Assignment 2

For each of the next **two problems**, only do those parts for which answers are in the back. Be sure to explain your answers.

1. Exercise 1, page 27

2. Exercise 2, page 27

3. Exercise 3, page 27.

4. Using the norms $||f||_\infty = \sup\{|f(s)| : s \in [0,1]\}$ and $||f||_1 = \int_0^1 |f(s)|ds\}$ on $D_\infty[0,1]$ as domain or range, is either ι or δ a bounded function?

There are really 4 questions here.

"Does there exist a real number M such that if $f \in D_\infty[0,1], ||f||_1 = 1$, then $||\iota(f)||_\infty \le M$".

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Then do 4 more by replacing ι with δ. Eight problems in all.

For each of these eight problems, you must either specify an M (e.g. $M = 13$) and prove the desired inequality, OR you must assume (proof by contradiction) that some unspecified number M works and specify an $f \in D_\infty[0,1]$ (depending on M) for which the inequality fails.