Math 122B Homework 3

Rad Mallari

April 28, 2022

1 Problem 1

Suppose f is a rational function of the form $f = P/Q$ where the polynomials P and Q satisfy $\deg Q - \deg P \geq 2$. Show that the sum of the residues of f is zero.

Proof.

□

2 Problem 2

Evaluate the following sums:

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$$

Proof.

□

3 Problem 3

Let U be an open set of the complex plane. Find conditios on U assuring that:

(a) The function $z \mapsto z^2$, $z \in U$, is one to one.

(b) The function $z \mapsto \cos(z)$, $z \in U$, is one to one.

Proof.

(a)

(b)

□