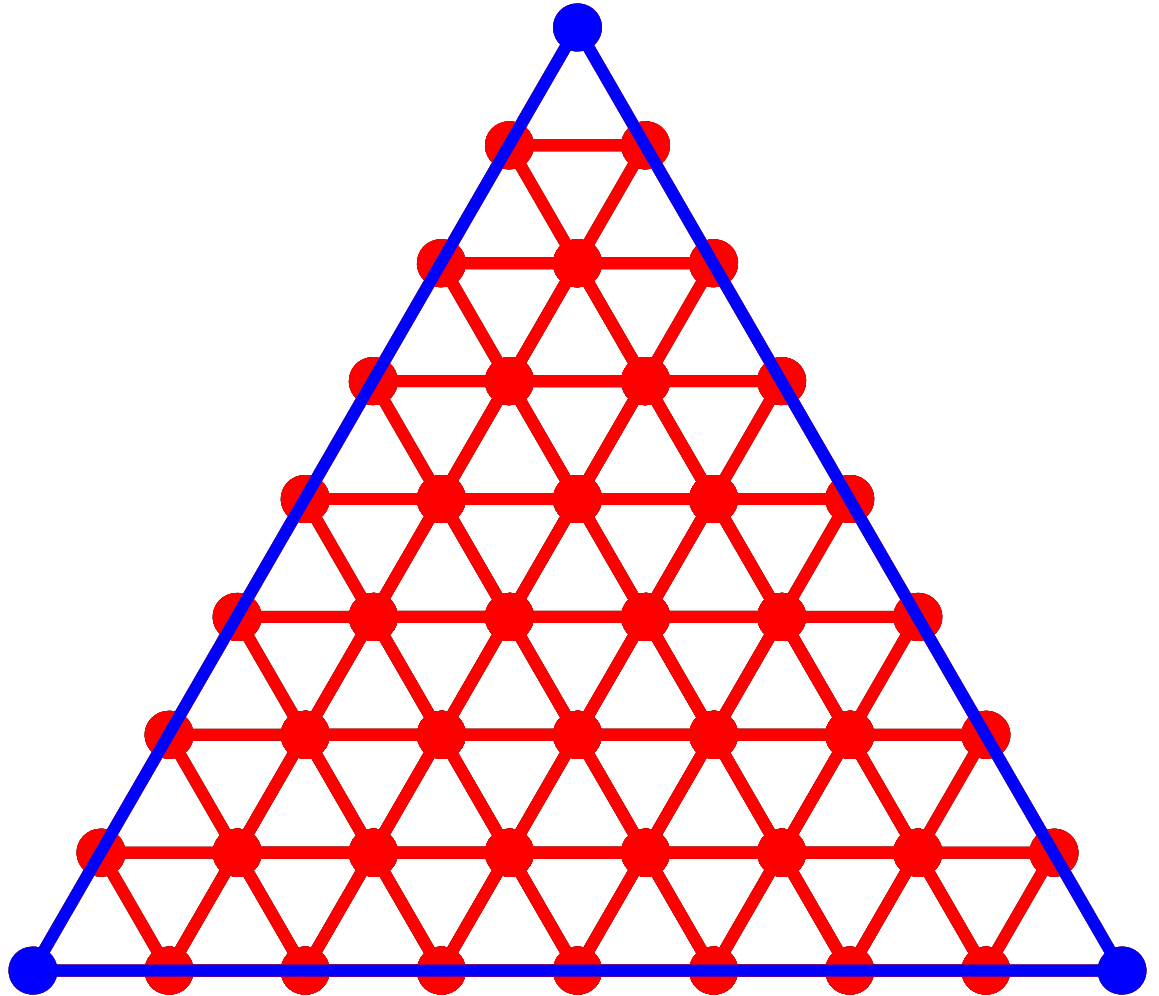
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Factor $7 = 1 \times 7$, factors 1-7

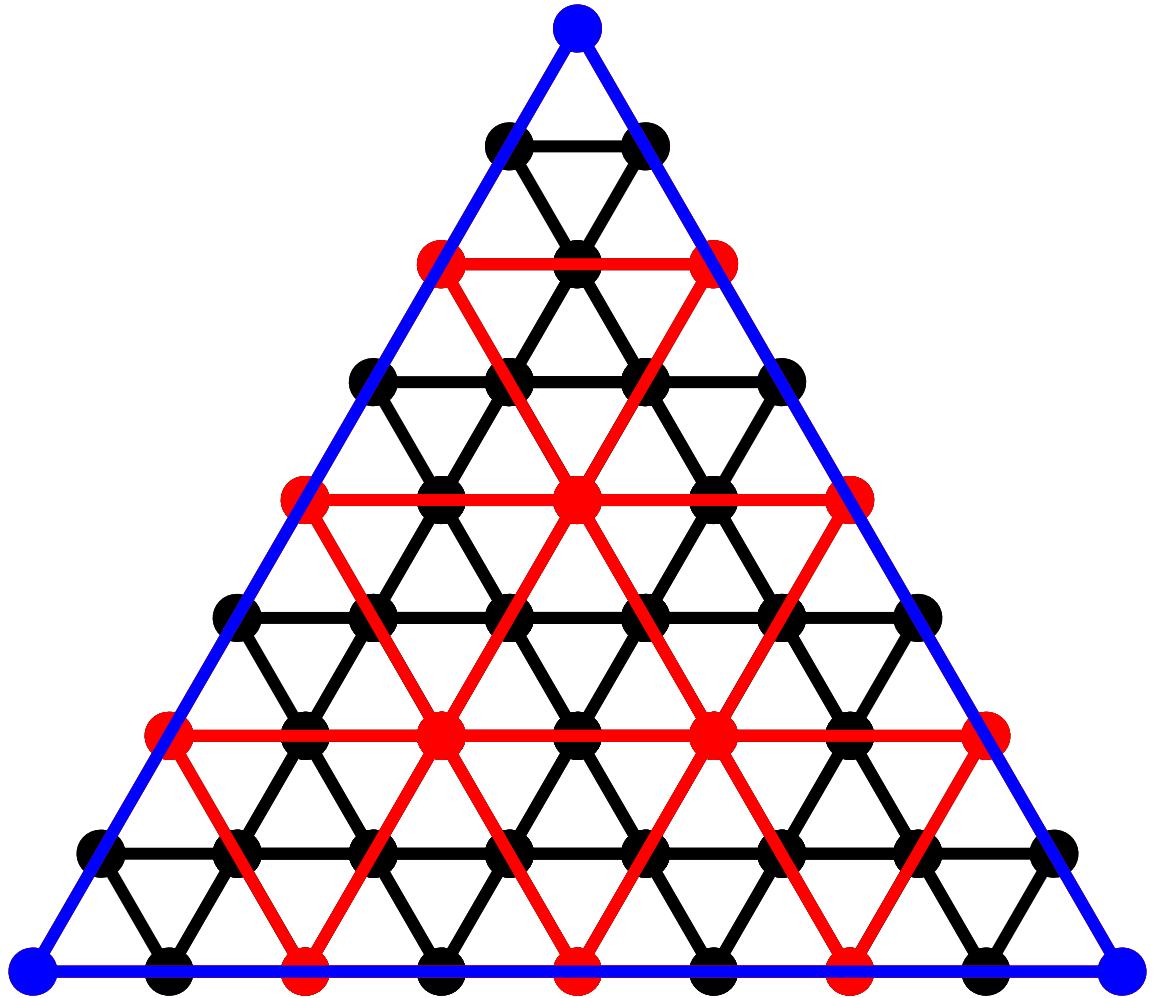


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Factor $8 = 1 \times 8$, factors 1-8

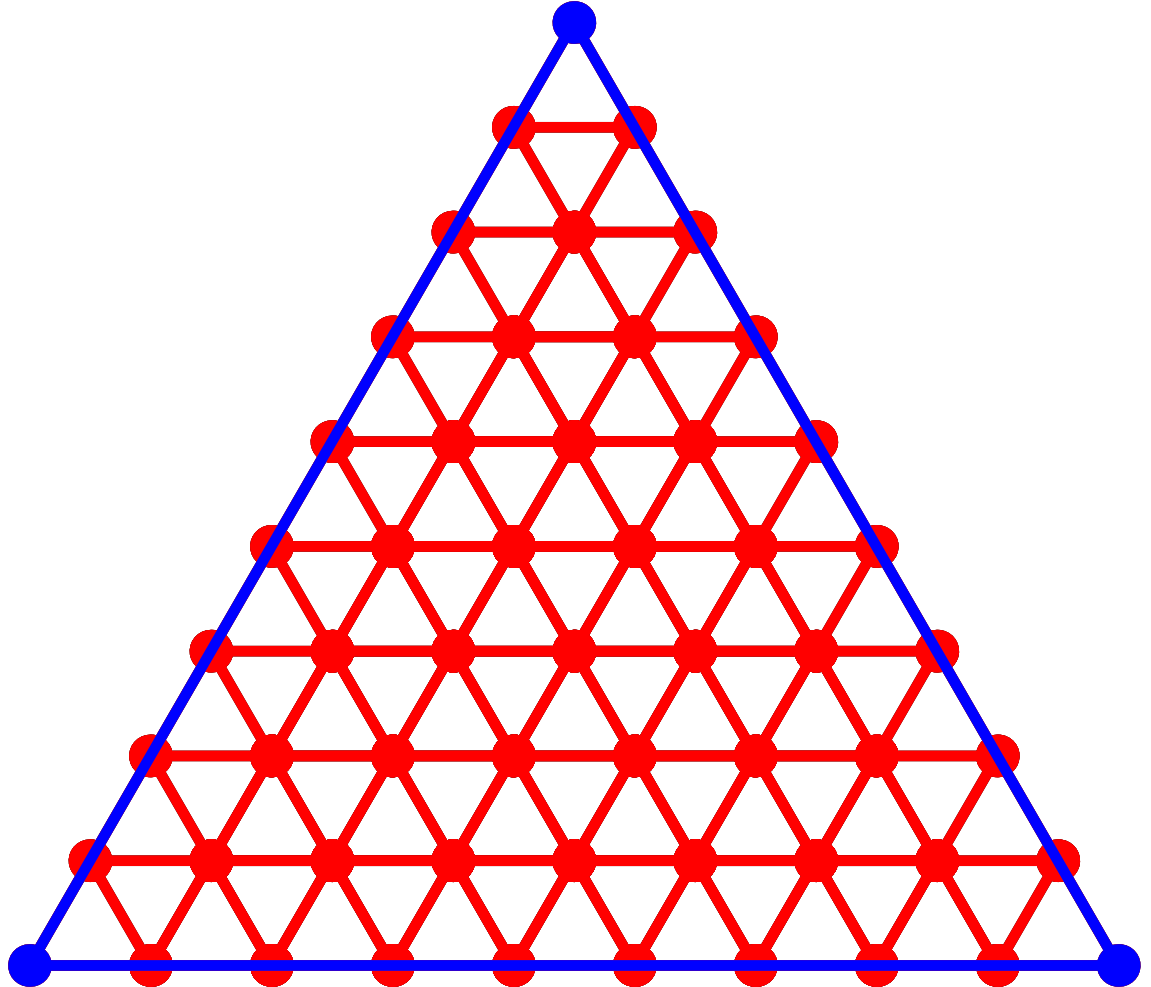


Factor $8 = 2 \times 4$, factors 2-4

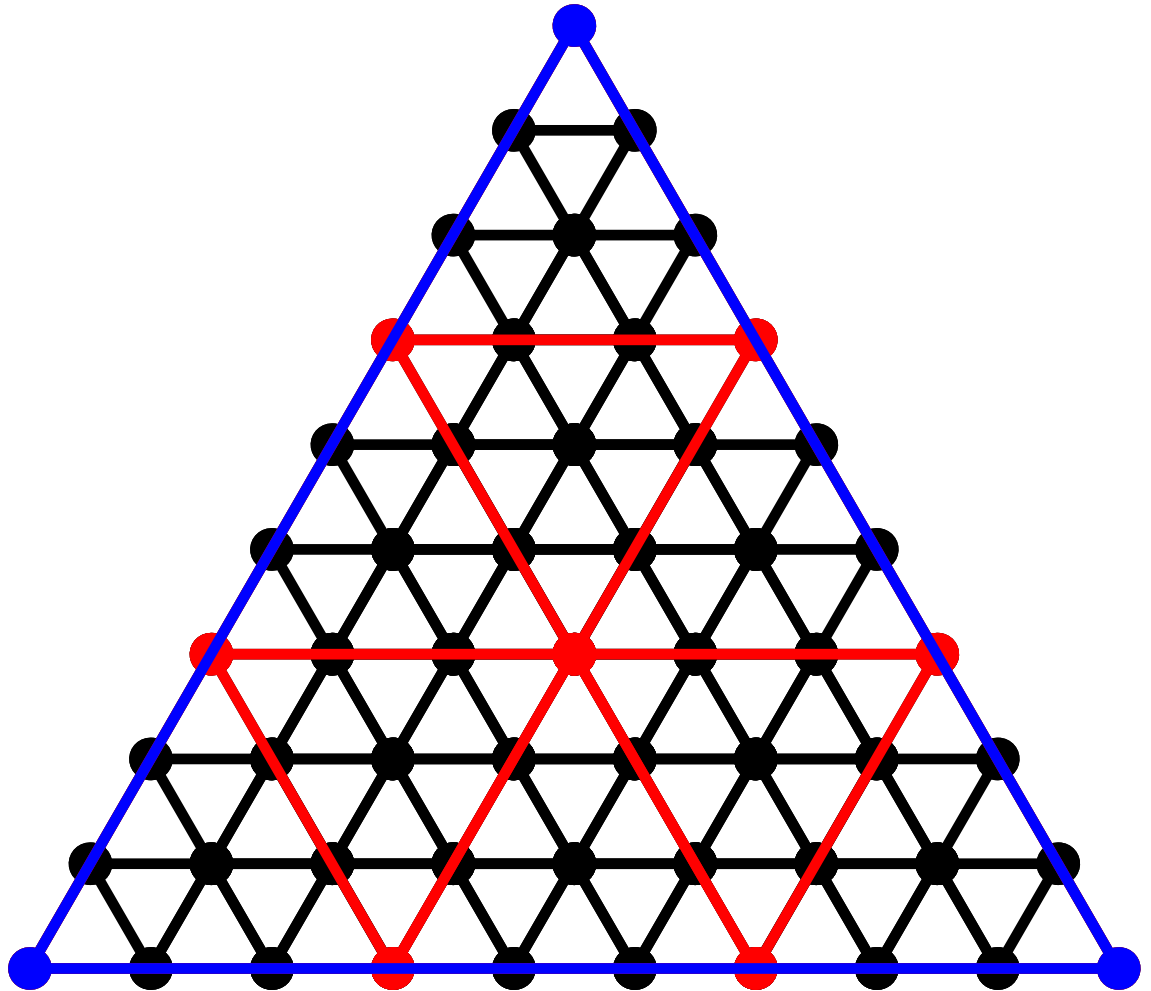


The Number 9

Factor $9 = 1 \times 9$, factors 1-9

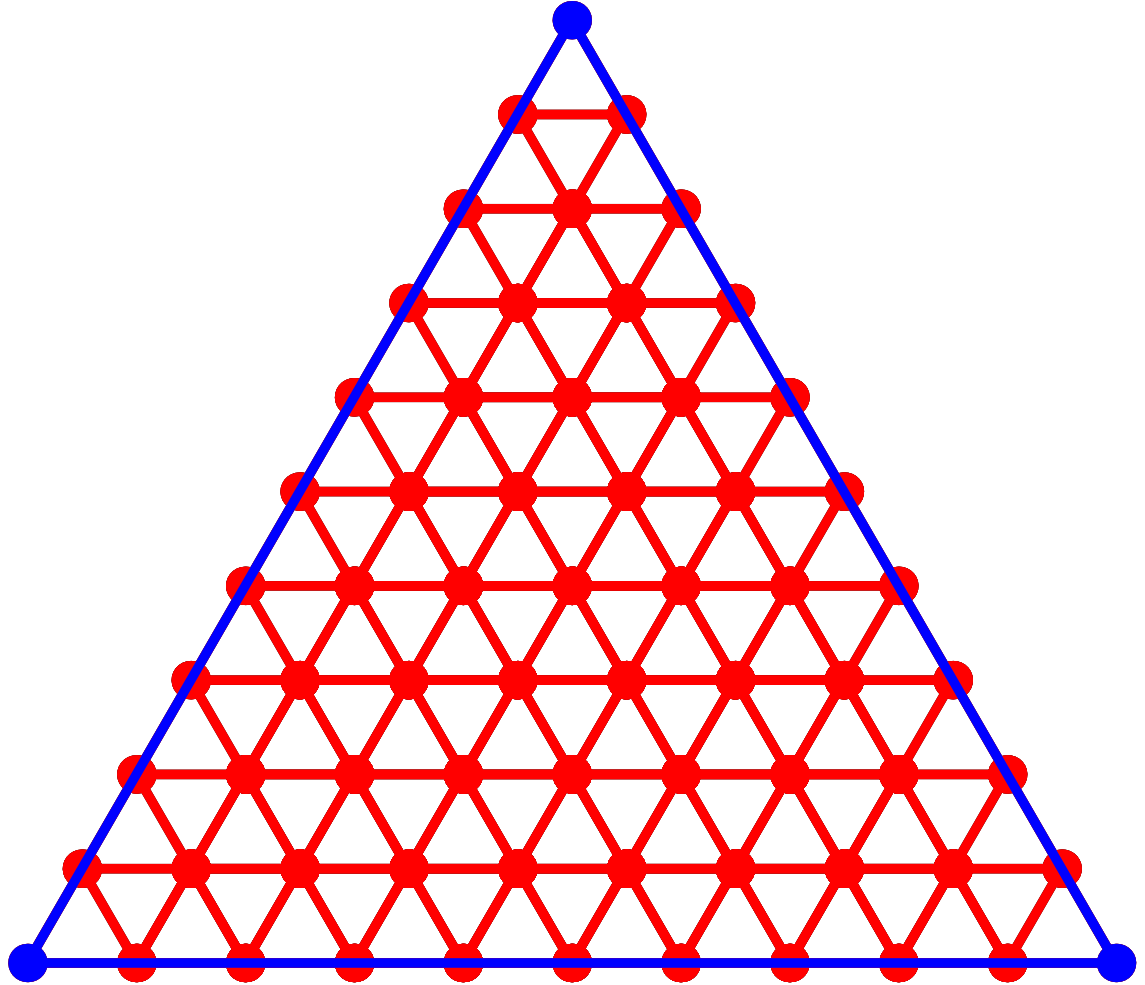


Factor $9 = 3 \times 3$, factors 3-3

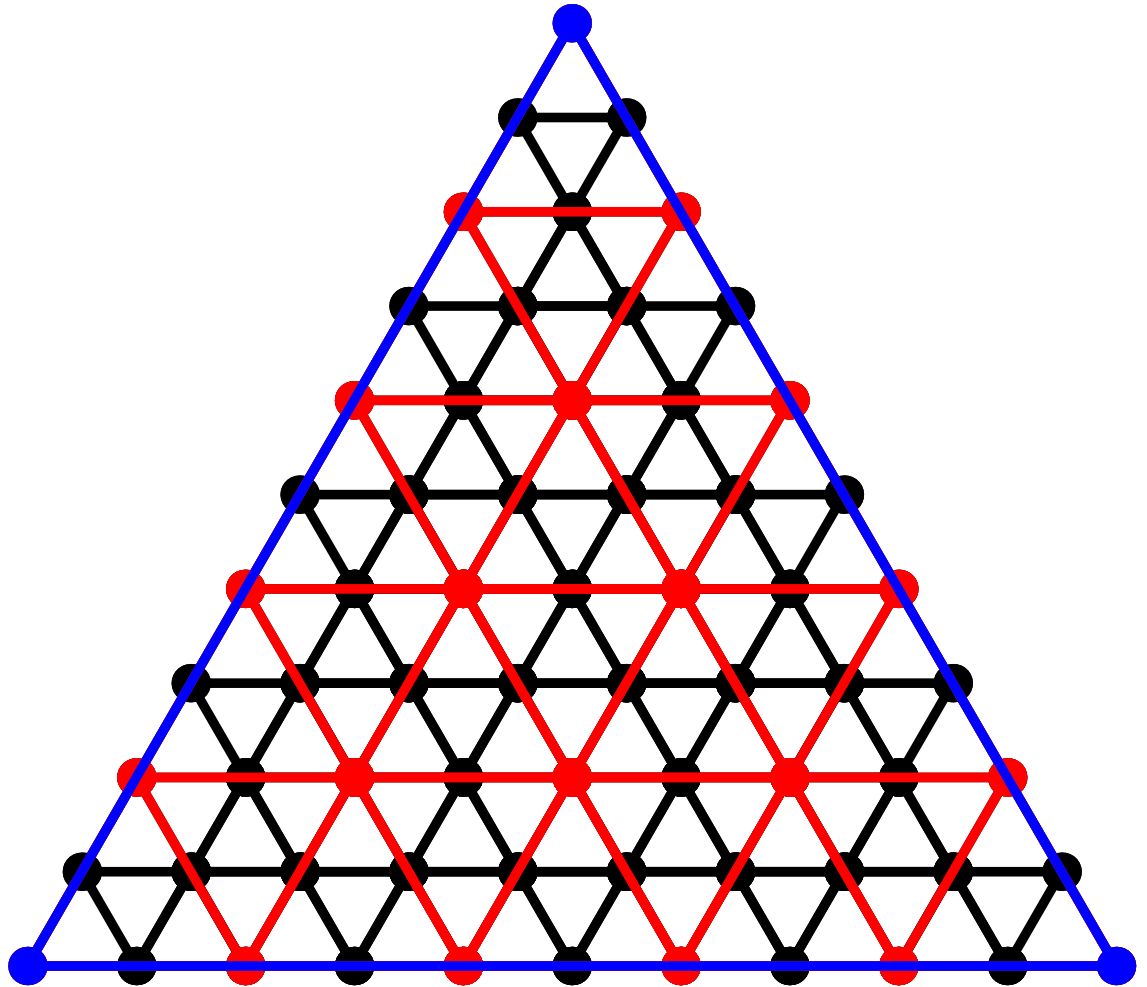


The Number 10

Factor $10 = 1 \times 10$, **factors** 1-10

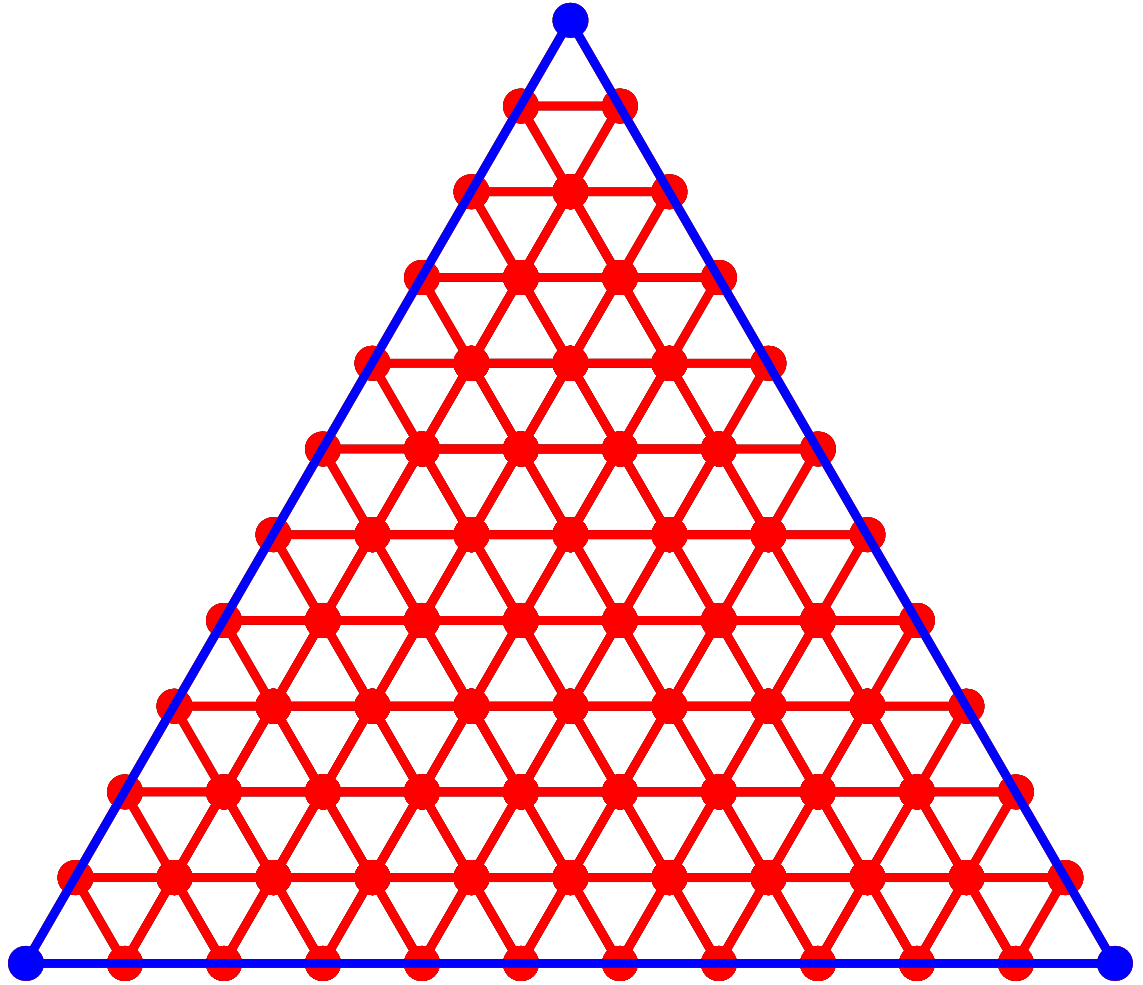


Factor $10 = 2 \times 5$, factors 2-5



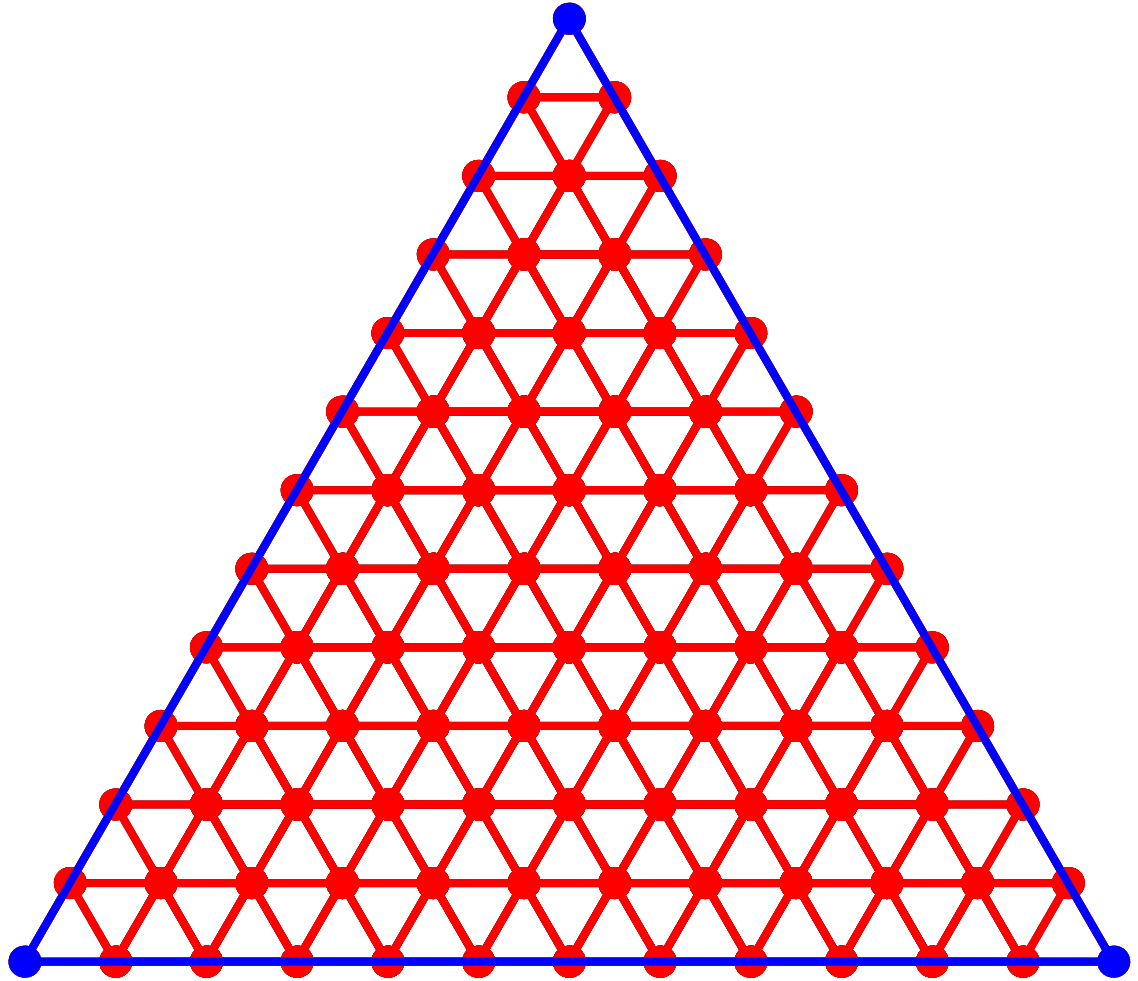
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Factor $11 = 1 \times 11$, **factors** 1-11

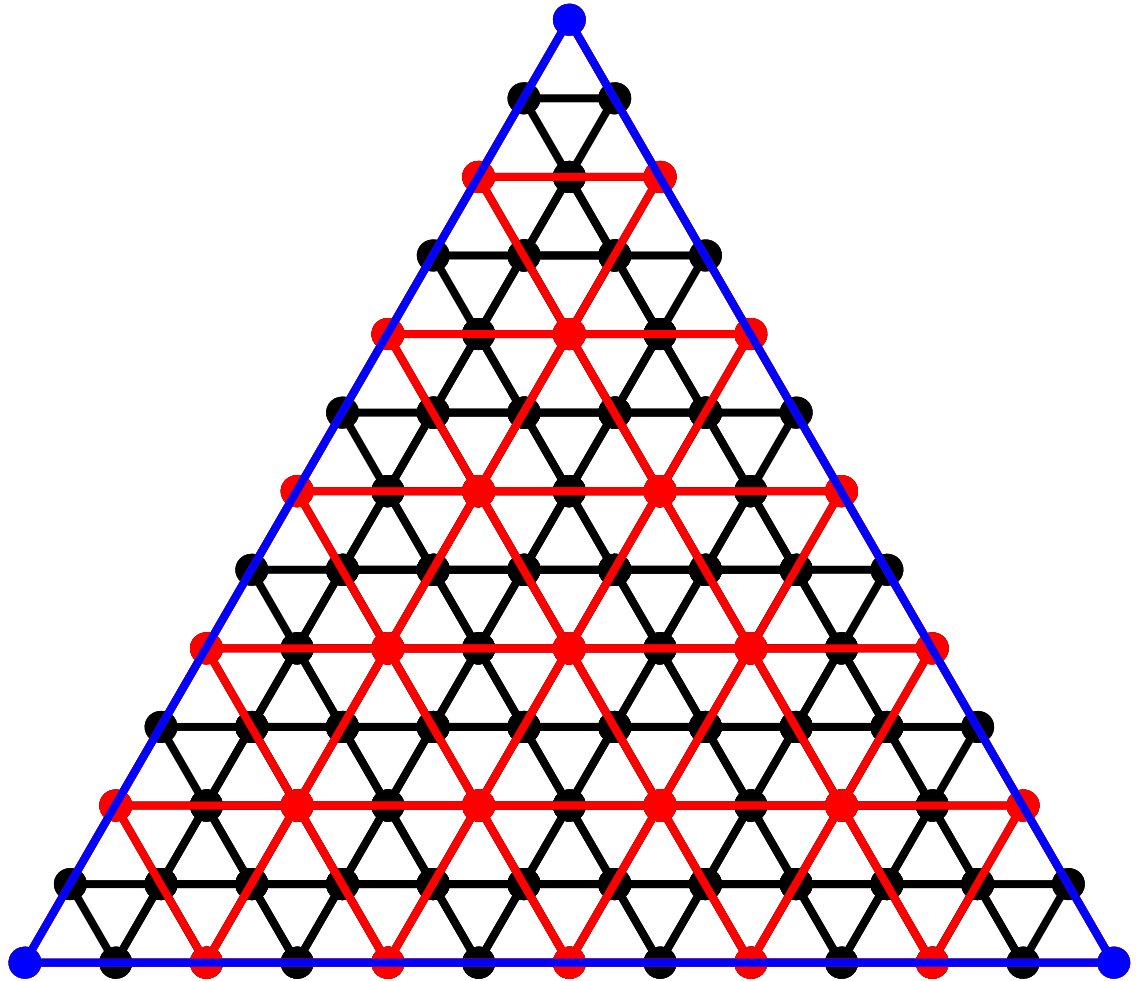


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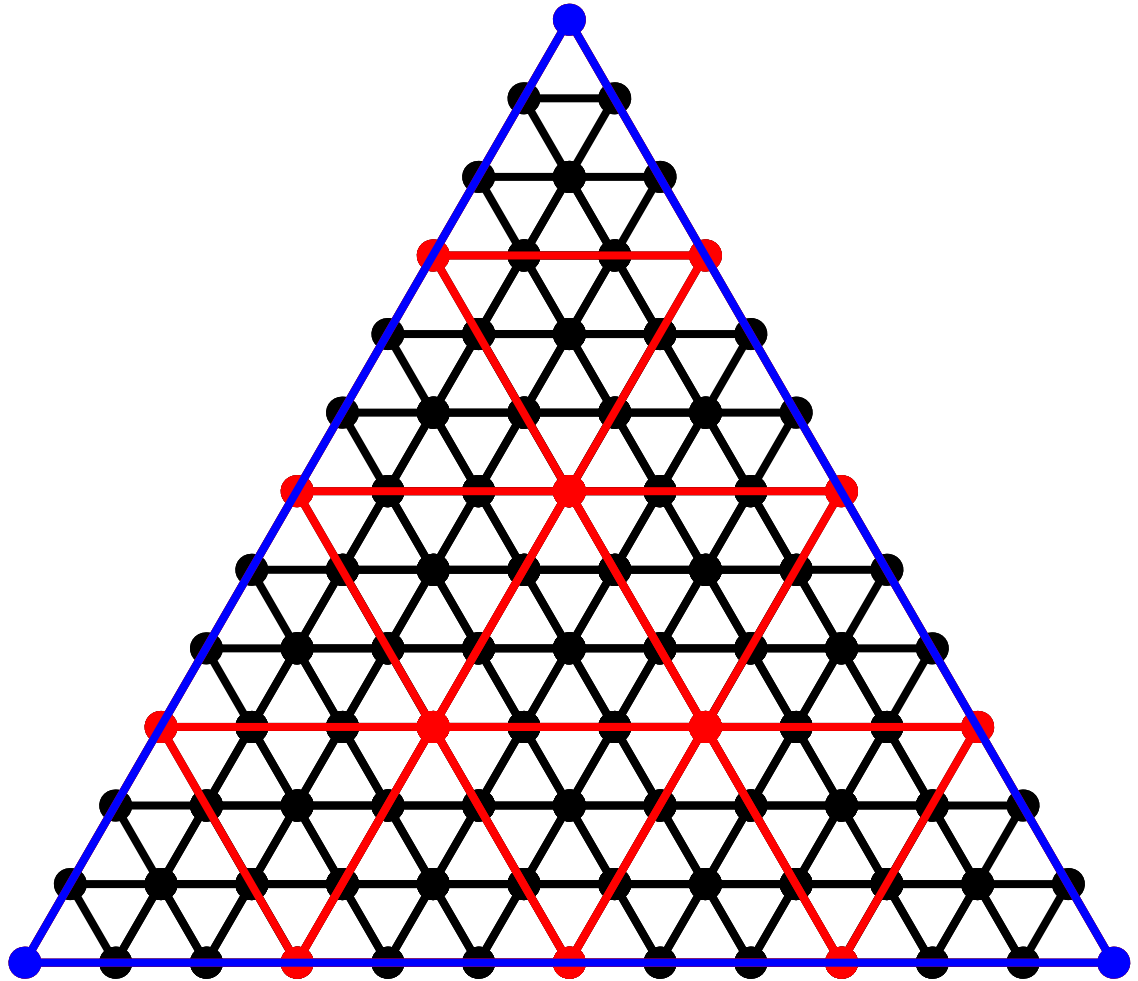
Factor $12 = 1 \times 12$, **factors 1-12**



Factor $12 = 2 \times 6$, factors 2-6

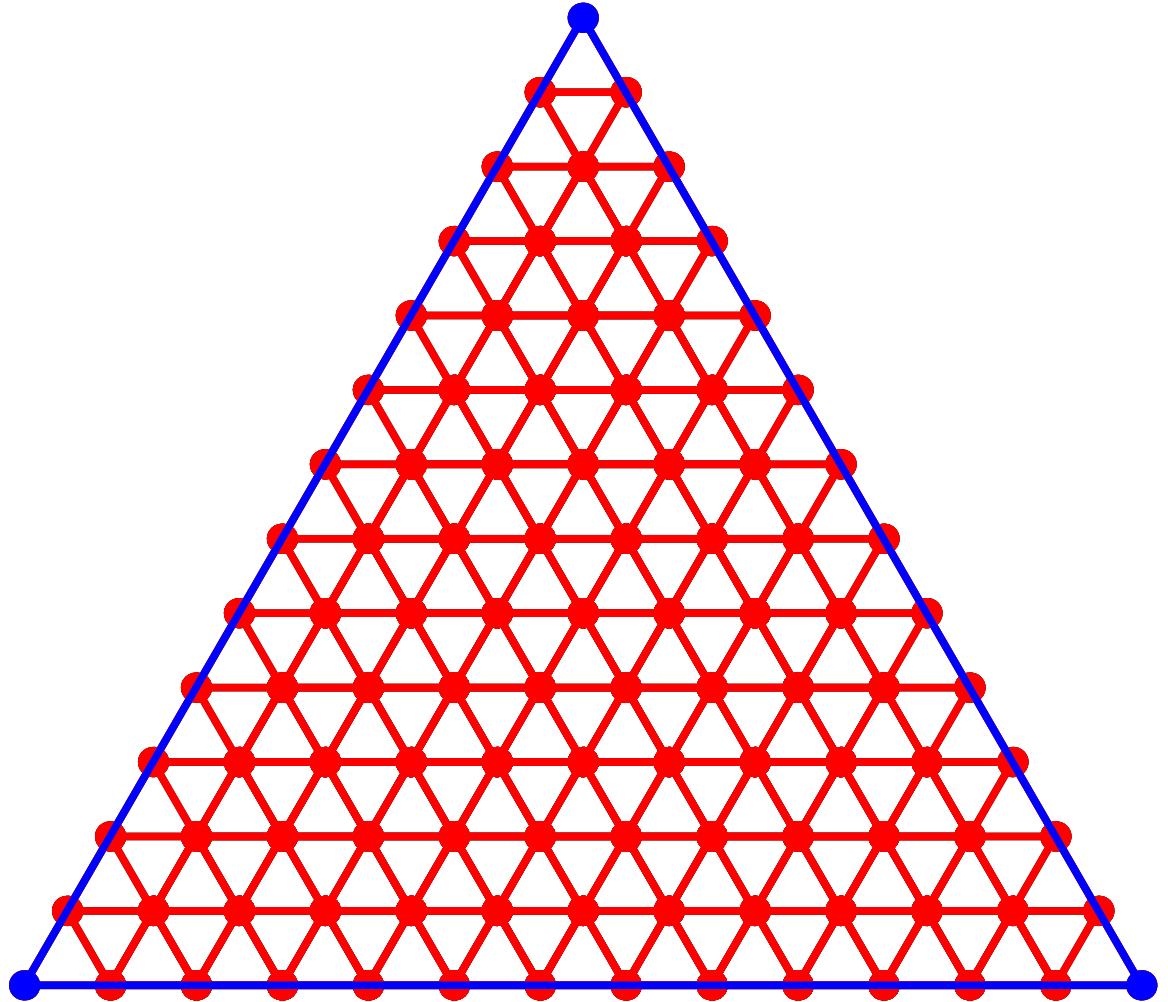


Factor $12 = 3 \times 4$, factors 3-4



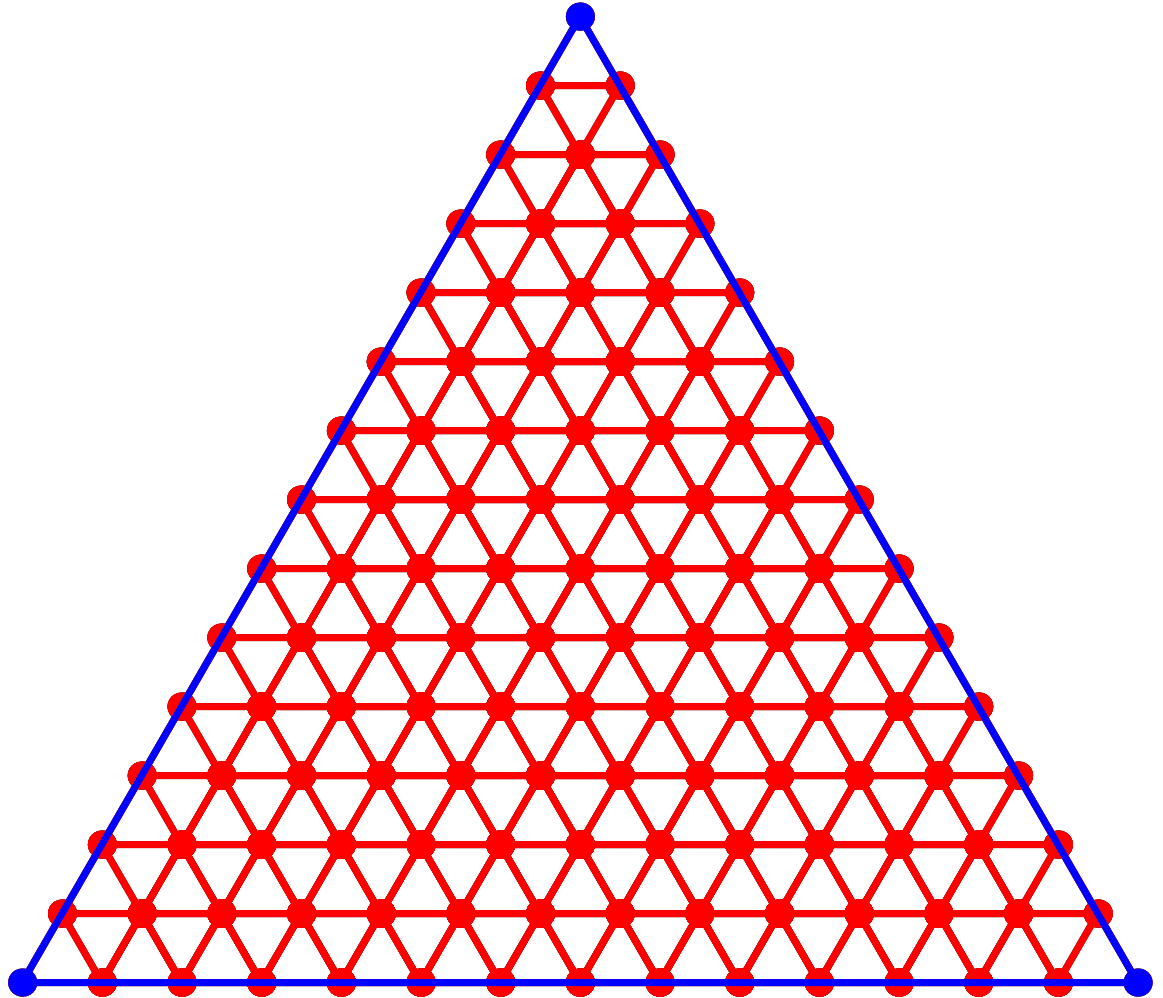
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Factor $13 = 1 \times 13$, **factors** 1-13

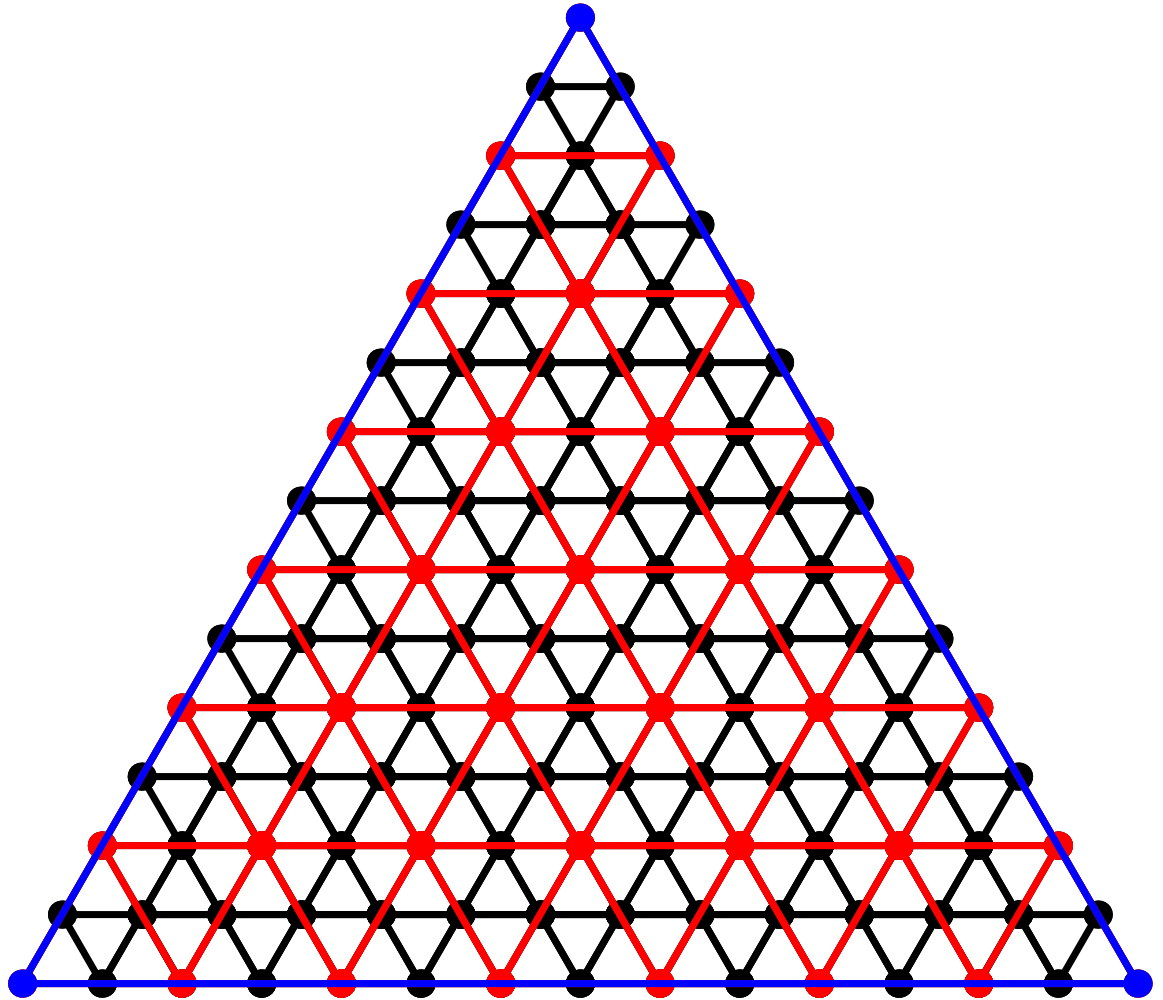


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Factor $14 = 1 \times 14$, **factors** 1-14

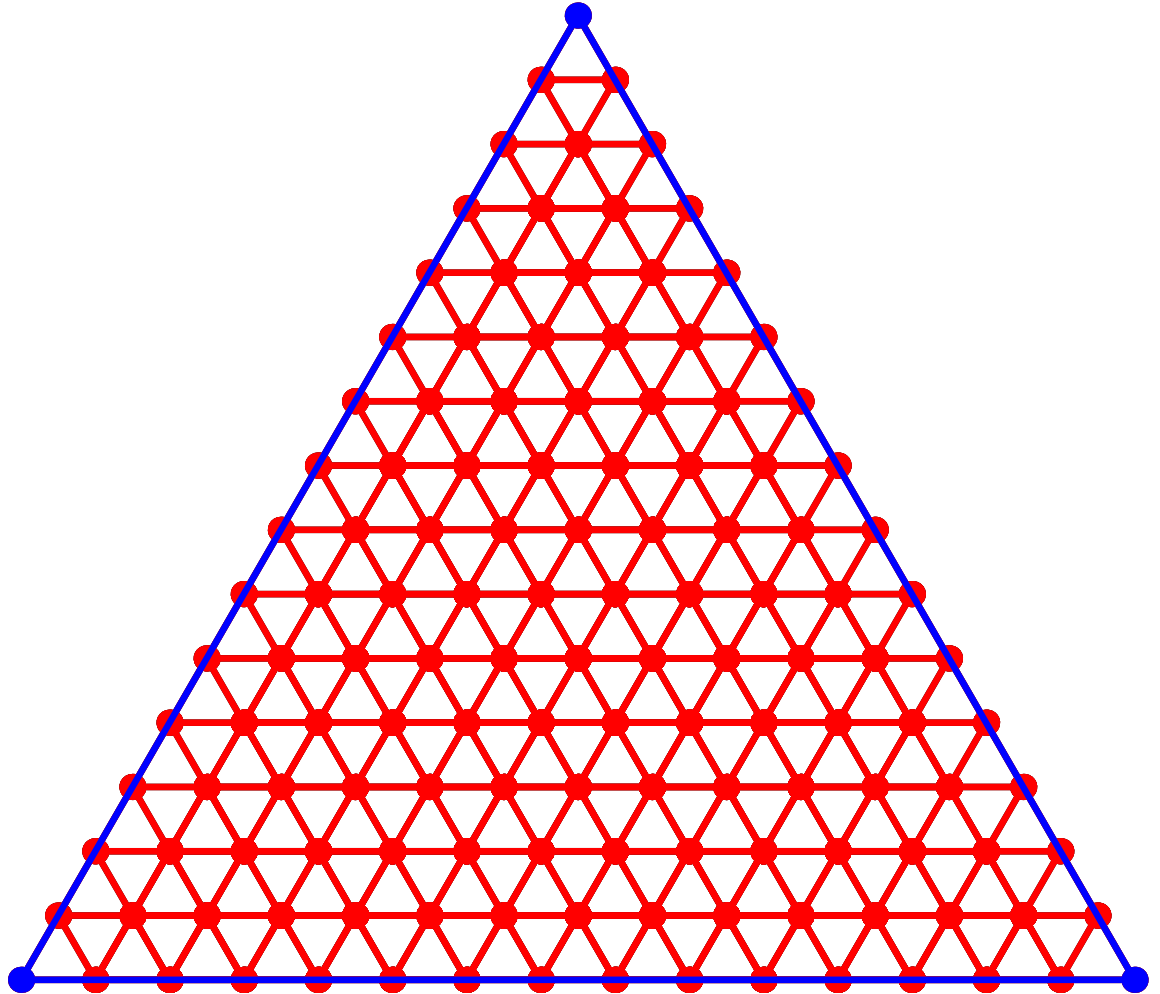


Factor $14 = 2 \times 7$, factors 2-7

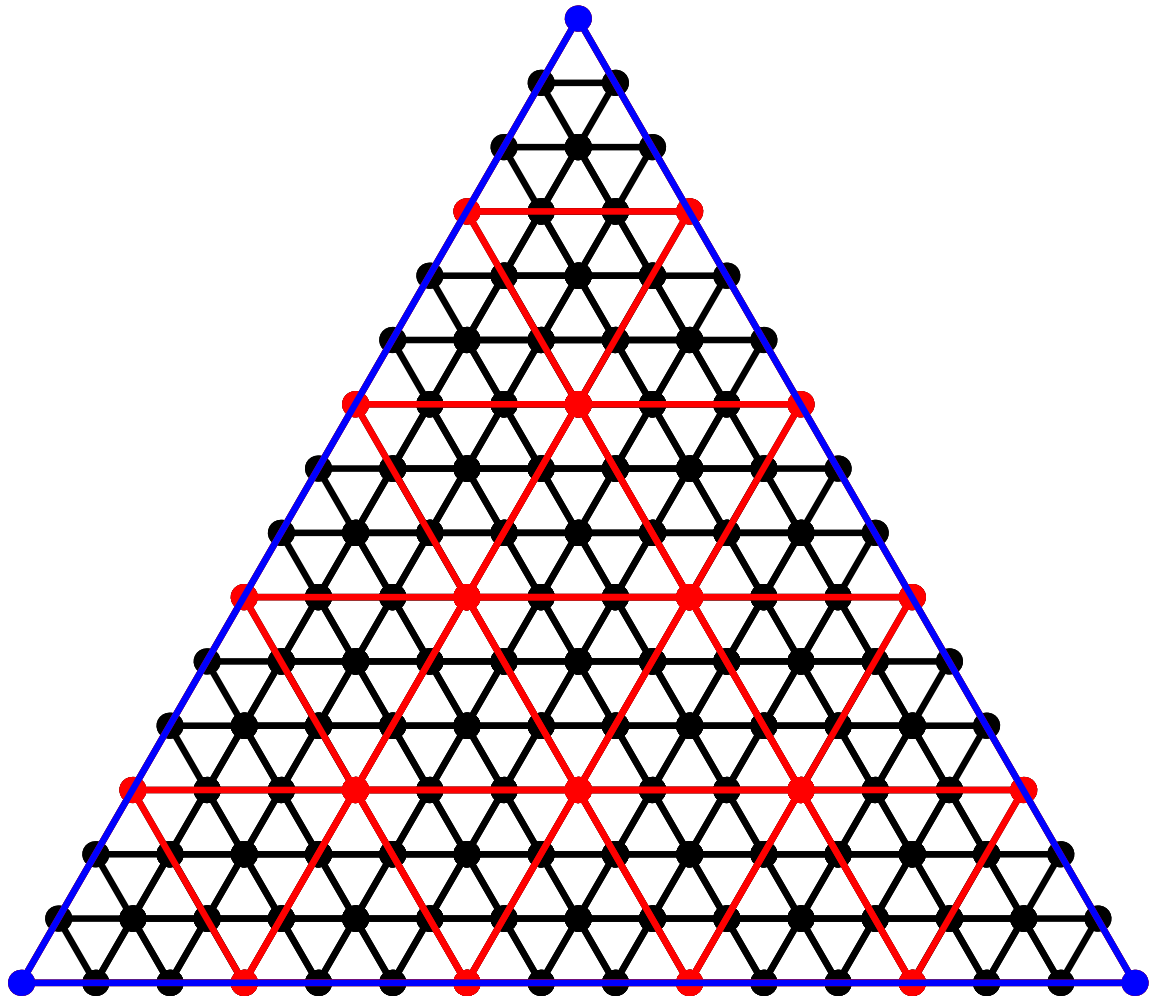


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Factor $15 = 1 \times 15$, **factors 1-15**

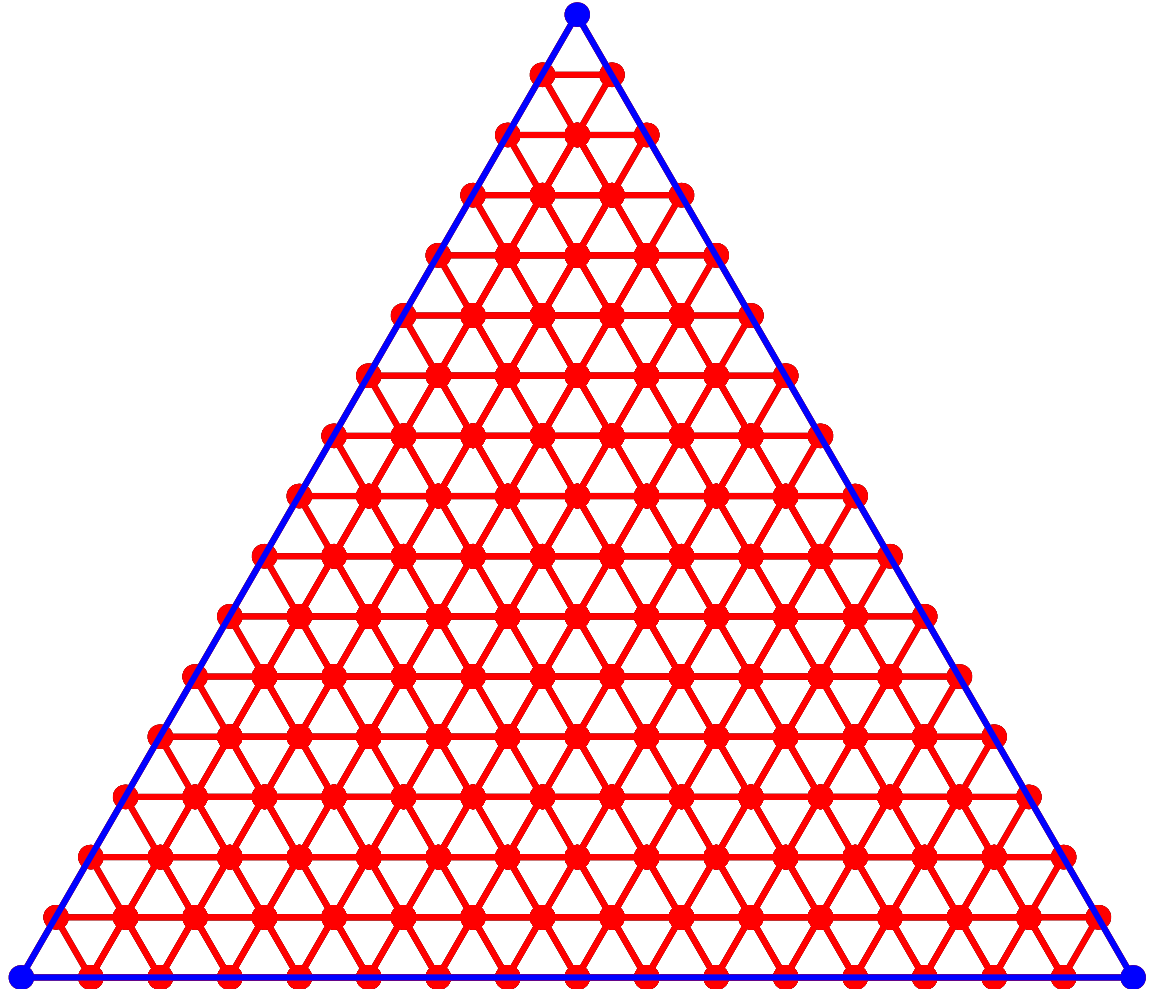


Factor $15 = 3 \times 5$, factors 3-5

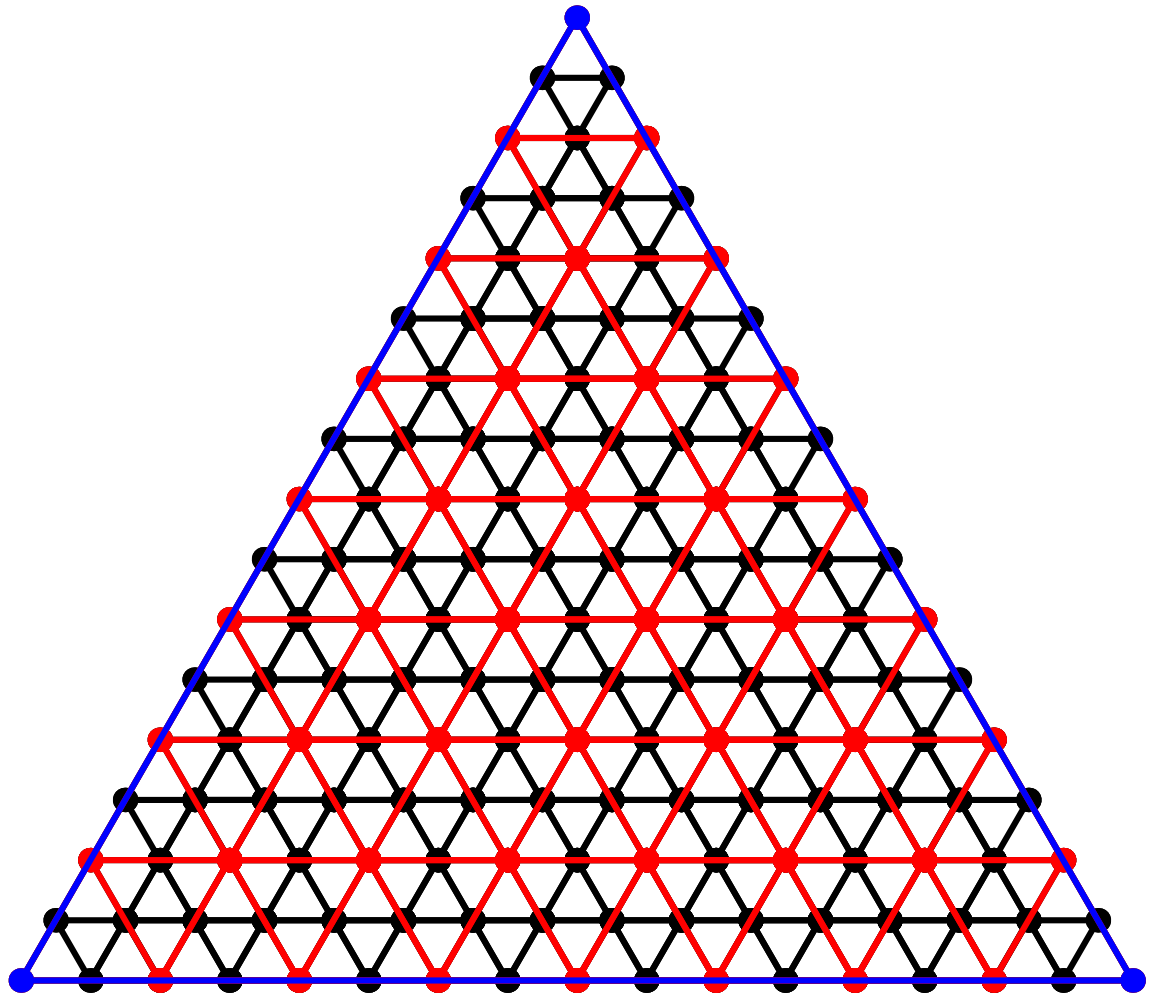


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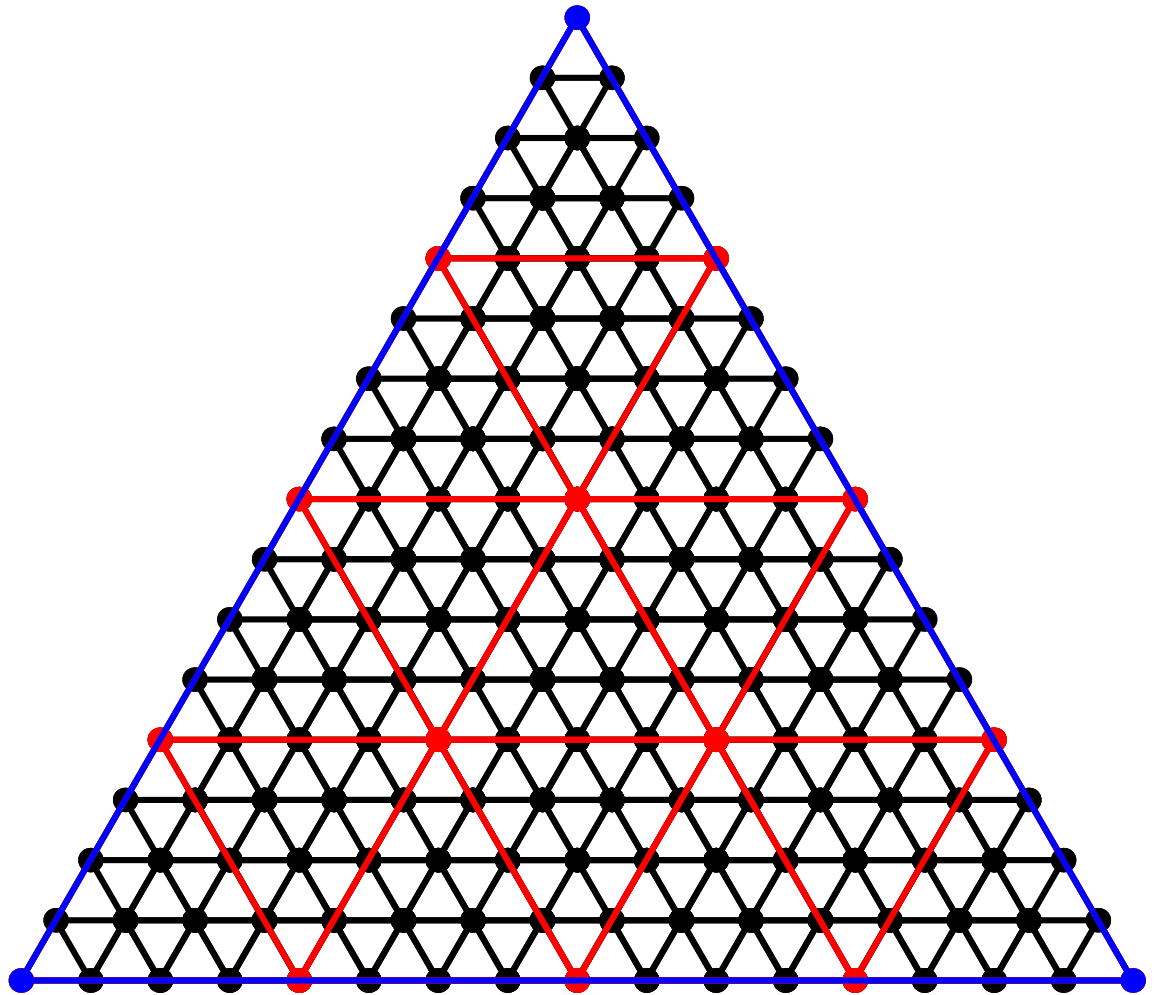
Factor $16 = 1 \times 16$, **factors 1-16**



Factor $16 = 2 \times 8$, factors 2-8

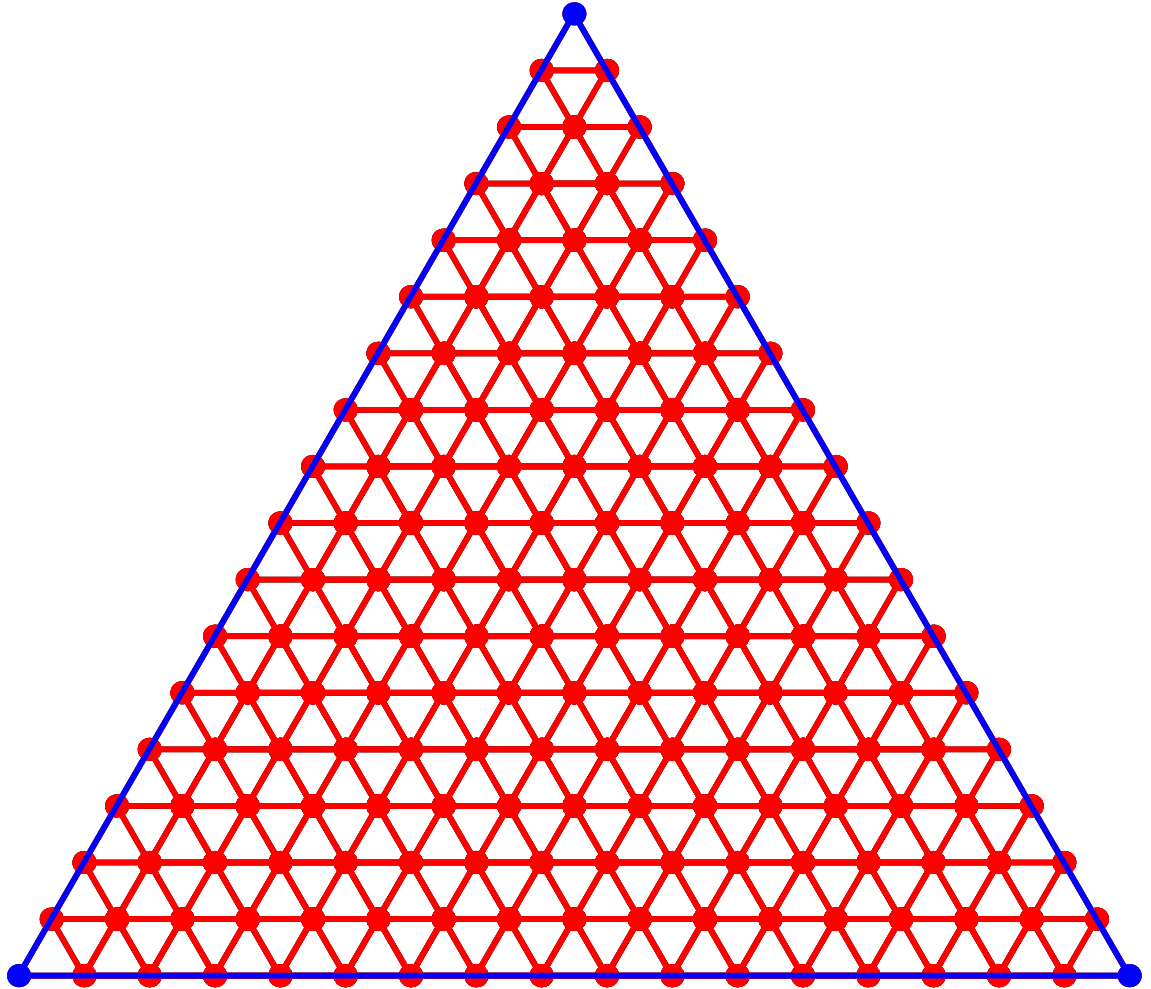


Factor $16 = 4 \times 4$, factors 4-4



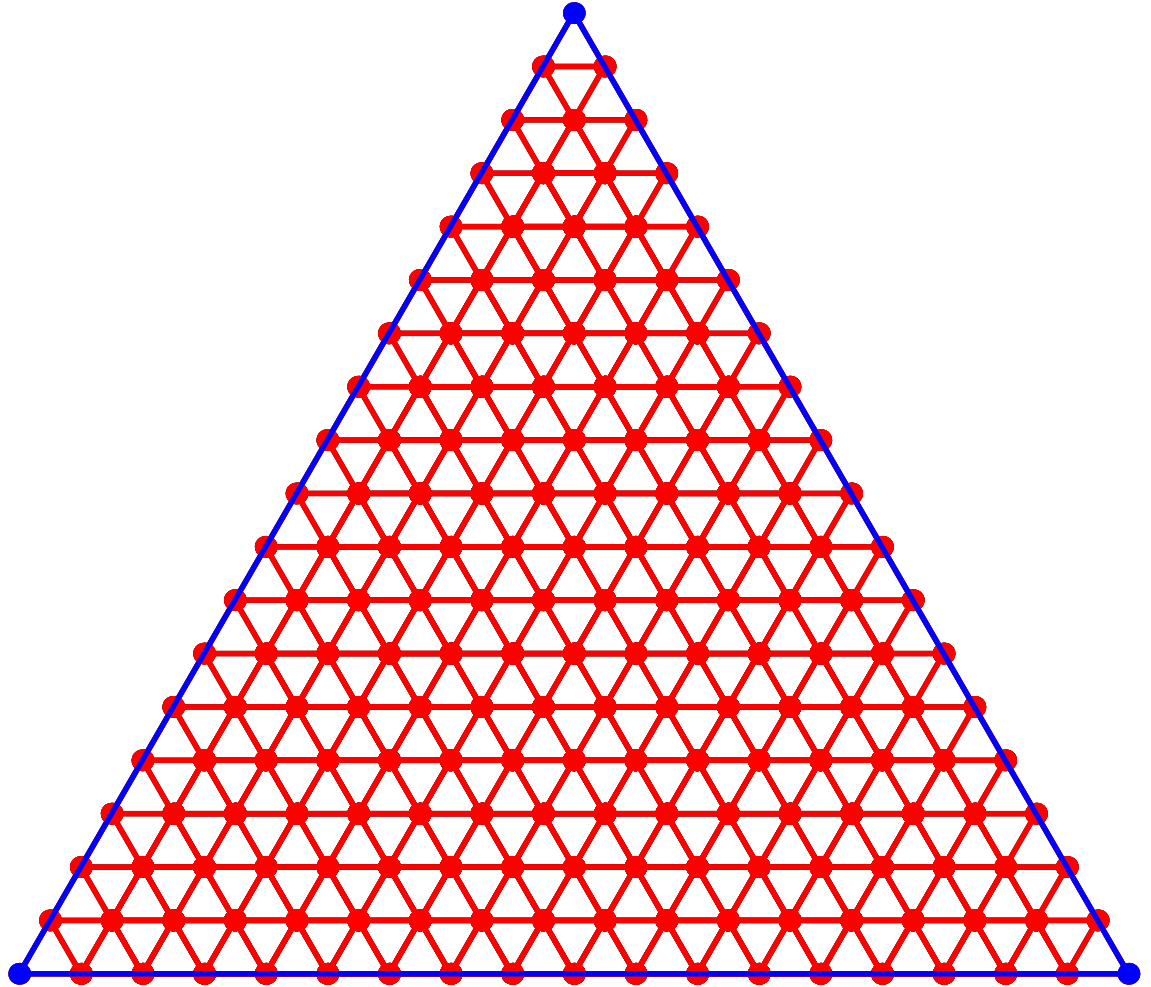
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Factor $17 = 1 \times 17$, factors 1-17

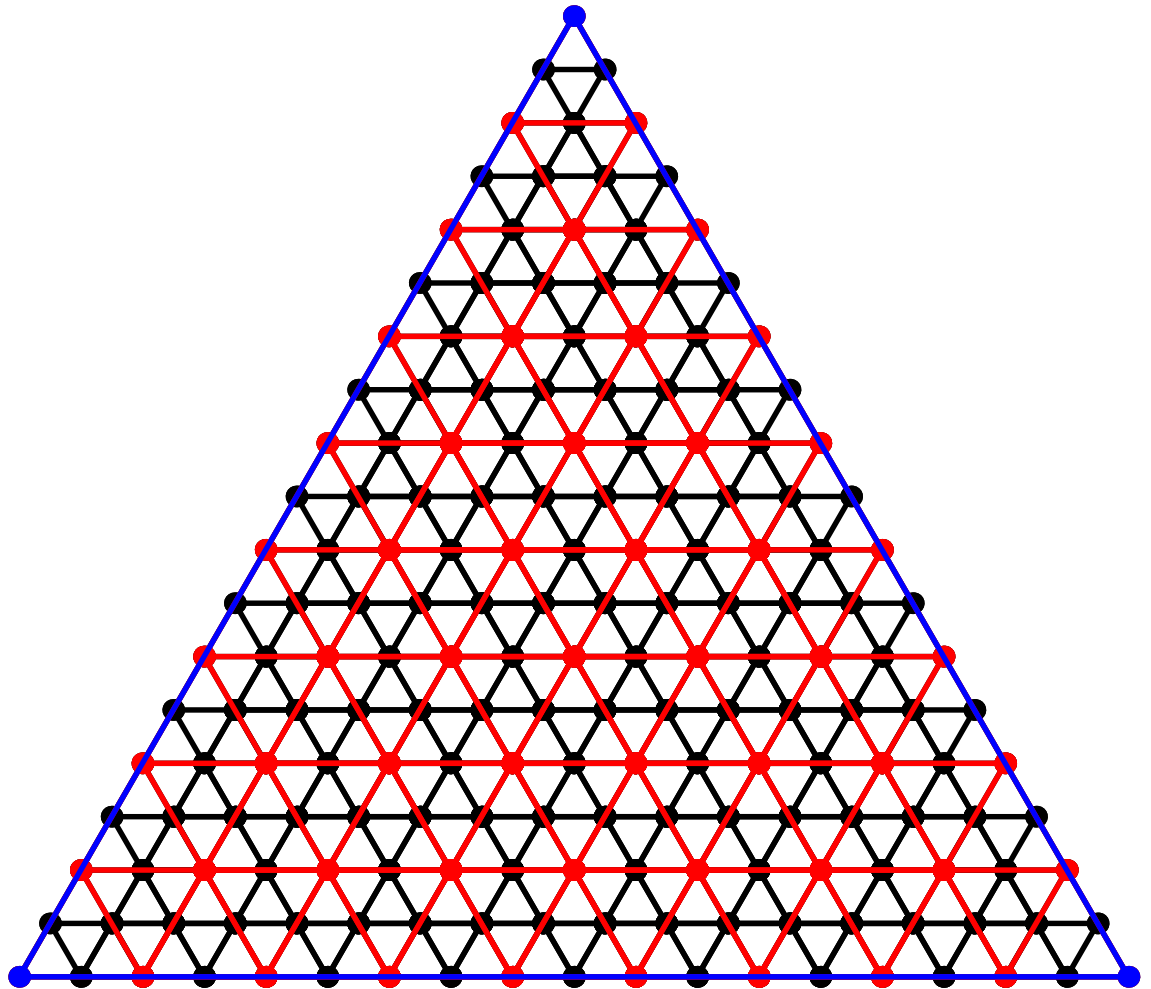


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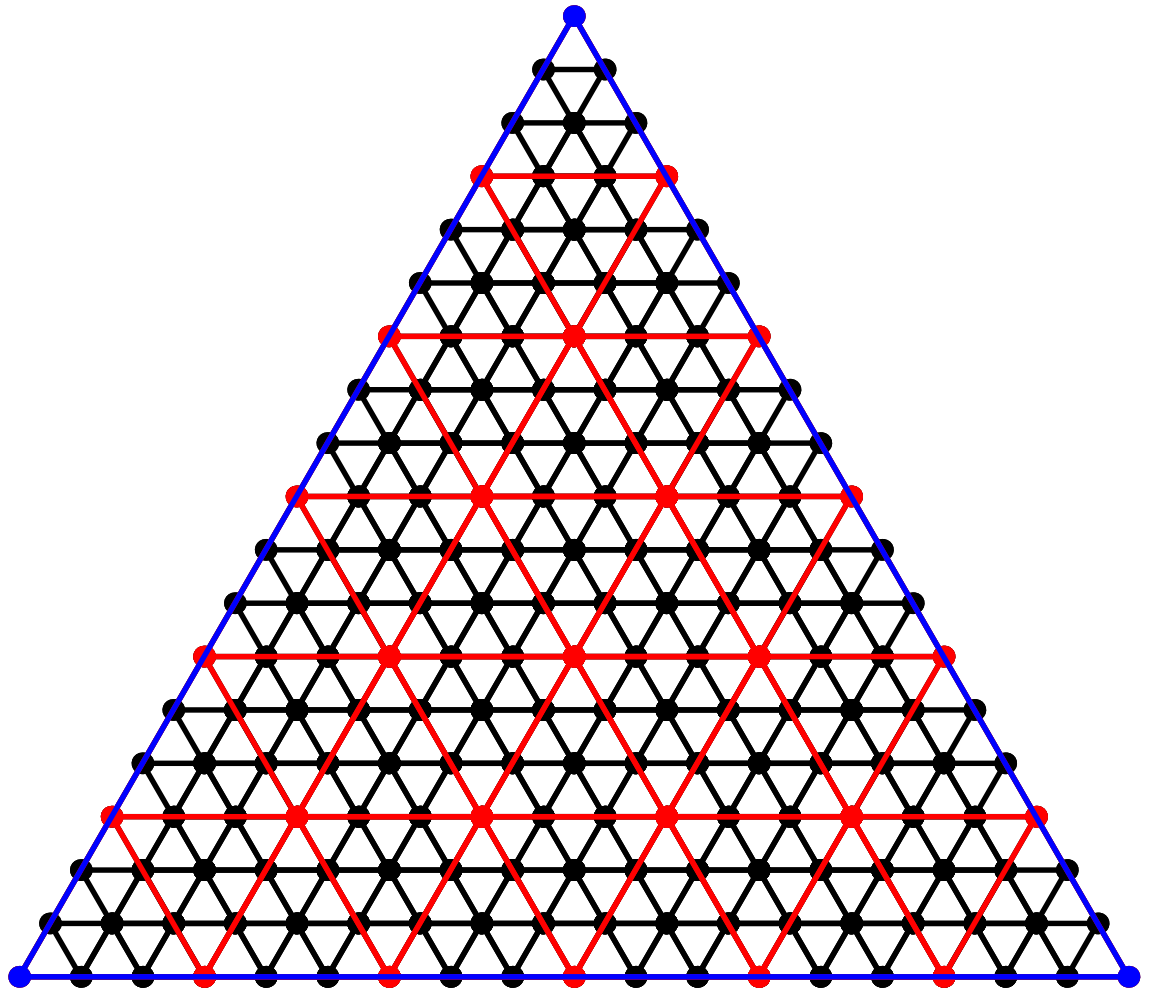
Factor $18 = 1 \times 18$, **factors 1-18**



Factor $18 = 2 \times 9$, factors 2-9

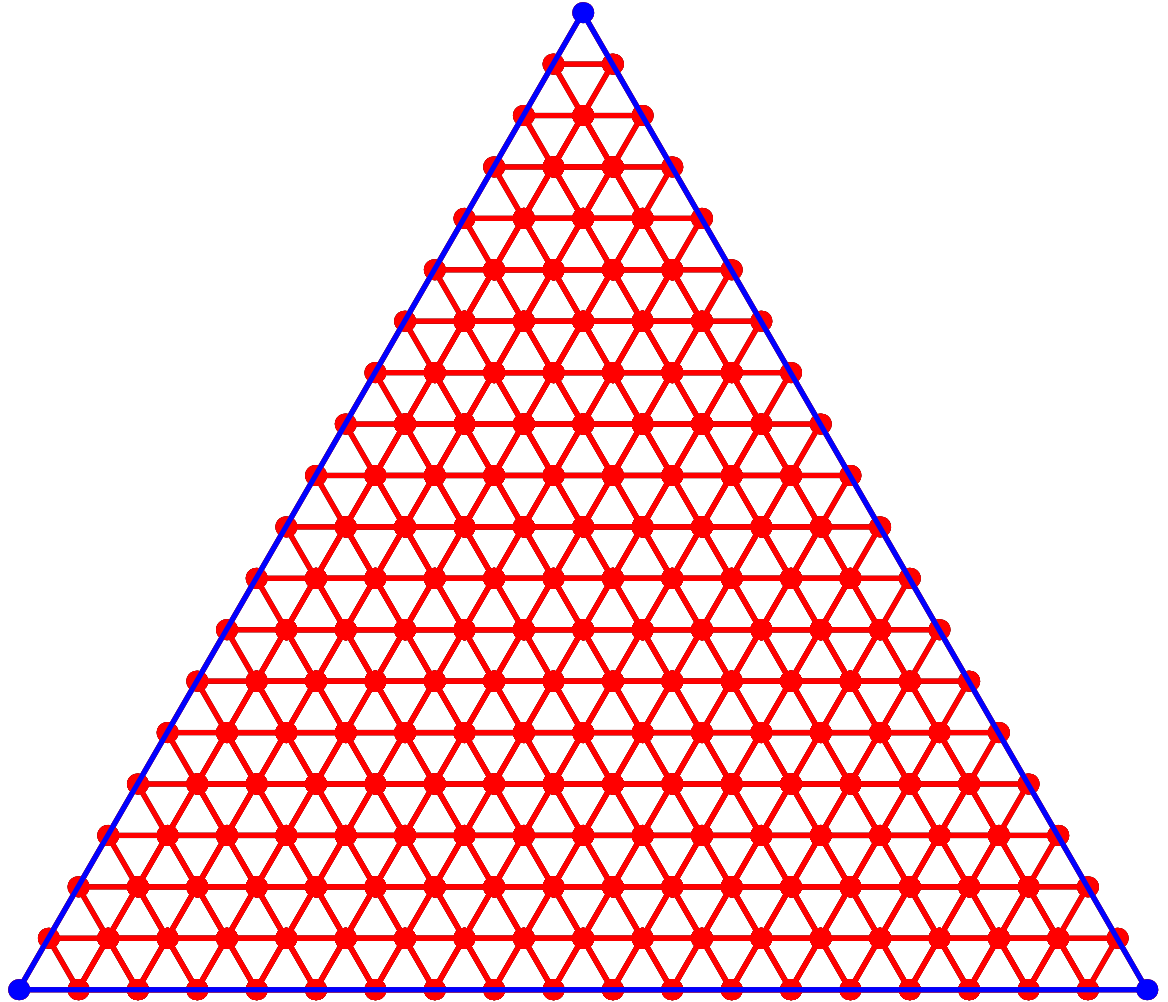


Factor $18 = 3 \times 6$, factors 3-6



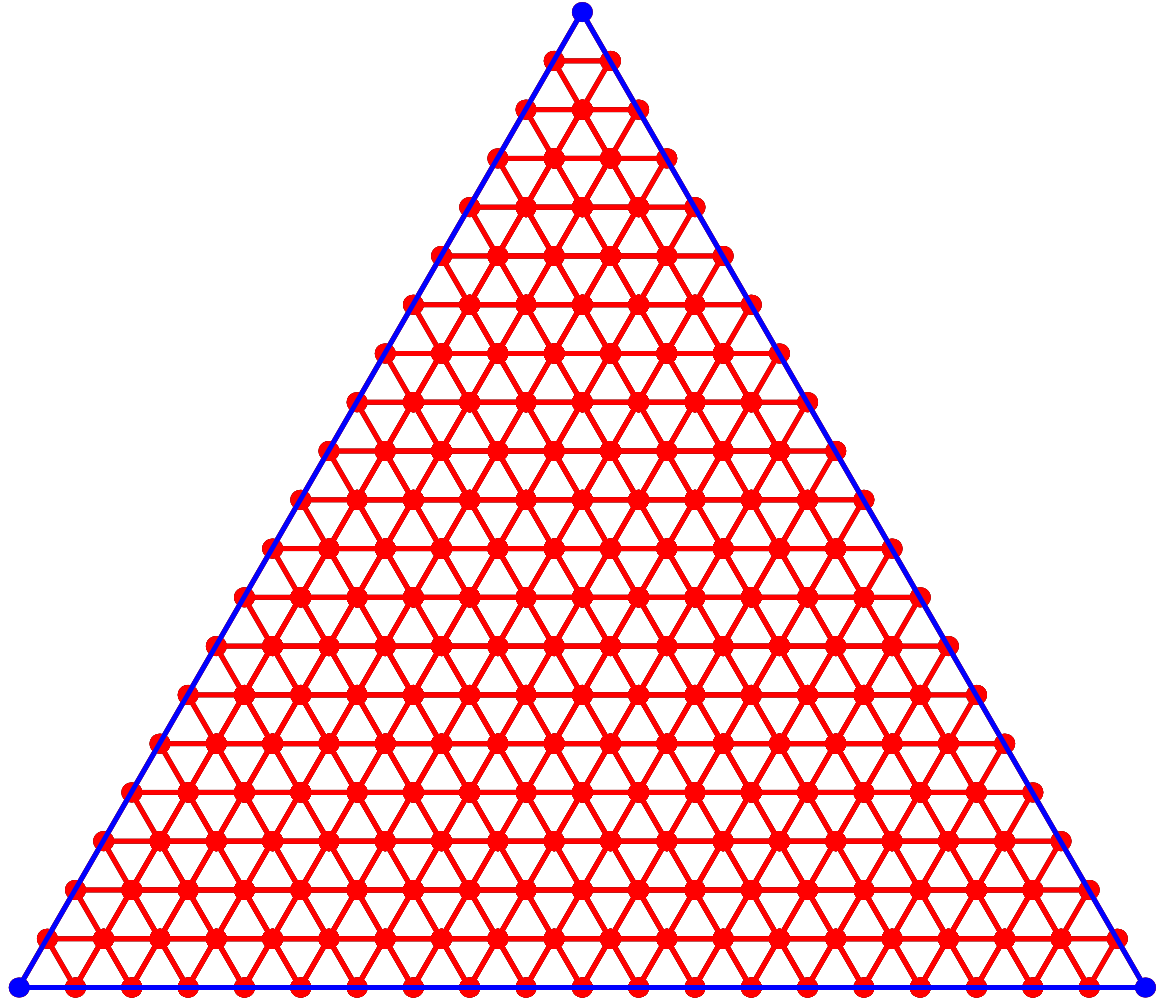
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Factor $19 = 1 \times 19$, factors 1-19

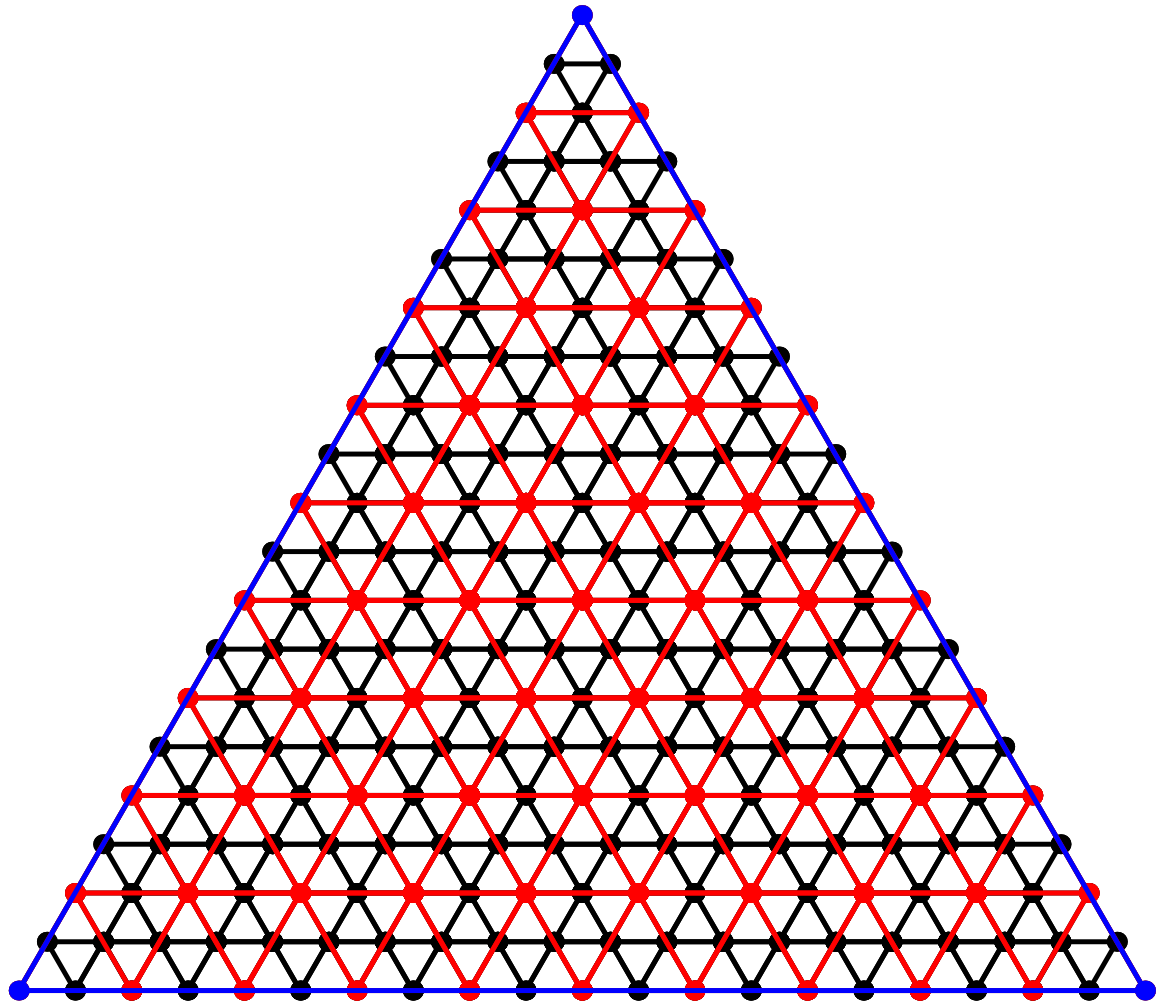


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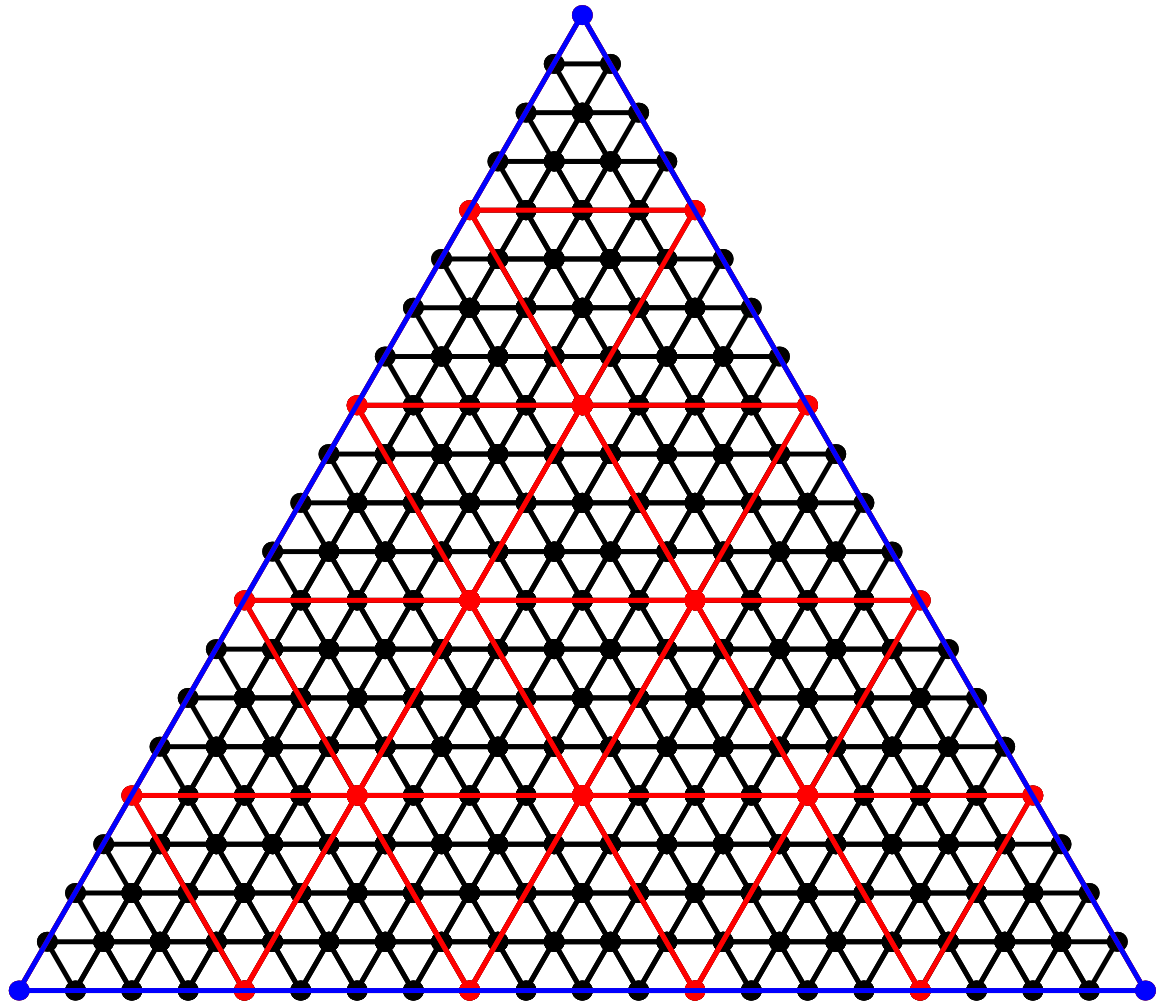
Factor $20 = 1 \times 20$, factors 1-20



Factor $20 = 2 \times 10$, **factors** 2-10

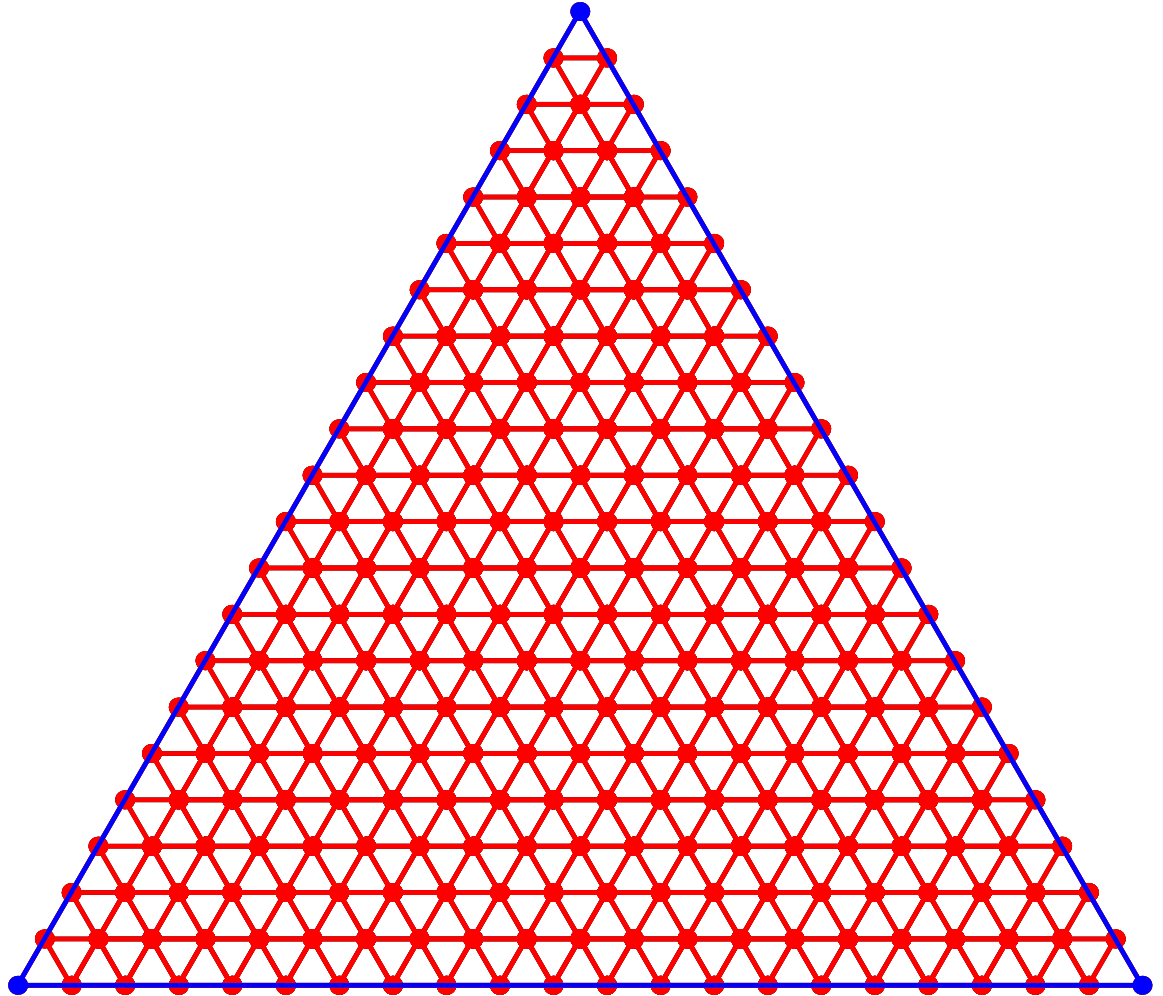


Factor $20 = 4 \times 5$, **factors** 4-5

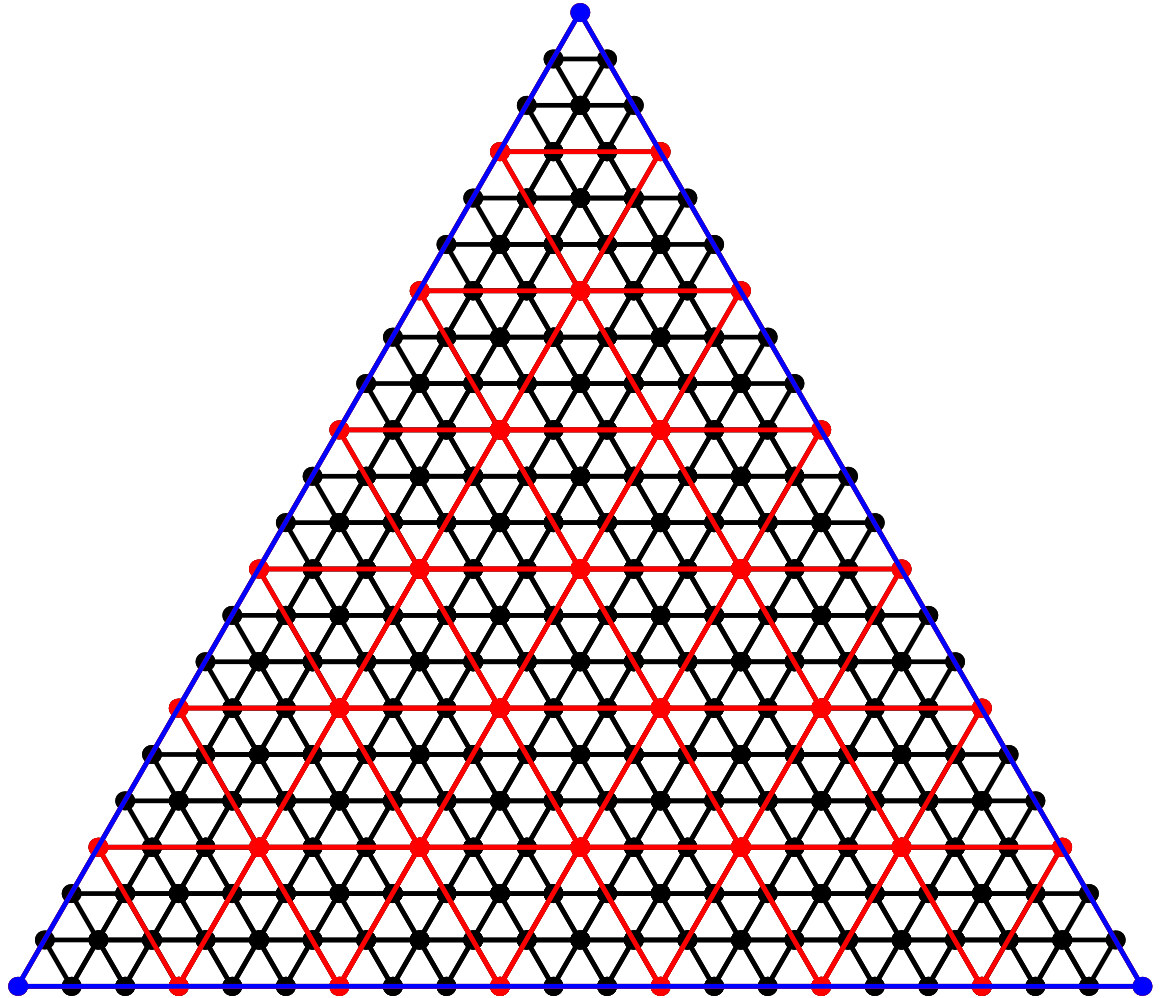


The Number 21

Factor $21 = 1 \times 21$, factors 1-21

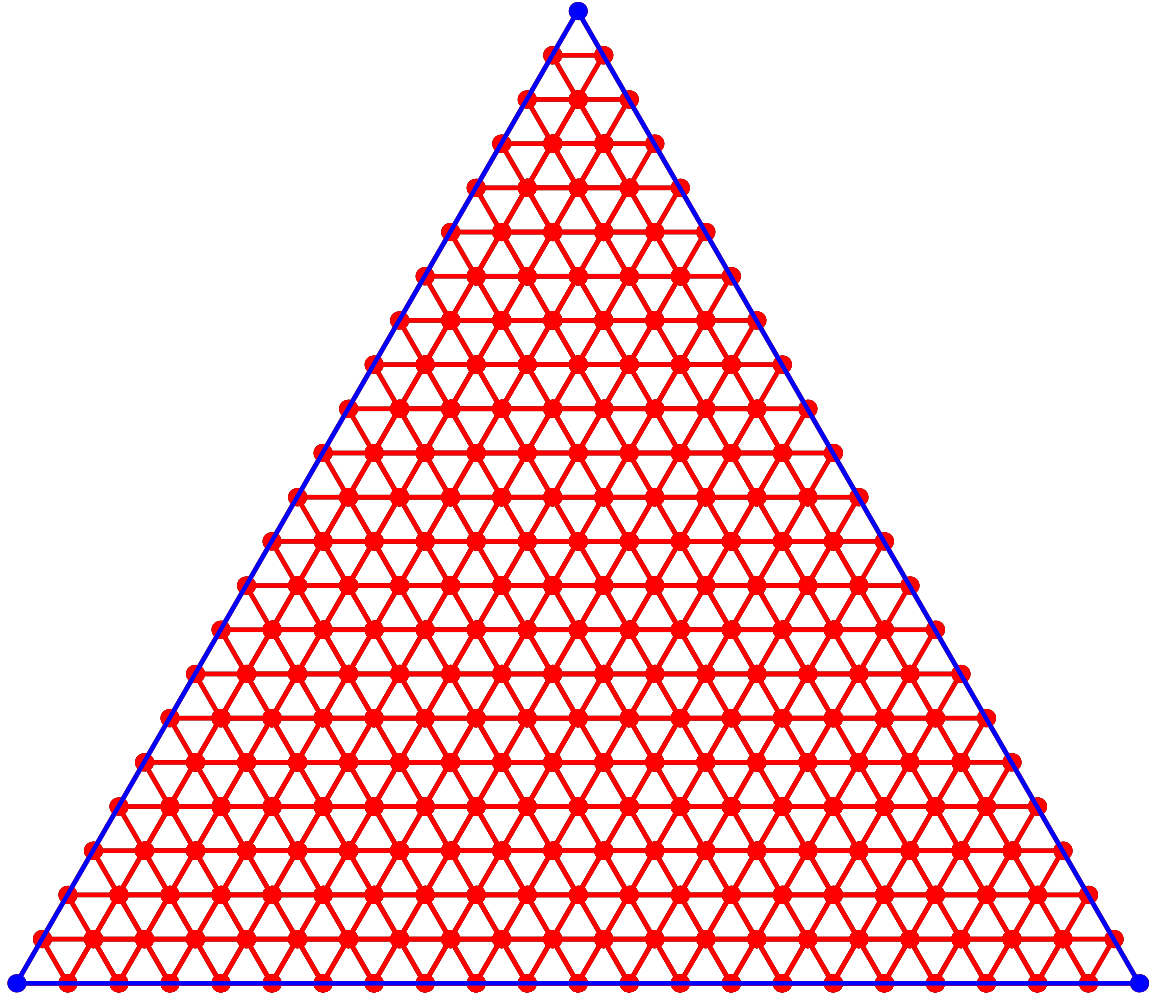


Factor $21 = 3 \times 7$, factors 3-7

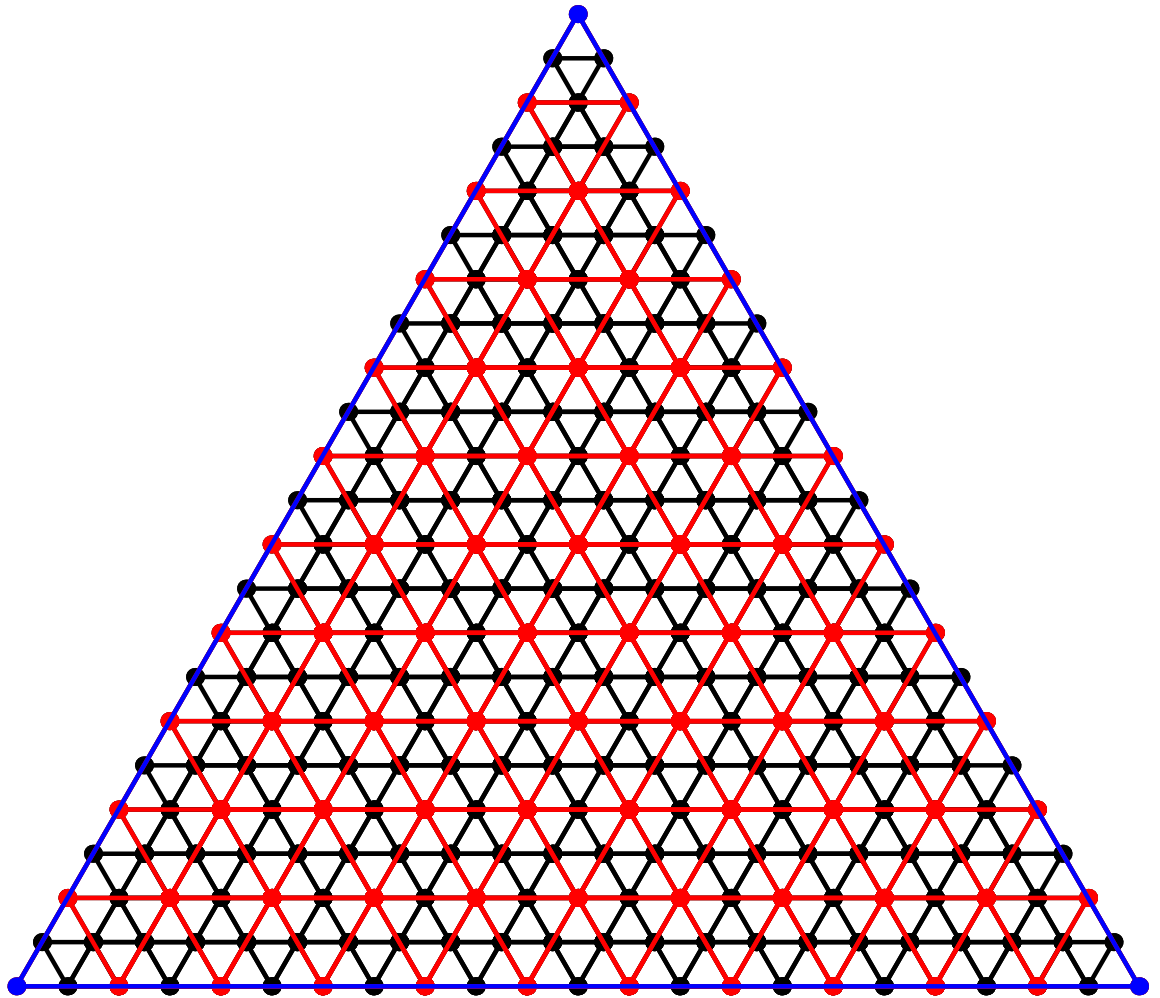


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Factor $22 = 1 \times 22$, factors 1-22

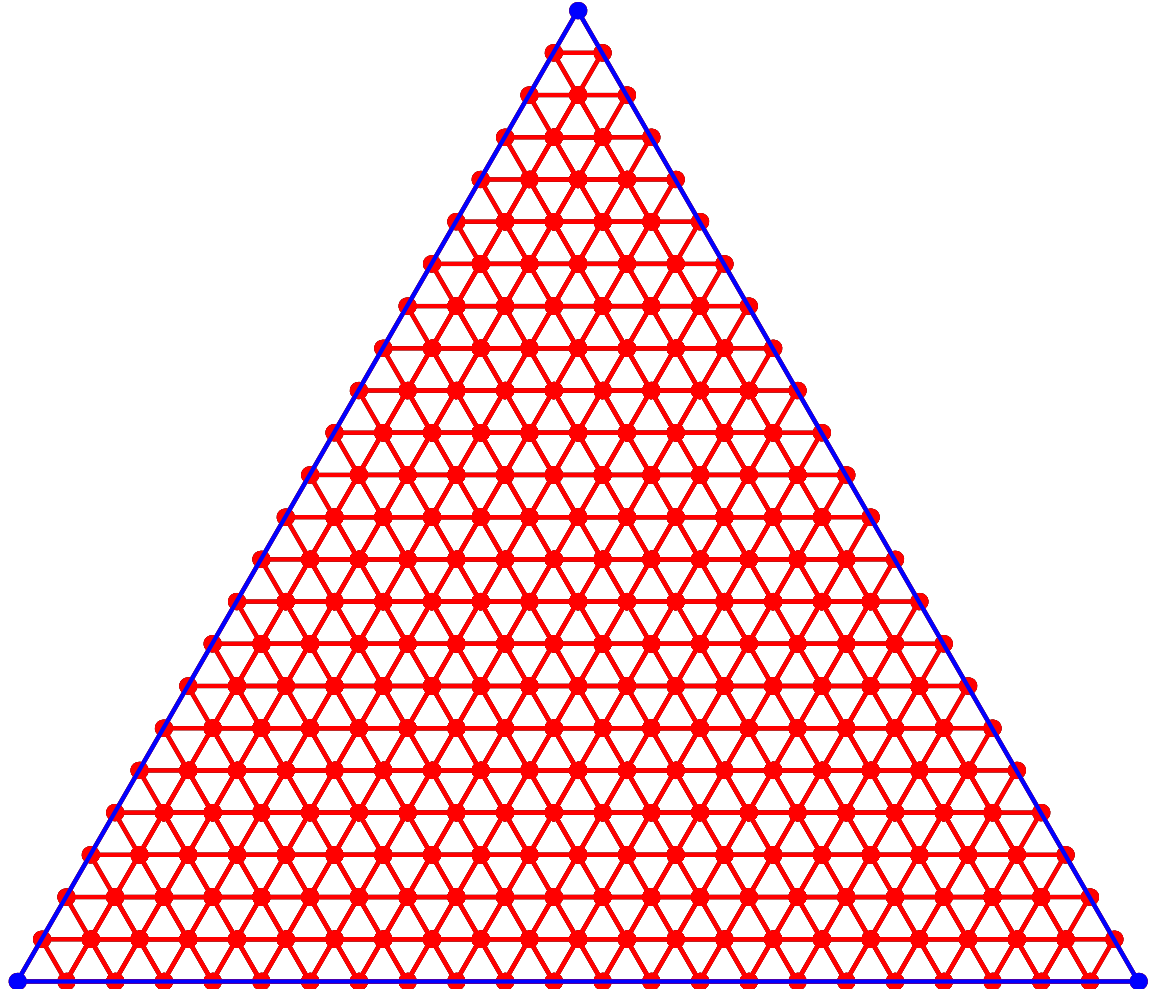


Factor $22 = 2 \times 11$, **factors** 2-11



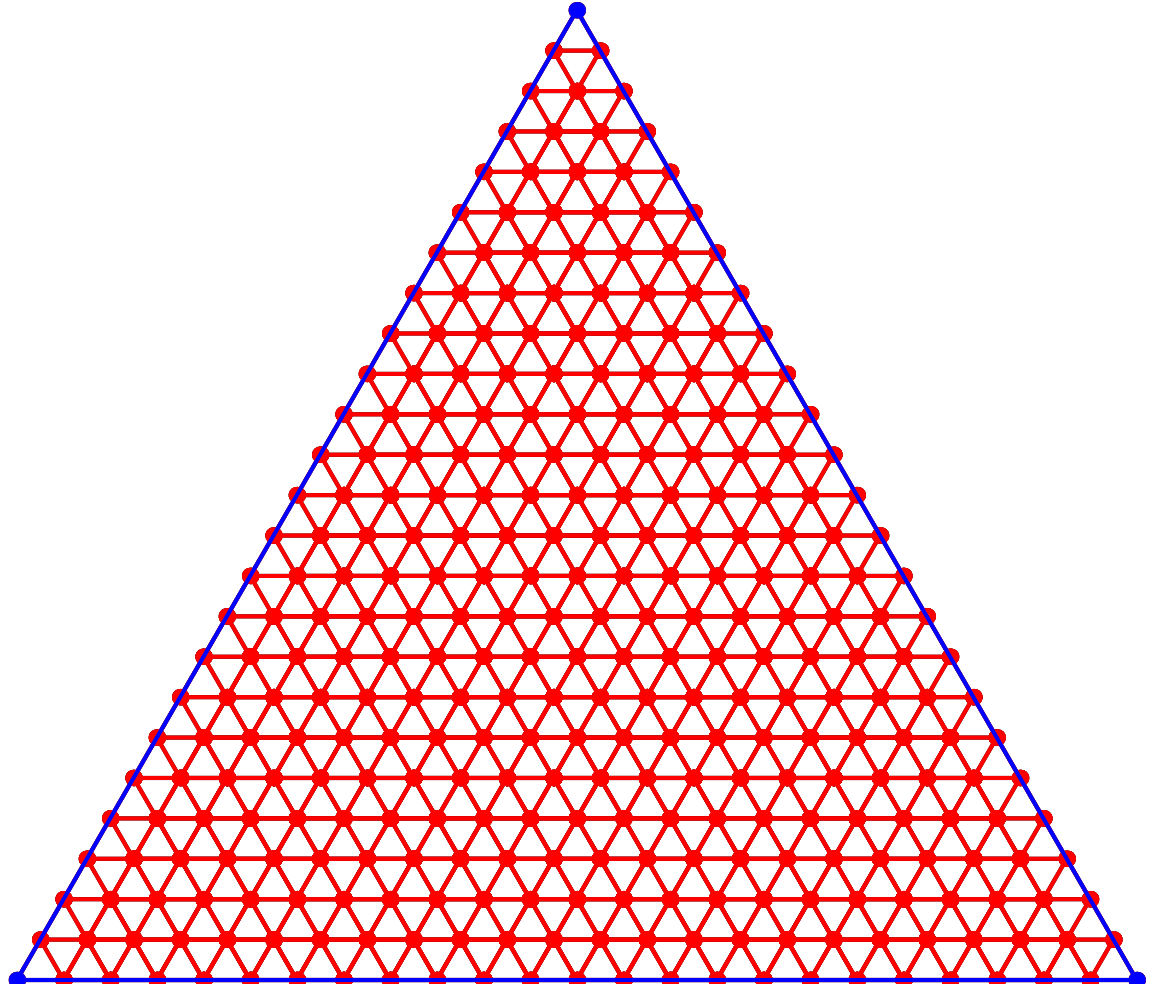
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Factor $23 = 1 \times 23$, factors 1-23

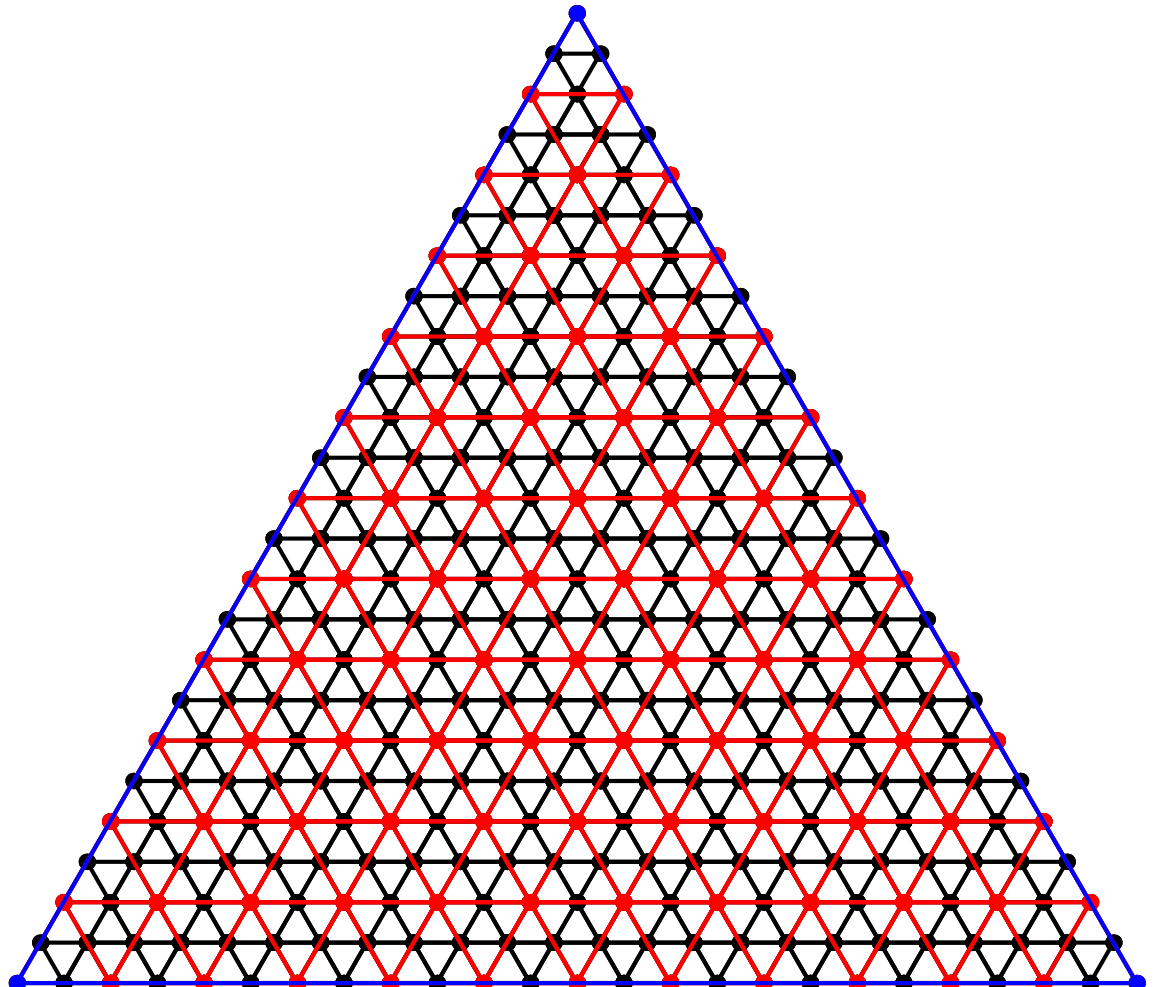


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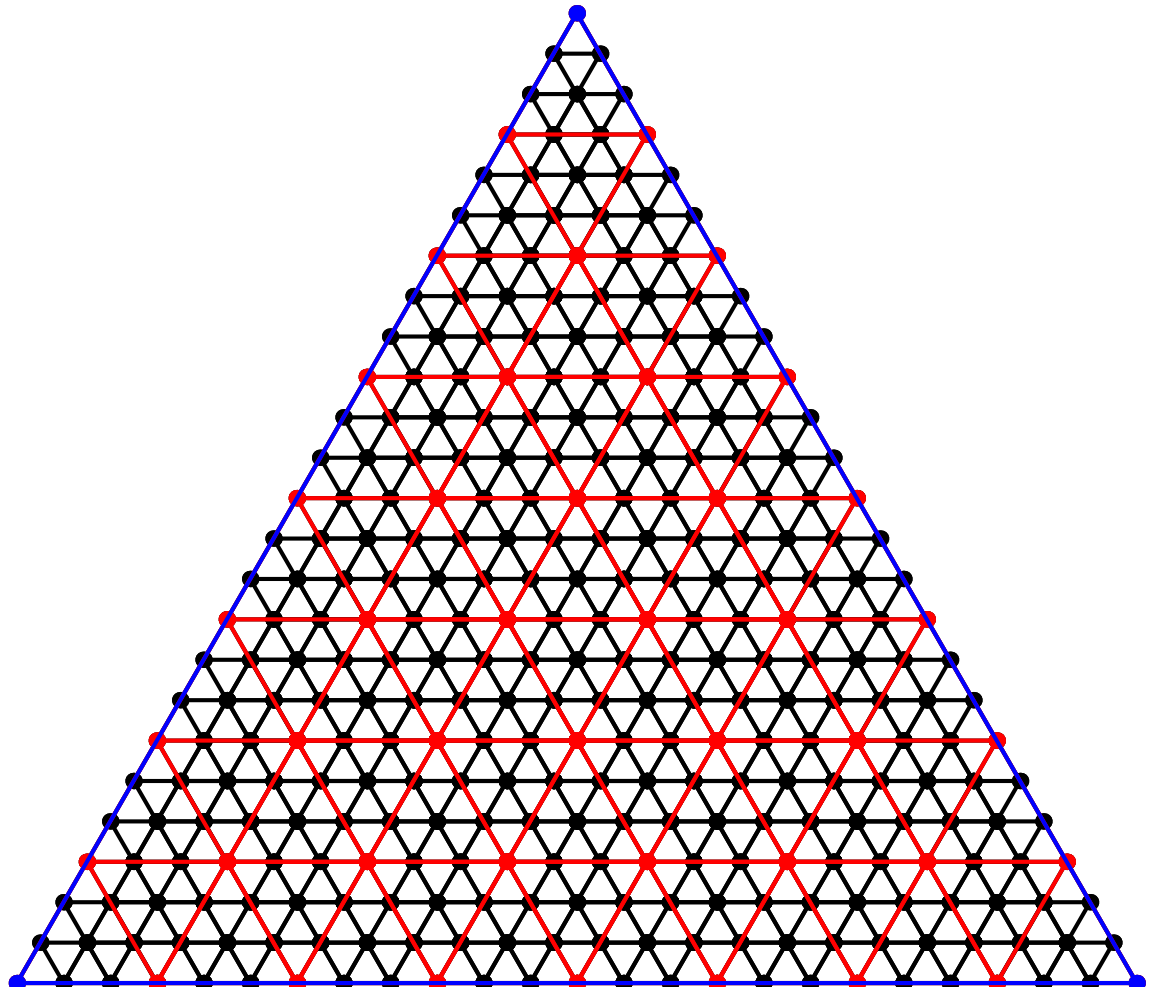
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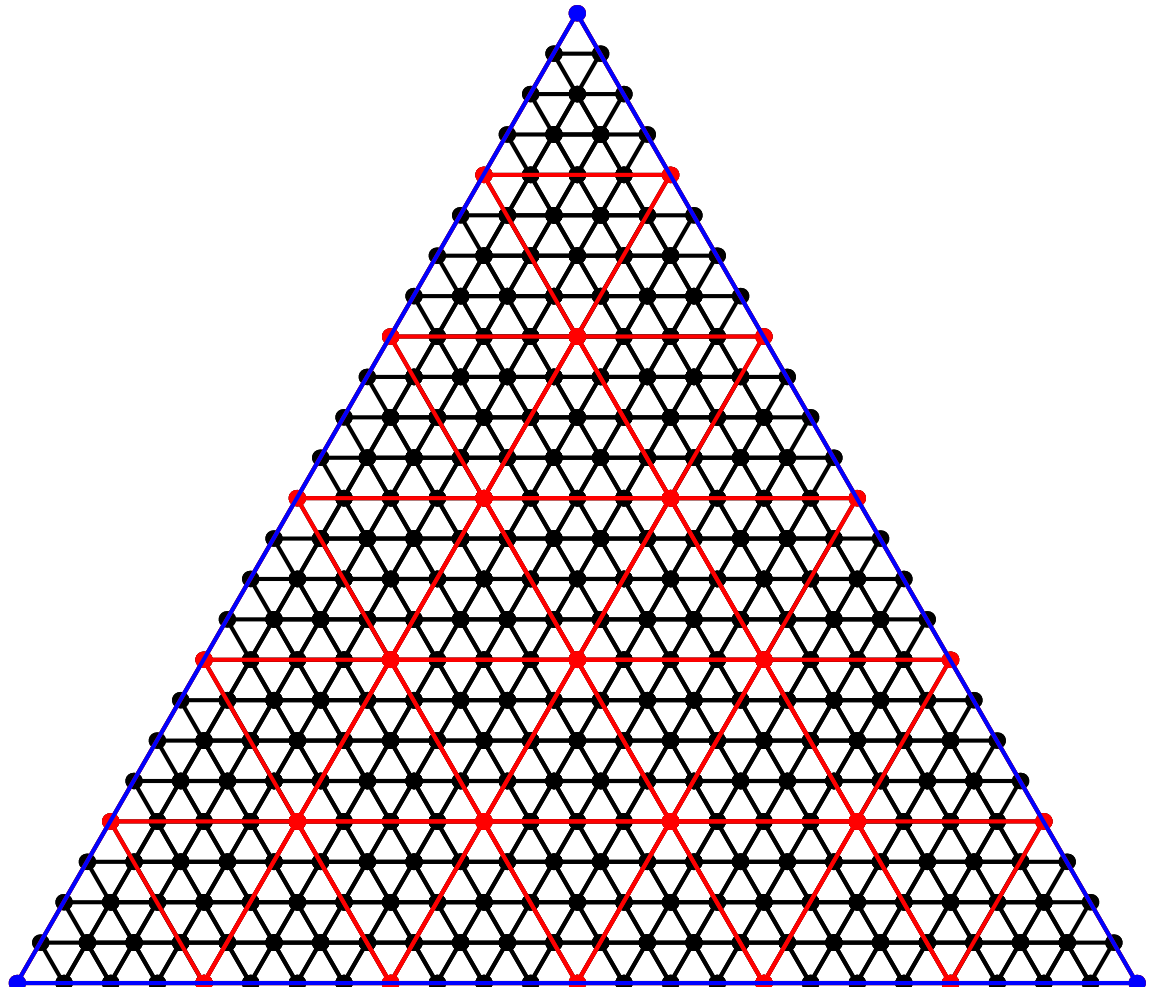
Factor $24 = 2 \times 12$, **factors** 2-12



Factor $24 = 3 \times 8$, factors 3-8



Factor $24 = 4 \times 6$, factors 4-6



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