The Buck Converter
Part 1

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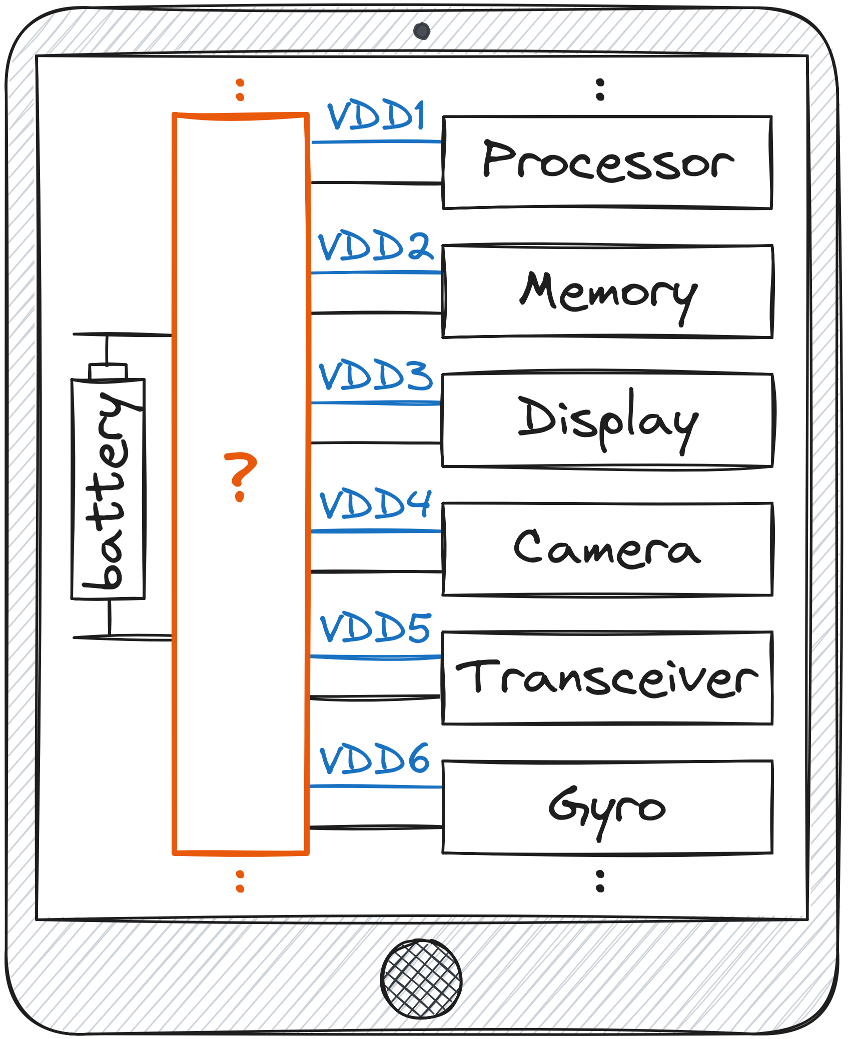
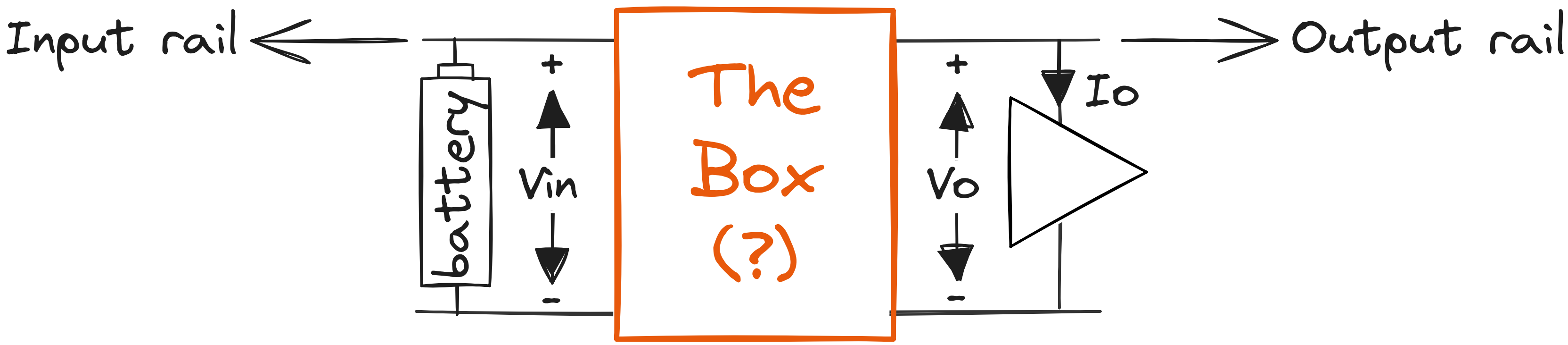
Given a single battery, how can we generate multiple and independent voltage rails to simultaneously power different subsystems?



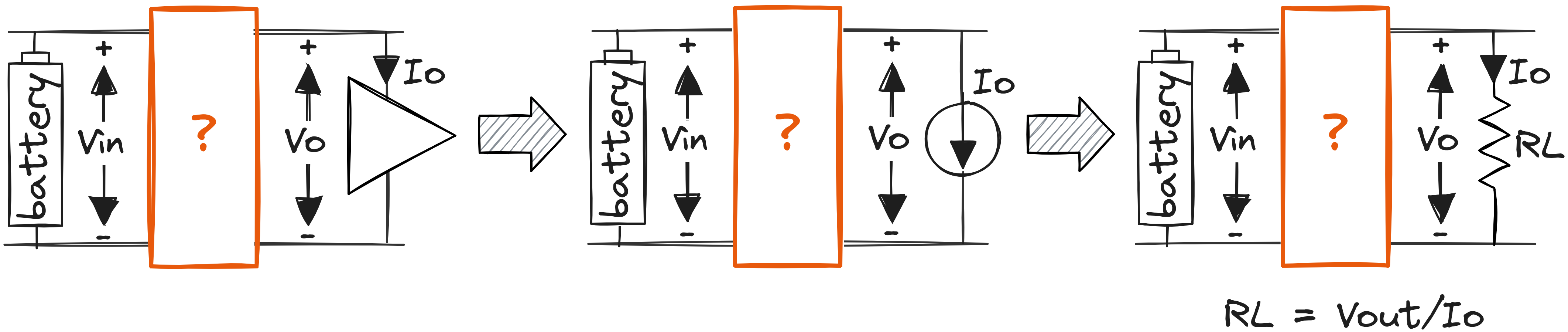
Let's start with a simple case:

A single Li-ion battery and a single circuit

Now the question is:



From the point of view of the output rail,
the circuit can be viewed as a DC current source Io,
or a resistive load RL drawing the equivalent DC current



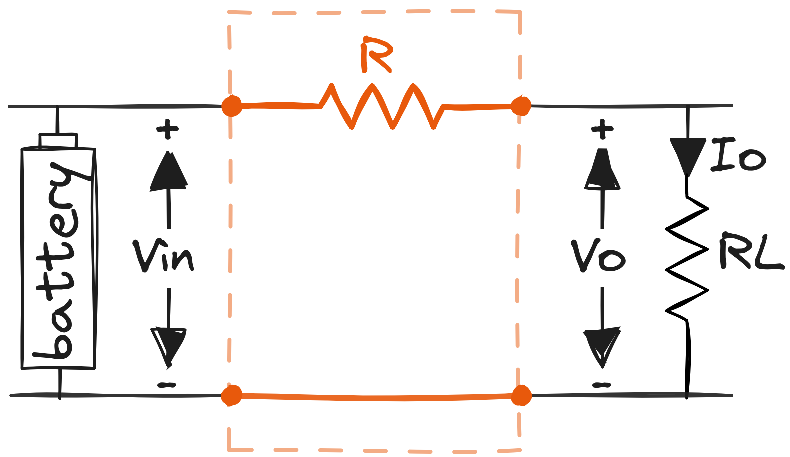
We focus on generating a Vo that is less than Vin

Using the resistive load model, the mystery box interfacing the battery to the circuit can be a simple resistor

The resistor and the load form a voltage-divider that steps-down the battery voltage to the desired output voltage

$$Vo = Vin \times RL/(R+RL) = Vin \times K$$

$$K = RL/(R+RL)$$



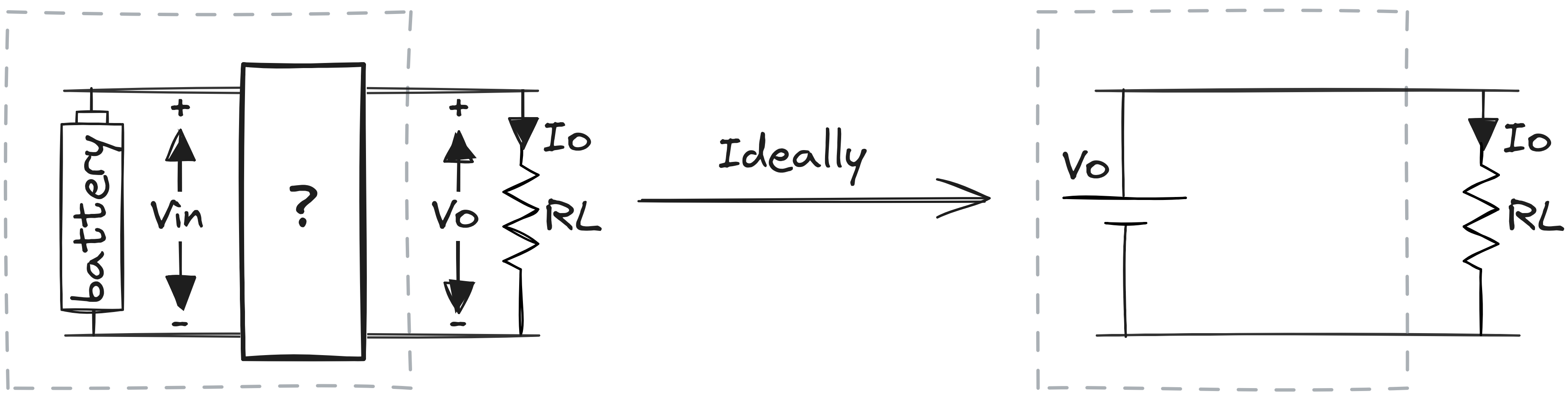
This is not a very good solution though ..

Issue #1: Output voltage is unregulated

We would like the output rail to be an ideal voltage source

An ideal voltage source is independent of load current, battery voltage, temperature .. etc

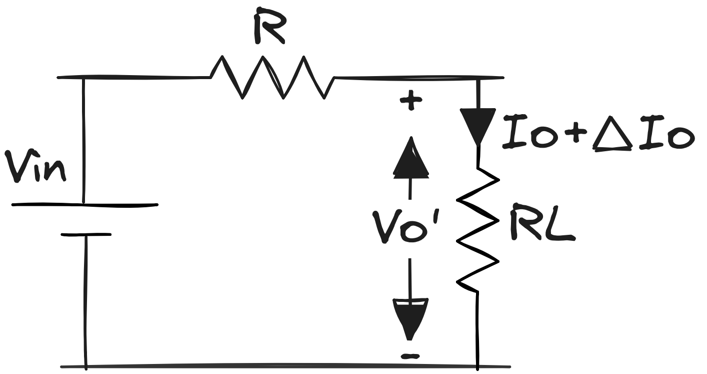
We call this a regulated voltage



Issue #1: Output voltage is unregulated (continued)

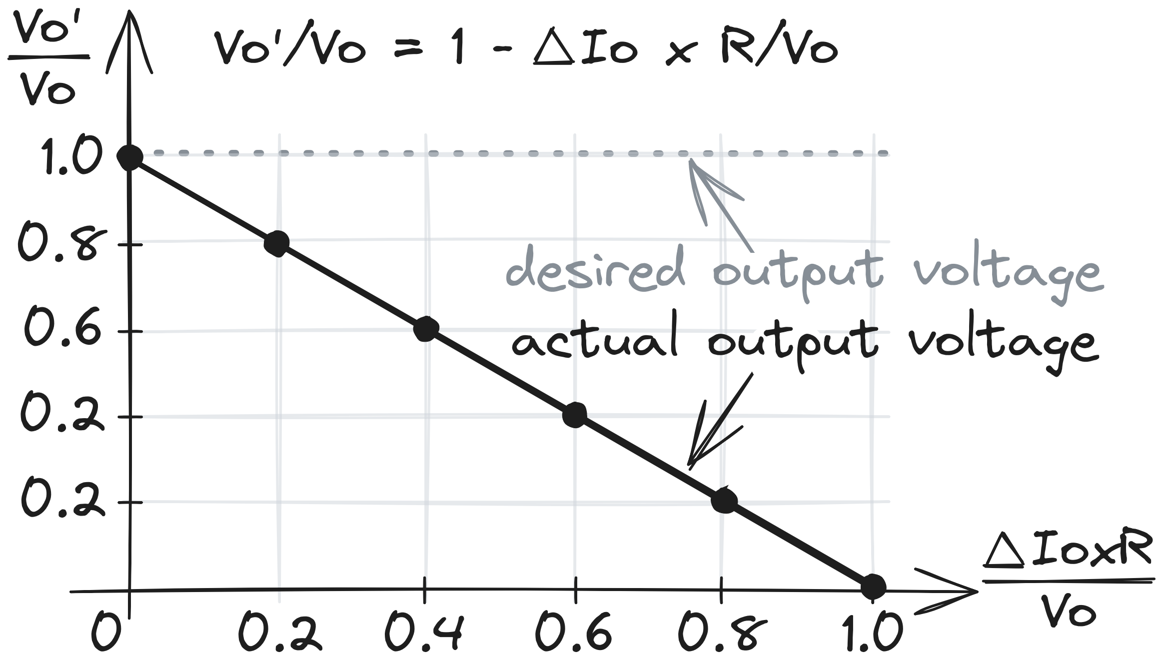
Load current is not always constant,
e.g. the same circuit can have a high power mode and low power mode

With a resistive divider, a ΔI_o change in load current causes a
causes a $\Delta I_o \times R$ change in output voltage



$$V_o' = V_{in} - (I_o + \Delta I_o) \times R$$

$$V_o' = \underbrace{V_{in} - I_o \times R}_{\text{Ideal } V_o \ @\ \Delta I_o = 0} - \underbrace{\Delta I_o \times R}_{\Delta V_o \text{ due to } \Delta I_o}$$

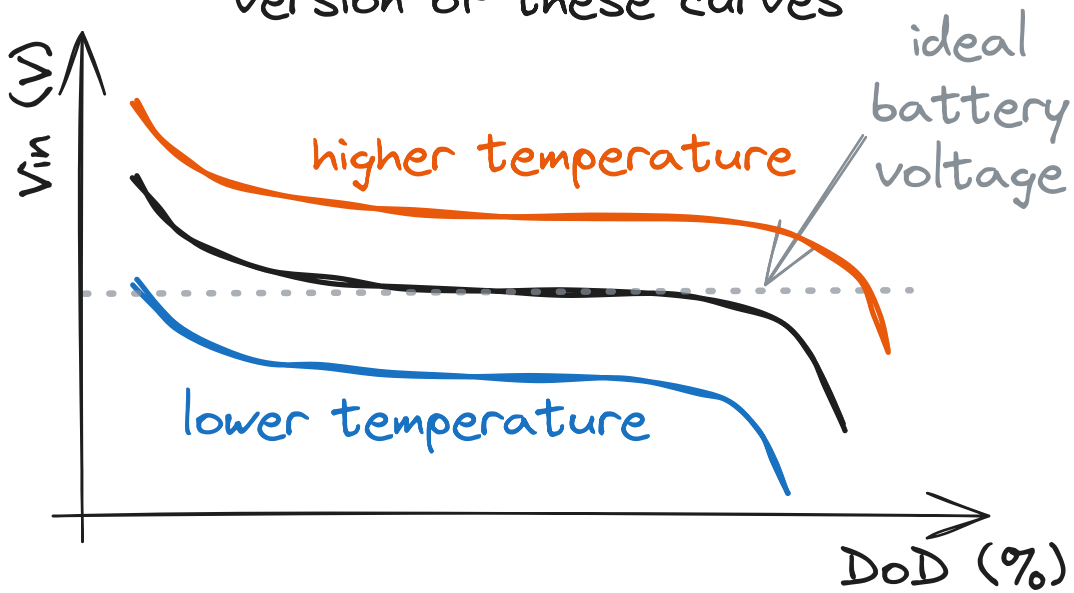


Issue #1: Output voltage is unregulated (continued)

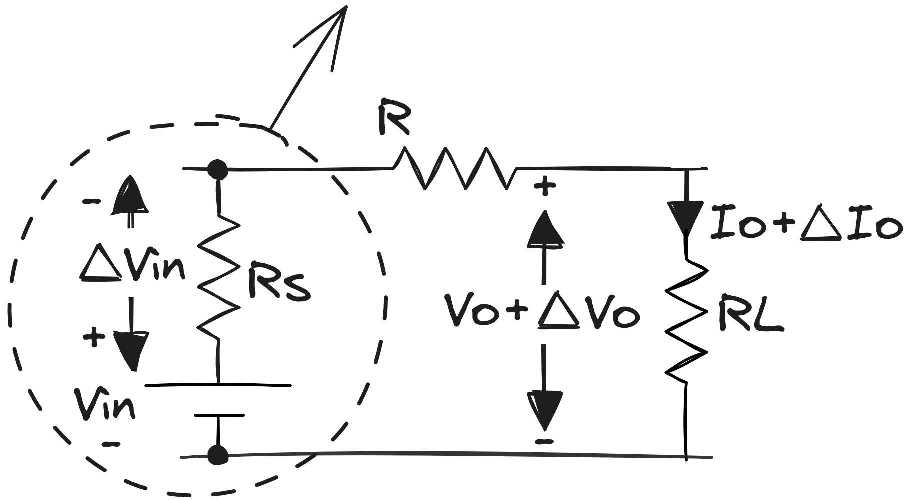
Battery voltage also varies with depth of discharge (DoD), temperature, and even load current

With a resistive divider, this variation directly shows at the output voltage

Vout is a scaled-down version of these curves



A real battery has internal resistance Rs



$\Delta Vin = \Delta Io \times Rs$

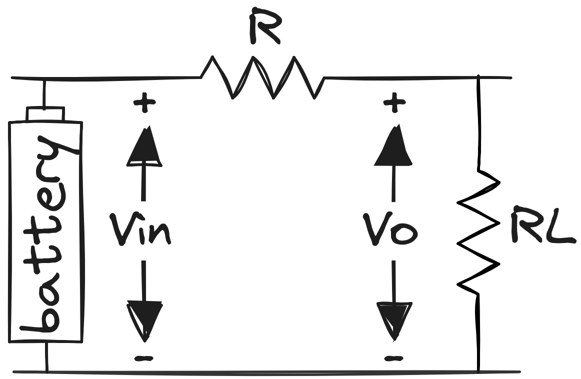
Issue #2: Poor efficiency

Ideally, we would like all power drawn form the battery to be put to "good use"

Good use = delivered to the load to run the circuit

This maximizes battery lifetime

What's the efficiency of a resistive divider?



Input power -> $P_{in} = V_{in}^2/(R+R_L)$

Output power -> $P_{out} = V_{out}^2/R_L$

Efficiency -> $\eta = P_{out}/P_{in} = V_{out}/V_{in} = K$

Issue #2: Poor efficiency (continued)

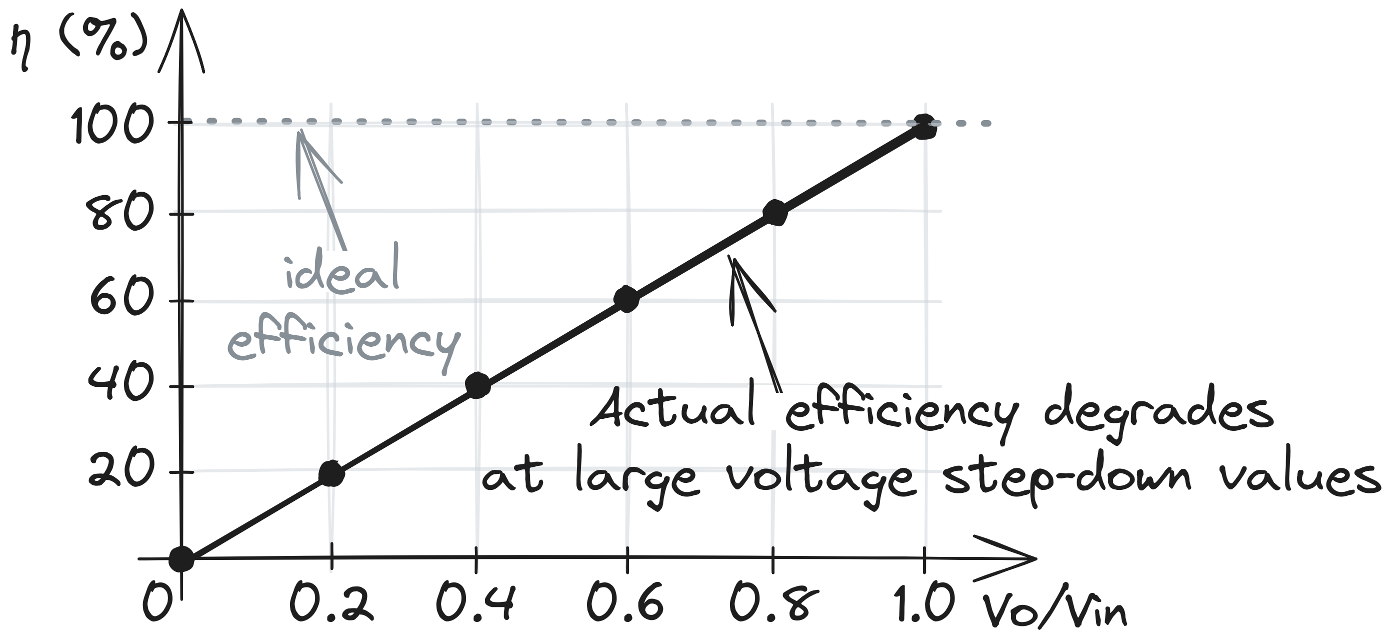
With a resistive divider, the series resistor used consumes significant power

Example:

V_{in} = 3.6V (typical Li-ion battery)

V_o = 0.9V (typical for digital circuits)

η = 20% -> 80% of power drawn from the battery is wasted!

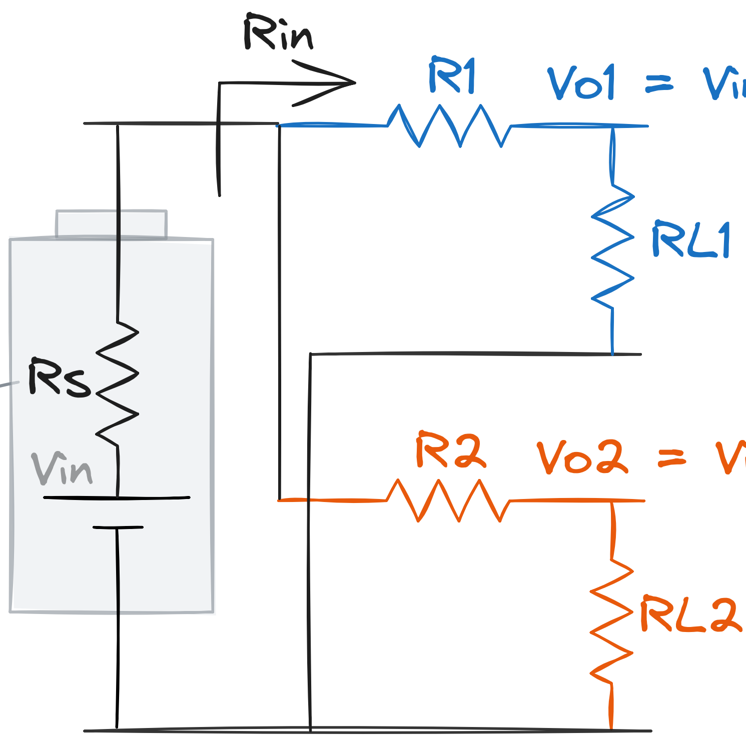


Issue #3: No isolation between rails

Ideally, two (or more) rails sharing a common battery should be independent
We call these isolated rails

With a resistive divider, the voltage of one rail
depends on the load of the other rail

$$R_{in} = \frac{(R1+RL1)(R2+RL2)}{R1+RL1+R2+RL2}$$



$$Vo1 = Vin \times \frac{R_{in}}{R_s+R_{in}} \times \frac{RL1}{R1+RL1}$$

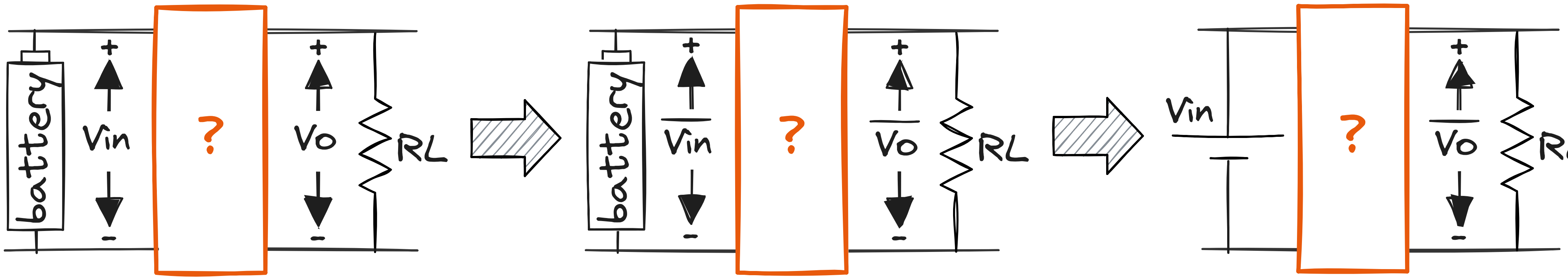
$$Vo2 = Vin \times \frac{R_{in}}{R_s+R_{in}} \times \frac{RL2}{R1+RL2}$$

If Rs = 0
(ideal battery),
the two output
rails would be
perfectly isolated

Now, let's find a better way to do things

We start by noting that DC voltage = average voltage

From this point of view, the job of the mystery box
is to change the average input voltage to an average output voltage



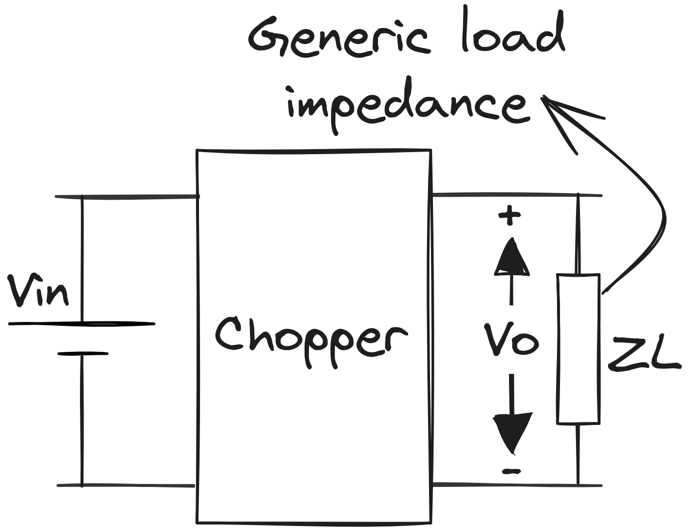
Assuming ideal battery

$\overline{Vin} = Vin$

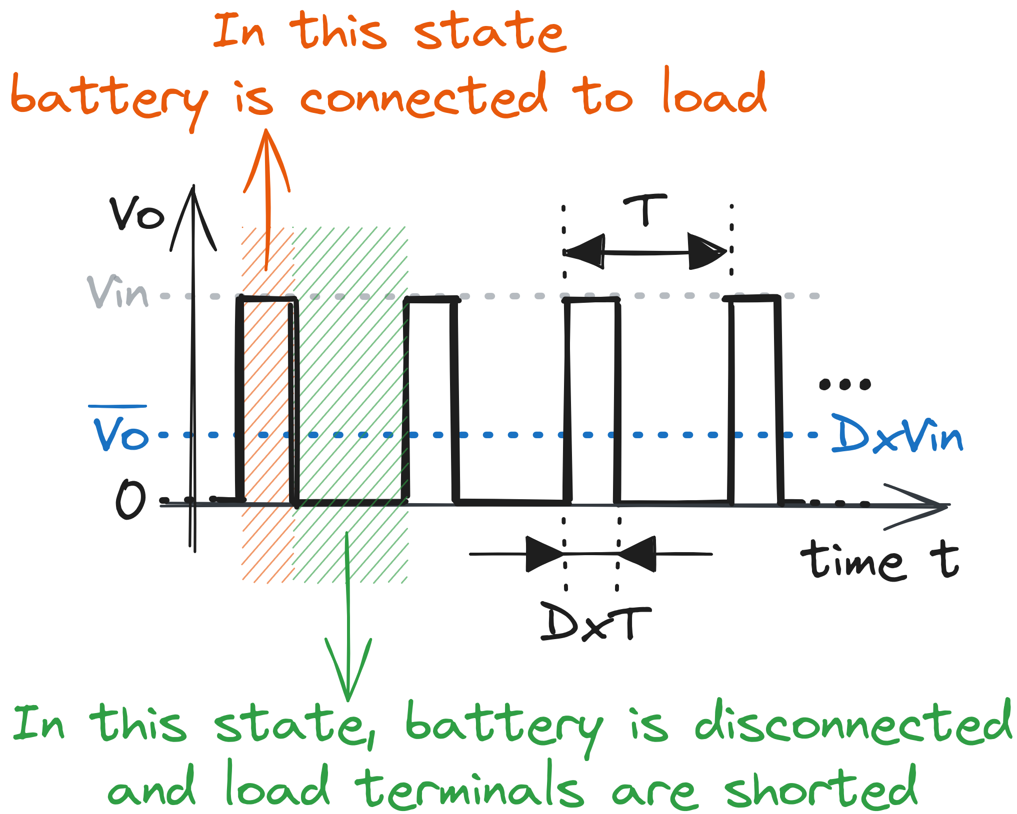
We can control the average voltage appearing across a load by duty cycling the battery connection to that load

The average output voltage is controlled by the switching duty cycle

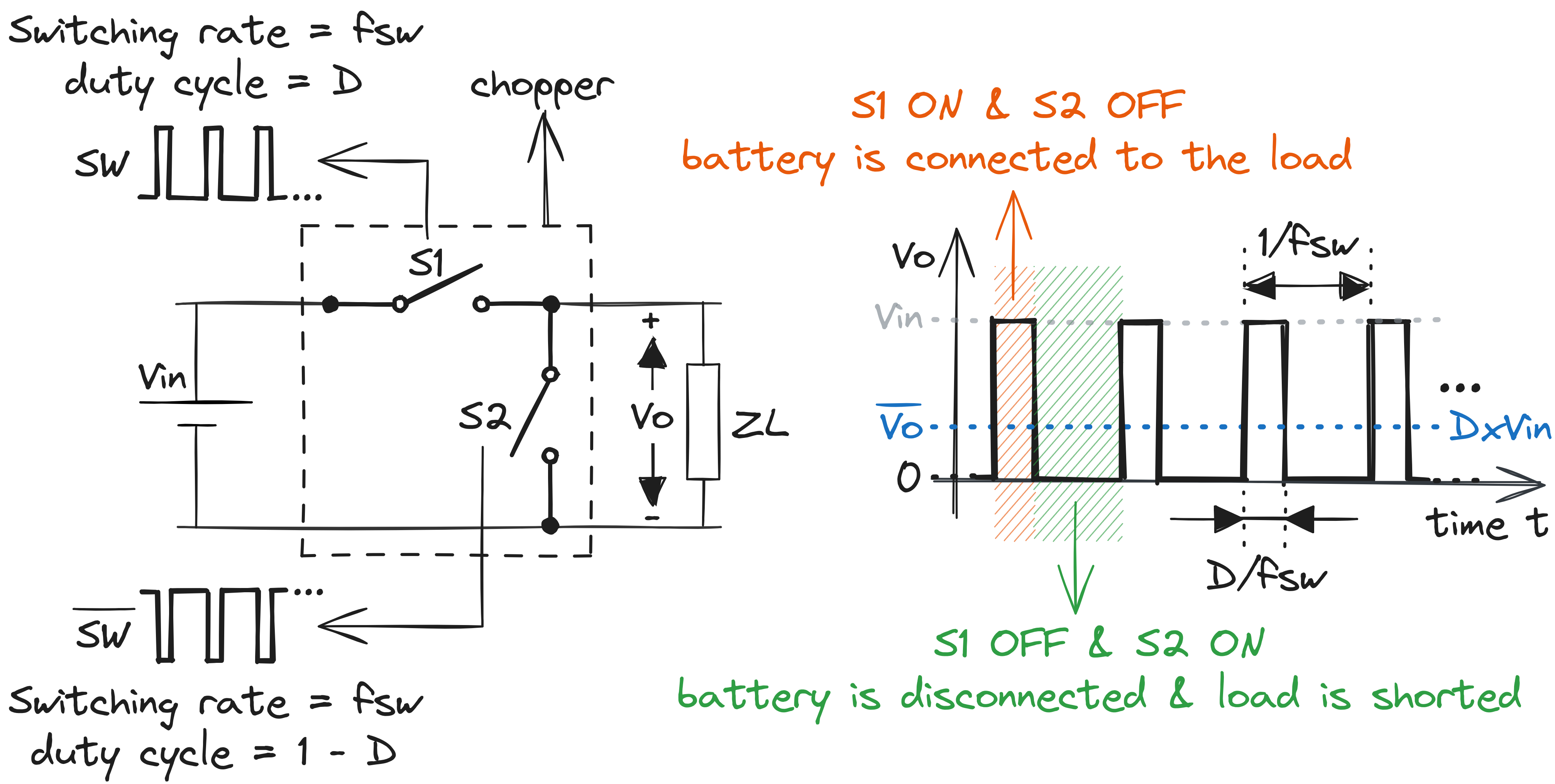
$$\overline{V_o} = D \times V_{in}$$



The chopper controls the flow of power from the battery to the load such that the input voltage appears "chopped" (duty cycled) across the load

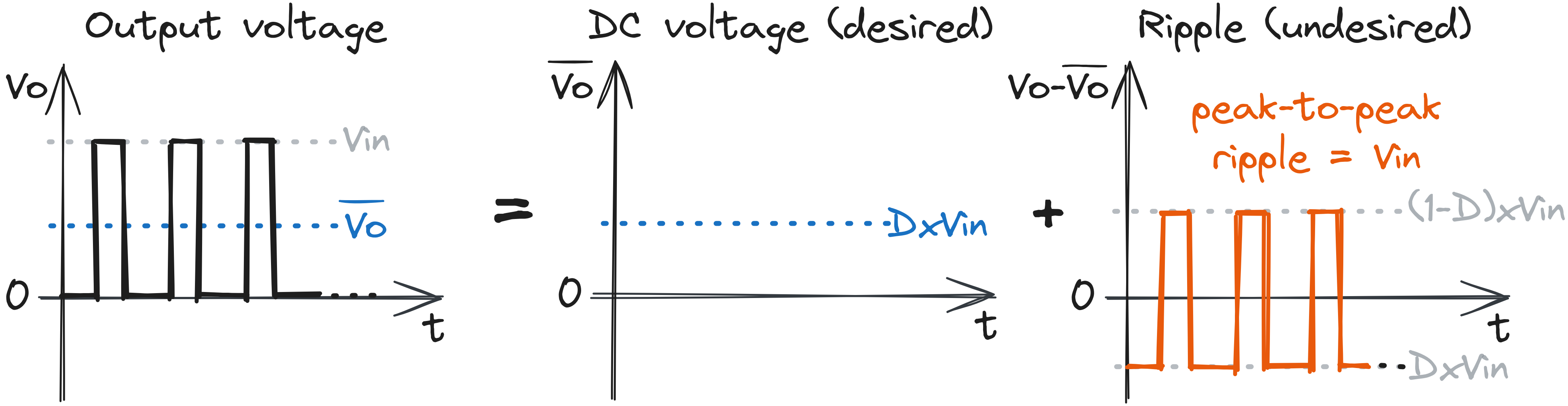


The chopper is simply a pair of switches controlled by complementary signals



Now, even though the output voltage has the desired DC value, it does not look anything like a fixed supply voltage that can power a circuit

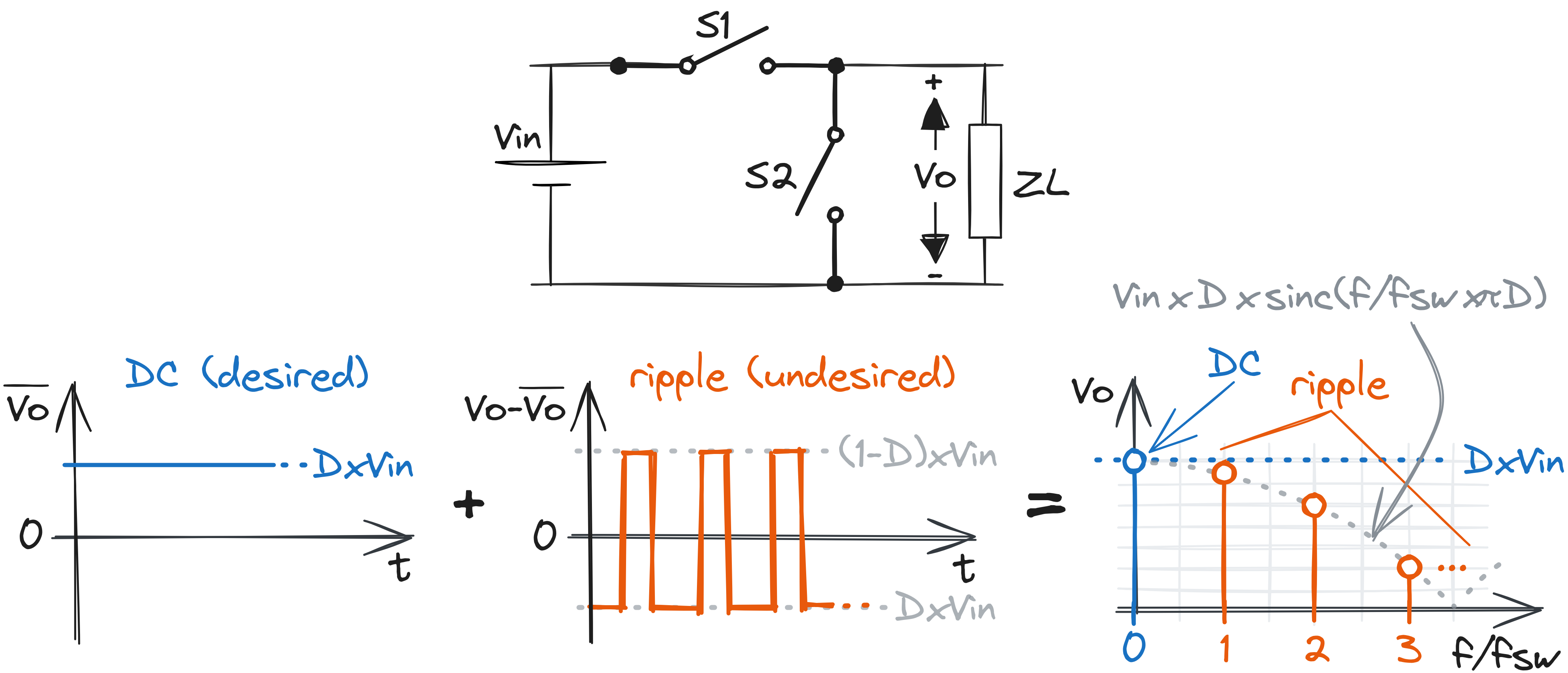
We can view this output voltage as the sum of two components: a desired DC component + undesired ripple



For example, a 0.9V rail generated from a 3.6V battery would have a 3.6V peak-to-peak ripple!

We need to get rid of the output ripple

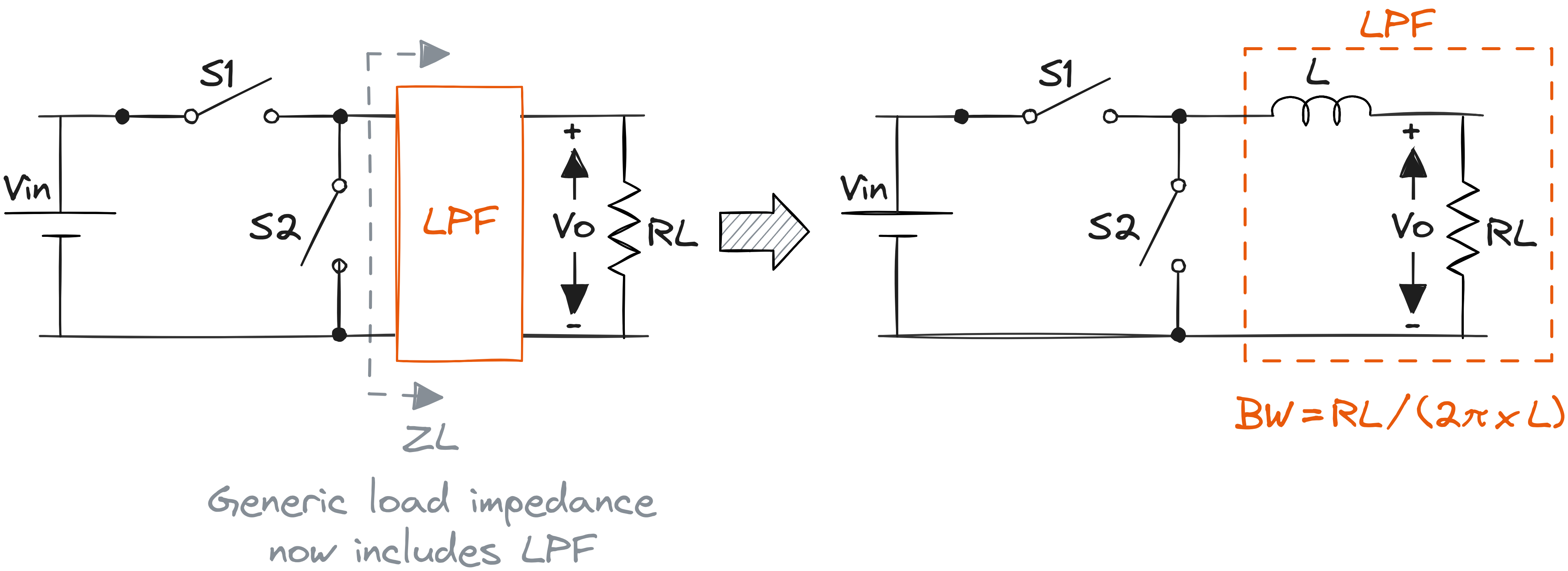
Examining the output voltage in the frequency domain,
we see that the desired signal is at DC,
while the ripple energy is at the harmonics of the switching frequency



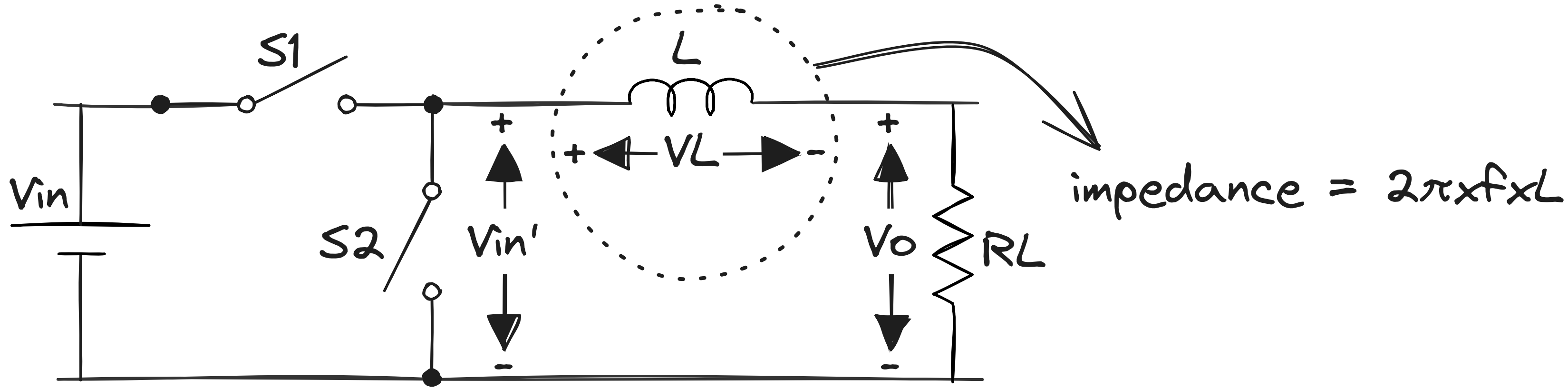
Based on this frequency separation, we can filter out the ripple

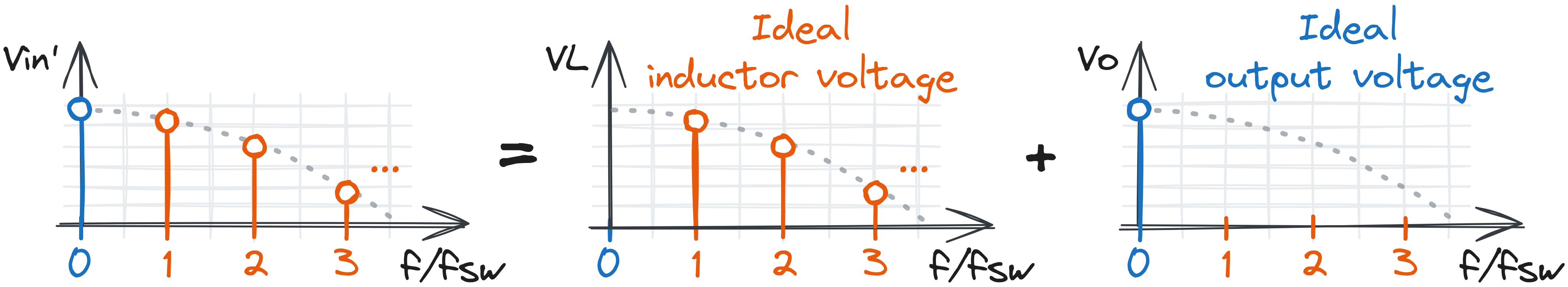
We add a low-pass filter (LPF) between the chopper and the load

A simple inductor in series with the load resistor forms a LPF



Ideally, the inductor is infinitely large,
so it has a zero impedance at DC and infinite impedance at all harmonics,
and the output voltage is a DC level with no ripple (as desired)

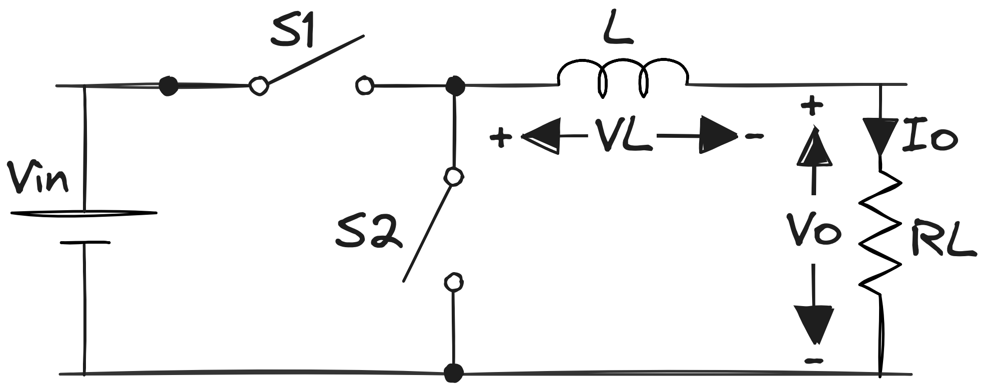




Of course, a practical inductor is finite and there's always ripple at the output
So, there's a tradeoff between output ripple and inductor size

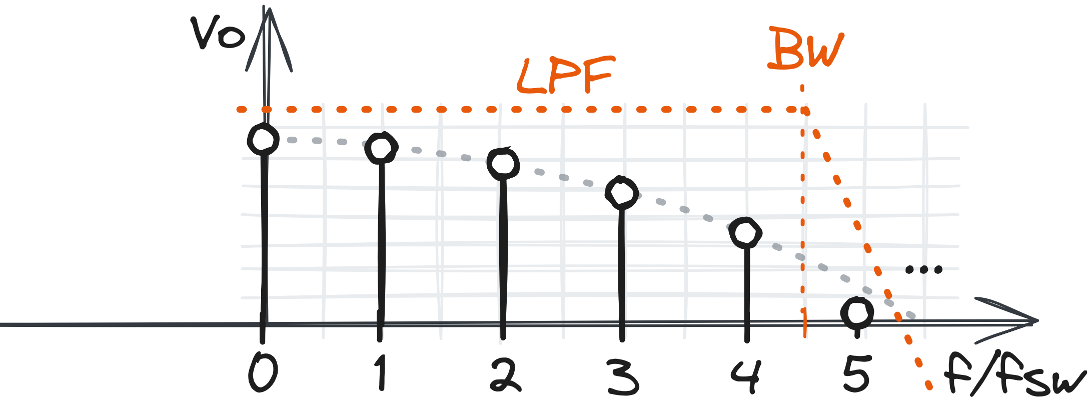
If L is small -> LPF BW > f_{sw} (τ < $1/f_{sw}$)

Peak-to-peak output ripple is still equal to Vin



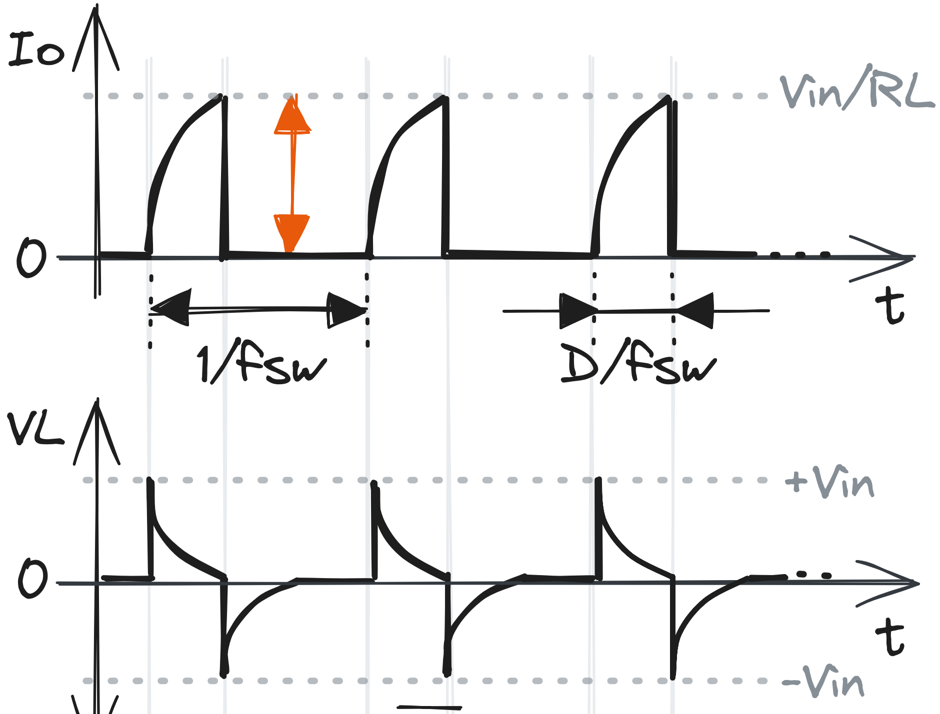
$BW = 1/(2\pi) \times R_L/L$

Time constant $\tau = L/R_L$



peak-to-peak output

ripple = $V_{in}/R_L \times R_L = V_{in}$

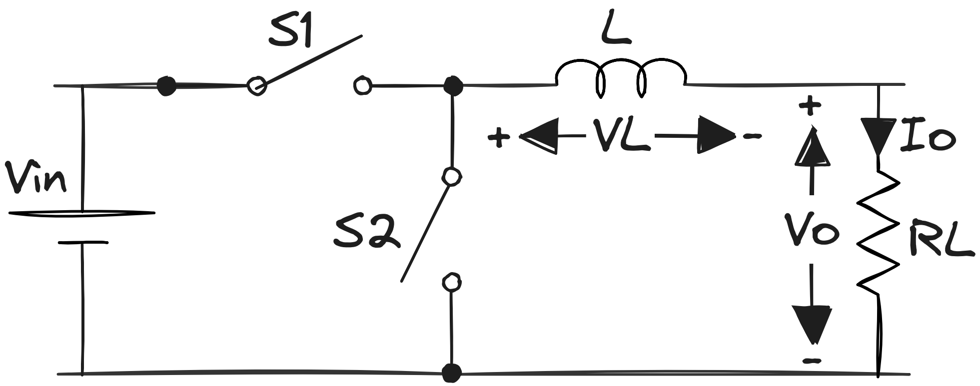


$\overline{V_L} = 0$

Average (DC) voltage across L = 0

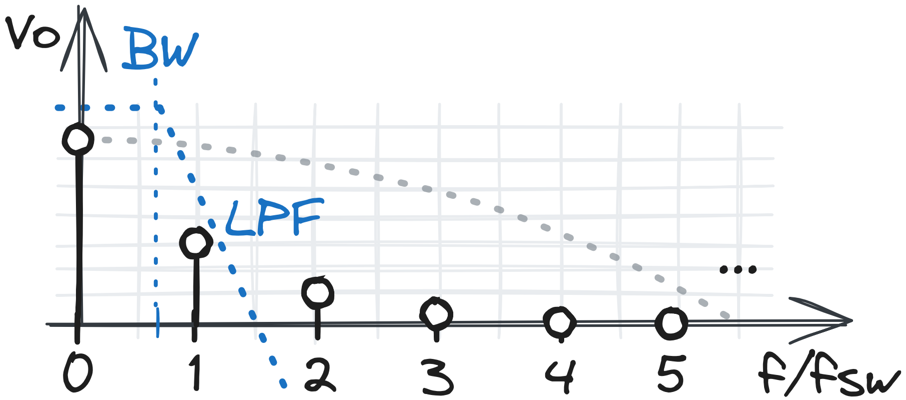
If L is large -> LPF BW < fsw (τ > 1/fsw)

Peak-to-peak output ripple is now << Vin

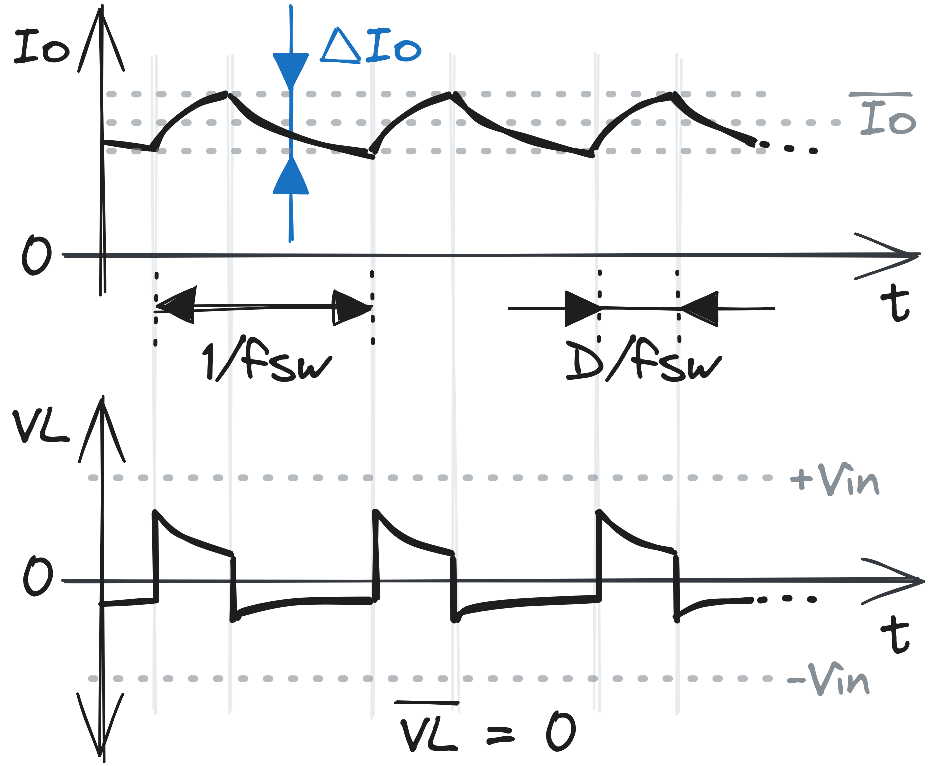


$BW = 1/(2\pi) \times RL/L$

Time constant $\tau = L/RL$



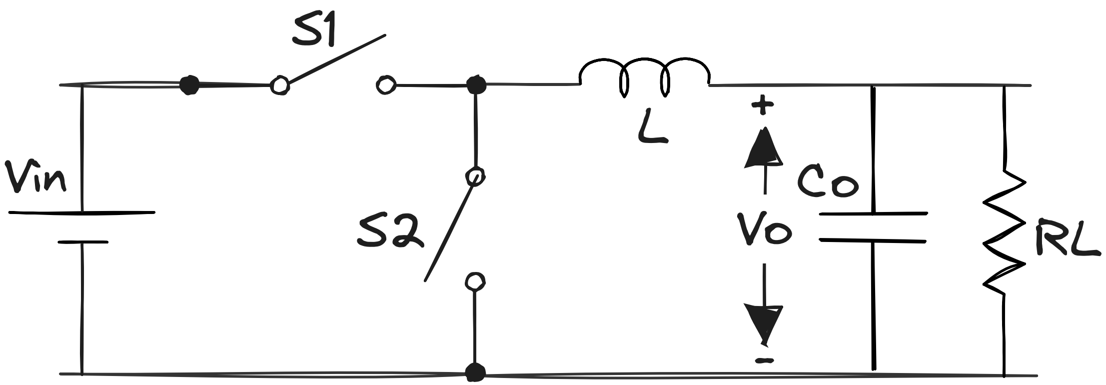
peak-to-peak output ripple = $\Delta Io \times RL$ << Vin

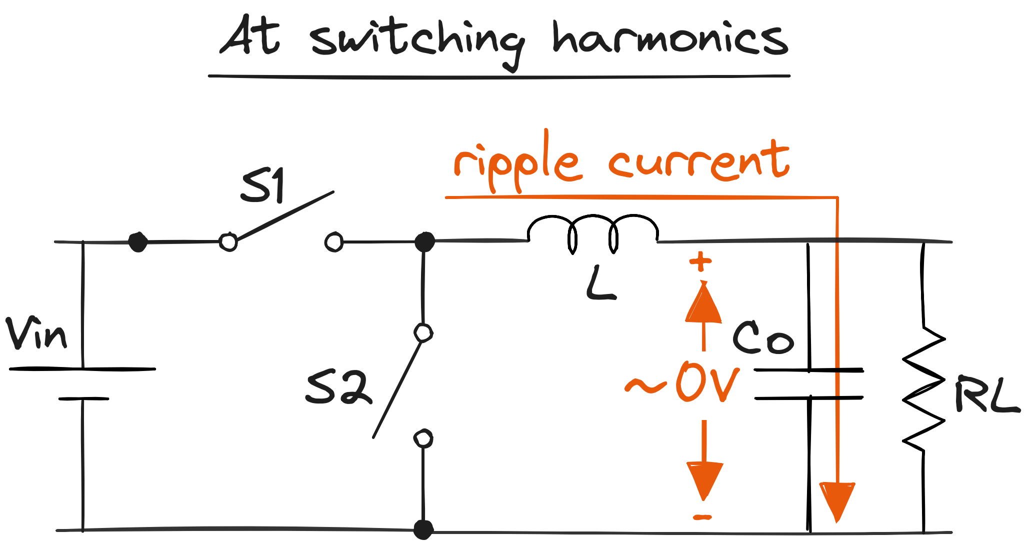


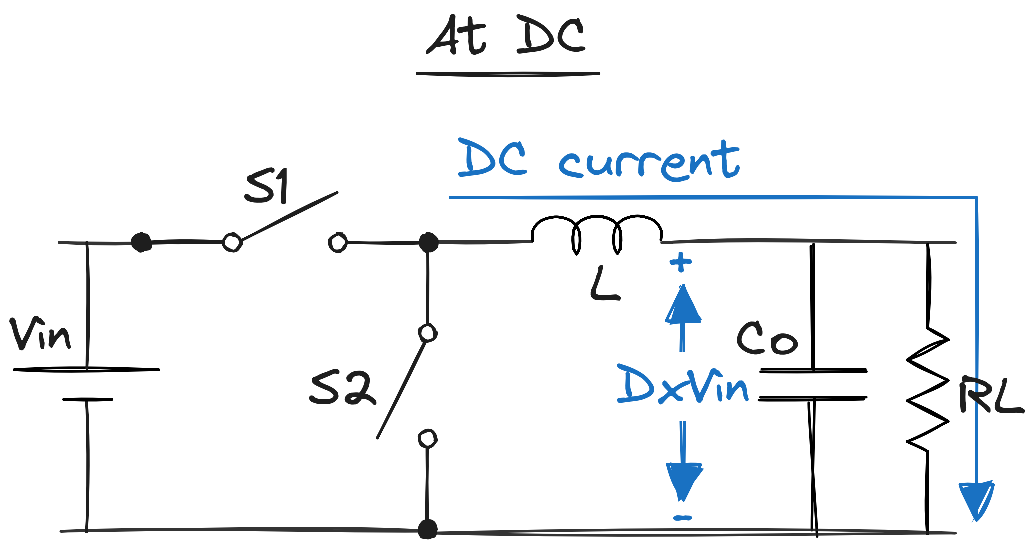
Average (DC) voltage across L = 0

We can reduce ripple further by adding a capacitor in shunt with the load

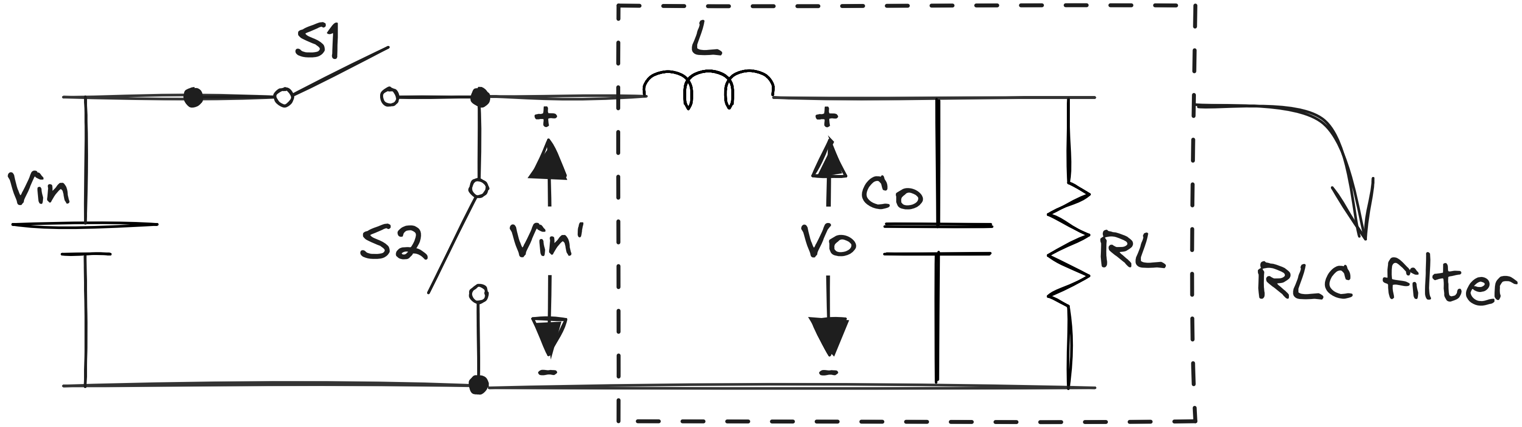
The capacitor creates a low impedance path at switching harmonics

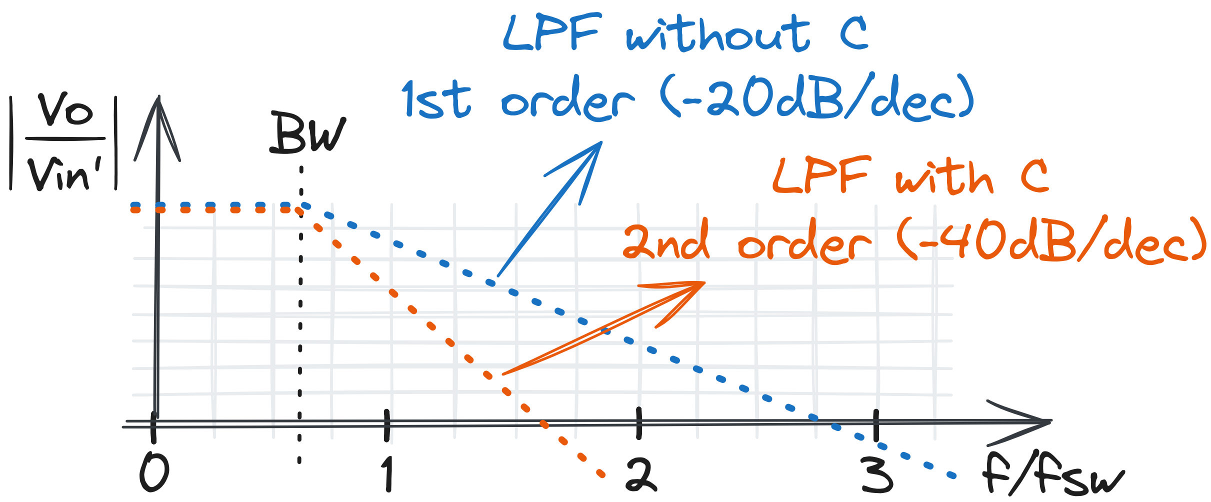






The benefit of adding the capacitor can also be explained in the frequency domain
Together with the inductor, the capacitor creates a second-order filter
The reduction in ripple is due to increased rejection at all switching harmonics



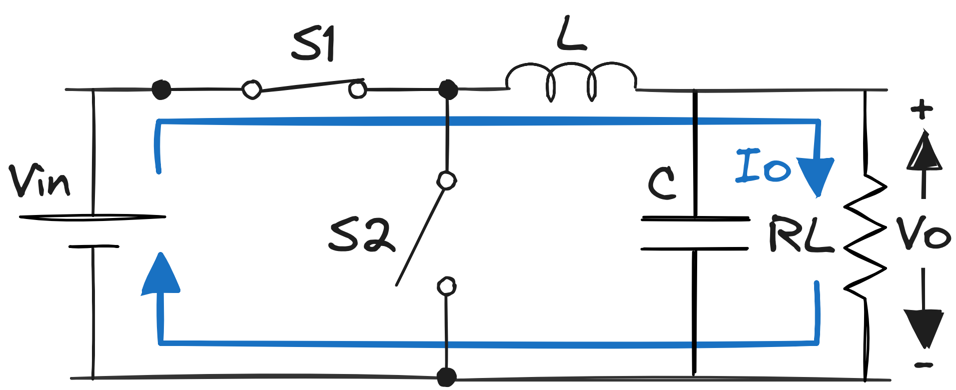


Power transfer perspective (1)

We can also understand the operation of the circuit based on power transfer

Phase 1: S1 ON & S2 OFF

In this phase, the battery is connected to the load through the inductor
So power is delivered to the load directly from the battery



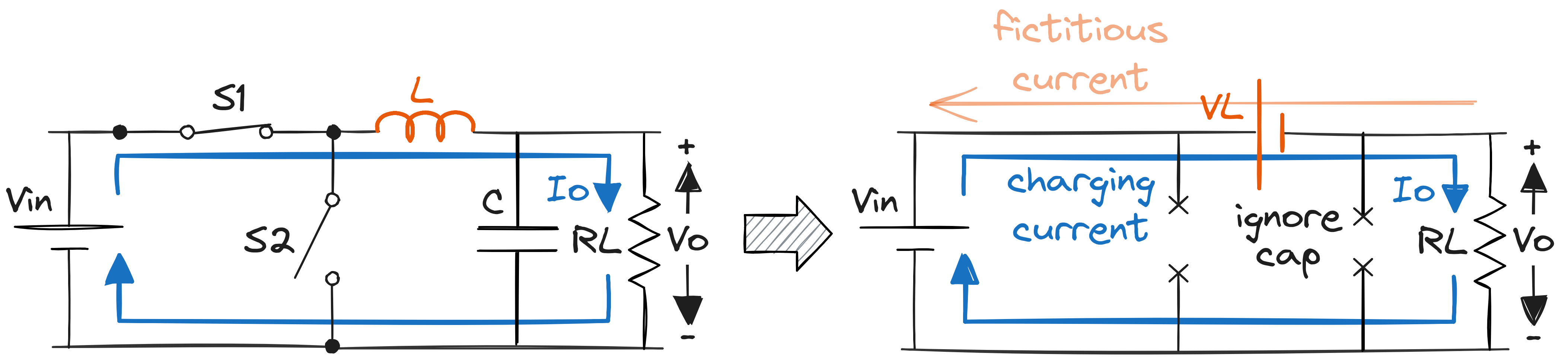
Because the inductor is in series with the load,
the same current "charges" the inductor
By "charges" we mean that the magnetic field is building up in the inductor
and energy is stored in that magnetic field

Power transfer perspective (2)

Phase 1: S1 ON & S2 OFF (continued)

The inductor behaves as a battery (back emf)
The polarity of this "back emf battery" is such that its fictitious current opposes the current that's causing the magnetic field to increase (Lenz's law)

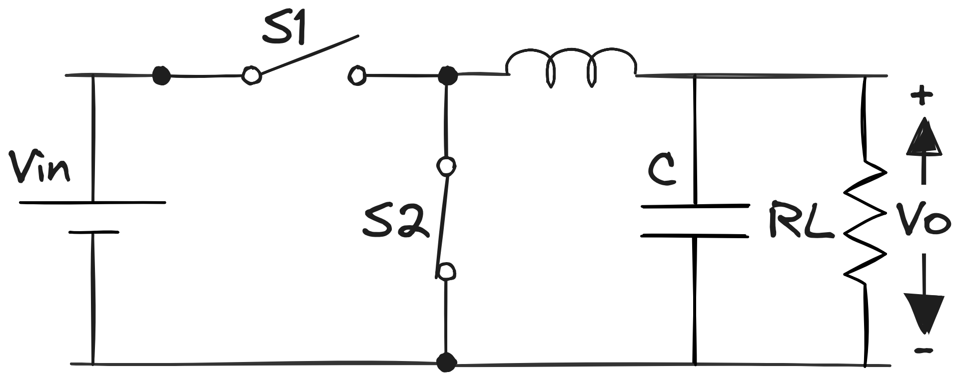
Alternatively, the polarity of this back emf battery is set up in such a way as if it's being charged by Vin, which is consistent with energy being stored in the magnetic field of the inductor



Power transfer perspective (3)

Phase 2: S1 OFF & S2 ON

In this phase, the battery is disconnected from the load
and no power can be delivered from the battery directly to the load



Because the power source (battery) is now disconnected,
the inductor starts discharging through S2
By "discharging" we mean that the energy stored in the magnetic field
of the inductor is now being released and the magnetic field is collapsing

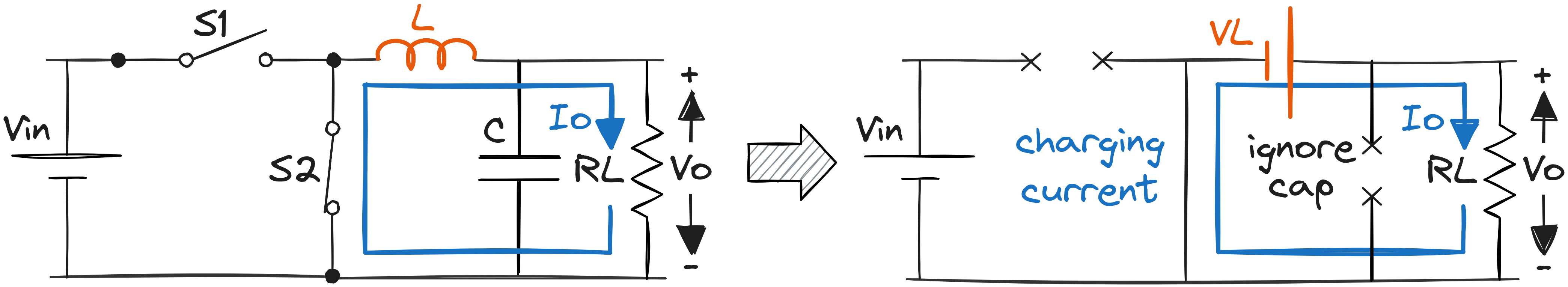
Power transfer perspective (4)

Phase 2: S2 OFF & S1 ON (continued)

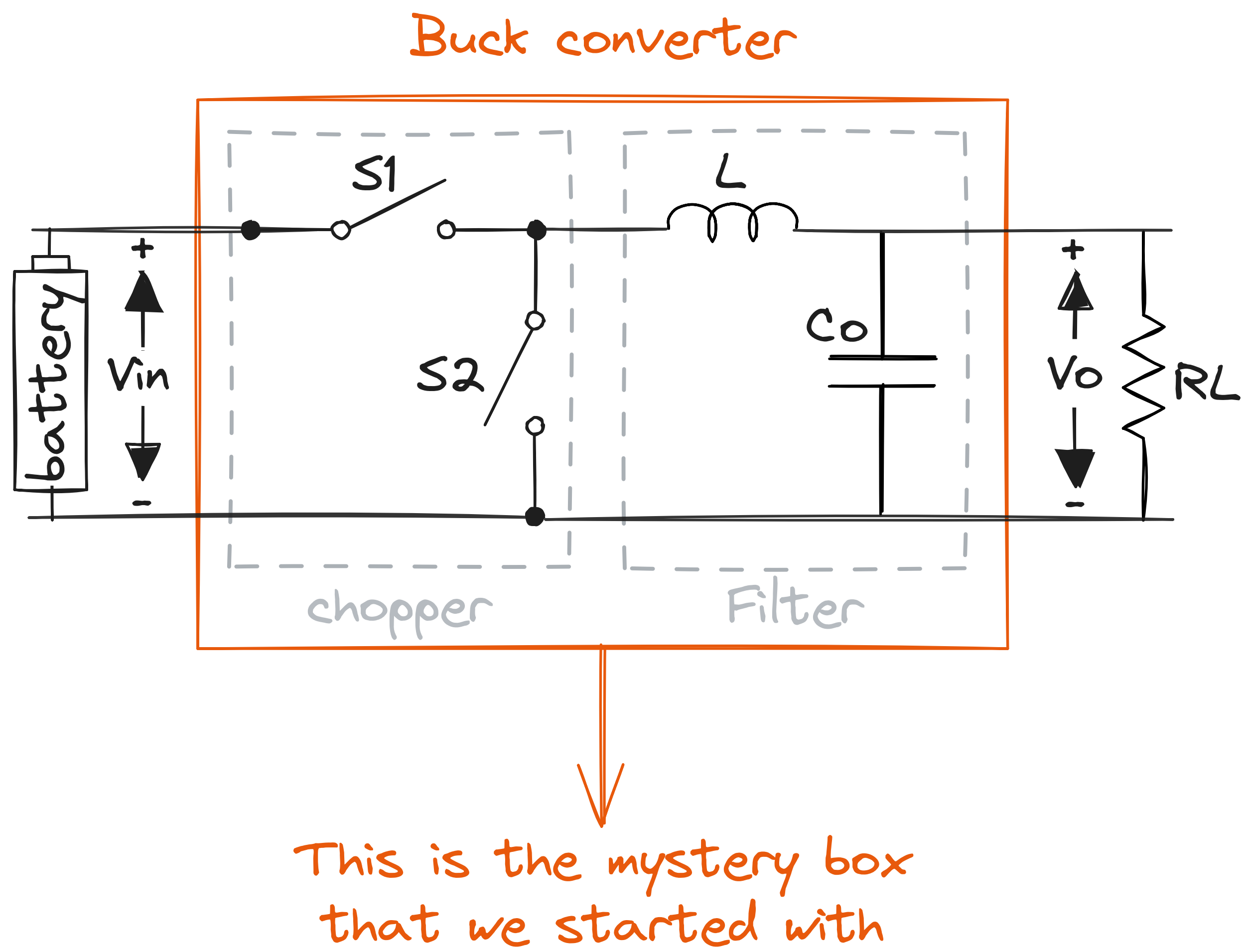
The inductor now reverses the polarity of its back emf
to generate a current that tries to sustain the collapsing magnetic field
In doing so, the inductor provides power to the load

In other words, the inductor behaves as a power source
releasing previously stored energy to the load

We can now see that the inductor ensures continuous flow of power
to the load even though the battery is not always connected to the load



The cascade of the chopper and the filter we've come up with so far forms the core of the famous buck converter



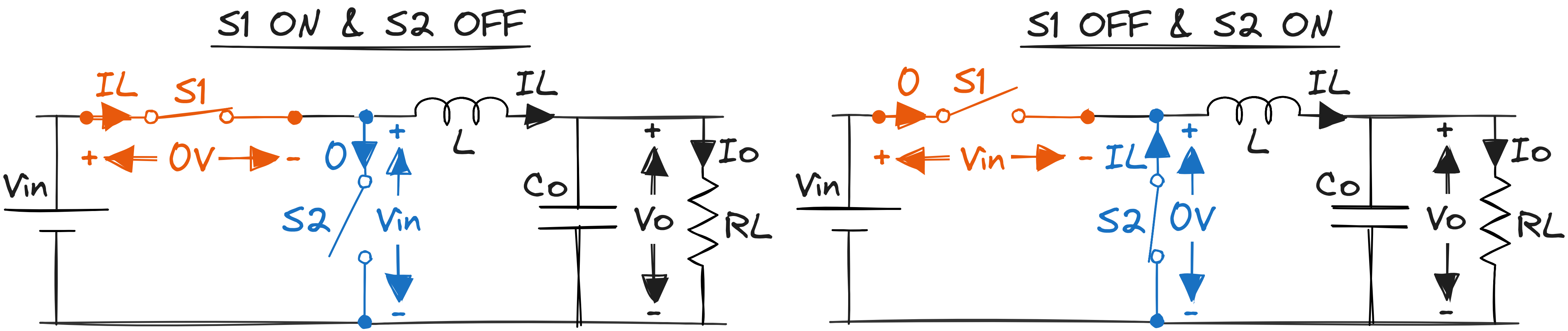
Before going further, let's take a step back and ask:
Is this switching configuration really better than a voltage divider?

It's better in one critical aspect: efficiency

Assuming ideal components, neither the chopper nor the filter consume any power

So, in principle, all power from battery is delivered to the load & efficiency is 100%

In practice, efficiency can be very high, but, of course, never 100%
(more on this later)

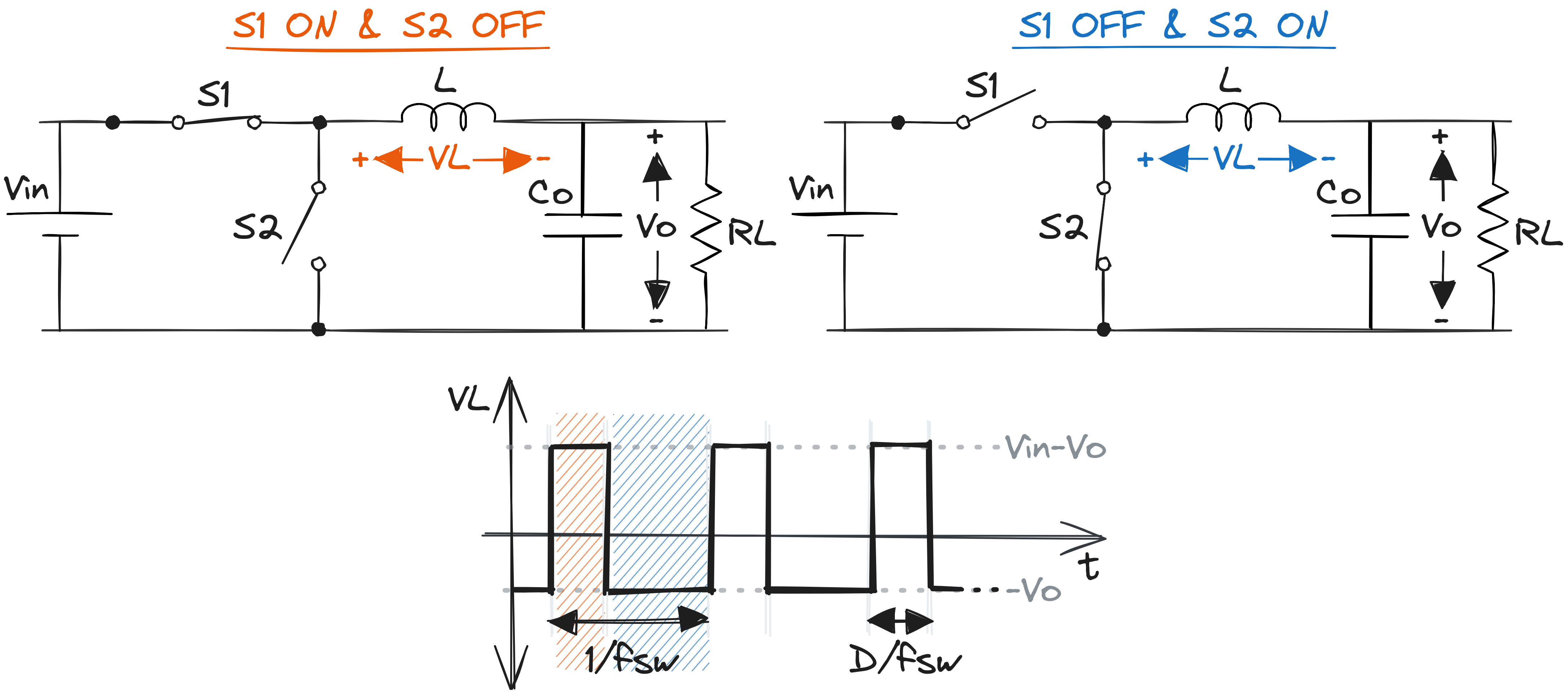


For S1 & S2, voltage x current product is always zero
For L and C, no real power is used, only reactive power

Input-output relationship (1)

Main simplifying assumption: with second-order filter, output ripple ~ 0

Output voltage Vo is purely DC

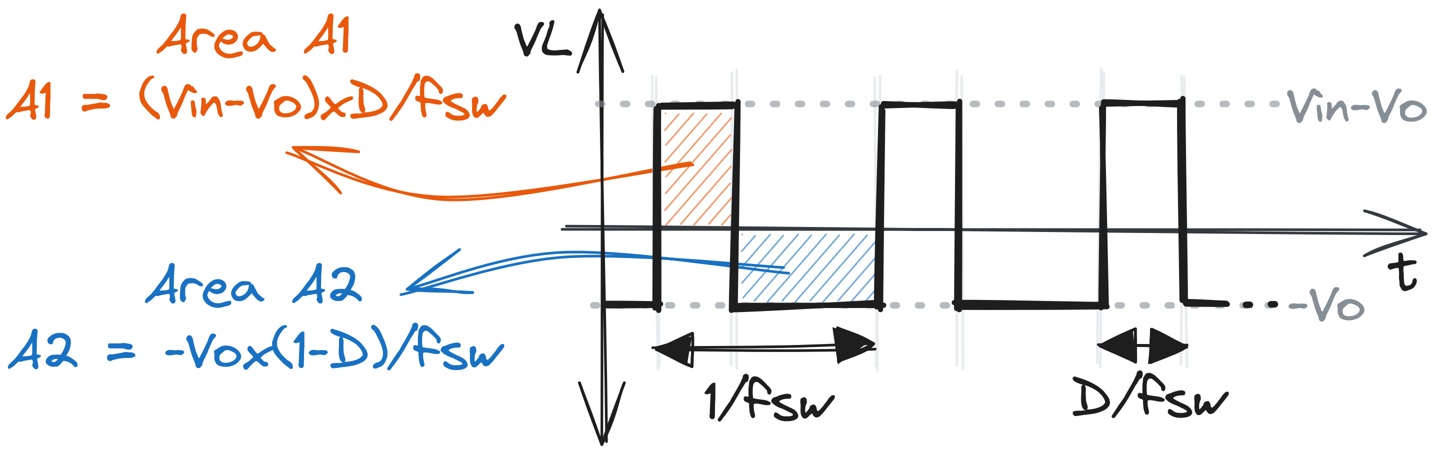


Input-output relationship (2)

By definition, DC (average) voltage drop across an ideal inductor = 0

This leads to the (expected) relationship between input and output voltage

$$\boxed{V_o = D \times V_{in}}$$



Average voltage across inductor = A1 + A2 = 0

$$(V_{in}-V_o) \times D/f_{sw} - V_o \times (1-D)/f_{sw} = 0$$

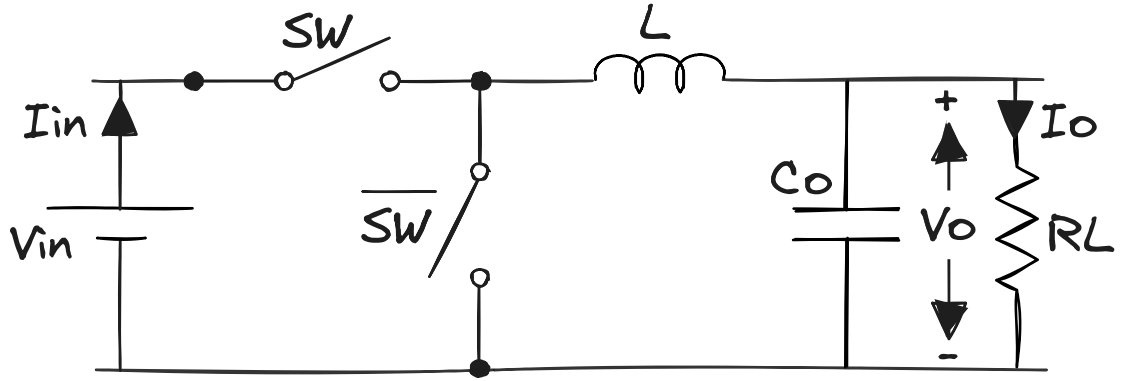
$$V_o = D \times V_{in}$$

Input-output relationship (3)

With 100% theoretical efficiency,

we can also find the relationship between input and output current

$$\boxed{I_{in} = D \times I_o}$$



Input power -> $P_{in} = V_{in} \times I_{in}$

Output power -> $P_{out} = V_o \times I_o$

Efficiency -> $\eta = P_{out}/P_{in} = (V_o/V_{in}) \times (I_o/I_{in}) = 1$

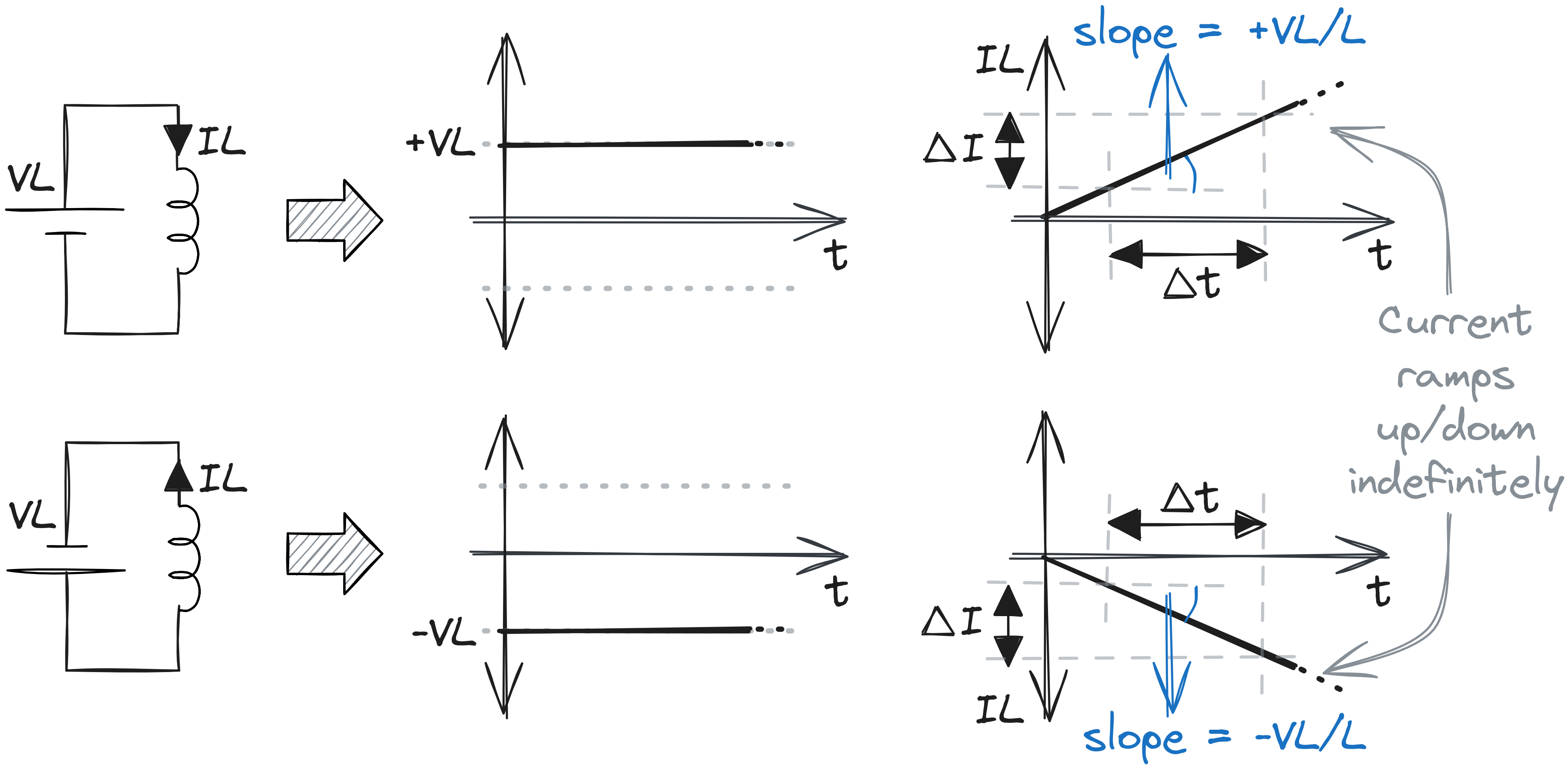
$I_o/I_{in} = V_{in}/V_o = 1/D$

stepping-down voltage by D means stepping-up current by 1/D

Sizing the inductor (1)

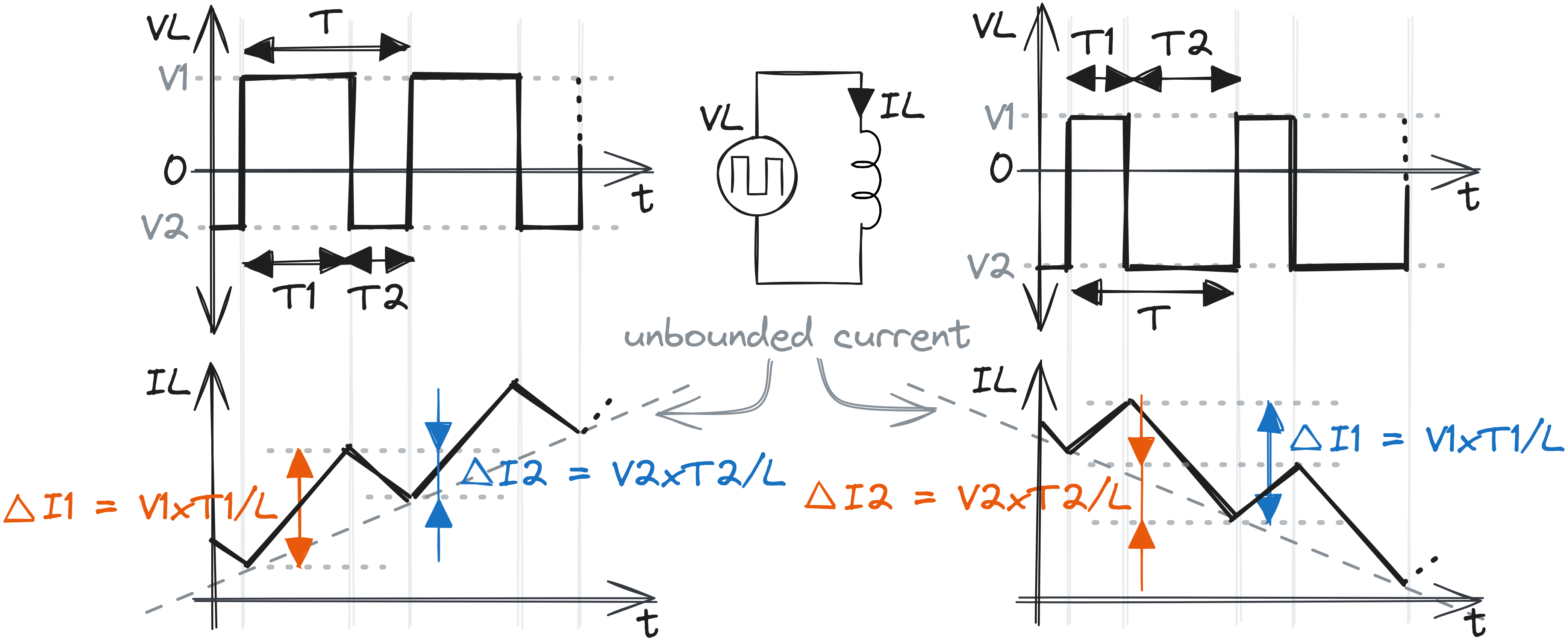
A DC voltage across an inductor generates a linear ramp current

$$VL = L \times dI/dt \quad \rightarrow \quad \Delta I = VL/L \times \Delta t$$



Sizing the inductor (2)

Consequently, a square wave voltage across an inductor generates a generally increasing or decreasing sawtooth current



Net current increase is ΔI every T

$\Delta I = \Delta I1 - \Delta I2 = (V1 \times T1 - V2 \times T2)/L$

Net current decrease is ΔI every T

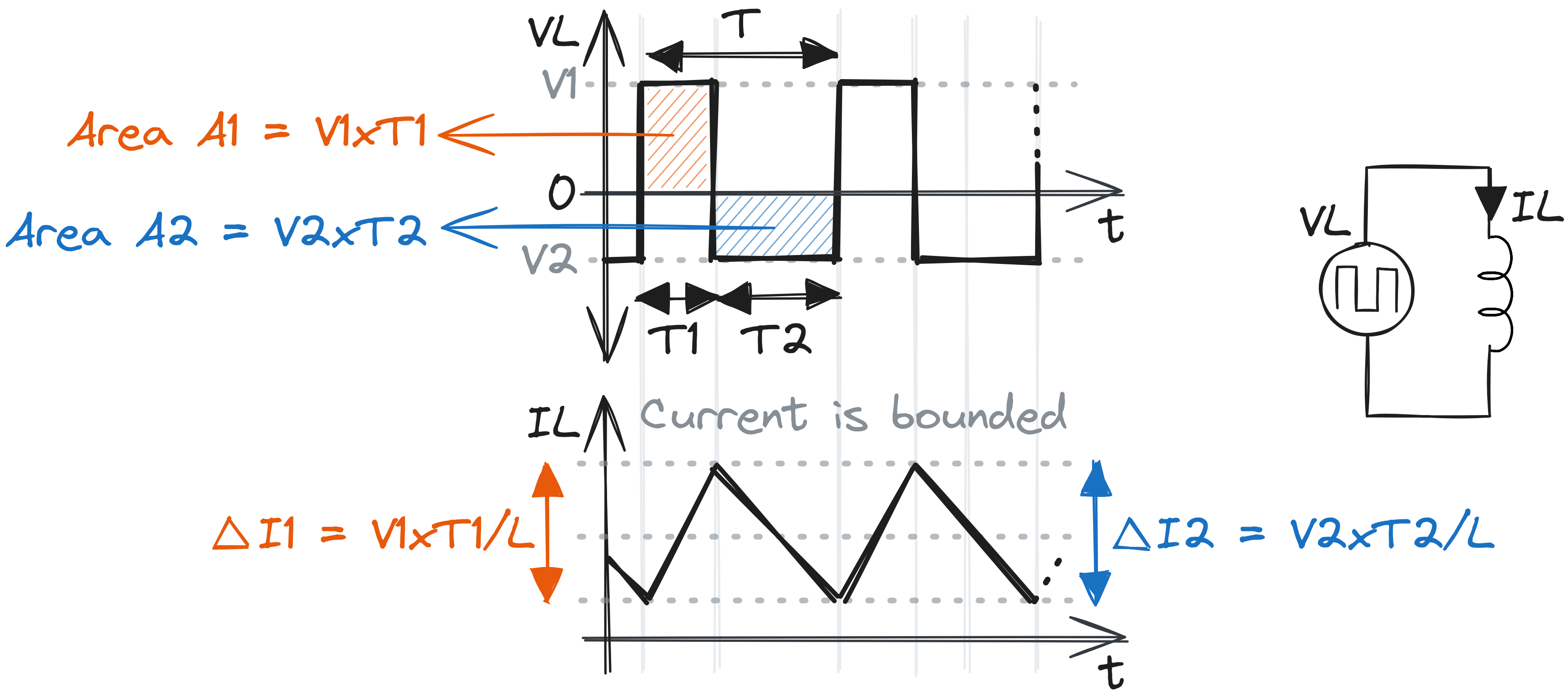
$\Delta I = \Delta I2 - \Delta I1 = (V2 \times T2 - V1 \times T1)/L$

Sizing the inductor (3)

We can now see that the net current change over one cycle can be zero if

$$V1 \times T1 = V2 \times T2$$

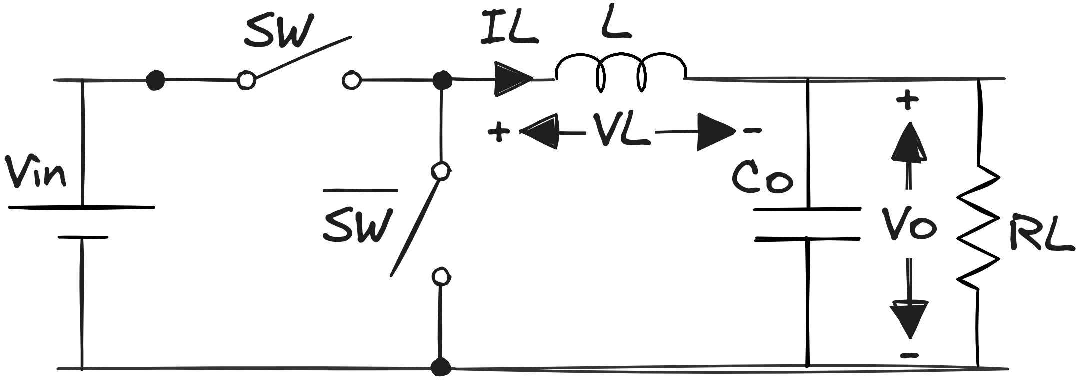
This is equivalent to the applied square wave voltage having a zero mean which is exactly the steady state condition we saw before for the buck converter

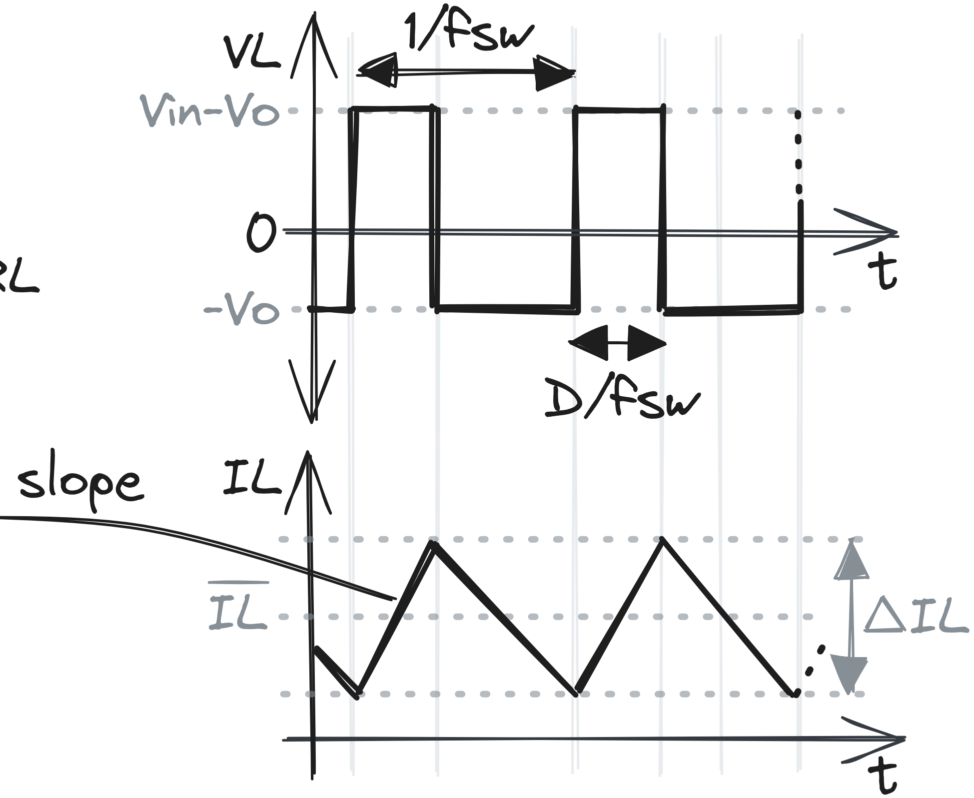


Sizing the inductor (4)

Applying this steady state condition to the buck converter, we can figure out the inductor size

$$\boxed{L = Vo/\Delta IL \times (1-D)/fsw}$$





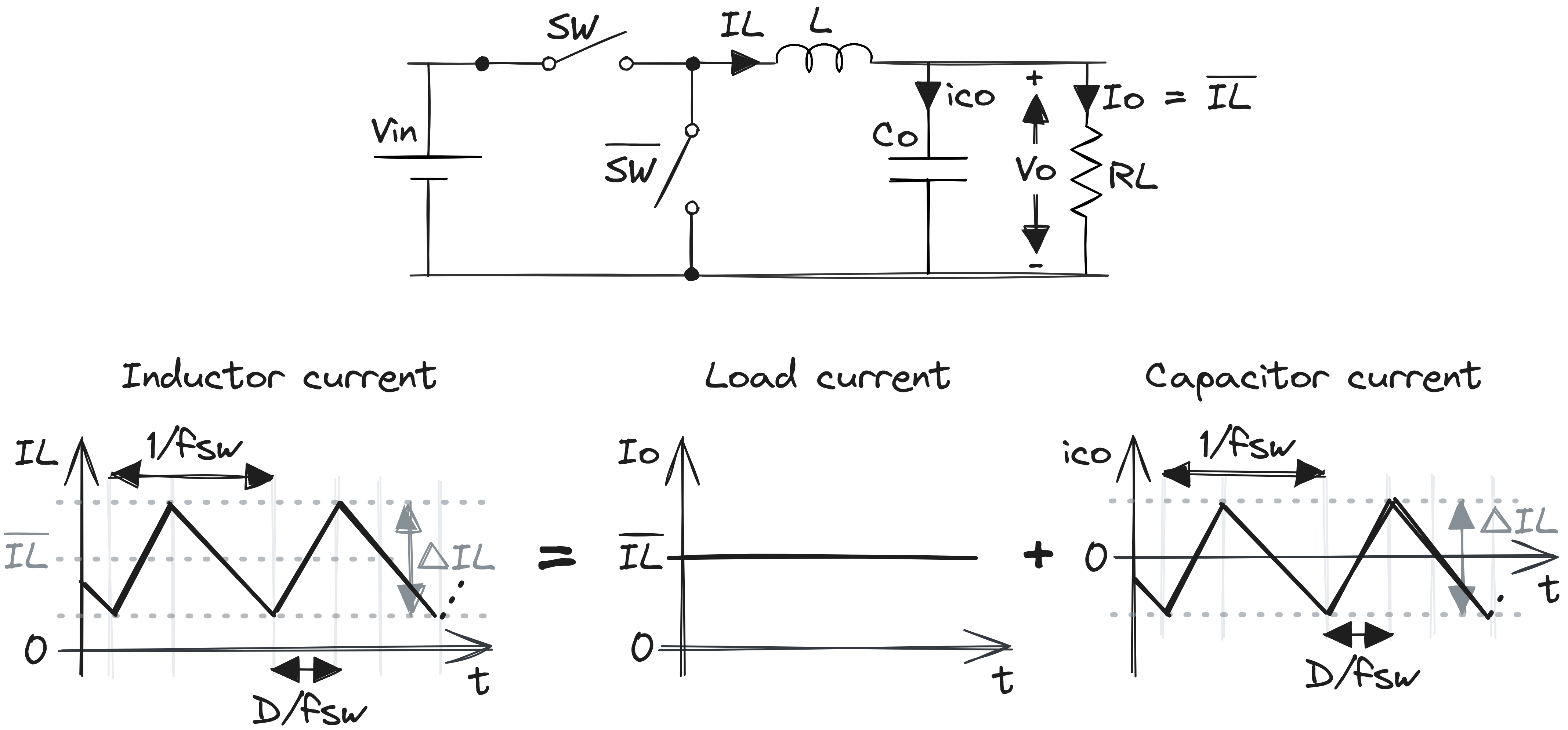
$$(Vin-Vo)/L = \Delta IL/(D/fsw)$$

$$Vo \times (1/D-1)/L = \Delta IL \times fsw/D$$

$$L = Vo/\Delta IL \times (1-D)/fsw$$

Sizing the output capacitor (1)

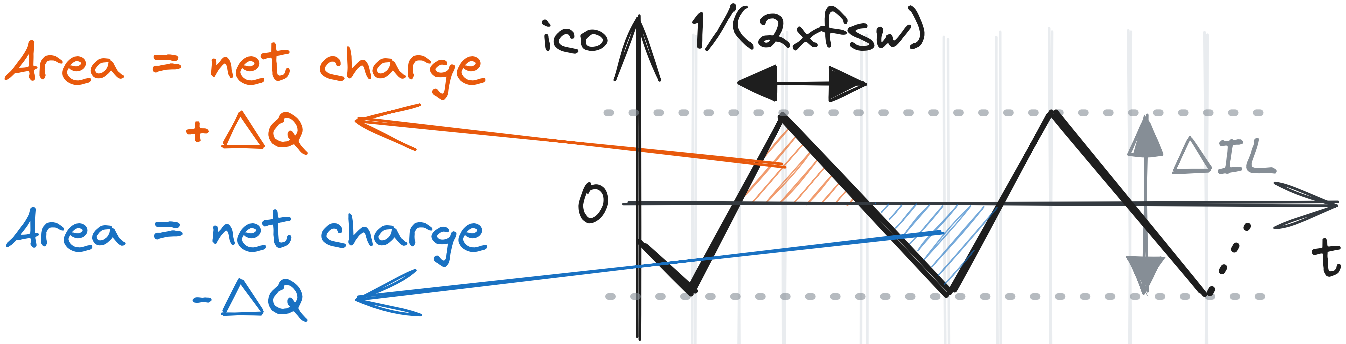
Main simplifying assumption: all ripple current flows in capacitor



Sizing the output capacitor (2)

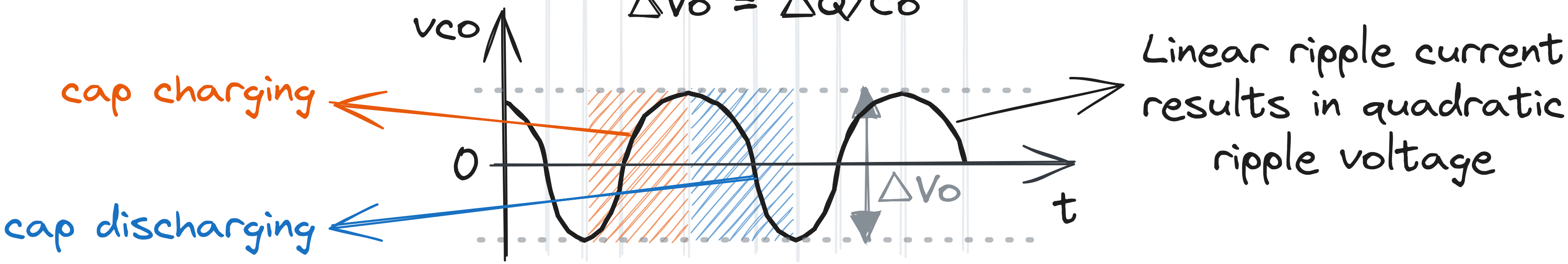
From charge calculations, we can size the output capacitor

$$\boxed{C_o = (\Delta I_L / \Delta V_o) \times 1/(8 \times f_{sw})}$$



$$\Delta Q = 1/2 \times 1/(2 \times f_{sw}) \times \Delta I_L/2$$

$$\Delta V_o = \Delta Q / C_o$$



Design Example

Requirements:

Battery voltage = 3.6V

Output rail voltage = 0.9V

Load current = 100mA

$$D = V_o/V_{in} = 0.9/3.6 = 0.25$$

Assuming a 20% current ripple

$$\Delta I_L = 0.2 \times \overline{I_L} = 0.2 \times 100mA = 20mA$$

Assuming fsw = 1MHz

$$L = V_o/\Delta I_L \times (1-D)/f_{sw} = (0.9/0.02) \times (1-0.25)/1e6 = 33uH$$

Assuming 1% ripple voltage

$$\Delta V_o = 0.01 \times V_o = 9mV$$

$$C_o = (\Delta I_L/\Delta V_o) \times 1/(8 \times f_{sw}) = (0.02/0.009) \times 1/(8 \times 1e6) = 278nF$$

Let's start with a simple case:

A single Li-ion battery and a single circuit

Now the question is: