

Math 108C Homework 1

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1.) Compute the product of the following matrices using the column-row definition of matrix multiplication

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \\ 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$$

Proof:

We can do this by **Proposition 5, Section 4.3** which yields:

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 3 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 0 \\ -6 & 3 & 0 \\ 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 1 \\ -4 & 3 & 2 \\ 2 & -1 & 0 \end{bmatrix}$$

3.) Express the product of AB in terms of

1. A and the columns of B .
2. B and the rows of A .

Proof:

The columns of AB can be expressed as A times the i th column of B .

That is, $AB = A[b_1, b_2, \dots, b_j] = [Ab_1, Ab_2, \dots, Ab_j]$ where b_i is the i th column of B .

2. The rows of AB can be expressed as the j th row of A times B , i.e.,

$$AB = \begin{bmatrix} a_1^T B \\ a_2^T B \\ \vdots \\ a_j^T B \end{bmatrix} \text{ where } a_j^T \text{ is the } j\text{th row of } A.$$

4.) Assume the following matrices are partitioned conformably for block multiplication.

Compute the product

$$\begin{bmatrix} I & 0 \\ -X & I \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Proof:

By **Theorem 2**, the product is given by:

$$\begin{bmatrix} AI & BI \\ -AX + CI & -BX + DI \end{bmatrix}$$

13.)(Matlab/Python)For block operations, it may be necessary to access or enter submatrices of a large matrix.

Describe the function or commands in Matlab that accomplish the following tasks.

Suppose A is 20×30 matrix.

(a) Display the submatrix of A from rows 15 to 20 and columns 5 to 10.

(b) Insert a 5×10 matrix B into A , beginning at row 10 and column 20.

(c) Create a 50×50 matrix of the form

$$B = \begin{bmatrix} A & 0 \\ 0 & A^T \end{bmatrix}$$

Proof:

We can use 2D list in Python to represent a matrix.

Therefore, a block matrix can be represented by a 2D list, with each element being another 2d list.

In [1]:

```
import numpy
big_matrix = [[f"A_{r+1}_{c+1}" for c in range(30)] for r in range(20)]
print("(a)")
for row in big_matrix[14:20]:
    print(row[4:10])

print("(b)")
matrix_B = [[f"B_{r+1}_{c+1}" for c in range(10)] for r in range(5)]
for i in range(10,20):
    big_matrix[i][20] = matrix_B
print("row 10, column 20 of matrix A:")
for x in big_matrix[10][20]:
    print(x)
print("row 15, column 20 of matrix A")
for x in big_matrix[15][20]:
    print(x)

print("(c)")
very_big_matrix = [[big_matrix for c in range(50)] for r in range(50)]
matrix_B = [[big_matrix for i in range(2)] for j in range(2)]
# too braindead to do this properly.
matrix_B[0][1] = 0
matrix_B[1][0] = 0
t_big_matrix = [list(x) for x in numpy.transpose(big_matrix)]
matrix_B[1][1] = t_big_matrix
for i in range(10,20):
    very_big_matrix[i][20]=matrix_B
print("row 15, column 20 of matrix A")
for x in very_big_matrix[15][20]:
    print(x)
```

```
(a)
['A_15.5', 'A_15.6', 'A_15.7', 'A_15.8', 'A_15.9', 'A_15.10']
['A_16.5', 'A_16.6', 'A_16.7', 'A_16.8', 'A_16.9', 'A_16.10']
['A_17.5', 'A_17.6', 'A_17.7', 'A_17.8', 'A_17.9', 'A_17.10']
['A_18.5', 'A_18.6', 'A_18.7', 'A_18.8', 'A_18.9', 'A_18.10']
['A_19.5', 'A_19.6', 'A_19.7', 'A_19.8', 'A_19.9', 'A_19.10']
['A_20.5', 'A_20.6', 'A_20.7', 'A_20.8', 'A_20.9', 'A_20.10']
(b)
row 10, column 20 of matrix A:
['B_1.1', 'B_1.2', 'B_1.3', 'B_1.4', 'B_1.5', 'B_1.6', 'B_1.7', 'B_1.8', 'B_1.9', 'B_1.10']
['B_2.1', 'B_2.2', 'B_2.3', 'B_2.4', 'B_2.5', 'B_2.6', 'B_2.7', 'B_2.8', 'B_2.9', 'B_2.10']
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row 15, column 20 of matrix A
['B_1.1', 'B_1.2', 'B_1.3', 'B_1.4', 'B_1.5', 'B_1.6', 'B_1.7', 'B_1.8', 'B_1.9', 'B_1.10']
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(c)
row 15, column 20 of matrix A
[[['A_1.1', 'A_1.2', 'A_1.3', 'A_1.4', 'A_1.5', 'A_1.6', 'A_1.7', 'A_1.8', 'A_1.9', 'A_1.10', 'A_1.11', 'A_1.12', 'A_1.13', 'A_1.14', 'A_1.15', 'A_1.16', 'A_1.17', 'A_1.18', 'A_1.19', 'A_1.20', 'A_1.21', 'A_1.22', 'A_1.23', 'A_1.24', 'A_1.25', 'A_1.26', 'A_1.27', 'A_1.28', 'A_1.29', 'A_1.30'], ['A_2.1', 'A_2.2', 'A_2.3', 'A_2.4', 'A_2.5', 'A_2.6', 'A_2.7', 'A_2.8', 'A_2.9', 'A_2.10', 'A_2.11', 'A_2.12', 'A_2.13', 'A_2.14', 'A_2.15', 'A_2.16', 'A_2.17', 'A_2.18', 'A_2.19', 'A_2.20', 'A_2.21', 'A_2.22', 'A_2.23', 'A_2.24', 'A_2.25', 'A_2.26', 'A_2.27', 'A_2.28', 'A_2.29', 'A_2.30'], ['A_3.1', 'A_3.2', 'A_3.3', 'A_3.4', 'A_3.5', 'A_3.6', 'A_3.7', 'A_3.8', 'A_3.9', 'A_3.10', 'A_3.11', 'A_3.12', 'A_3.13', 'A_3.14', 'A_3.15', 'A_3.16', 'A_3.17', 'A_3.18', 'A_3.19', 'A_3.20', 'A_3.21', 'A_3.22', 'A_3.23', 'A_3.24', 'A_3.25', 'A_3.26', 'A_3.27', 'A_3.28', 'A_3.29', 'A_3.30'], ['A_4.1', 'A_4.2', 'A_4.3', 'A_4.4', 'A_4.5', 'A_4.6', 'A_4.7', 'A_4.8', 'A_4.9', 'A_4.10', 'A_4.11', 'A_4.12', 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