Quadrature Sampling

Mac Radigan

1 Preliminaries

1.1 Context of Discussion

```
#!/usr/bin/octave -q
%% parameters.m
%%
%% Specifies the paramters used throughout this discussion.

  function y = parameters()

    fs    = 1e4;        % [Hz]        sampling frequency
    fc    = 3e2;        % [Hz]        carrier frequency
    fi    = 1e2;        % [Hz]        intermediate frequency
    A     = 1.0;        % [unitless] gain for signal of interest
    fk    = fc + 1e2;  % [Hz]        frequency for signal of interest
    phi   = 0.45 * pi; % [rad]       phase angle for signal of interest
    T     = 5e-2;       % [s ]        duration
    t     = 0:1/fs:T;  % [s ]        time vector

    y = struct(   ...
      'fs',  fs,  ...
      'fc',  fc,  ...
      'fi',  fi,  ...
      'A',   A,   ...
      'fk',  fk,  ...
      'phi', phi, ...
      'T',   T,   ...
      't',   t    ...
      );

  end % parameters

%% *EOF*
```

1.2 Even and Odd Functions

1.2.1 Definition of Even and Odd Functions

even function:

$$f(x) = f(-x) \tag{1}$$

odd function:

$$f(-x) = -f(x) \tag{2}$$

1.2.2 Trigonometric Symmetries

$$cos(x) = cos(-x) \tag{3}$$

$$sin(-x) = -sin(x) \tag{4}$$

1.3 Inner Products

conjugate symmetry

$$\langle \cdot, \cdot \rangle : \mathbb{V} \times \mathbb{V} \mapsto \mathbb{F} \tag{5}$$

$x, y, z \in \mathbb{V}$ and $\alpha \in \mathbb{F}$

conjugate symmetry

$$\langle x, y \rangle = \overline{\langle y, x \rangle} \tag{6}$$

linearity

$$\begin{gathered} \langle \alpha x, y \rangle = \alpha \langle x, y \rangle \\ \langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle \end{gathered} \tag{7}$$

positive-definateness

$$
\begin{aligned}
\langle x, x \rangle &\geq 0 \\
\langle x, x \rangle = 0 &\leftrightarrow x = 0
\end{aligned}
\tag{8}
$$

1.3.1 Inner Product Defined

For the space of points in $\mathbb{R}^n$, the inner product is defined as

$$
\begin{aligned}
\left\langle \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\rangle &\triangleq x^{\mathrm{T}} y \\
&= \sum_{k=1}^{n} x_k y_k \\
&= x_1 y_1 + \cdots + x_n y_n
\end{aligned}
\tag{9}
$$

For the space of continuous complex functions on the interval $[a, b]$, i.e. $\mathbb{C}\left[a, b\right]$, the inner product is defined as

$$
\langle f, g \rangle \triangleq \int_a^b f\left(t\right) \overline{g\left(t\right)} dt \tag{10}
$$

1.3.2 Properties of Inner Products

$$
\langle x, x \rangle = \overline{\langle x, x \rangle} \tag{11}
$$

$$
\begin{aligned}
\langle x, -x \rangle &= -1 \langle x, x \rangle \\
&= \overline{-1} \langle x, x \rangle \\
&= \langle x, -x \rangle
\end{aligned}
\tag{12}
$$

1.4 Trigonomic Identities

1.4.1 Angle Sum and Differences

$$
\begin{aligned}
\sin\left(\alpha \pm \beta\right) &= \sin\alpha \cos\beta \pm \cos\alpha \sin\beta \\
\cos\left(\alpha \pm \beta\right) &= \cos\alpha \cos\beta \mp \sin\alpha \sin\beta
\end{aligned}
\tag{13}
$$

1.4.2 Identities Involving Complex Exponentials

Euler's formula:

$$e^{ix} = \cos x + i \sin x \tag{14}$$

Written in terms of cosine and sine:

$$\begin{aligned} \cos x &= \frac{e^{ix} + e^{-ix}}{2} \\ \sin x &= \frac{e^{ix} - e^{-ix}}{2i} \end{aligned} \tag{15}$$

1.5 Fourier Transform

1.5.1 Fourier Transform Defined

forward transform:

$$\mathbb{F}\{s(t)\} = S(f) \triangleq \int_{-\infty}^{+\infty} s(t)\, e^{-2\pi i f t} dt$$

inverse transform:

$$\mathbb{F}^{-1}\{S(f)\} = s(t) \triangleq \int_{-\infty}^{+\infty} S(f)\, e^{2\pi i t f} df$$

1.6 Fourier Transform as the Superposition of Inner Products

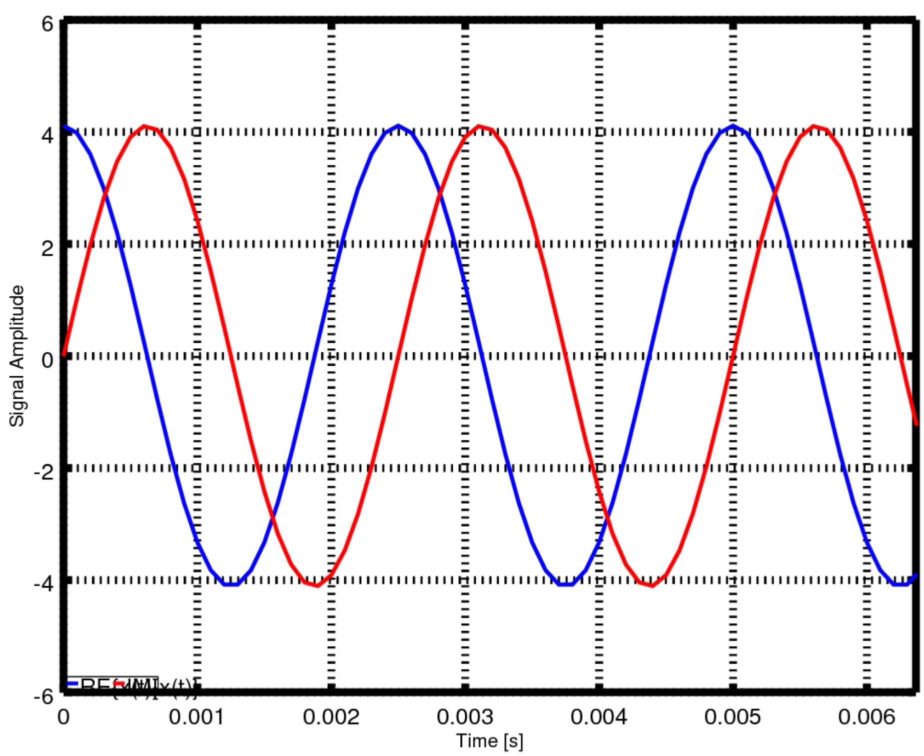
1.7 Fourier Analysis of a Single Frequency Component

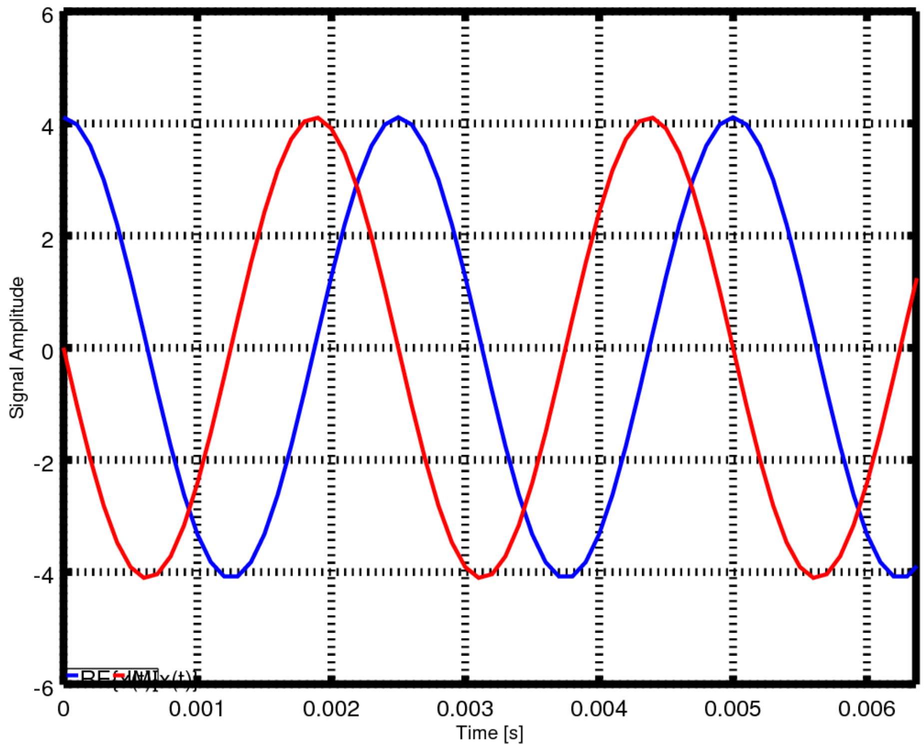
1.7.1 Fourier Transform of Analytic Signals, Single Frequency

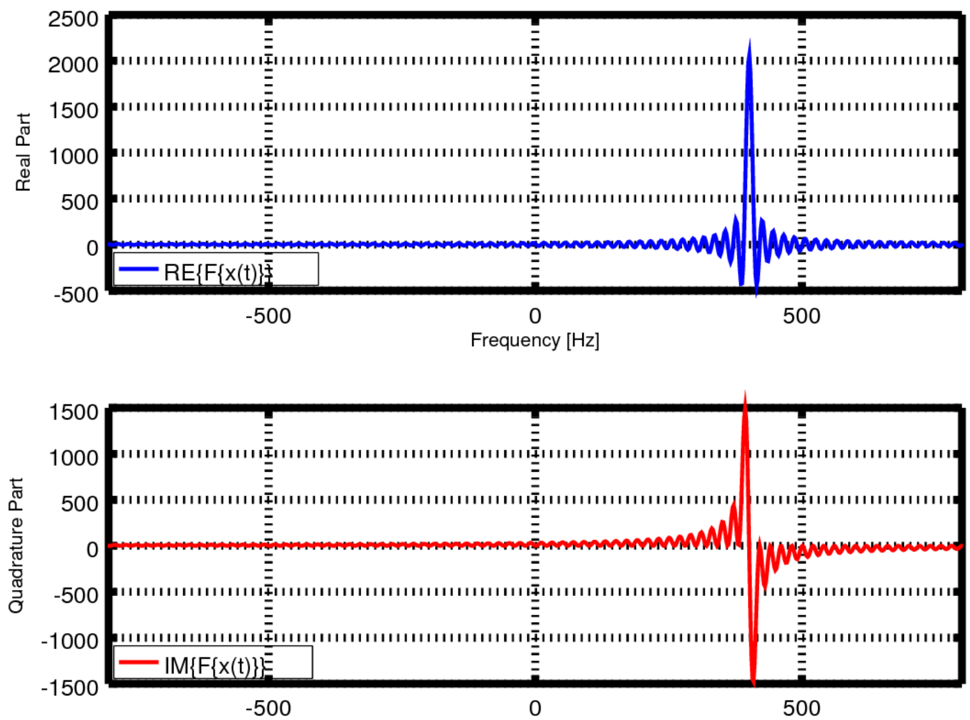
$$\begin{aligned} \mathbb{F}\{e^{2\pi i f_k t + \phi_k}\} &= \int_{-\infty}^{+\infty} e^{2\pi i f_k t + \phi_k} e^{-2\pi i f t} dt \\ &= \int_{-\infty}^{+\infty} e^{2\pi i (f_k - f) t} e_k^{\phi} dt \\ &= 2\pi e^{-i\phi_k} \delta(f - f_k) \end{aligned}$$

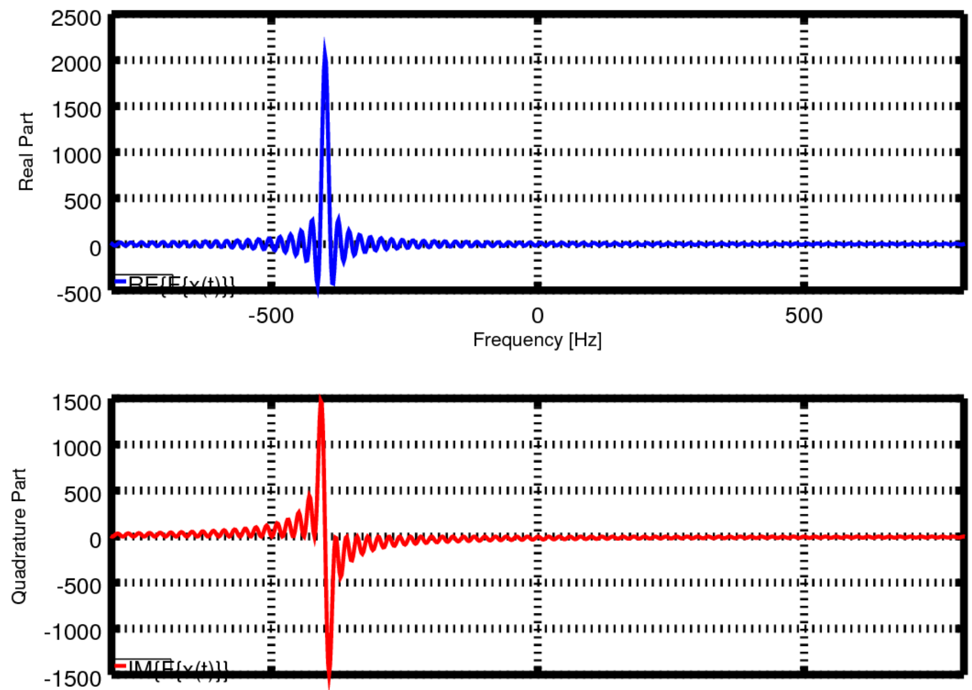
1.7.2 Fourier Transform of Real Signals, Single Frequency

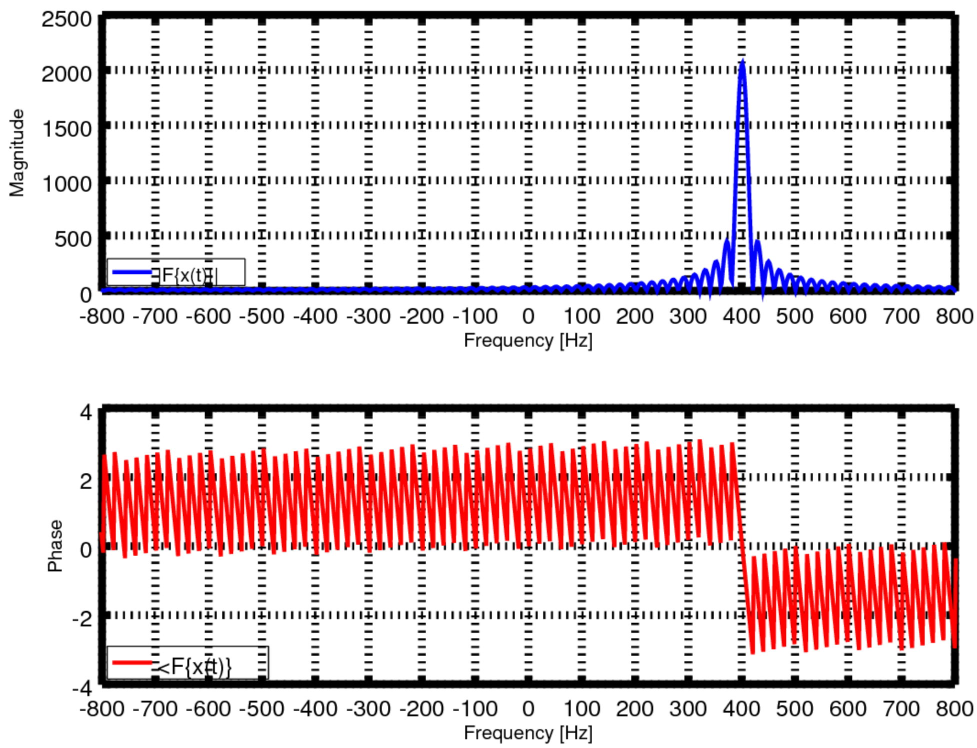
$$
\begin{aligned}
\mathbb{F}\{\mathbb{RE}\{e^{2\pi i f_k t+\phi_k}\}\} &= \mathbb{F}\{\mathbb{RE}\{\cos\left(2\pi i f_k t+\phi_k\right)+i\sin\left(2\pi i f_k t+\phi_k\right)\}\} \\
&= \mathbb{F}\{\cos\left(2\pi i f_k t+\phi_k\right)\} \\
&= \mathbb{F}\{\frac{e^{2\pi i f_k t+\phi_k}+e^{-2\pi i f_k t+\phi_k}}{2}\} \\
&= \frac{1}{2}\left[\int_{-\infty}^{+\infty} e^{2\pi i f_k t+\phi_k} e^{-2\pi i f t} dt + \int_{-\infty}^{+\infty} e^{-2\pi i f_k t+\phi_k} e^{-2\pi i f t} dt\right] \\
&= \frac{1}{2}\left[\int_{-\infty}^{+\infty} e^{2\pi i (f_k - f) t} e_k^{\phi} dt + \int_{-\infty}^{+\infty} e^{2\pi i (-f_k - f) t} e_k^{\phi} dt\right] \\
&= 2\pi\left[e^{-i\phi_k}\delta\left(f+f_k\right)+e^{i\phi_k}\delta\left(f-f_k\right)\right]
\end{aligned}
$$



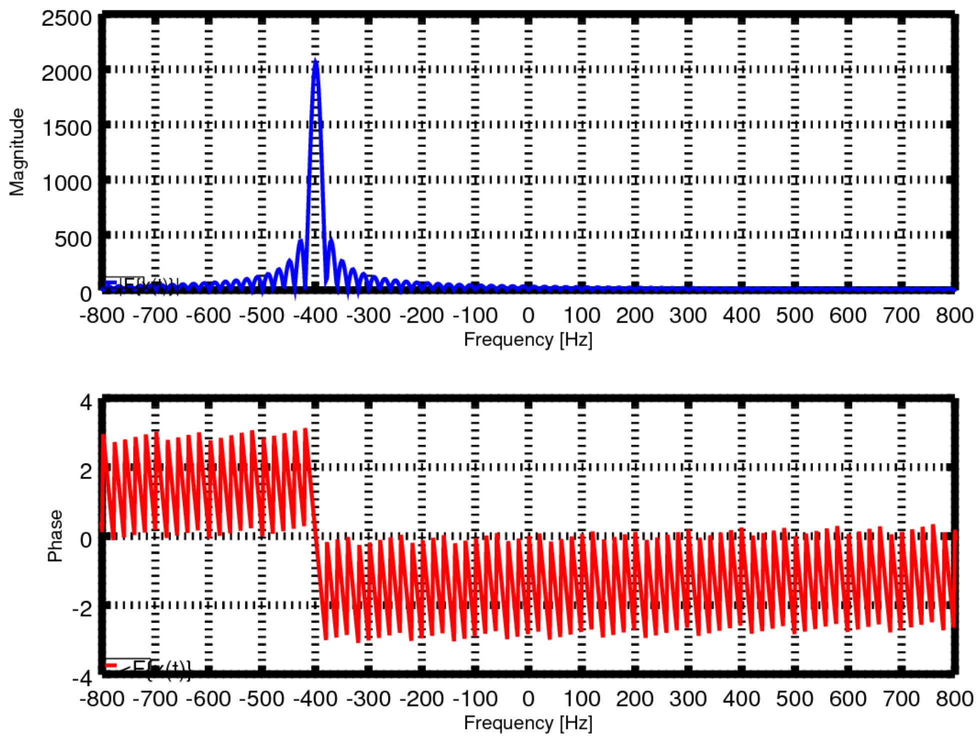

Signal Amplitude
Time [s]
6
4
2
0
-2
-4
-6
0
0.001
0.002
0.003
0.004
0.005
0.006

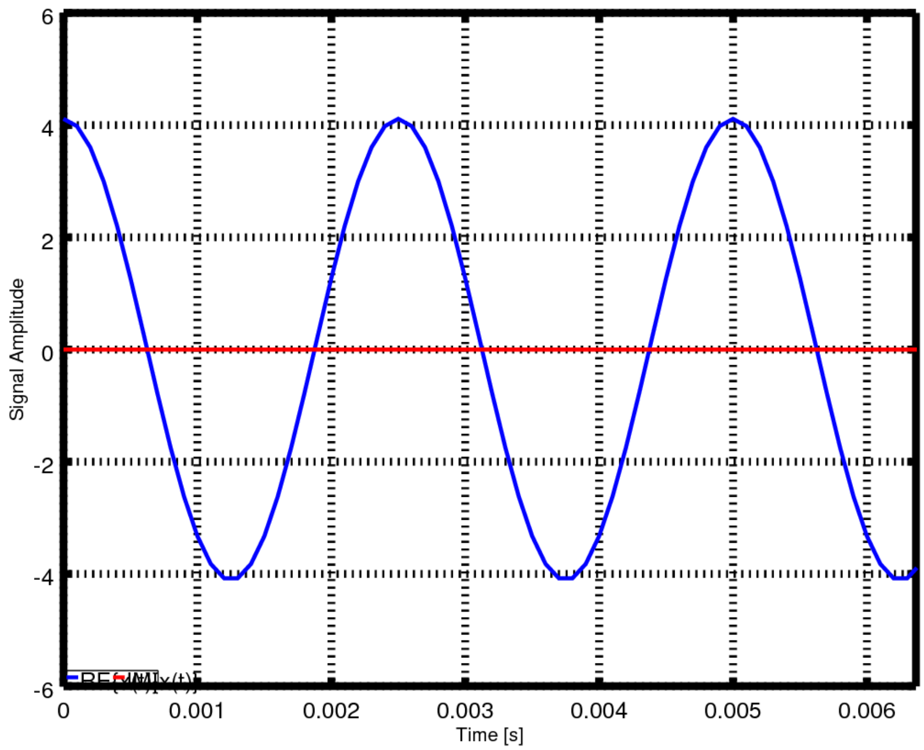

2500
2000
1500
1000
500
0
-500
Real Part
RE{F{x(t)}}
-500
0
500
Frequency [Hz]
1500
1000
500
0
-500
-1000
-1500
Quadrature Part
IM{F{x(t)}}
-500
0
500

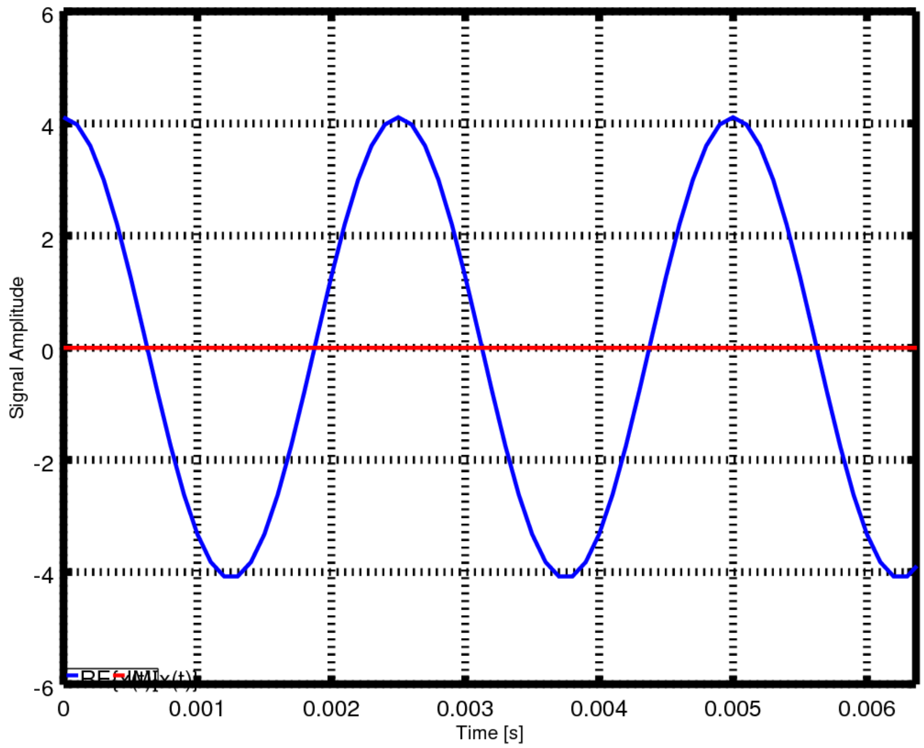

2500
2000
1500
1000
500
0
-500
Real Part
RE{F{x(t)}}
-500
0
500
Frequency [Hz]
1500
1000
500
0
-500
-1000
-1500
Quadrature Part
IM{F{x(t)}}
-500
0
500

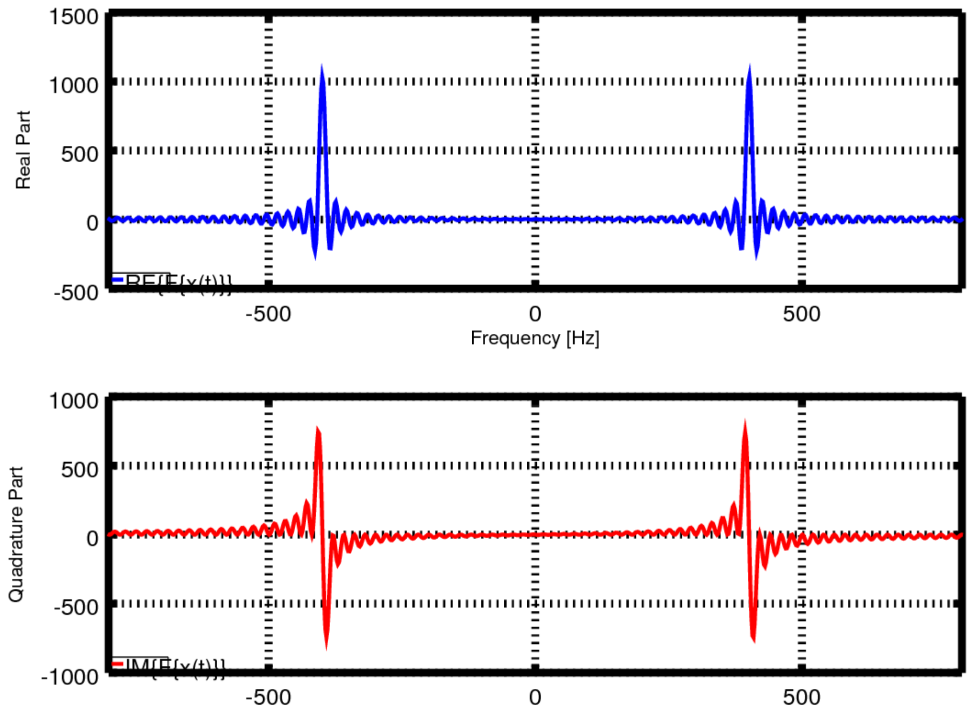


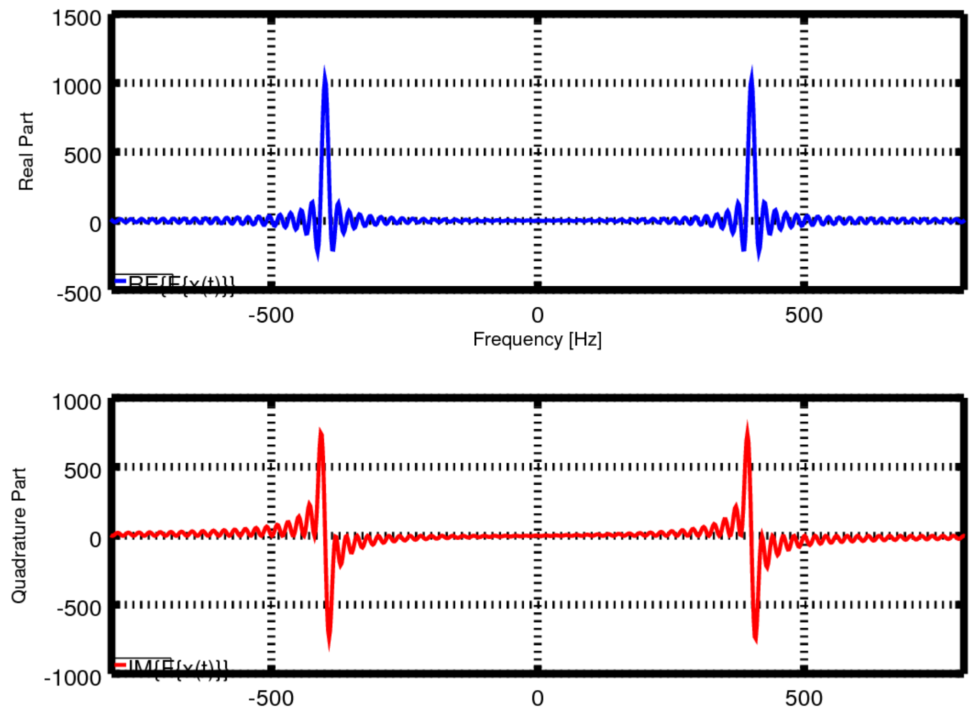
2500
2000
1500
1000
500
0
Magnitude
|F{x(t)}|
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]
4
2
0
-2
-4
Phase
<F{x(t)}
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]


Magnitude
Frequency [Hz]
Phase
Frequency [Hz]

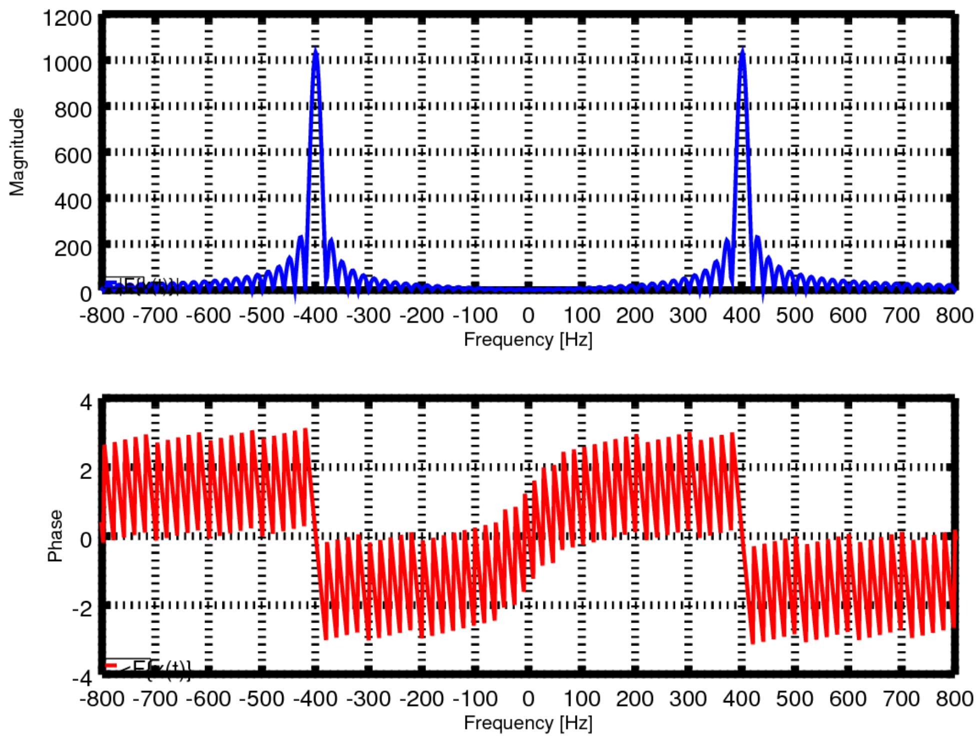

6
4
2
0
-2
-4
-6
Signal Amplitude
0
0.001
0.002
0.003
0.004
0.005
0.006
Time [s]

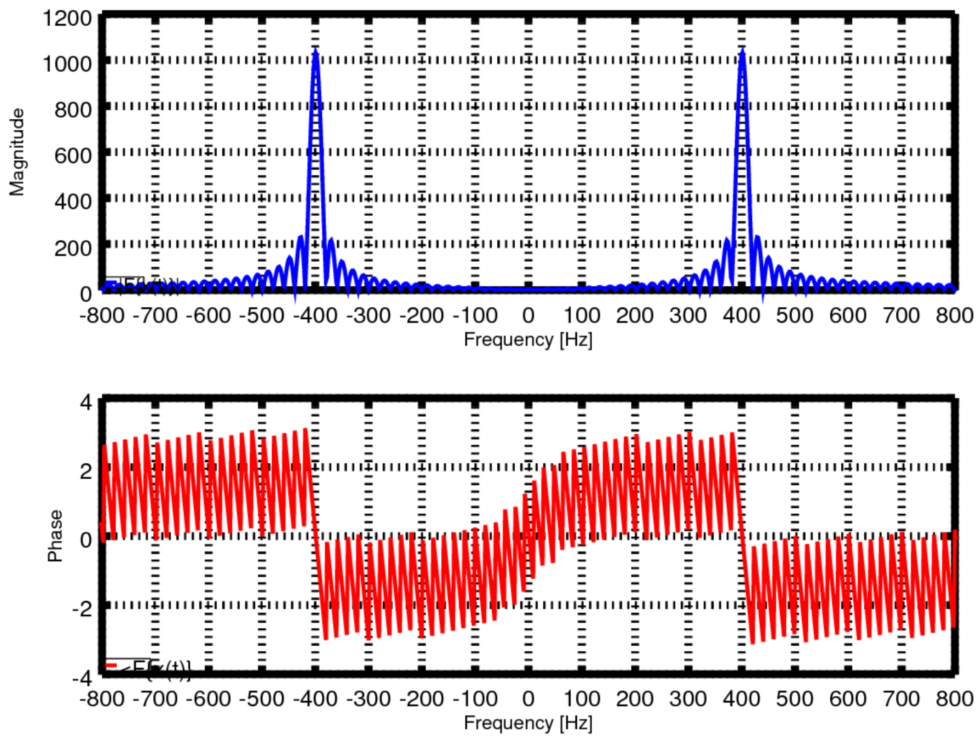

Signal Amplitude
Time [s]
6
4
2
0
-2
-4
-6
0
0.001
0.002
0.003
0.004
0.005
0.006

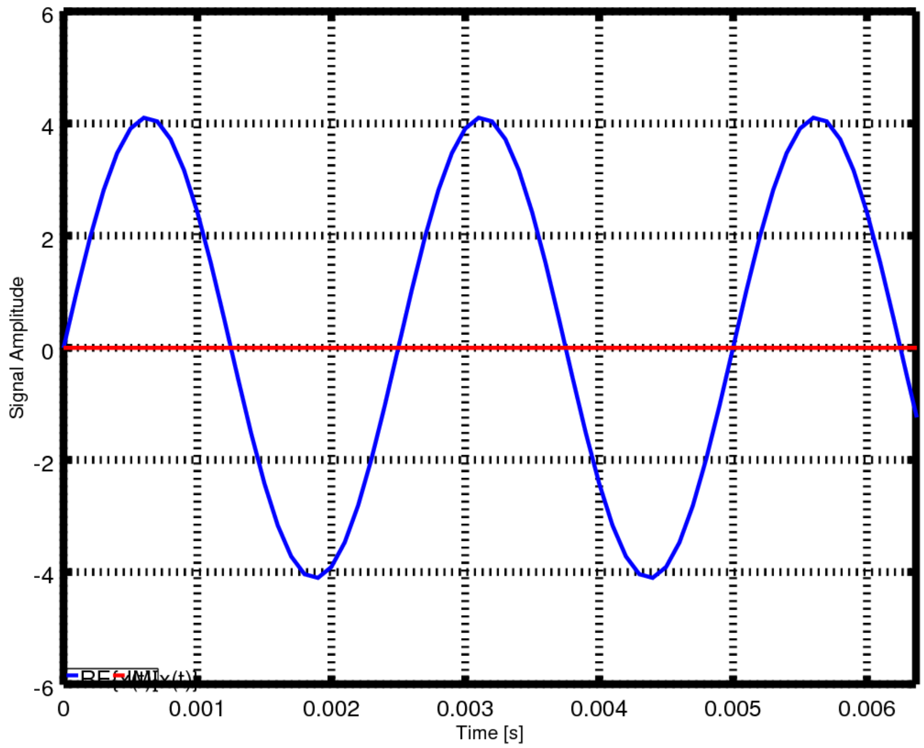

1500
1000
500
0
-500
Real Part
-500
0
500
Frequency [Hz]
RE{F{x(t)}}
1000
500
0
-500
-1000
Quadrature Part
-500
0
500
IM{F{x(t)}}

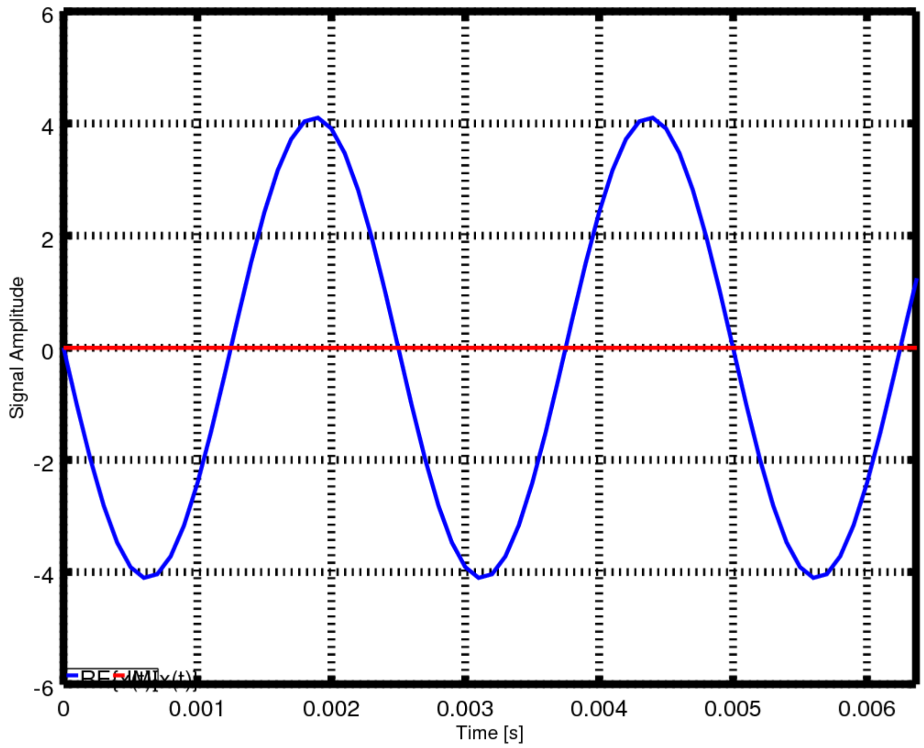


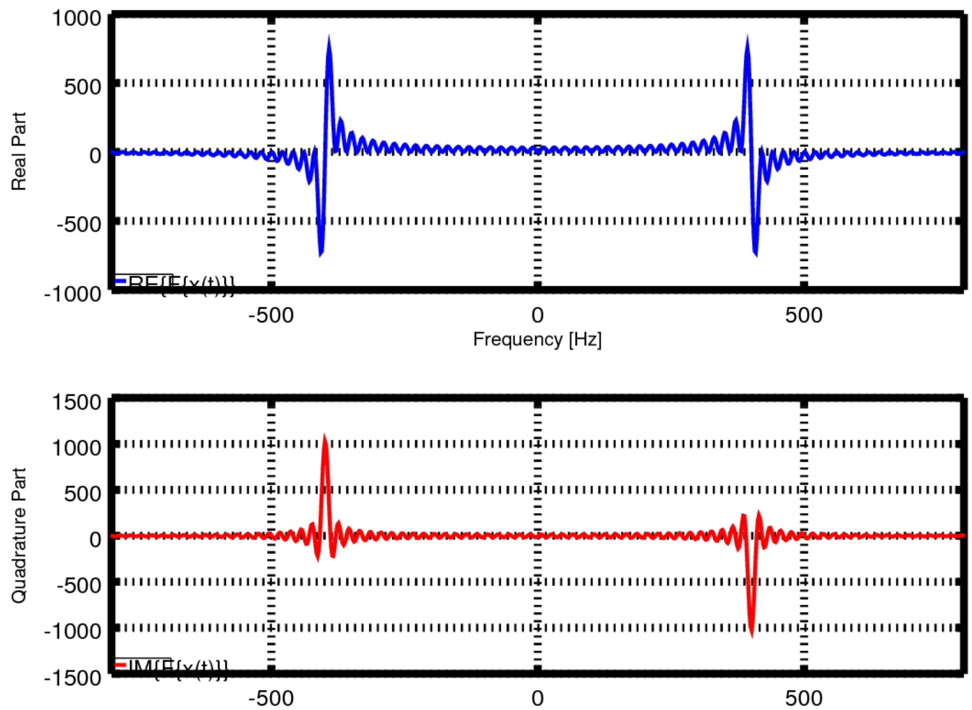
Real Part
1500
1000
500
0
-500
RE{F{x(t)}}
-500
0
500
Frequency [Hz]
Quadrature Part
1000
500
0
-500
-1000
IM{F{x(t)}}
-500
0
500

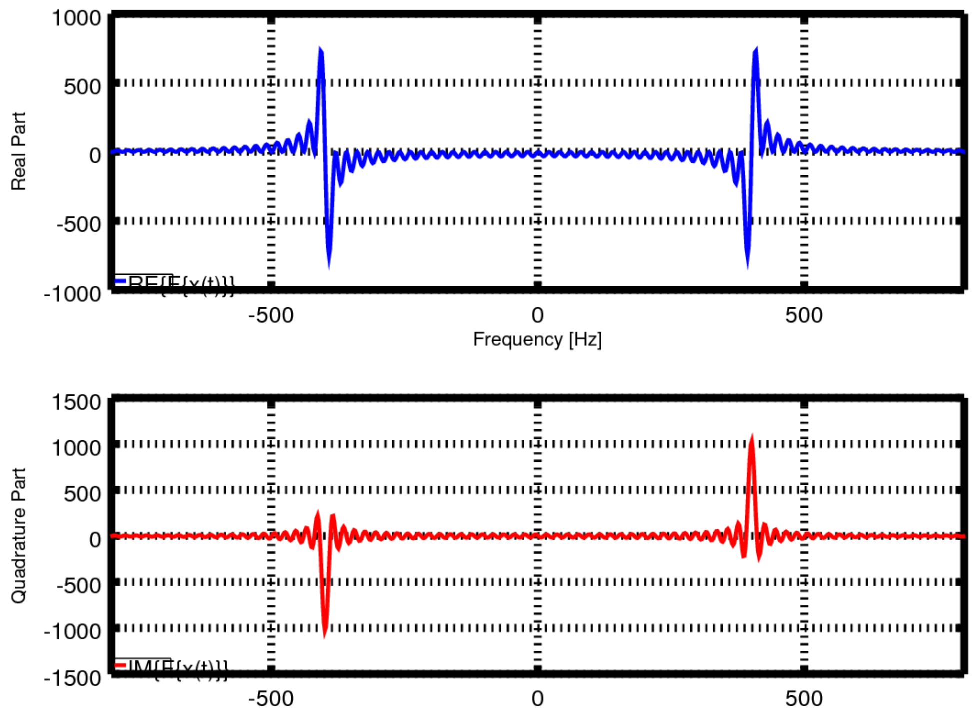

Magnitude
1200
1000
800
600
400
200
0
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]
Phase
4
2
0
-2
-4
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]

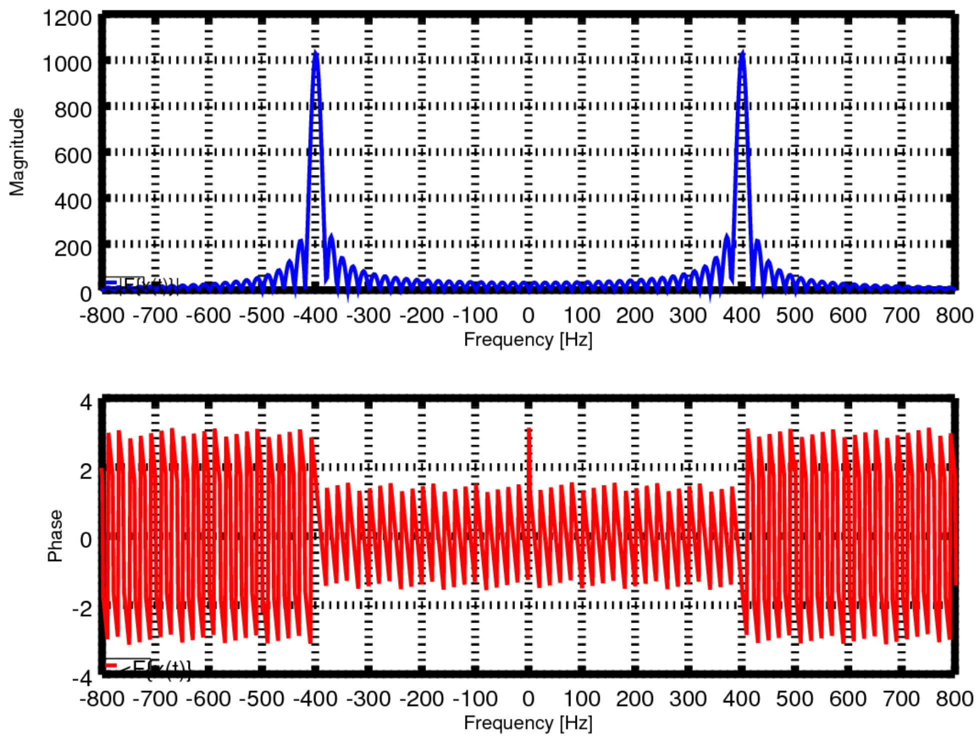

Magnitude
1200
1000
800
600
400
200
0
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]
Phase
4
2
0
-2
-4
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]

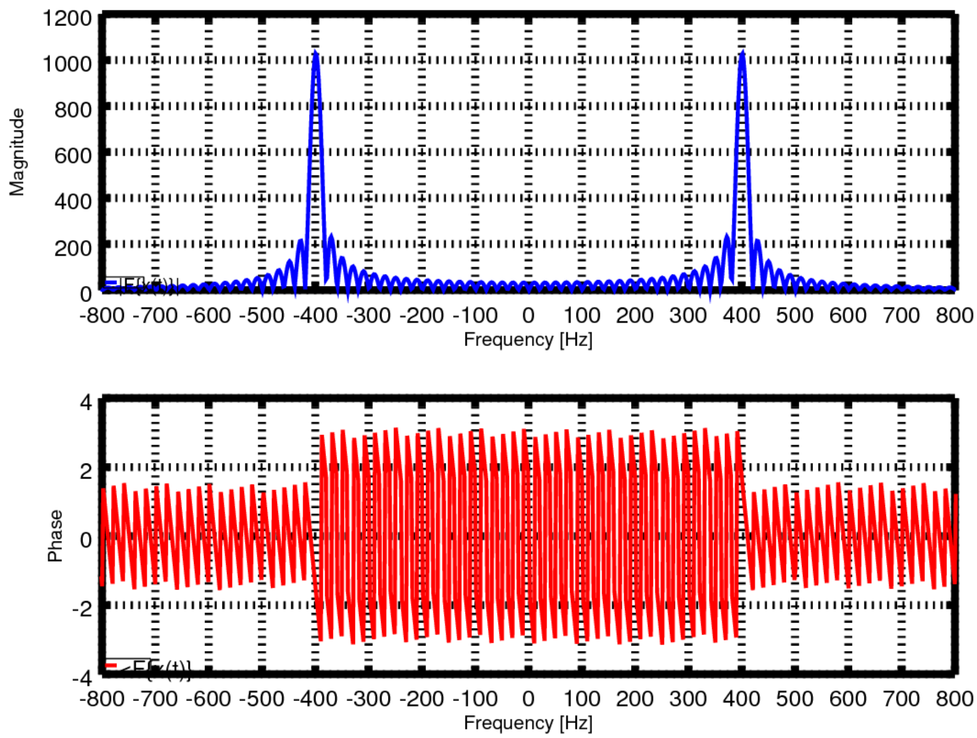

6
4
2
0
-2
-4
-6
Signal Amplitude
0
0.001
0.002
0.003
0.004
0.005
0.006
Time [s]


6
4
2
0
-2
-4
-6
Signal Amplitude
0
0.001
0.002
0.003
0.004
0.005
0.006
Time [s]


1000
500
0
-500
-1000
Real Part
RE{F{x(t)}}
-500
0
500
Frequency [Hz]
1500
1000
500
0
-500
-1000
-1500
Quadrature Part
IM{F{x(t)}}
-500
0
500


1000
500
0
-500
-1000
Real Part
RE{F{x(t)}}
-500
0
500
Frequency [Hz]
1500
1000
500
0
-500
-1000
-1500
Quadrature Part
IM{F{x(t)}}
-500
0
500


Magnitude
1200
1000
800
600
400
200
0
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]
Phase
4
2
0
-2
-4
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]




```
#!/usr/bin/octave -q
%% fft_tone.m
%%
%% Displays the time and frequency domain plot of a single tone.

  function fft_tone(fs, t, fc, A, fk, phi, f, tag)

    x = f(A*exp(1i*2*pi*fk*t + phi));

    T2   = abs(fk/fs/2/pi);

    %% Fast Fourier Transform
    N = 10 * 2^nextpow2(numel(x));
    x_f = fftshift(fft(x, N));
    if mod(N,2), k=-N/2:N/2-1; else k=-(N-1)/2:(N-1)/2; end
    T = N/fs;
    n_f = k/T;

```

```
    x_f      = fftshift(fft(x, N));
    x_f_max  = max(abs(x_f));
    xrange = sort( (2*fk*[-1 +1]) );

    %% frequency domain spectral plot
    ax = figure(201);
      title('Spectrum');
      subplot(2,1,1);
        plot(n_f, abs(x_f), ...
          'linewidth', 2, 'color', 'b');
        ylabel('Magnitude');
        xlabel('Frequency [Hz]');
        xlim(xrange);
        grid on;
        set(gca,'XTick',xrange(1):100:xrange(end));
        legend({'|F\{x(t)\}|'}, 'location', 'southwest', 'orientation', 'horizontal');
        set(gca, 'linewidth', 4, 'fontsize', 12)
      subplot(2,1,2);
        plot(n_f, angle(x_f), 'linewidth', 2, 'color', 'r');
        ylabel('Phase');
        xlabel('Frequency [Hz]');
        xlim(xrange);
        grid on;
        set(gca,'XTick',xrange(1):100:xrange(end));
        set(gca, 'linewidth', 4, 'fontsize', 12)
        legend({'<F\{x(t)\}'}, 'location', 'southwest', 'orientation', 'horizontal');
      set(ax, 'visible', 'off');
      saveas(ax, sprintf('../figures/%s_spectrum_%s.png', mfilename, tag));

    %% frequency domain I and Q plot
    ax = figure(203);
      title('Frequency Domain');
      subplot(2,1,1);
        plot(n_f, real(x_f), 'linewidth', 2, 'color', 'b');
        xlabel('Frequency [Hz]');
        ylabel('Real Part');
        xlim(xrange);
        grid on;
        set(gca, 'linewidth', 4, 'fontsize', 12)
        legend({ 'RE\{F\{x(t)\}\}' }, 'location', 'southwest', 'orientation', 'horizontal');
      subplot(2,1,2);
        xlabel('Frequency [Hz]');
        plot(n_f, imag(x_f), 'linewidth', 2, 'color', 'r');
        ylabel('Quadrature Part');
        xlim(xrange);
```

```
        grid on;
        set(gca, 'linewidth', 4, 'fontsize', 12)
        legend({ 'IM\{F\{x(t)\}\}' }, 'location', 'southwest', 'orientation', 'horizontal');
      set(ax, 'visible', 'off');
      saveas(ax, sprintf('../figures/%s_fft_%s.png', mfilename, tag));

    %% time domain plot
    ax = figure(203);
      title('Time Domain');
      subplot(1,1,1);
        plot(t, real(x), 'linewidth', 2, 'color', 'b');
        xlabel('Time [s]');
        hold on;
        plot(t, imag(x), 'linewidth', 2, 'color', 'r');
        ylabel('Signal Amplitude');
        xlabel('Time [s]');
        grid on;
        xlim([t(1) T2]);
        set(gca, 'linewidth', 4, 'fontsize', 12)
        legend({ 'RE\{x(t)\}','IM\{x(t)\}'}, 'location', 'southwest', 'orientation', 'horizo
      set(ax, 'visible', 'off');
      saveas(ax, sprintf('../figures/%s_%s.png', mfilename, tag));

  end % fft_tone

 %o = parameters();
 %fft_tone(...);

%% *EOF*
```

1.8 Hilbert Transform

1.8.1 Hilbert Transform Defined

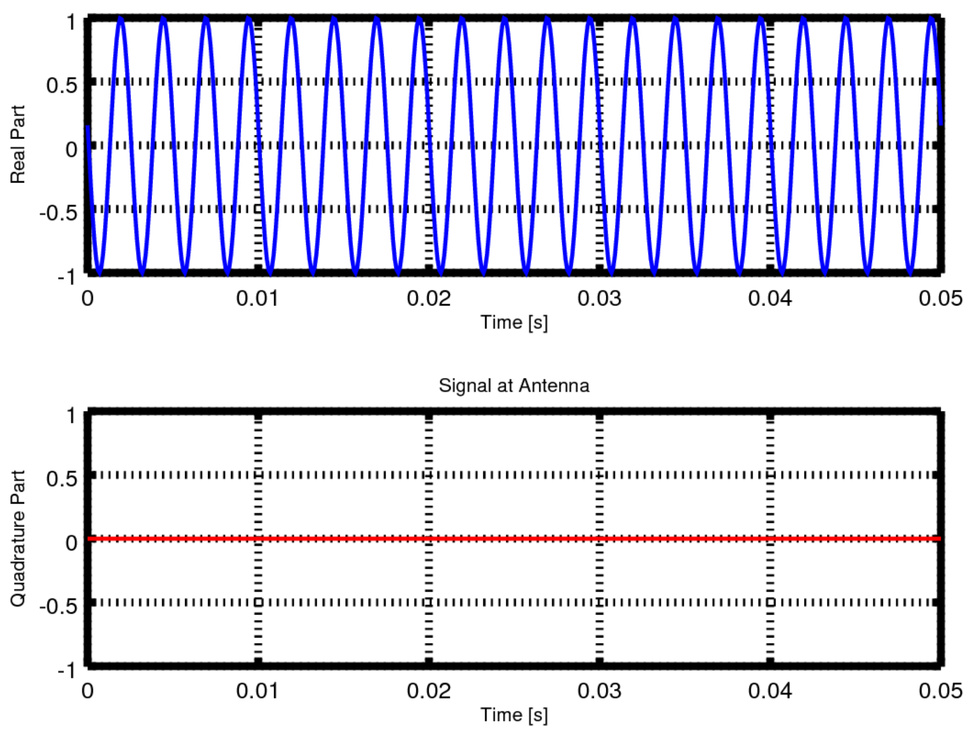
$$
\begin{aligned}
H\{g(t)\} &= g(t) \\
&= \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{g(\tau)}{t-\tau} d\tau \\
&= \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{g(t-\tau)}{\tau} d\tau
\end{aligned}
$$

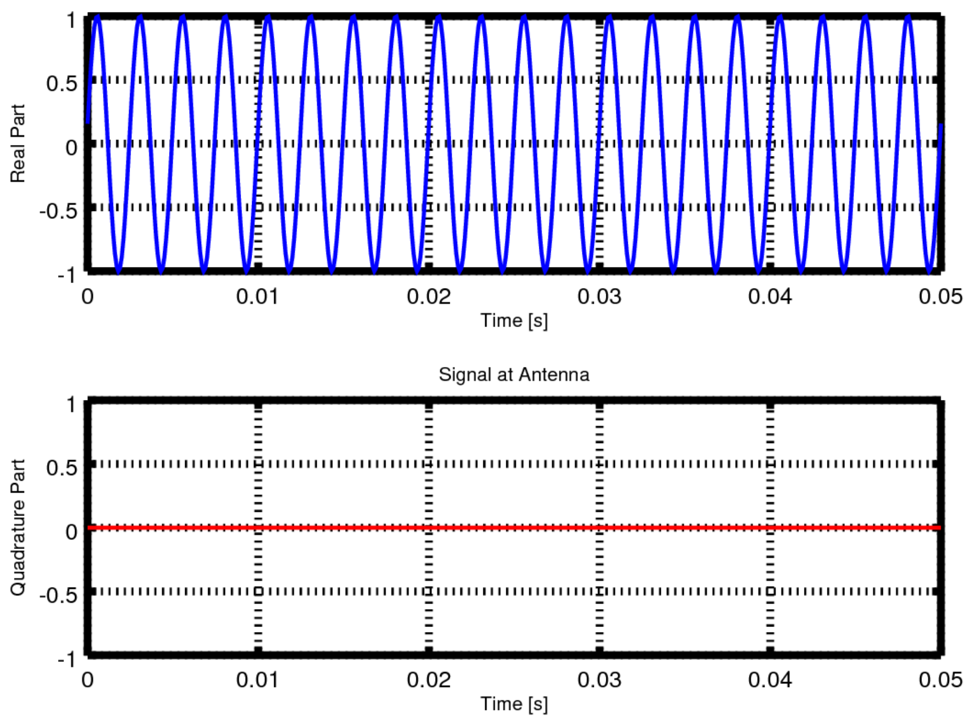
1.8.2 Hilbert Tranfsorm Defined in Terms of the Fourier Transform

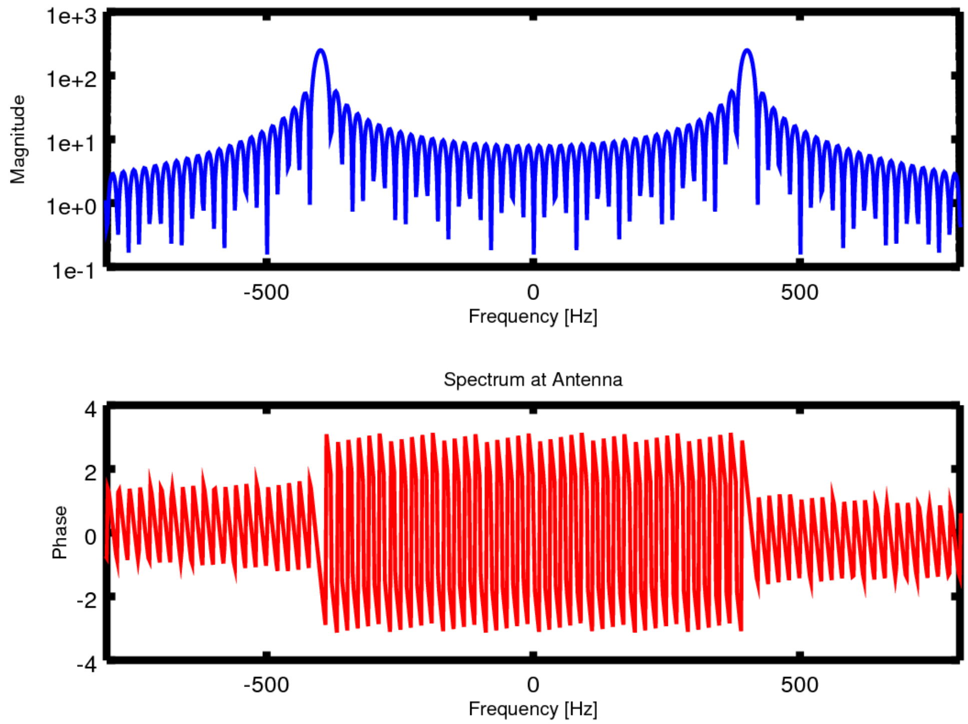
2 Receiver Antenna

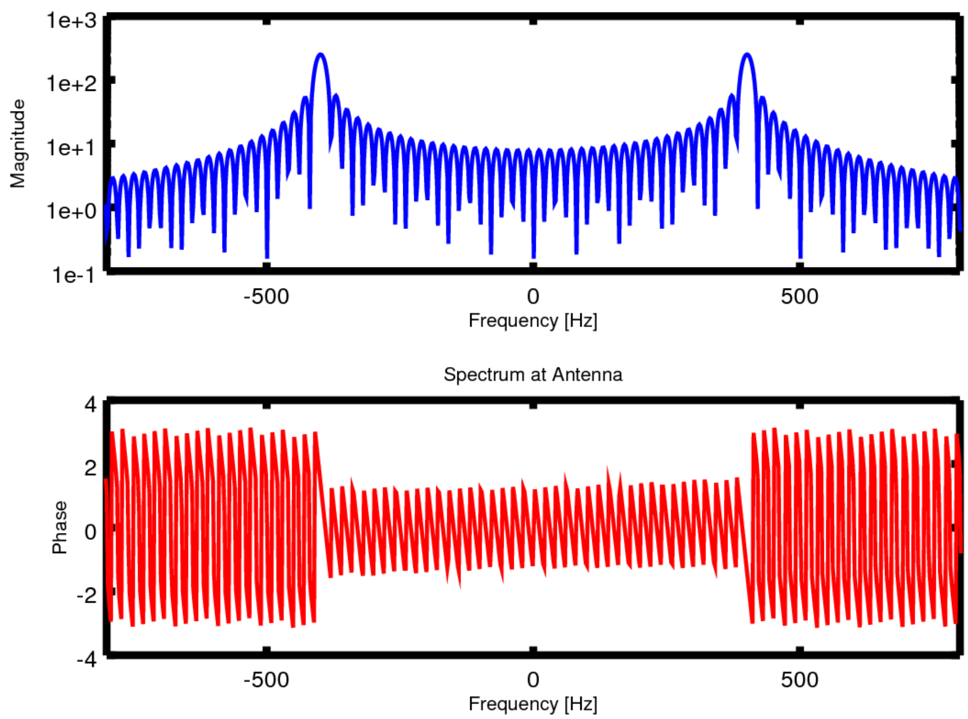
$$x_{+}(t) = \cos(2\pi f_k + \phi_k) \tag{16}$$

$$x_{-}(t) = \cos(-2\pi f_k + \phi_k) \tag{17}$$




1
0.5
0
-0.5
-1
Real Part
0
0.01
0.02
0.03
0.04
0.05
Time [s]
Signal at Antenna
1
0.5
0
-0.5
-1
Quadrature Part
0
0.01
0.02
0.03
0.04
0.05
Time [s]


Magnitude
1e+3
1e+2
1e+1
1e+0
1e-1
-500
0
500
Frequency [Hz]
Spectrum at Antenna
Phase
4
2
0
-2
-4
-500
0
500
Frequency [Hz]




```
#!/usr/bin/octave -q
%% rcv_antenna.m
%%
%% Displays the time domain signal as seen by the antenna.

  function rcv_antenna(fs, t, fc, A, fk, phi, tag)

    x = A*cos(2*pi*fk*t + phi); % pure real signal

    %% time domain plot
    ax = figure(301);
      subplot(2,1,1);
        plot(t, real(x), ...
          'linewidth', 2, 'color', 'b');
        ylabel('Real Part');
        xlabel('Time [s]');
        grid on;
        set(gca, 'linewidth', 4, 'fontsize', 12)
```

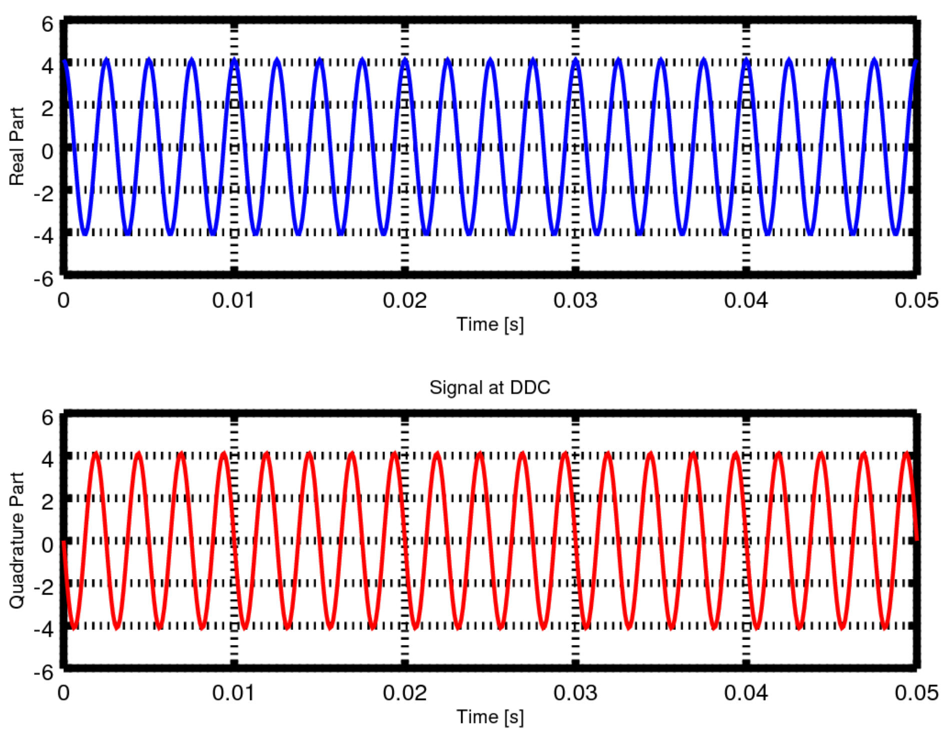
```
   subplot(2,1,2);
    plot(t, imag(x), ...
     'linewidth', 2, 'color', 'r');
    title('Signal at Antenna');
    ylabel('Quadrature Part');
    xlabel('Time [s]');
    grid on;
    set(gca, 'linewidth', 4, 'fontsize', 12)
   set(ax, 'visible', 'off');
   saveas(ax, sprintf('../figures/%s_%s.png', mfilename, tag));

  %% Fast Fourier Transform
  N = 10 * 2^nextpow2(numel(x));
 %x_f = fftshift(fft(x, N))/N;
  x_f = fftshift(fft(x, N));
  if mod(N,2), k=-N/2:N/2-1; else k=-(N-1)/2:(N-1)/2; end
  T = N/fs;
  n_f = k/T;

  %% frequency domain plot
  ax = figure(302);
   xrange = sort( (2*fk*[-1 +1]) );
   subplot(2,1,1);
    semilogy(n_f, abs(x_f), ...
     'linewidth', 2, 'color', 'b');
    ylabel('Magnitude');
    xlabel('Frequency [Hz]');
    xlim(xrange);
    set(gca, 'linewidth', 4, 'fontsize', 12)
   subplot(2,1,2);
    plot(n_f, angle(x_f), ...
     'linewidth', 2, 'color', 'r');
    title('Spectrum at Antenna');
    ylabel('Phase');
    xlabel('Frequency [Hz]');
    xlim(xrange);
    set(gca, 'linewidth', 4, 'fontsize', 12)
   set(ax, 'visible', 'off');
   saveas(ax, sprintf('../figures/%s_freq_%s.png', mfilename, tag));

 end % rcv_antenna

%o = parameters();
%rcv_antenna(...);
```

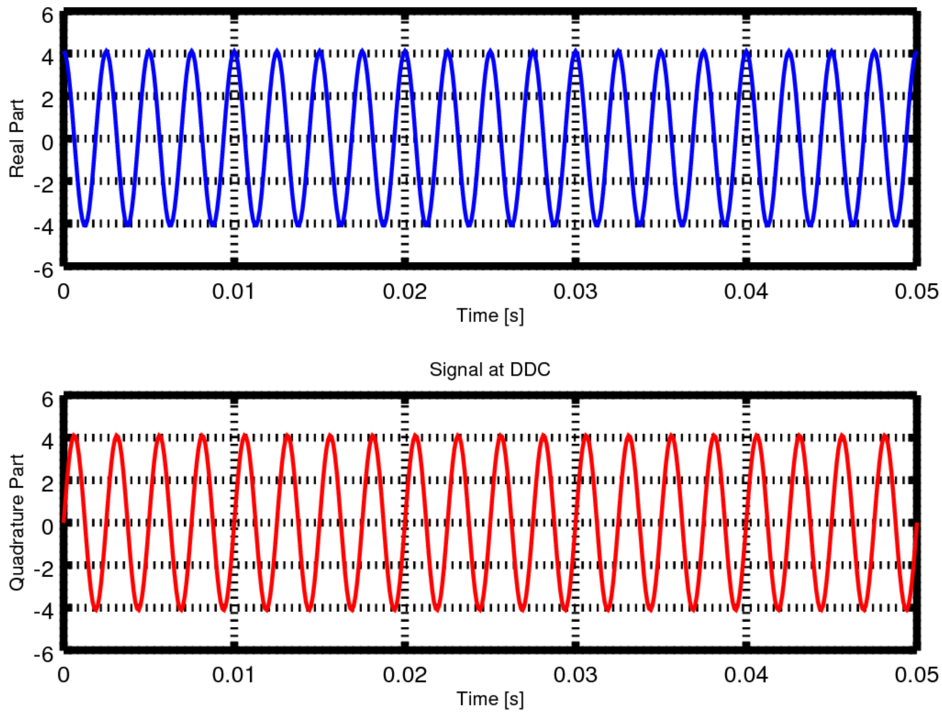
```
%% *EOF*
```

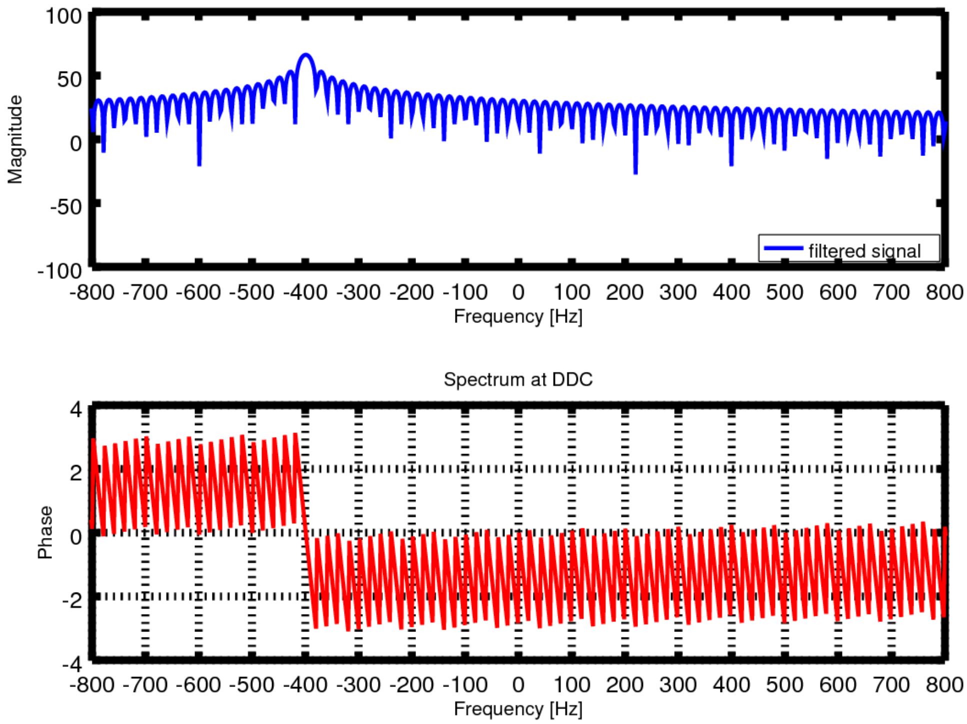
3 Analytic Signal

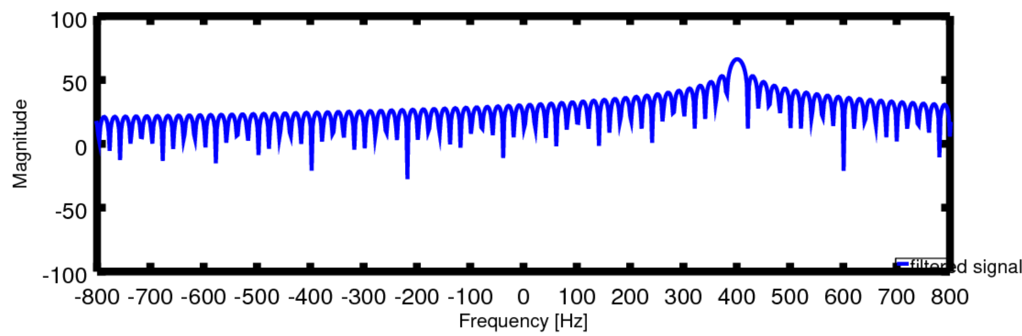
$$\begin{aligned} x_{+}\left(t\right) &= e^{+i2\pi f_k t+\phi_k} \\ &= \cos\left(2\pi f_k+\phi_k\right)+i\sin\left(2\pi f_k+\phi_k\right) \end{aligned} \tag{18}$$

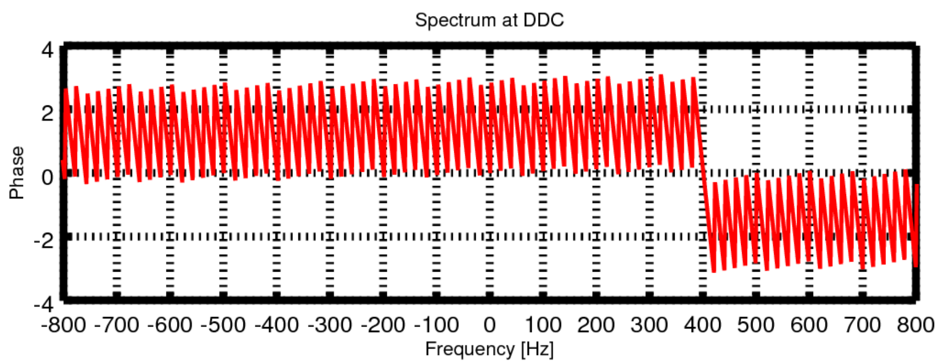
$$\begin{aligned} x_{-}\left(t\right) &= e^{-i2\pi f_k t+\phi_k} \\ &= \cos\left(-2\pi f_k+\phi_k\right)+i\sin\left(-2\pi f_k+\phi_k\right) \\ &= \cos\left(2\pi f_k+\phi_k\right)-i\sin\left(2\pi f_k+\phi_k\right) \end{aligned} \tag{19}$$




6
4
2
0
-2
-4
-6
Real Part
0
0.01
0.02
0.03
0.04
0.05
Time [s]
Signal at DDC
Quadrature Part
Time [s]


100
50
0
-50
-100
Magnitude
filtered signal
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]
Spectrum at DDC
4
2
0
-2
-4
Phase
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]



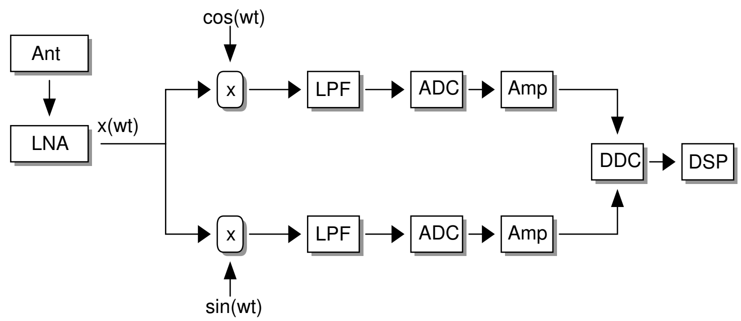


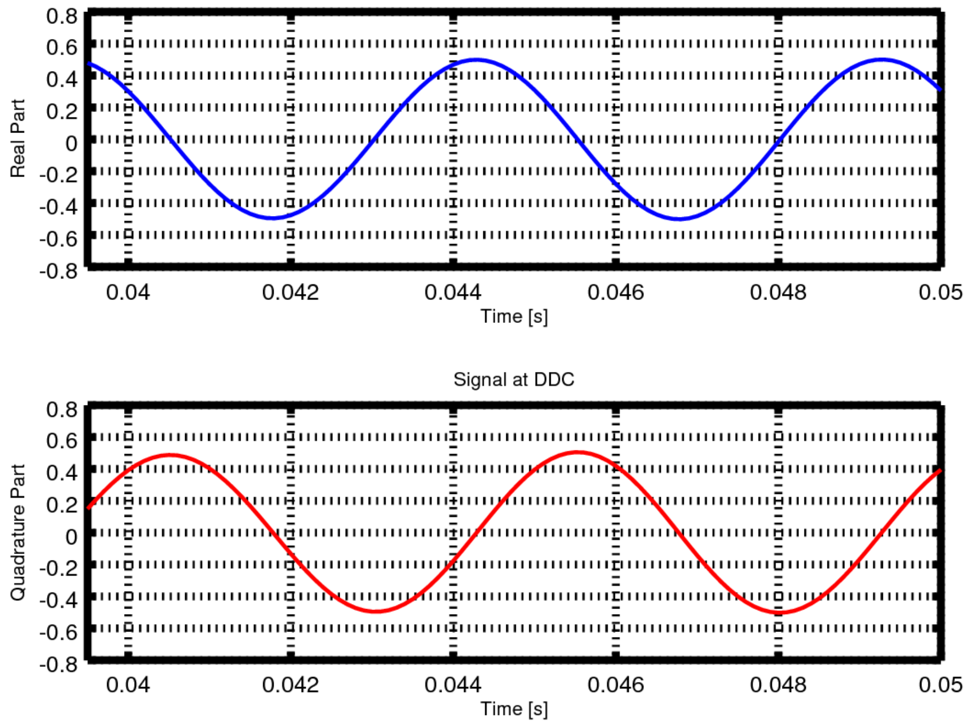

```
#!/usr/bin/octave -q
%% rcv_ddc.m
%%
%% Displays the time domain signal as seen by the DDC.

  function rcv_analytic(fs, t, fc, fi, A, fk, phi, tag)

    %% Receiver Channels
    x_a = A*exp(-1i*2*pi*fk*t + phi);      % analytic signal

    %% time domain plot
    ax = figure(321);
      subplot(2,1,1);
        plot(t, real(x_a), 'linewidth', 2, 'color', 'b');
        ylabel('Real Part');
        xlabel('Time [s]');
        grid on;
        xlim([t(1) t(end)]);
```

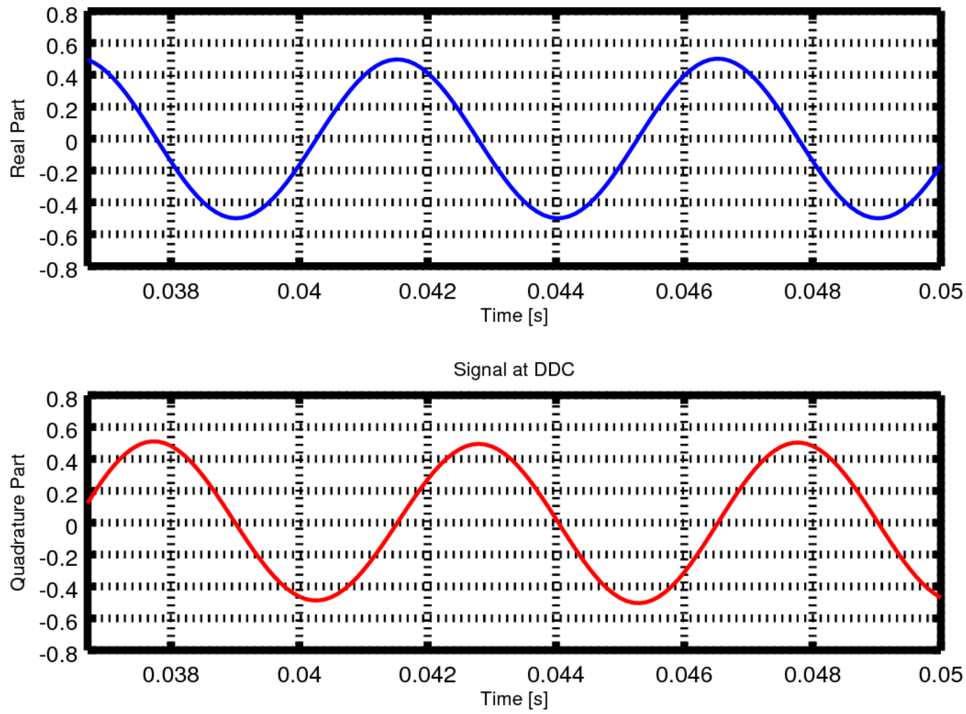
```
        set(gca, 'linewidth', 4, 'fontsize', 12)
      subplot(2,1,2);
        plot(t, imag(x_a), 'linewidth', 2, 'color', 'r');
        title('Signal at DDC');
        ylabel('Quadrature Part');
        xlabel('Time [s]');
        grid on;
        xlim([t(1) t(end)]);
        set(gca, 'linewidth', 4, 'fontsize', 12)
      set(ax, 'visible', 'off');
      saveas(ax, sprintf('../figures/%s_%s.png', mfilename, tag));

    %% Fast Fourier Transform
    N = 10 * 2^nextpow2(numel(x_a));
    x_f = fftshift(fft(x_a, N));
    if mod(N,2), k=-N/2:N/2-1; else k=-(N-1)/2:(N-1)/2; end
    T = N/fs;
    n_f = k/T;

    %% frequency domain plot
    ax = figure(322);
      xrange = sort( (2*fk*[-1 +1]) );
      subplot(2,1,1);
        plot(n_f, 20*log10(abs(x_f)), ...
          'linewidth', 2, 'color', 'b');
        ylabel('Magnitude');
        xlabel('Frequency [Hz]');
        xlim(xrange);
       %grid on;
        set(gca,'XTick',xrange(1):100:xrange(end));
        legend({'filtered signal', 'low pass filter response'}, ...
          'location', 'southeast');
        set(gca, 'linewidth', 4, 'fontsize', 12)
      subplot(2,1,2);
        plot(n_f, angle(x_f), 'linewidth', 2, 'color', 'r');
        title('Spectrum at DDC');
        ylabel('Phase');
        xlabel('Frequency [Hz]');
        xlim(xrange);
        grid on;
        set(gca,'XTick',xrange(1):100:xrange(end));
        set(gca, 'linewidth', 4, 'fontsize', 12)
      set(ax, 'visible', 'off');
      saveas(ax, sprintf('../figures/%s_freq_%s.png', mfilename, tag));
```

```
  end % rcv_analytic

 %o = parameters();
 %rcv_analytic(...);

%% *EOF*
```

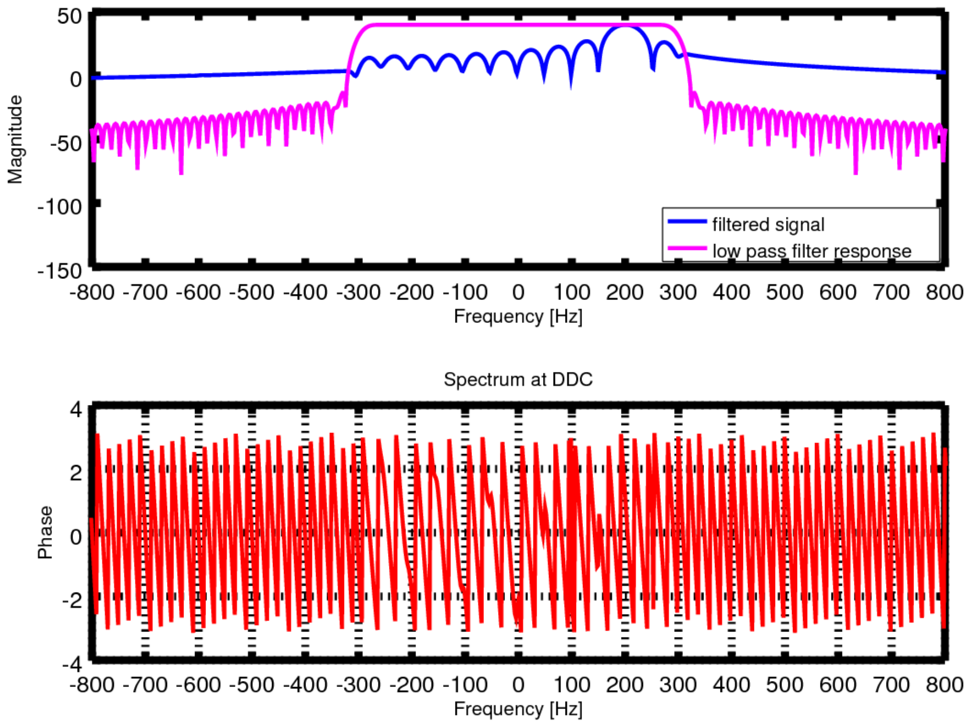
4 Quadrature Sampling

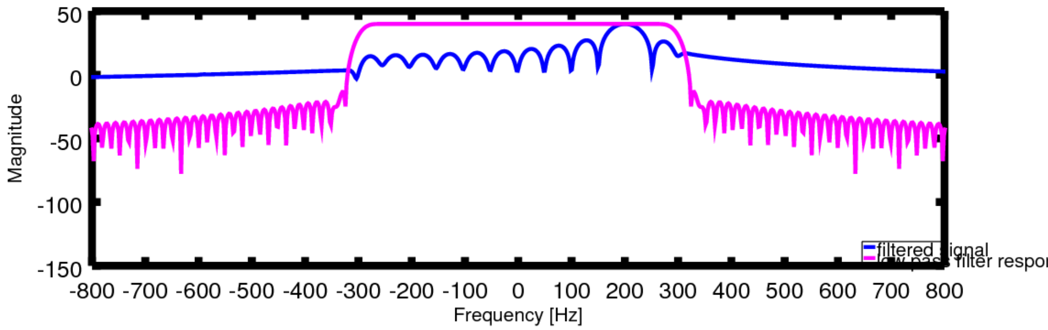


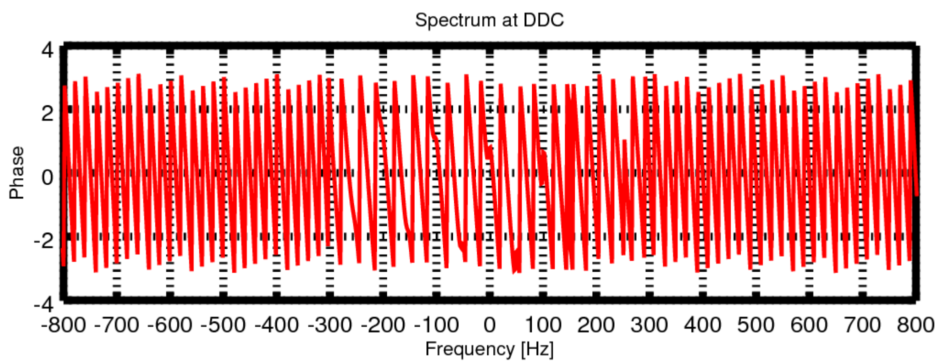

0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Real Part
0.04
0.042
0.044
0.046
0.048
0.05
Time [s]
Signal at DDC
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Quadrature Part
0.04
0.042
0.044
0.046
0.048
0.05
Time [s]



Real Part
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0.038
0.04
0.042
0.044
0.046
0.048
0.05
Time [s]
Signal at DDC
Quadrature Part
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0.038
0.04
0.042
0.044
0.046
0.048
0.05
Time [s]


50
0
-50
-100
-150
Magnitude
filtered signal
low pass filter response
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]
Spectrum at DDC
4
2
0
-2
-4
Phase
-800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800
Frequency [Hz]






```
#!/usr/bin/octave -q
%% rcv_ddc.m
%%
%% Displays the time domain signal as seen by the DDC.

  function rcv_ddc(fs, t, fc, fi, A, fk, phi, tag)

    %% Receiver Channels
    x         = A*cos(2*pi*fk*t + phi);          % signal at antenna
    fd        = -1 * abs(fc - fi);               % mixer frequency
    x_i       = x .* cos(2*pi*fd*t);             % real channel
    x_q       = x .* sin(2*pi*fd*t);             % quadrature channel

    %% Low-Pass Filter
    fcut      = 3.0 * fi;                        % [Hz    ] cutoff frequency
    wcut      = fcut/(fs/2);                     % [f norm] cutoff frequency (normalized)
    attn      = 40;                              % [dB    ] attenuation
    ord       = (attn*fs)/(22*(1.1*fcut-fcut)); % [#     ] FIR filter order
```

```
    if mod(ceil(ord),2)
      ord = ceil(ord) + 1;
    else
      ord = ceil(ord);
    end
    hb        = fir1(ord, wcut, 'low');
    ha        = [1];
    x_i_lpf   = filter(hb, ha, x_i);    % real channel low pass filter
    x_q_lpf   = filter(hb, ha, x_q);    % quadrature channel low pass filter

    %% Analytic Signal
    x_a_full = x_i     + 1i*x_q;
    x_a      = x_i_lpf + 1i*x_q_lpf;   % analytic signal

    %% time-alignment to account for filter response delay
    t0        = 0.03;                   % [s     ] filter response time
    ks        = find(t>t0);
    ks        = ks(1) + find(real(x_a(ks)) == max(real(x_a(ks))));  % cosine peak
    t0        = t(ks(1));               % snap to cosine peak
    ks        = find(t>t0);

    %% time domain plot
    ax = figure(311);
      subplot(2,1,1);
        plot(t(ks), real(x_a(ks)), 'linewidth', 2, 'color', 'b');
        ylabel('Real Part');
        xlabel('Time [s]');
        grid on;
        xlim([t(ks(1)) t(ks(end))]);
        set(gca, 'linewidth', 4, 'fontsize', 12)
      subplot(2,1,2);
        plot(t(ks), imag(x_a(ks)), 'linewidth', 2, 'color', 'r');
        title('Signal at DDC');
        ylabel('Quadrature Part');
        xlabel('Time [s]');
        grid on;
        xlim([t(ks(1)) t(ks(end))]);
        set(gca, 'linewidth', 4, 'fontsize', 12)
      set(ax, 'visible', 'off');
      saveas(ax, sprintf('../figures/%s_%s.png', mfilename, tag));

    %% Fast Fourier Transform
    N = 10 * 2^nextpow2(numel(x_a));
    x_f = fftshift(fft(x_a, N));
    if mod(N,2), k=-N/2:N/2-1; else k=-(N-1)/2:(N-1)/2; end
```

```
    T = N/fs;
    n_f = k/T;

    x_f_full = fftshift(fft(x_a_full, N));
    x_f_max  = max(abs(x_f));

    x_lpf = abs(fftshift(fft(hb,N)));
    scale = x_f_max / max(x_lpf);
    x_lpf = x_lpf * scale;
    f_lpf = (-0.5:1/N:0.5-1/N)*fs;

    %% frequency domain plot
    ax = figure(312);
      xrange = sort( (2*fk*[-1 +1]) );
      subplot(2,1,1);
       %plot(n_f, 20*log10(abs(x_f_full)), 'linewidth', 2, 'color', 'k-');
       %hold on;
        plot(n_f, 20*log10(abs(x_f)), ...
          'linewidth', 2, 'color', 'b');
        hold on;
        plot(f_lpf, 20*log10(x_lpf), ...
          'linewidth', 2, 'color', 'm', 'linestyle', '-');
        ylabel('Magnitude');
        xlabel('Frequency [Hz]');
        xlim(xrange);
       %grid on;
        set(gca,'XTick',xrange(1):100:xrange(end));
        legend({'filtered signal', 'low pass filter response'}, ...
          'location', 'southeast');
        set(gca, 'linewidth', 4, 'fontsize', 12)
      subplot(2,1,2);
        plot(n_f, angle(x_f), 'linewidth', 2, 'color', 'r');
        title('Spectrum at DDC');
        ylabel('Phase');
        xlabel('Frequency [Hz]');
        xlim(xrange);
        grid on;
        set(gca,'XTick',xrange(1):100:xrange(end));
        set(gca, 'linewidth', 4, 'fontsize', 12)
      set(ax, 'visible', 'off');
      saveas(ax, sprintf('../figures/%s_freq_%s.png', mfilename, tag));

    %% Low-Pass Filter response
    frange = 3.0*fi * [-1 +1];
    ax = figure(313);
```

```
      subplot(1,1,1);
        plot((-0.5:1/N:0.5-1/N)*fs,20*log10(abs(fftshift(fft(hb,N)))), ...
          'linewidth', 2, 'color', 'r');
        xlim([frange(1) frange(end)]);
        set(gca, 'linewidth', 4, 'fontsize', 12)
      set(ax, 'visible', 'off');
      saveas(ax, sprintf('../figures/%s_lpf_%s.png', mfilename, tag));

  end % rcv_ddc

 %o = parameters();
 %rcv_ddc(...);

%% *EOF*
```

5 Demo

```
#!/usr/bin/octave -q
%% run_demo.m
%%
%% Runs the complete demo for all sections.

  function my_demo(o)

    %% positive frequency (+f_k)
    tag = 'pos';
    fk = +1 * o.fk;
    fft_tone(    o.fs, o.t, o.fc,       o.A, fk, o.phi, @(x)x, sprintf('%s_cplx', tag));
    fft_tone(    o.fs, o.t, o.fc,       o.A, fk, o.phi, @real, sprintf('%s_real', tag));
    fft_tone(    o.fs, o.t, o.fc,       o.A, fk, o.phi, @imag, sprintf('%s_imag', tag));
    rcv_antenna( o.fs, o.t, o.fc,       o.A, fk, o.phi, tag);
    rcv_ddc(     o.fs, o.t, o.fc, o.fi, o.A, fk, o.phi, tag);
    rcv_analytic(o.fs, o.t, o.fc, o.fi, o.A, fk, o.phi, tag);

    %% negative frequency (-f_k)
    tag = 'neg';
    fk = -1 * o.fk;
    fft_tone(    o.fs, o.t, o.fc,       o.A, fk, o.phi, @(x)x, sprintf('%s_cplx', tag));
    fft_tone(    o.fs, o.t, o.fc,       o.A, fk, o.phi, @real, sprintf('%s_real', tag));
    fft_tone(    o.fs, o.t, o.fc,       o.A, fk, o.phi, @imag, sprintf('%s_imag', tag));
    rcv_antenna( o.fs, o.t, o.fc,       o.A, fk, o.phi, tag);
    rcv_ddc(     o.fs, o.t, o.fc, o.fi, o.A, fk, o.phi, tag);
```

```
    rcv_analytic(o.fs, o.t, o.fc, o.fi, o.A, fk, o.phi, tag);

  end % my_demo

  pkg load signal;
  o = parameters();
  my_demo(o);

%% *EOF*
```