Math 122B Homework 4

Rad Mallari

May 5, 2022

1 Problem 1

Verify directly that the function $f(z) = z^2 + z + 1$ is injective in a neighborhood of the point $z = 0$.
Proof. A function $f(z)$ is locally one to one around z_0 if it is analytic at z_0 and $f'(z_0) \neq 0$. Clearly $f(z)$ is analytic at 0, meanwhile $f'(0) = 0^2 + 0 + 1 \neq 0$, therefore $f(z)$ is injective about $z = 0$. $\square$

2 Problem 2

Find the image of the square $|\Re z| < 1$, $|\Im z| < 1$, under the exponential map $z \mapsto e^z$.
Proof. Letting $z = x+iy$, we can rewrite our exponential map as $e^z = e^x e^{iy}$. Defining z this way, we know that $\Re z = x$, meanwhile $\Im z = y$. Therefore, we have a square region $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. Now we can rewrite our mapping as $w = \rho e^{i\phi}$, where $\rho = e^x$ and $\phi = y$. This tells us that our mapping is a circle around 0 with radius 1. $\square$

3 Problem 3

Describe a conformal map of the infinite band $-2 < \Re z < 1$, onto the unit disk.
Proof.

$\square$

4 Problem 4

Find all conformal mappings $h(z)$ from the upper-half plane to itself satisfying $f(i) = i$.
Proof.

$\square$