

# Math 108C Homework 3

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1.) Show that the nullspace of the matrix  $AB$  contains the null space of  $B$ .

**Proof:**  
By definition  $\text{null}(AB) = \{x \in \mathbb{R}^n : AB(x) = 0\}$ .  
Now suppose  $x \in \text{null}(B)$ , then this implies that  $Bx = 0$ .  
Applying  $A$  yields  $A(Bx) = 0 \implies (AB)x = 0$ .  
This result is exactly the definition of  $\text{null}(AB)$  so the null space of  $AB$  contains null space of  $B$ .

3.) Show that the Gram matrix  $A^T A$  has the same null space as  $A$ .

**Proof:**  
Taking an arbitrary  $x \in \mathbb{R}^n$ , where  $x \in \text{null}(A^T A) \implies A^T A x = 0$ , then  $x^T A^T A x = 0$ .  
Multiplying  $x^T$  yields  $0 = x^T A A^T x = (A^T x^T)(Ax) \implies Ax = 0$ . Therefore  $x \in \text{null}(A)$ .

8.) Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 0 & 1 \end{bmatrix}$$

Find a CR decomposition of  $A$  and use it to provide a basis for  $\text{col}(A)$  and  $\text{col}(A^T)$ .  
What are the dimensions of those two spaces? Is the factorization that you found the only possibility? Explain.

**Proof:**  
A CR decomposition of  $A$  is given by:  
$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
  
This factorization provides us with a basis for column space of  $A$  which is:

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

This has dimension of  $2 \times 2$ . Meanwhile, a basis for the column space of  $A^T$  is give to us as:

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Which has a dimension of  $2 \times 4$ . Since the row reduction, which is essential to the process of CR factorization, is unique.  
The CR factorization is the only possibility.

11.) A social researcher is interested in the relationship between income and happiness.  
They survey 500 people (see excel file) whose income ranges from \$15k to \$75k and ask them to rank their happiness on a scale from 1 to 10. (Note: the income is presented in the excel sheet divided by 10,000).  
1. Plot the data and determine by inspection if there is a linear relationship between income and happiness.  
2. Compute the line that best fit the provided data.  
3. Graph the data with the linear regression line.  
4. Use the linear model to estimate the happiness of a person whose income is \$60,000.  
<br> &nbsp;&nbsp;&nbsp;<b>5. <br> Is it safe to use this model to predict the happiness of a person whose income is \$80,000?

**Proof:**  
1.

```
In [4]: import csv
import matplotlib.pyplot as plt
from pathlib import Path
from pprint import pprint

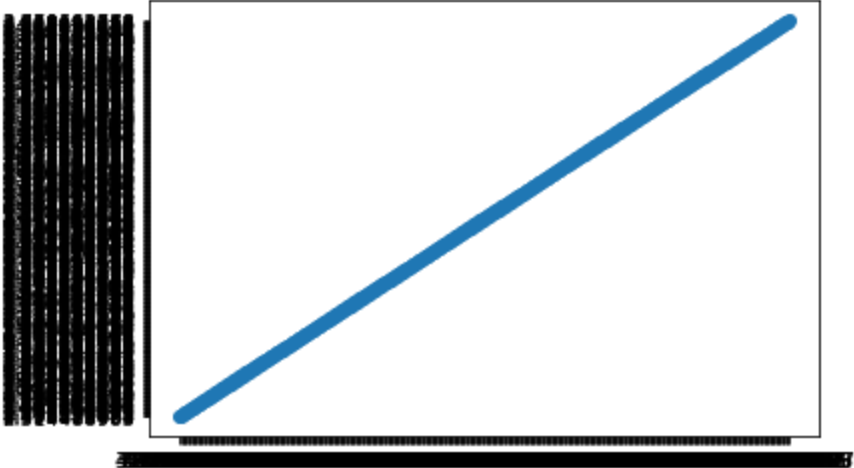
data_file_path = Path(r"./HW3 Income Data.csv").resolve()

with data_file_path.open("r") as f:
    csv_data = csv.reader(f)
    header = next(csv_data)
    result = [row for row in csv_data]
income = [row[1] for row in result]
happiness = [row[2] for row in result]

plt.scatter(income, happiness)

# ax.set_xlabel("Happiness (1 to 10)")
# ax.set_ylabel("Income (in $10,000)")
```

Out[4]: <matplotlib.collections.PathCollection at 0x129347cf388>



- 2. did not have time to finish
- 3. did not have time to finish
- 4. did not have time to finish
- 5. did not have time to finish