Extra Credit

Math 119a

Due: August 7th, 2023

Instructions: Provide complete justifications for the following results reproduced from the course textbook. Any missing details from the textbook should be provided and all proofs must be written in your own words after careful study.

1. Provide a complete proof of the Jordan Canonical form. That is, show that any finite-dimensional $\mathbb{C}$-vector space V can be expressed as the direct sum of generalized eigenspaces of an arbitrarily chosen operator $T \in L(V)$. Show that T can be uniquely written as the sum of a semisimple and nilpotent operator. Moreover, one can pick a basis of V such that the matrix representing the nilpotent operator consists of elementary nilpotent blocks.

2. Show that the set of operators with n distinct eigenvalues is dense and open in $L(\mathbb{R}^n)$.

3. Show that the set of operators giving rise to contractions is open but not dense in $L(\mathbb{R}^n)$.

4. Show that the set of operators giving rise to hyperbolic flows is open and dense in $L(\mathbb{R}^n)$.

References

- "Differential Equations, Dynamical Systems, and Linear Algebra" (M. Hirsch and S. Smale)