The (Micro) Genetic Algorithm - generally and specifically

This is project report for OMfE course. In this report:

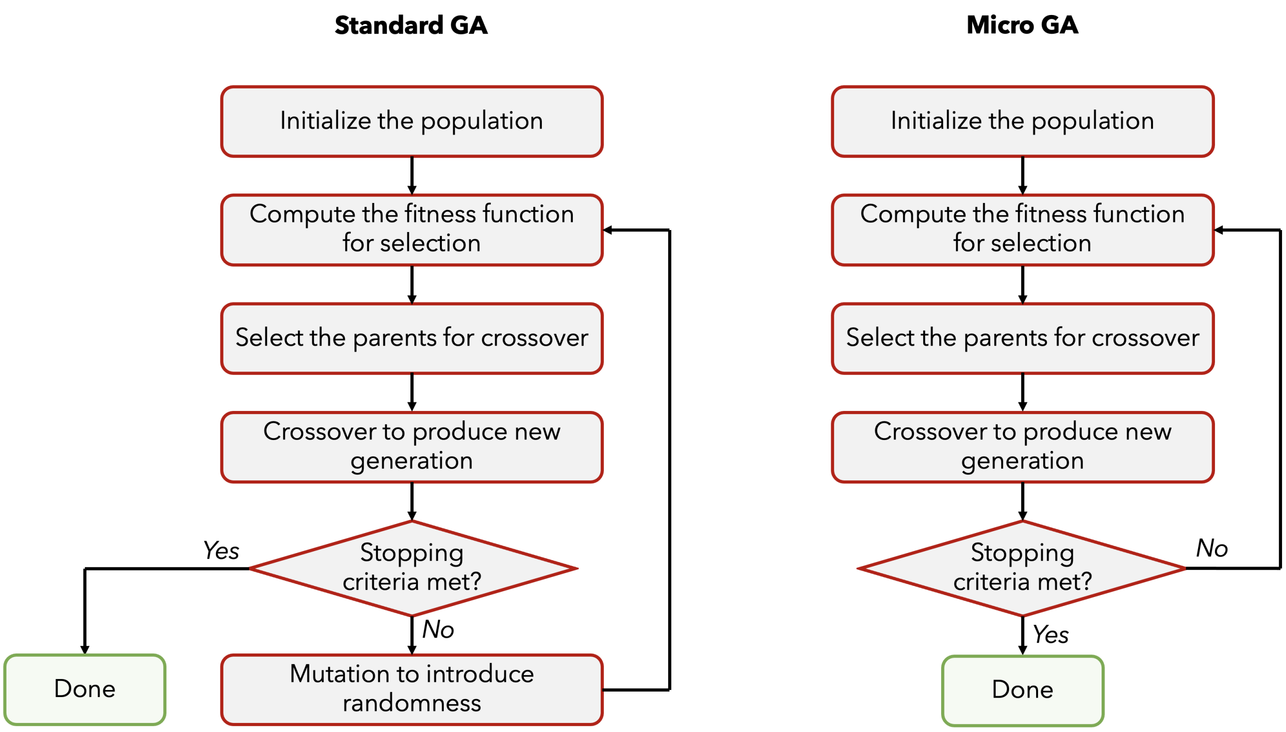
1. The implementation of the Genetic Algorithm
2. Problem #1 - Optimal allocation of resources
3. Problem #2 - Optimizing the design and conditions for an aircraft
4. Problem #3 - Optimizing the power of an engine

Our complete implementation can be found here: https://github.com/radimurban/opt_methods_project/tree/main

1. The (Micro) Genetic Algorithm

The genetic algorithm is a method for solving optimization problems that is based on natural selection. The genetic algorithm repeatedly improves a population of individuals. At each iteration, the GA selects parents from the current population of individuals (*ranking*), which then produce the children for the next generation (*breeding*) using crossover and mutation (if this is omitted, we talk about *Micro GA*). Over many generations, the population converges to an optimal solution by replacing the weakest individuals by the children.

This process can be easily described using the following flowchart:



To summarize: **Population** is the set of all the **individuals** (also called **chromosomes**). Each chromosome carries some information about itself stored and segmented into **genes**.

We implement the Micro GA on a high level by specifying the necessary aspects of the algorithm.

Population Initilization

Random population of N individuals p_n (initialization), where p_n is a bitstring of length L.

```
public Population(int size) {
    individuals = new Chromosome[size];
    random = new Random();
    for (int i = 0; i < size; i++) {
        int[] genes = new int[10];
        for (int j = 0; j < genes.length; j++) {
            genes[j] = random.nextInt(2);
        }
        individuals[i] = new Chromosome(genes);
    }
}
```

Fitness Function

This is the metric for choosing the parents and individual to be replaced by the child(ren). In this implementation the fitness function is the number of bits in individual that are 1.

$$F(p_n) = \sum_{i=0}^{L-1} p_{n_i} \quad \text{where } p_{n_i} \text{ is the i-th bit in } p_n$$

```
private double evaluate(Chromosome individual) {
        // This implementation simply returns the sum of the genes
        int[] genes = individual.getGenes();
        double sum = 0;
        for (int gene : genes) {
                sum += gene;
        }
        return sum;
}
```

Parents Selection

Generally speaking, there are many ways to select parents. In this simple implementation, we use the tournament methods. To choose one parent, we randomly choose x individuals from the population and select the one with the highest fitness function.

```
public Chromosome select(){
  Chromosome best = null;
  for (int i = 0; i < 5; i++) {
      Chromosome individual = individuals[random.nextInt(individuals.length)];
      if (best == null || individual.getFitness() > best.getFitness()) {
          best = individual;
      }
  }
  return best;
}
```

Children Generation

We choose to implement **one-point crossover**, i.e. randomly generate a midpoint $\in [0; L]$ and generate child by taking bits from *0* to *midpoint* from *parent1* and the rest from *parent2*.

```
public Chromosome crossover(Chromosome other) {
    int[] childGenes = new int[genes.length];
    int midpoint = random.nextInt(genes.length);
```

```
        for (int i = 0; i < midpoint; i++) {
            childGenes[i] = genes[i];
        }
        for (int i = midpoint; i<genes.length; i++) {
          childGenes[i] = other.genes[i];
        }
        return new Chromosome(childGenes);
    }
```

We then replace (**updating the population**) this new child with the weakest individual (individual) with the lowest fitness function.

```
public void replaceWorst(Chromosome child) {
    int worstIndex = 0;
    double worstFitness = Double.MAX_VALUE;
    for (int i = 0; i < individuals.length; i++) {
        if (individuals[i].getFitness() < worstFitness) {
            worstIndex = i;
            worstFitness = individuals[i].getFitness();
        }
    }
    individuals[worstIndex] = child;
}
```

Stopping Criteria

For this implementation we only simply check pre-defined number of generations (=iterations).

```
while (generation <= MAX_GENERATIONS) {

    /*
     * Main Algorithm Loop
    */

    generation++;
}
```

2. Optimal allocation of resources

A practical and real-world example of optimizing resource allocation in a supply chain to minimize costs and maximize efficiency could be in the production of a consumer electronics product, such as a smartphone. In this example, the supply chain involves several stages, including the sourcing of raw materials, manufacturing of components, assembly of the final product, and distribution to retailers.
We decided to implement the following micro genetic algorithm to solve the problem of optimal resource allocation to minimize costs and maximize efficiency. In our example we simplify this to determining the optimal number of raw materials to purchase and which products to manufacture from the available resources.

Population

There are two classes, `Resource` and `Product`. Each population contains n resources and m products. Each resource and product object is represented mainly through a vector (array) with integer entries. This approach is known as **value encoding**. A member of the class `Resource` represents all the resources we can acquire, the total cost of these resources and the fitness score. A product that the company can make is represented by the class `Product`. Each product has information about the selling price and the resources necessary to produce it. The price of a product must be at least the cost of the needed resources (otherwise it does not make sense economically).

Both of these classes are managed by a `Manager`. The main purpose of the `Manager` class is to make testing easier. You only have to create one `Manager` object with the corresponding arguments and it takes care of all the initialization for you. There is also the possibility to enter the mutation rate for testing purposes to see how a micro genetic algorithm compares to a full genetic algorithm. Each Manager object has all the functions for the (micro)GA algorithm as well as important properties.

Fitness Function

Each `Resource` is evaluated by a fitness function that assigns a fitness score. Since we live in a capitalist society, **our goal is to maximize the profit while minimizing the cost**. We also want to produce as much as possible so we encourage filling up the warehouse with useful parts. The fitness is computed as follows:

$$F = \frac{\mathrm{maximal\_profit}}{\mathrm{total\_cost}}(\mathrm{npw} - \mathrm{nu\_penalty})$$

where:

- npw - **n**umber of **p**arts in the **w**arehouse
- $\mathrm{nu\_penalty}$ - **penalty** for parts **n**ot **u**sed in manufacturing the products

F is the quotient of the profit and the cost. Maximizing the profit or minimizing the cost leads to higher fitness score. Multiplying with the overall available resources penalized with the number of unneeded resources.
The `evalProfit` function computes the profit from products made out of the available resources. It starts with the most expensive product first and when there are no more enough resources it continues with the next most expensive product. The resources that are not left at the end are not enough to create any of the products and are used as a penalty since the resources have cost but no profit.

```
public double[] evalProfit(Resource res) {
        Arrays.sort(products, Comparator.comparingDouble(Product::getPrice).reversed());
        double profit = 0.0;
        int[] localCopy = Arrays.copyOf(res.getResources(), res.getResources().length);

        for (int i = 0; i < products.length; i++) {
                while(isAvailable(products[i].getNeededResources(), localCopy)) {
                        Product product = products[i];
                        Arrays.setAll(localCopy, j -> localCopy[j] -
product.getNeededResources()[j]);
                        profit += product.getPrice();
                }
        }
        double penalty = Arrays.stream(localCopy).sum();
        return new double[]{profit, penalty};
}

private boolean isAvailable(int[] needed, int[] available) {
        assert needed.length == available.length;
        for (int i = 0; i < available.length; i++) {
                if (available[i] - needed[i] < 0) {
                        return false;
                }
        }
        return true;
}
```

Parent Selection

We choose two parents in a **tournament selection** process. Both parents are Resource objects maximizing the fitness function among 5 randomly selected Resources. Selecting one parent will look like this:

```
public Resource select() {
        Resource best = null;
        for (int i = 0; i < 5; i++) {
                Resource res = resources[random.nextInt(n)];
                if (best == null || res.getFitness() > best.getFitness()) {
                        best = res;
                }
        }
        return best;
}
```

Generating Children

We use **two-point crossover**. For this problem, the multi-point crossover makes more sense than a single point crossover. All the products require different types and amounts of resources and it is better to better to mix up the current values so there is more room for new combinations. The optimal number of crossover points depend also on the nature of the products. As an example, if each product only required one unit of one resource then also a single point crossover could be sufficient. The more intrigue the products get the more you have to adjust the number of crossover points. In the comparison with the standard genetic algorithm this makes sense. For the mGA we can only use the values that we get during the initialization and they cannot be mutated. This also means that a higher number of first generation resources is better (and necessary) for mGA because there are more values to choose from during crossover. For this implementation. it is also necessary to keep track of the new cost and the number of resources. Upon creating the child we check in a do-while loop that the sum of the resources is under the limit.

```
public Resource crossover(Resource parent1, Resource parent2) {
        int[] childResources = new int[n];
        int crossoverPoint1 = random.nextInt(n);
        int crossoverPoint2 = random.nextInt(n - crossoverPoint1) + crossoverPoint1;
        double varCost = 0.0;

        for (int i = 0; i < crossoverPoint1; i++) {
            childResources[i] = parent1.getResources()[i];
            varCost += childResources[i] * prices[i];
        }
        for (int i = crossoverPoint1; i < crossoverPoint2; i++) {
            childResources[i] = parent2.getResources()[i];
            varCost += childResources[i] * prices[i];
        }
        for (int i = crossoverPoint2; i < n; i++) {
            childResources[i] = parent1.getResources()[i];
            varCost += childResources[i] * prices[i];
        }

        return new Resource(childResources, varCost);
    }
```

Sample Results

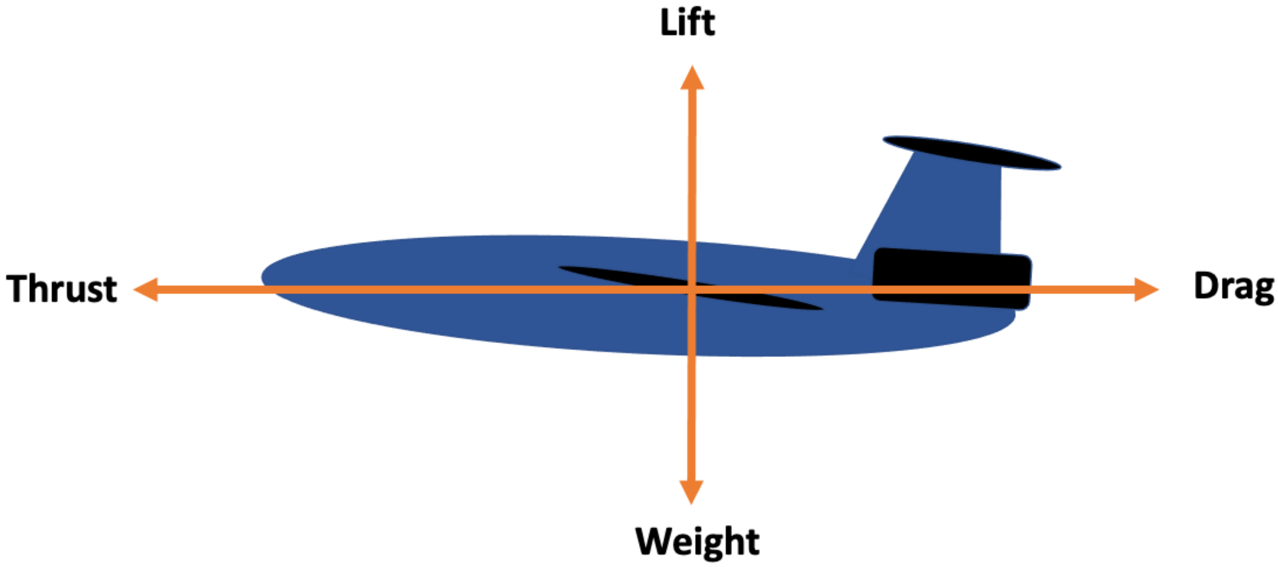
```
Generation: 0
Best allocation: [12, 14, 7]
Cost: 41.0
Profit: 115.5
Penalty: 5.0
Fitness: 78.8780487804878
```

```
Generation: 1
Best allocation: [12, 14, 16]
Cost: 59.0
Profit: 162.0
Penalty: 10.0
Fitness: 87.86440677966101

Generation: 2
Best allocation: [12, 14, 16]
Cost: 59.0
Profit: 162.0
Penalty: 10.0
Fitness: 87.86440677966101
...
...
...
Generation: 98
Best allocation: [12, 14, 16]
Cost: 59.0
Profit: 162.0
Penalty: 10.0
Fitness: 87.86440677966101

Generation: 99
Best allocation: [12, 14, 16]
Cost: 59.0
Profit: 162.0
Penalty: 10.0
Fitness: 87.86440677966101

Generation: 100
Best allocation: [12, 14, 16]
Cost: 59.0
Profit: 162.0
Penalty: 10.0
Fitness: 87.86440677966101
```

As a bonus, add graph/plot of micro vs. full simulations

3. Find the optimal design and conditions for an aircraft

We model this problem as having a few components&conditions for which we want to maximize the lift force. To limit the ranges of the components, we assume we are modeling this problem on an airliner (e.g. Boeing 777 and similar).



Population

Let's assume that each candidate solution (i.e., *chromosome*) in the population is represented by a vector $p = (V, S, \alpha, e, AR) \in \mathbb{R}^5$ of design variables that define a part of an aircraft. (**Value Encoding**) The dimensions correspond to:

- V is the speed of the aircraft $(m/s) \in [50; 350]$
- S is the wing area $(m^2) \in [60; 200]$
- α is the angle of attack (rad) $\in [0; 0.43]$
- e is the Oswald efficiency factor $\in]0; 1]$
- AR is the wing aspect ratio $\in [5; 15]$

Fitness function

Each chromosome is evaluated by a fitness function that computes its performance and assigns a fitness score. In this case we want to compute the maximum lift.

Lift (L): The lift generated by the plane depends on the air density, the speed of the aircraft, and the geometry of the wing. The lift can be computed using the following formula:

$$L = \frac{1}{2}\rho V^2 S \frac{(2\pi \cdot \alpha)}{(1 + (\pi \cdot e \cdot AR))}$$

where:

- ρ is the air density (kg/m^3) -> Assume constant at $1.293 kg/m^3$

Parent Selection

We will choose two parents. Both as chromosome maximizing the fitness function among 10 randomly selected chromosomes (**Tournament Selection**)
Some randomness might make sense because for example of different conditions for the plane (meaning, the chromosome maximizing the fitness function might not maximize it in all conditions).

Selecting one parent will look like this:

```
Chromosome best = null;
    for (int i = 0; i < 10; i++) {
```

```
        Chromosome individual = individuals[random.nextInt(individuals.length)];
        if (best == null || individual.getFitness() > best.getFitness()) {
            best = individual;
        }
    }
return best;
```

Generating Children

We generate children as follows. To optimize but also to keep randomness in the process we *randomly* mix the genes of the two parents. (**Uniform Crossover** with randomly generated bit-mask). Here, *bit-mask* is just a bit-vector determining which parent the child inherits the value from.

```
public Chromosome[] crossover(Chromosome other) {
    double[] childGenes = new double[genes.length];
    double[] otherChildGenes = new double[genes.length];

    for (int i = 0; i < childGenes.length; i++) {
        boolean mask = random.nextBoolean();

        // Mask bit stays unchanged for both assignments => children inherit from different
parents
        childGenes[i] = mask ? this.getGenes()[i] : other.getGenes()[i];
        otherChildGenes[i] = mask ? other.getGenes()[i] : this.getGenes()[i];

    }

    Chromosome first_child = new Chromosome(childGenes);
    Chromosome second_child = new Chromosome(otherChildGenes);
    Chromosome[] children = {first_child, second_child};

    return children;
}
```

We then replace the worst individuals (lowest fitness score) with just generated children.

```
// Crossover
Chromosome[] children = parent1.crossover(parent2);

// Replace x worst individuals with x children
for (Chromosome child : children){
    population.replaceWorst(child);
}
```

Stopping Criteria

We will pre-define the number of generations we want to optimize over and abort after achieving this number. We observe, that in 9 from 10 runs, the algorithm converges around 75th generation/iteration, i.e. the algorithms settles on a final value at around 75th iteration.

Sample Results

Following result was obtained by having `POPULATION_SIZE = 150` and stopping criteria constant `MAX_GENERATIONS = 100`.

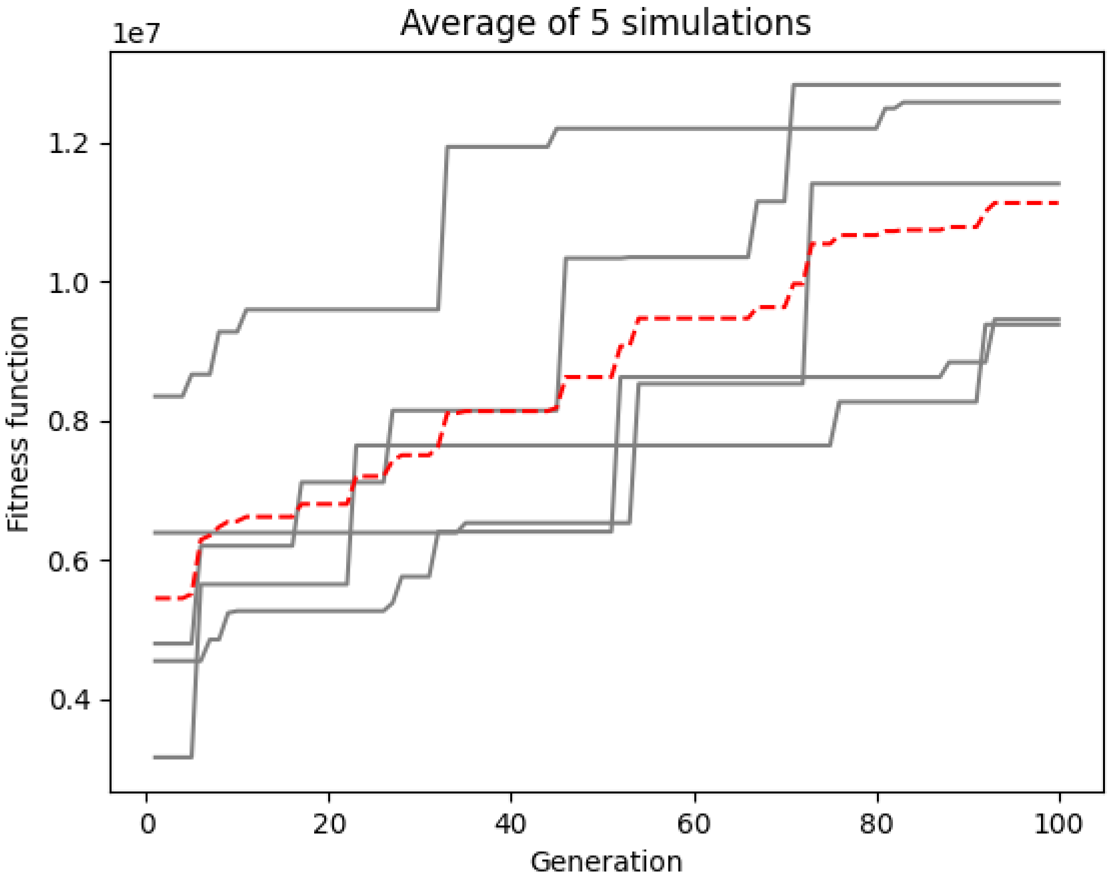
```
Generation: 1
Best fitness: 8121325.844336658
```

```
Generation: 2
Best fitness: 8121325.844336658
Generation: 3
Best fitness: 8121325.844336658
...
...
...
Generation: 73
Best fitness: 9273070.677375384
Generation: 74
Best fitness: 9273070.677375384
Generation: 75
Best fitness: 9273070.677375384
Generation: 76
Best fitness: 1.2647211860884406E7

...
...

Generation: 96
Best fitness: 1.2647211860884406E7
Generation: 97
Best fitness: 1.2647211860884406E7
Generation: 98
Best fitness: 1.2647211860884406E7
Generation: 99
Best fitness: 1.2647211860884406E7
Generation: 100
Best fitness: 1.2647211860884406E7

------Component--------|----Value----

Speed of the aircraift:   345.61
Wing area:                189.95
Angle of attack:          0.41
Oswald eff. factor:       0.12
Wing aspect ratio:        5.29
```

Result is returning the maximized Lift L in *Newtons (N)*. At the end we also extract the parameters for which the fitness function was maximized.

We can reason that the delievered result is somewhat reasonable by realizing that a loaded airplane can weigh *up to* $600.000kg$ corresponding rouhgly to needed lift of at least $6MN$. At generation 100, the best configuration has a lift force of roughly $9.5MN$ (computed average of 10 simulations). Hence in the range of reasonable results. In real life achieving this lift by an airplane might be impossible and also unnecessary - it's more about a half of our computed maximum. Result is coming from simplifying and theoretizing about the components and their ranges.

How quickly the algorithm (in terms of generations) finds increases the found maximum of the list can be seen the following plot of the average lift over 5 independent simulations.



4. Optimizing the power of an engine

Optimizing the mean effective pressure(MEP), stroke, bore and revolutions per minute to achieve maximal power output of an engine.

Population

Let's assume that each candidate solution (i.e., chromosome) in the population is represented by a vector $p = (\text{MEP}, \text{stroke}, \text{bore}, \text{revs})$ of design variables that define a part of an aircraft. The type of encoding used to represent the population in the algorithm is called value encoding, since we have specific values for every gene. The dimensions correspond to:

- MEP is a measure of the average pressure exerted by the gases in the combustion chamber of an engine during the power stroke. $\in [170; 280]$ measured in (psi)
- Strokelength is the distance that the piston travels in the cylinder between the top dead center (TDC) and the bottom dead center (BDC) positions. $\in [0.27; 0.3]$ measured in (ft)
- Bore is the diameter of the cylinder in which the piston moves $\in [2.9; 3.5]$ measured in (in)
- Revs refer to the number of times an engine's crankshaft rotates in a given period of time. $\in [0; 1]$ measured in (rpm)
  For the example we choose a specific range of values that represent the specification of a typical diesel engine.

Fitness function

Each member of the population is evaluated using a fitness function that computes the power output of engine. We will predefine the number of cylinders, which is also a part of the formula. In this case we want to the power of an engine (in kW).

$$Power(p) = \frac{(\text{Number of cylinders}) \cdot \text{MEP} \cdot \text{Strokelength} \cdot (\frac{\pi}{4}) \cdot (\text{Bore}^2) \cdot \text{Revs}}{2 \cdot 33000}$$

Parent Selection

We use a **Roulette Wheel Selection**, where the individuals with better fitness have greater probability to be selected. The method randomly chooses an individual by spinning a "roulette wheel" based on the assigned probabilities. The following code is used to achieve this:

```
private static int selectParent(double[] fitness) {
        double totalFitness = 0.0;
        for (double f : fitness) {
            totalFitness += f;
        }

        double rand = random.nextDouble() * totalFitness;
        int index = 0;
        while (rand > 0) {
            rand -= fitness[index];
            index++;
        }
        index--;

        return index;
    }
```

Generating Children

We generate children by randomly mixing up the genes of parents (**Uniform Crossover with randomly generated mask**):

```
private static double[] crossover(double[] parent1, double[] parent2) {
        double[] offspring = new double[4];
        for (int i = 0; i < 4; i++) {
            offspring[i] = random.nextBoolean() ? parent1[i] : parent2[i];
        }

        return offspring;
    }
```

Stopping Criteria

Maximal number of generations will be pre-defined. As one can see in the next section, the algorithm converges very fast to a result and then doesn't change much for the rest of the iterations.

Sample Results

After letting the program run once we obtained the following results:

```
Current generation: 0 Power: 172.30875761283218
Current generation: 1 Power: 179.41164990040235
Current generation: 2 Power: 176.32772827419709
Current generation: 3 Power: 180.43241667622289
Current generation: 4 Power: 181.07112755762162
Current generation: 5 Power: 181.07112755762162
Current generation: 6 Power: 176.0395650676611
Current generation: 7 Power: 179.7047058207012
```

```
Current generation: 8 Power: 182.55219091640774
Current generation: 9 Power: 180.99903878634007
Current generation: 10 Power: 182.72063746713687

...

Current generation: 20 Power: 185.77070982524214
Current generation: 21 Power: 192.74861000371254
Current generation: 22 Power: 192.74861000371254
Current generation: 23 Power: 191.36705308728457

...

Current generation: 97 Power: 192.74861000371254
Current generation: 98 Power: 192.74861000371254
Current generation: 99 Power: 192.74861000371254

Best engine parameters:
-------------------------------------------------
Mean effective pressure: 279.5649404443828
Stroke:                  0.2996102145516404
Bore:                    3.4885951446065233
Revs:                    3972.3257121139554
Power:                   192.74861000371254
```

We can see the resulting sizes of the optimal engine. If we calculate the displacement of the engine in this case, where we predefined that the engine has 4 cylinders, we get about 136 cubic inches, which is about 2.2 liters. This power output for the displacement of the engine seems very reasonable.

The algorithm converges very fast to a result and then doesn't change much for the rest of the iterations.