Euclid's Elements Of Geometry

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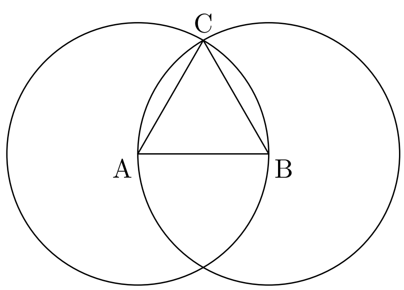
Book 1

Proposition 1

To construct an equilateral triangle on a given finite straight-line.



Let AB be the given finite straight line.

So it is required to contruct an equilateral triangle on the straight-line AB.

Let the circle BCD with center A and radius AB have been drawn, and again let the circle ACE with center B and radius BA have been drawn. And let the straight-lines CA and CB have been joined from the point C, where the circles cut one another, to the points A and B (repsectively). $\square$

Proposition 2

To place a straight-line equal to a given straight-line at a given point (as an extremity). Let A be the given point, BC the given straight-line. So it is required to place a straight-line at point A equal to the given straight-line BC.

For let the straight-line AB have been jointed from point A to point B, and let the euqilateral triangle DAB have been constructed upon it And let the straight-lines AE and BF have been produced in a straight-line with DA and DB (repectively). And let the circle GCH with center B and radius BC have been drawn, and again let the circle GKL with center D and radius DG have been drawn.

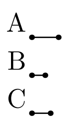
Therefore, since the point B is the center (of the circle) GCH, BC is equal to BG. Again, since the point D is the center of the circle GKL, DL is equal to DG. And within these, DA is equal to DB. Thus, the remainder AL is equal to the remainder BG. But BC ws shown (to be) equal to BG. Thus, AL and BC are each euqal to BG. But things equal to the same thing are also equal to one another. Thus, AL is also equal to BC.

Thus, the straight-line AL, equal to the given straight-line BC, has been placed at the point A. (Which is) the very thing it was required to do. □

Book 6

Proposition 12

To find a fourth (straight-line) proportional to three given straight-lines.



Let A, B, and C be the given straight-lines. So it is required to find a fourth (straight-line) proportional to A, B, and C.

Let the two straight-lines DE and DF be set out encompassing the [random] angle EDF. And let DG be made equal to A, and GE to B, and, further, let EF have been drawn throught (point) E parallel to it [Prop 1.31].

Therefore, since GH has been drawn parallel to one of the sides EF of triangle DEF, thus as DG is to GE, so DH (is) to HF [Prop. 6.2]. And DG (is) equal to A, and GE to B, and DH to C. Thus, as A is to B, so C (is) to HF.

Thus, a fourth (straight-line), HF, has been found (which is) proportional to the three given straight-lines, A, B, and C. (Which is) the very thing it was required to do. □

Book 9

Proposition 20

The (set of all) prime numbers is more than any assigned multitude of prime numbers.

Let A, B, C be the assigned prime numbers. I say the (set of all) prime numbers is more numerous than A, B, C.

For let the least number measured by A, B, C have been taken, and let it be DE [Prop. 7.36]. And let the unit DF have been added to DE. So EF is either prime or not. Let it, first of all, be prime. Thus, the (set of) prime numbers A, B, C, EF, (which is) more numerous than A, B, $C#, has been found.

And so let EF be not prime. Thus, it is measured by some prime (number) [Prop. 7.31]. Let it be measured by the prime number G. I say that G is not the same number as any of A, B, C. For, if possible, let it be (the same). And A, B, C (all) measure DE. Thus, G will also measure DE. And it also measured EF. (So) G will also measure the remainder, unit DF (despite) being a number [Prop. 7.28]. The very thing (is) absurd. Thus, G is not the same as one of A, B, C. And it was assumed (to be) prime. Thus, the (set of) prime numbers A, B, C, G, (which is) more numerous than the assigned multitude (of prime numbers), A, B, C, has been found. (Which is) the very thing it was required to show. □