Time-Frequency Notes

Mac Radigan

Fourier Expansion

$$s(t) = \frac{1}{\sqrt{2\pi}} \int S(\omega) e^{j\omega t} d\omega$$

$$S(t) = \frac{1}{\sqrt{2\pi}} \int s(t) e^{-j\omega t} dt$$

Mean Frequency

$$\langle \omega \rangle = \int \omega |S(\omega)|^2 d\omega$$

$$= \int s^*(t) \frac{1}{j} \frac{d}{dt} s(t) \, dt$$

Mean Square Frequency

$$\langle \omega^2 \rangle = \int \omega^2 |S(\omega)|^2 d\omega$$

$$= \int s^*(t) \left(\frac{1}{j} \frac{d}{dt} \right)^2 s(t) \, dt$$

$$= \int |\frac{d}{dt} s(t)|^2 dt$$

Root Mean Square Bandwidth

$$B^2 = \langle \sigma_\omega^2 \rangle = \int (\omega - \langle \omega \rangle)^2 |S(\omega)|^2 d\omega$$

$$= \int s^*(t) \left(\frac{1}{j} \frac{d}{dt} - \langle \omega \rangle \right)^2 s(t) \, dt$$

$$= \int |\left(\frac{1}{j} \frac{d}{dt} - \langle \omega \rangle \right) s(t)|^2 dt$$

Ambiguity Function

$$A\left(\tau, f\right) = \int_0^T s_1\left(t\right) s_2^*\left(t + \tau\right) e^{-j2\pi ft} dt$$

Analytic Signal

$$s_a\left(t\right) = \frac{2}{\sqrt{2\pi}} \int_0^\infty S_{RE}\left(\omega\right) e^{j\omega t} d\omega$$

Uncertainty

$$T^2 = \sigma_t^2 = \int \left(t - \langle t \rangle\right)^2 \left|s\left(s\right)\right|^2 dt$$

$$B^2 = \sigma_\omega^2 = \int \left(\omega - \langle \omega \rangle\right)^2 \left|S\left(\omega\right)\right|^2 d\omega$$

$$TB \geq \frac{1}{2}$$

$$\sigma_t \sigma_\omega \geq \frac{1}{2}\sqrt{1 + 4Cov_{t\omega}^2}$$