Math 104B Homework 4

Rad Mallari

May 9, 2022

1 Problem 1

Find α so that

$$A = \begin{bmatrix} \alpha & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 4 \end{bmatrix}$$

is positive definite.
Proof.

$\square$

2 Problem 2

Let A and B be $n \times n$ matrices. Prove that

$$||A + B||_\infty \leq ||A||_\infty + ||B||_\infty$$

Proof.

$\square$

3 Problem 3

Let

$$A = \begin{bmatrix} 2 & 1 & -10 \\ 1 & 2 & 1 \\ -5 & 1 & 4 \end{bmatrix}$$

Find $||A||_1$ and $||A||_\infty$.
Proof.

$\square$

4 Problem 4

Let S be a nonsingular $n \times n$ matrix and let $||\cdot||$ be an induced matrix norm. Prove that $||S^{-1}AS||$ defines a matrix norm for all $n \times n$ matrices A.
Proof.

$\square$

5 Problem 5

Let I be the $n \times n$ identity matrix. Prove that $||I|| = 1$ for all induced norms.
Proof.

$\square$

6 Problem 6

Prove that the condition number of a nonsingular $n \times n$ matrix A is at least 1, i.e. $1 \leq ||A||||A^{-1}||$, for all induced matrix norms.
Proof.

$\square$

7 Problem 7

Let

$$A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

Find $||A||_2$.
Proof.

$\square$

8 Problem 8

Compute the condition number $\kappa_1(A) = ||A||_1||A^{-1}||_1$ for

$$A = \begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix}$$

Proof.

$\square$

9 Problem 9

Prove that the ocndition number satisfies the proper $\kappa(\lambda A) = \kappa(A)$ for all nonzero λ, scalar.

Proof.

$\square$