Time-Frequency Notes

Mac Radigan

Fourier Expansion

$$s\left(t\right) = \frac{1}{\sqrt{2\pi}} \int S\left(\omega\right) e^{j\omega t} d\omega$$

$$S\left(t\right) = \frac{1}{\sqrt{2\pi}} \int s\left(t\right) e^{-j\omega t} dt$$

Mean Frequency

$$\begin{aligned} \langle\omega\rangle &= \int \omega |S\left(\omega\right)|^2 d\omega \\ &= \int s^*\left(t\right) \frac{1}{j} \frac{d}{dt} s\left(t\right) dt \end{aligned}$$

Mean Square Frequency

$$\begin{aligned} \langle\omega^2\rangle &= \int \omega^2 |S\left(\omega\right)|^2 d\omega \\ &= \int s^*\left(t\right) \left(\frac{1}{j} \frac{d}{dt}\right)^2 s\left(t\right) dt \\ &= \int |\frac{d}{dt} s\left(t\right)|^2 dt \end{aligned}$$

Root Mean Square Bandwidth

$$\begin{aligned} B^2 = \langle\sigma_\omega^2\rangle &= \int \left(\omega - \langle\omega\rangle\right)^2 |S\left(\omega\right)|^2 d\omega \\ &= \int s^*\left(t\right) \left(\frac{1}{j} \frac{d}{dt} - \langle\omega\rangle\right)^2 s\left(t\right) dt \\ &= \int |\left(\frac{1}{j} \frac{d}{dt} - \langle\omega\rangle\right) s\left(t\right)|^2 dt \end{aligned}$$

Ambiguity Function

$$A(\tau, f) = \int_{0}^{T} s_1(t)\, s_2^*(t+\tau)\, e^{-j2\pi f t} dt$$

Analytic Signal

$$s_a(t) = \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} S_{RE}(\omega)\, e^{j\omega t} d\omega$$

Uncertainty

$$T^2 = \sigma_t^2 = \int (t - \langle t \rangle)^2 \left| s(s) \right|^2 dt$$

$$B^2 = \sigma_\omega^2 = \int (\omega - \langle \omega \rangle)^2 \left| S(\omega) \right|^2 d\omega$$

$$TB \geq \frac{1}{2}$$

$$\sigma_t \sigma_\omega \geq \frac{1}{2} \sqrt{1 + 4Cov_{t\omega}^2}$$