Directional Couplers

Shadi Youssef

April 11, 2020

1 Basic operation

A (bi)directional coupler discerns between forward and reflected waves, which allows us, among other things, to evaluate the input (or output) matching of a device under test (DUT).

Fig. 1(a) shows an ideal bidirectional coupler connected to a DUT whose input match, relative to Z_0, is given by the reflection coefficient Γ_{DUT}. The forward wave couples to one of the auxiliary ports (P_3), but is completely isolated from the other (P_4), and vice versa for the reflected wave. The coupling factor C determines the power levels measured at the coupled ports, P_{FC} and P_{RC}, for the forward and reflected signals, respectively

$$P_{\text{FC}} = P_{\text{IN}}/C \tag{1}$$

$$P_{\text{RC}} = P_{\text{IN}} \cdot \left|\Gamma_{\text{DUT}}\right|^2 /C \tag{2}$$

Thus, the ratio of (2) to (1) provides a direct measure of Γ_{DUT}

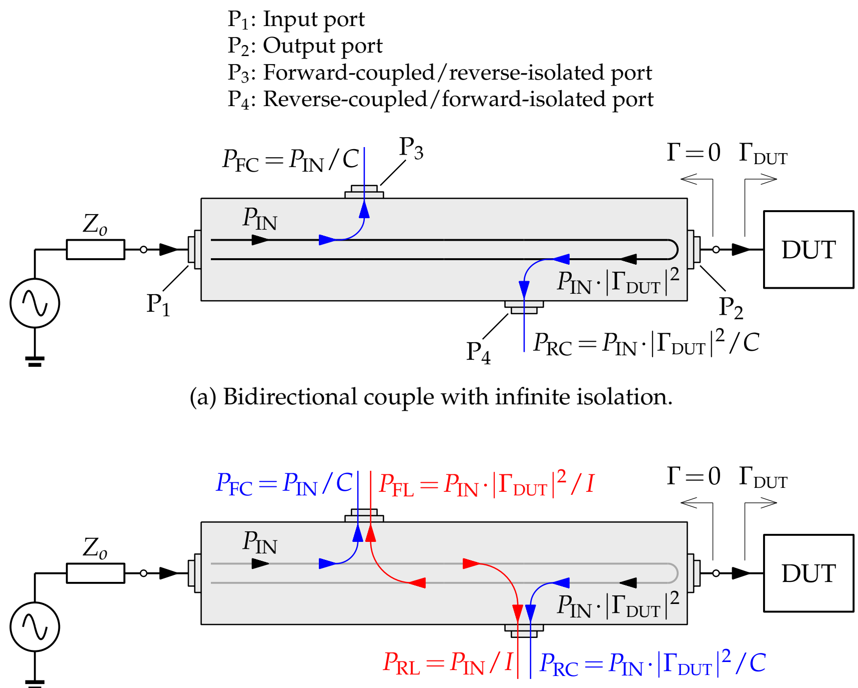
$$\left|\Gamma_{\text{DUT}}\right| = \sqrt{\frac{P_{\text{RC}}}{P_{\text{FC}}}} \tag{3}$$

2 Measurement Error

In reality, however, the forward and reflected signals leak to their respective isolated ports (P_4 and P_3, respectively), due to finite isolation I, which results in measurement error. Fig. 1(b) shows the forward leakage signal P_{FL} and the reverse leakage signals P_{RL}

$$P_{\text{FL}} = P_{\text{IN}} \cdot \left|\Gamma_{\text{DUT}}\right|^2 /I \tag{4}$$

$$P_{\text{RL}} = P_{\text{IN}}/I \tag{5}$$



(a) Bidirectional couple with infinite isolation.

(b) Bidirectional coupler with finite isolation.

Figure 1: Signal flow in a bidirectional coupler.

Since the forward and reflected signals are correlated, the total power measured at each of the coupled ports is the voltage sum squared of the coupled and leakage signals. That is

$$P_F = (\sqrt{P_{\mathrm{FC}}} \pm \sqrt{P_{\mathrm{FL}}})^2 \tag{6}$$

$$P_R = (\sqrt{P_{\mathrm{RC}}} \pm \sqrt{P_{\mathrm{RL}}})^2 \tag{7}$$

where P_F is the total forward power level measured at P_3 and P_R is the total reflected power levels measured at P_4, and $\pm$ accounts for in-phase and out-of-phase interference between the forward and reflected signals, respectively, thus providing a maximum-minimum range of measurement. By substituting (1) and (4) in (6), and (2) and (5) in (7)

$$P_F = P_{\mathrm{FC}}\left(1 \pm \sqrt{\frac{P_{\mathrm{FL}}}{P_{\mathrm{FC}}}}\right)^2 = P_{\mathrm{FC}}\left(1 \pm \sqrt{\frac{I}{C}} \cdot |\Gamma_{\mathrm{DUT}}|\right)^2 \tag{8}$$

$$P_R = P_{\mathrm{RC}}\left(1 \pm \sqrt{\frac{P_{\mathrm{RL}}}{P_{\mathrm{RC}}}}\right)^2 = P_{\mathrm{RC}}\left(1 \pm \sqrt{\frac{I}{C}} \cdot \frac{1}{|\Gamma_{\mathrm{DUT}}|}\right)^2 \tag{9}$$

We can now define the directivity of the directional coupler

$$D = I/C \tag{10}$$

and use (10) to re-write (8) and (9) as

$$P_F = P_{\mathrm{FC}} \left(1 \pm \frac{|\Gamma_{\mathrm{DUT}}|}{\sqrt{D}}\right)^2 \tag{11}$$

$$P_R = P_{\mathrm{RC}} \left(1 \pm \frac{1}{\sqrt{D}} \cdot \frac{1}{|\Gamma_{\mathrm{DUT}}|}\right)^2 \tag{12}$$

The bracketed terms in (11) and (12) represent power measurement error due to finite directivity of the coupler, and, as would be expected, both reduce to unity for infinite directivity. Thus, directivity is a measure of how well a coupler distinguishes between the forward and reverse waves. Note that measurement error is dependent on the reflection coefficient being measured. A better matched device (lower $|\Gamma_{\mathrm{DUT}}|$) improves the accuracy of forward measurement, but increases the error of the reverse measurement. This makes sense because attempting to measure a better matched device means a weaker reflected signal, which has less impact on forward measurement, but makes reverse measurement more susceptible to forward wave leakage. This is graphically represented in Fig. 2.

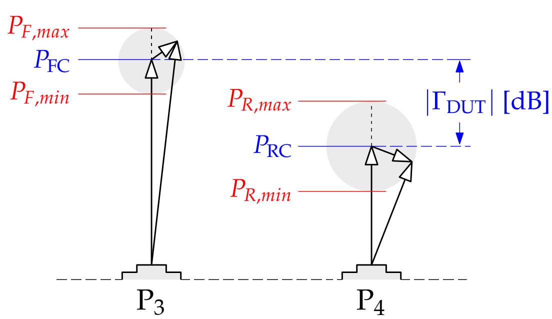


Figure 2: Forward and reflected measured power levels in a directional coupler with finite directivity. The gray regions indicate measurement uncertainty.

The measured reflection coefficient $|\Gamma_{meas}|$ is the ratio of (12) to (11)

$$|\Gamma_{meas}| = \sqrt{\frac{P_R}{P_F}} = \sqrt{\frac{P_{\mathrm{RC}}}{P_{\mathrm{FC}}}} \cdot \frac{1 \pm \frac{1}{\sqrt{D}} \cdot \frac{1}{|\Gamma_{\mathrm{DUT}}|}}{1 \pm \frac{1}{\sqrt{D}} \cdot |\Gamma_{\mathrm{DUT}}|} \tag{13}$$

and by using (3) in (13) and re-arranging terms, the error in reflection coefficient measurement due to finite directivity ε_D is

$$\varepsilon_D = \frac{|\Gamma_{meas}|}{|\Gamma_{\mathrm{DUT}}|} = \sqrt{\frac{P_R/P_F}{P_{\mathrm{RC}}/P_{\mathrm{FC}}}} = \frac{1 \pm \frac{1}{\sqrt{D}} \cdot \frac{1}{|\Gamma_{\mathrm{DUT}}|}}{1 \pm \frac{1}{\sqrt{D}} \cdot |\Gamma_{\mathrm{DUT}}|} \tag{14}$$

The measurement error is then confined within the boundaries ε_{D+} and ε_{D-}

$$
\begin{aligned}
\varepsilon_{D+} &= \frac{1 + \frac{1}{\sqrt{D}} \cdot \frac{1}{|\Gamma_{\mathrm{DUT}}|}}{1 - \frac{1}{\sqrt{D}} \cdot |\Gamma_{\mathrm{DUT}}|} \\
\varepsilon_{D-} &= \frac{1 - \frac{1}{\sqrt{D}} \cdot \frac{1}{|\Gamma_{\mathrm{DUT}}|}}{1 + \frac{1}{\sqrt{D}} \cdot |\Gamma_{\mathrm{DUT}}|}
\end{aligned}
\tag{15}
$$

By plotting (15), we get measurement uncertainty regions as shown in Fig. 3. For example, measuring a typical reflection coefficient in the order of 0.3 (90% power delivered to the DUT), the curves in Fig. 3 tell us we need a coupler directivity $>$ 25dB to keep measurement error within $\pm$2dB.

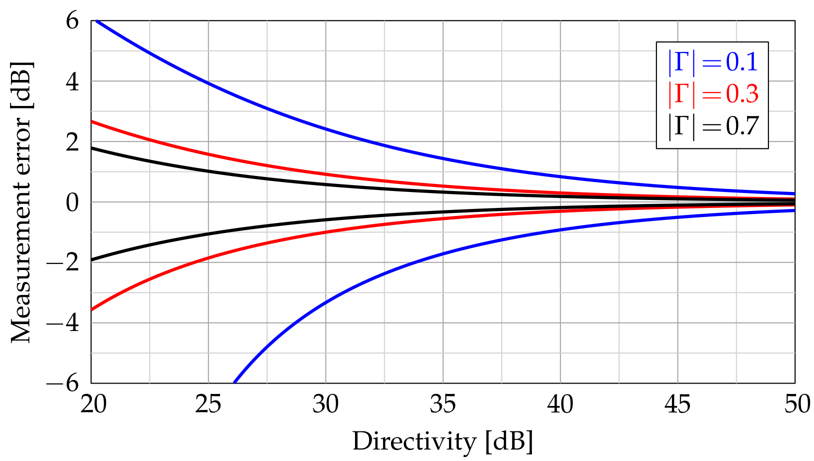


Figure 3: Reflection coefficient measurement error as a function of coupler's directivity.