Math 122B Homework 5

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1 Problem 1

Find a conformal mapping of the upper half-plane onto a equilateral triangle.
Proof. Similar to **Section 13.3** in the book, we can use

$$\begin{aligned} f(z) = \int_0^z f'(\zeta)d\zeta &= \int_0^z \frac{1}{(\zeta-1)^{\frac{2}{3}}(\zeta+1)^{\frac{2}{3}}}d\zeta \\ &= \int_0^z \frac{1}{(\zeta^2-1)^{\frac{2}{3}}} \end{aligned} \tag{1}$$

□

2 Problem 2

Suppose f is an entire function mapping a rectangle to a rectangle. Prove that $f(z) = az + b$, where $a \neq 0$ and b are complex numbers.
Proof.

□

3 Problem 3

Let R be an open, simply connected subset of the complex plane. Let $z_1, z_2 \in R$. Prove that there exists a conformal mapping of R onto R which takes z_1 into z_2.
Proof.

□

4 Problem 4

Let R be an open, simply connected domain, different than the entire complex $\mathbb{C}$. Prove that there exists no conformal mapping of $\mathbb{C}$ onto R.

Proof. Since R is an open, simply connected domain, it must be the case that there exists a comformal map f from R to our domain D by the Riemann Mapping theorem. Say there exists a map g from $\mathbb{C}$ to R, then $f \circ g$ is a comformal map from C to D, and since it is bounded, it must be constant. This tells us that g is not injective, thereby contradiction. $\square$