Math 122B Homework 6

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1 Problem 1

Let $a > 0$ be a positive parameter. Find the image of the exterior of the unit circle by the comformal mapp

$$w = az + \frac{1}{z}$$

Proof.

□

2 Problem 2

Let U be a simply connected domain, different than the entire complex plane. Let $z_0 \in U$. Let G denote the class of all analytic functions $g : U \to D$ satisfying $g'(z_0) > 0$. Show that

$$\sup_{g \in G} g'(z_0) < \infty$$

and that the supremum is attained by a function $f \in G$. Prove f is one-to-one.

Proof.

□

3 Problem 3

Let u be a harmonic function. Show that u^2 is harmonic if and only if u is constant.

Proof.

□

4 Problem 4

Suppose the function $f = u+iv$ is analytic. Prove uv is a harmonic function. Give exampels of two harmonic functions U, V with the properties that UV is not harmonic.

Proof.

□