

Math 119A Homework 1

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Notation. Let $D_\infty[0, 1]$ be the set of infinitely differentiable, real-valued functions on the real interval $[0, 1]$. Let $L = \{f \in D_\infty[0, 1] : f(0) = 0\}$

1 Problem 1

Does $\delta(f)(s) = f'(s)$ define an injective and surjective function from $L \rightarrow L$?

Proof.

□

2 Problem 2

Same question for $\iota(f)(t) = \int_0^t f(s)ds$.

Proof.

□

3 Problem 3

Are ι and δ inverse functions? I.e. does $\iota(\delta(f)) = \delta(\iota(f)) = f$ for all $f \in L$?

Proof.

□

4 Problem 4

Look at the solutions to some problems in Chapter 1 (on p. 343 of the text). Prove or disprove that these solutions are correct. Be sure to explain what you are doing in terms of the meaning of the words in the problems. Of course this problem is really several problems.

Proof.

□

5 Problem 5

Let $A = [a_{ij}]$ be an $n \times n$ diagonal matrix, that is, $a_{ij} = 0$ if $i \neq j$. Show that the differential equation

$$x' = Ax$$

has a unique solution for every initial condition.

Proof.

□

6 Problem 6

Find two different matrices A, B such that the curve

$$x(t) = (e^t, 2e^{2t}, 4e^{2t})$$

satisfies both the differential equations

$$x'Ax \quad \text{and} \quad x' = Bx$$

Proof.

□