Category Theory notes

Mac Radigan

Kleisli Triple $(M, unit, \star)$

type constructor (M)

$$Mt \tag{1}$$

unit function $(unit)$

$$t \rightarrow (Mt) \tag{2}$$

bind function $(\star)$

$$(Mt) \rightarrow (t \rightarrow (Mu)) \rightarrow (Mu) \tag{3}$$

Monad Laws

left unit

$$\text{unit } a \star \lambda b.m = n\,[a/b] \tag{4}$$

right unit

$$m \star \lambda a.\text{ unit } a = m \tag{5}$$

associative

$$m \star (\lambda a.n \star \lambda b.o) = (m \star \lambda a.n) \star \lambda b.o \tag{6}$$

Category

A *category* **C** consists of

- a collection of objects: A , B , C , ...
- a collection of arrows: f , g , h , ...
- for each arrow f objects $\mathrm{dom}(f)$ and $\mathrm{cod}(f)$ called the *domain* and *codomain* of f. If $\mathrm{dom}(f) = A$ and $\mathrm{cod}(f) = B$, we also write $f : A \to B$,
- given $f : A \to B$ and $g : B \to C$, so that $\mathrm{dom}(g) = \mathrm{cod}(f)$, there is an arrow $g \circ f : A \to C$,
- an arrow $\mathbf{1}_A : A \to A$ for every object A of $\mathbf{C}$,

such that

(Associative law) for every $f : A \to B$, $g : B \to C$ and $h : C \to C$ we have

$$h \circ (g \circ f) = (h \circ g) \circ f,$$

(Unit laws) for every $f : A \to B$ we have

$$f \circ \mathbf{1}_A = f = \mathbf{1}_B \circ f.$$

Composition

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[dr, "g\circ f"'] & B \arrow[d, "g"'] \arrow[dr, "g\circ h"] & \\
 & C \arrow[r, "h"'] & D
\end{tikzcd}$$