

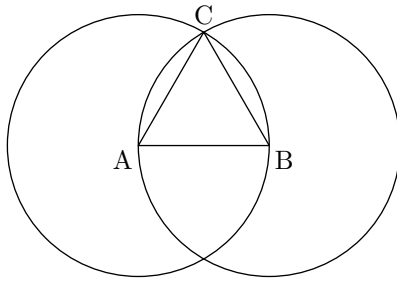
# Euclid's Elements Of Geometry

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## Book 1

### Proposition 1

To construct an equilateral triangle on a given finite straight-line.



Let  $AB$  be the given finite straight line.

So it is required to construct an equilateral triangle on the straight-line  $AB$ .

Let the circle  $BCD$  with center  $A$  and radius  $AB$  have been drawn, and again let the circle  $ACE$  with center  $B$  and radius  $BA$  have been drawn. And let the straight-lines  $CA$  and  $CB$  have been joined from the point  $C$ , where the circles cut one another, to the points  $A$  and  $B$  (repectively).  $\square$  ## Proposition 2

To place a straight-line equal to a given straight-line at a given point (as an extremity). Let  $A$  be the given point,  $BC$  the given straight-line. So it is required to place a straight-line at point  $A$  equal to the given straight-line  $BC$ .

For let the straight-line  $AB$  have been jointed from point  $A$  to point  $B$ , and let the euilateral triangle  $DAB$  have been constructed upon it And let the straight-lines  $AE$  and  $BF$  have been produced in a straight-line with  $DA$  and  $DB$  (repectively). And let the circle  $GCH$  with center  $B$  and radius  $BC$  have been drawn, and again let the circle  $GKL$  with center  $D$  and radius  $DG$  have been drawn.

Therefore, since the point  $B$  is the center (of the circle)  $GCH$ ,  $BC$  is equal to  $BG$ . Again, since the point  $D$  is the center of the circle  $GKL$ ,  $DL$  is equal to




$DG$ . And within these,  $DA$  is equal to  $DB$ . Thus, the remainder  $AL$  is equal to the remainder  $BG$ . But  $BC$  was shown (to be) equal to  $BG$ . Thus,  $AL$  and  $BC$  are each equal to  $BG$ . But things equal to the same thing are also equal to one another. Thus,  $AL$  is also equal to  $BC$ .

Thus, the straight-line  $AL$ , equal to the given straight-line  $BC$ , has been placed at the point  $A$ . (Which is) the very thing it was required to do.  $\square$

## Book 6

### Proposition 12

To find a fourth (straight-line) proportional to three given straight-lines.

A   
 B   
 C 

Let  $A$ ,  $B$ , and  $C$  be the given straight-lines. So it is required to find a fourth (straight-line) proportional to  $A$ ,  $B$ , and  $C$ .

Let the two straight-lines  $DE$  and  $DF$  be set out encompassing the [random] angle  $EDF$ . And let  $DG$  be made equal to  $A$ , and  $GE$  to  $B$ , and, further, let  $EF$  have been drawn through (point)  $E$  parallel to it [Prop 1.31].

Therefore, since  $GH$  has been drawn parallel to one of the sides  $EF$  of triangle  $DEF$ , thus as  $DG$  is to  $GE$ , so  $DH$  (is) to  $HF$  [Prop. 6.2]. And  $DG$  (is) equal to  $A$ , and  $GE$  to  $B$ , and  $DH$  to  $C$ . Thus, as  $A$  is to  $B$ , so  $C$  (is) to  $HF$ .

Thus, a fourth (straight-line),  $HF$ , has been found (which is) proportional to the three given straight-lines,  $A$ ,  $B$ , and  $C$ . (Which is) the very thing it was required to do.  $\square$

## Book 9

### Proposition 20

The (set of all) prime numbers is more than any assigned multitude of prime numbers.

Let  $A$ ,  $B$ ,  $C$  be the assigned prime numbers. I say the (set of all) prime numbers is more numerous than  $A$ ,  $B$ ,  $C$ .

For let the least number measured by  $A$ ,  $B$ ,  $C$  have been taken, and let it be  $DE$  [Prop. 7.36]. And let the unit  $DF$  have been added to  $DE$ . So  $EF$  is either

prime or not. Let it, first of all, be prime. Thus, the (set of) prime numbers  $A$ ,  $B$ ,  $C$ ,  $EF$ , (which is) more numerous than  $A$ ,  $B$ ,  $C$ , has been found.

And so let  $EF$  be not prime. Thus, it is measured by some prime (number) [Prop. 7.31]. Let it be measured by the prime number  $G$ . I say that  $G$  is not the same number as any of  $A$ ,  $B$ ,  $C$ . For, if possible, let it be (the same). And  $A$ ,  $B$ ,  $C$  (all) measure  $DE$ . Thus,  $G$  will also measure  $DE$ . And it also measured  $EF$ . (So)  $G$  will also measure the remainder, unit  $DF$  (despite) being a number [Prop. 7.28]. The very thing (is) absurd. Thus,  $G$  is not the same as one of  $A$ ,  $B$ ,  $C$ . And it was assumed (to be) prime. Thus, the (set of) prime numbers  $A$ ,  $B$ ,  $C$ ,  $G$ , (which is) more numerous than the assigned multitude (of prime numbers),  $A$ ,  $B$ ,  $C$ , has been found. (Which is) the very thing it was required to show.  $\square$