Digial Image Warping notes

Mac Radigan

Polynomial Transformations

Inferring polynomial coefficients with a pseudoinverse solution using a second-degree approximation

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdots \\ \cdots \\ u_M \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 & x_1^2 & y_1^2 \\ 1 & x_2 & y_1 & x_2 y_2 & x_2^2 & y_2^2 \\ 1 & x_3 & y_1 & x_3 y_3 & x_3^2 & y_3^2 \\ & & & \cdots & & \\ & & & \cdots & & \\ 1 & x_M & y_M & x_M y_M & x_M^2 & y_M^2 \end{bmatrix} = \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \\ a_{20} \\ a_{02} \end{bmatrix}$$

and

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \cdots \\ \cdots \\ v_M \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 & x_1^2 & y_1^2 \\ 1 & x_2 & y_1 & x_2 y_2 & x_2^2 & y_2^2 \\ 1 & x_3 & y_1 & x_3 y_3 & x_3^2 & y_3^2 \\ & & & \cdots & & \\ & & & \cdots & & \\ 1 & x_M & y_M & x_M y_M & x_M^2 & y_M^2 \end{bmatrix} = \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \\ a_{20} \\ a_{02} \end{bmatrix}$$

Written in matrix form

$U = WA$

$V = WB$

This gives us

$W^\intercal U = W^\intercal W A$

Expanding the notation

$$\begin{bmatrix} \sum u \\ \sum xu \\ \sum yu \\ \sum xyu \\ \sum x^2u \\ \sum y^2u \end{bmatrix} = \begin{bmatrix} M & \sum x & \sum y & \sum xy & \sum x^2 & \sum y^2 \\ \sum x & \sum x^2 & \sum xy & \sum x^2y & \sum x^3 & \sum xy^2 \\ \sum y & \sum xy & \sum y^2 & \sum xy^2 & \sum x^2y & \sum y^3 \\ \sum xy & \sum x^2y & \sum xy^2 & \sum x^2y^2 & \sum x^3y & \sum xy^3 \\ \sum x^2 & \sum x^3 & \sum x^2y & \sum x^3y & \sum x^4 & \sum x^2y^2 \\ \sum y^2 & \sum xy^2 & \sum y^3 & \sum xy^3 & \sum x^2y^2 & \sum y^4 \end{bmatrix} = \begin{bmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \\ a_{20} \\ a_{02} \end{bmatrix}$$

Solving for A and B, we have

$A = (W^\intercal W)^{-1} W^\intercal U$

$B = (W^\intercal W)^{-1} W^\intercal V$