IQ Mismatch in Radio Transceivers

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1 Mismatch Model: LO Domain

Ideally, a quadrature local oscillator (LO) signal, $s_{\text{LO,ideal}}(t)$, is given by

$$s_{\text{LO,ideal}}(t) = e^{-\jmath k(\omega_{\text{LO}} t + \phi_{\text{LO}})} \tag{1}$$

where ω_{LO} is the LO frequency in radians per second, ϕ_{LO} is an arbitrary initial phase, and $k = \pm 1$ denotes the rotation direction of the complex sinusoid (-1 for a positive LO frequency and $+1$ for a negative LO frequency). Such an ideal LO is circular because (1) represents a vector that traces a circular path over time.

In the presence of IQ mismatch, however, circularity is no longer true because the in-phase (I) and quadrature (Q) components of the LO signal are unequal in amplitude and not in perfect quadrature. This can be expressed as

$$\begin{aligned} s_{\text{LO}}(t) = {} & (1 + \Delta A_I)\cos(\omega_{\text{LO}} t + \phi_{\text{LO}} + \Delta\phi_I) \\ & - \jmath k(1 + \Delta A_Q)\sin(\omega_{\text{LO}} t + \phi_{\text{LO}} + \Delta\phi_Q) \end{aligned} \tag{2}$$

where ΔA and $\Delta\phi$ represent the amplitude and phase mismatches, respectively, and the I/Q subscripts denote the I/Q branches. The impact of IQ mismatch on circularity is shown in Fig. 1 By expanding the sine and cosine terms in (2) using Euler's formula and grouping terms, $s_{\text{LO}}(t)$ can be simplified to

$$s_{\text{LO}}(t) = A_{\text{IQ}}(k) \cdot e^{+\jmath(\omega_{\text{LO}} t + \phi_{\text{LO}})} + A^*_{\text{IQ}}(-k) \cdot e^{-\jmath(\omega_{\text{LO}} t + \phi_{\text{LO}})} \tag{3}$$

and coefficients $A_{\text{IQ}}(k)$ and $A^*_{\text{IQ}}(-k)$ capture IQ mismatch

$$A_{\text{IQ}}(k) = \frac{1}{2}\Big((1 + \Delta A_I)e^{+\jmath\Delta\phi_I} - k(1 + \Delta A_Q)e^{+\jmath\Delta\phi_Q}\Big) \tag{4}$$

The expression in (3) shows that, in the presence of IQ mismatch, the LO signal becomes the sum of two complex sinusoids, a desired and an image, both rotating at a rate of ω_{LO}, but in opposite directions.

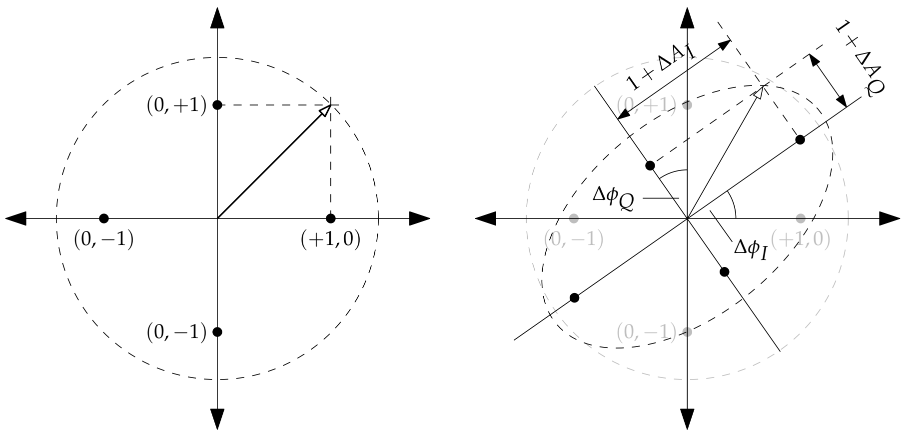

Figure 1: LO signal without (left) and with (right) IQ mismatch.

1.1 Transmitter

A radio transmitter utilizes a positive frequency LO to up-convert a baseband signal to RF. From (1), a positive frequency LO corresponds to $k = -1$, and we can substitute that in (4) to find the desired and image coefficients in the presence of IQ mismatch

$$\begin{aligned} A_{\mathrm{LO},desired} &= A_{\mathrm{IQ}}(-1) = \frac{1}{2}\Big((1+\Delta A_I)e^{+\jmath\Delta\phi_I} + (1+\Delta A_Q)e^{+\jmath\Delta\phi_Q}\Big) \\ A_{\mathrm{LO},image} &= A^*_{\mathrm{IQ}}(+1) = \frac{1}{2}\Big((1+\Delta A_I)e^{-\jmath\Delta\phi_I} - (1+\Delta A_Q)e^{-\jmath\Delta\phi_Q}\Big) \end{aligned} \tag{5}$$

and, given a complex baseband signal $s(t)$, the complex up-converted signal $s_{\mathrm{TX}}(t)$ is

$$s_{\mathrm{TX}}(t) = A_{\mathrm{IQ}}(-1)\ s(t)\cdot e^{+\jmath(\omega_{\mathrm{LO}}t+\phi_{\mathrm{LO}})} + A^*_{\mathrm{IQ}}(+1)\ s^*(t)\cdot e^{-\jmath(\omega_{\mathrm{LO}}t+\phi_{\mathrm{LO}})} \tag{6}$$

Figure 2 shows the up-conversion process in the frequency domain for a direct conversion transmitter with IQ mismatch. The LO image due to IQ mismatch results in a signal image that degrades the signal-to-noise ratio (SNR) of the transmitted signal[1]. The image rejection ratio of the transmitter, IRR_{TX} , is given by

$$IRR_{\mathrm{TX}} = \left|\frac{A_{\mathrm{IQ}}(+1)}{A_{\mathrm{IQ}}(-1)}\right|^2 \tag{7}$$

Note that the SNR is independent of both the carrier and LO signal levels, so the only way to improve the signal quality is to improve IQ matching.

[1]For a digital radio transmitter, we usually talk about error vector magnitude (EVM), which is a measure of error in the transmitted constellation. The relation between root-mean square EVM and SNR is straightforward: $SNR \approx 1/EVM^2_{rms}$

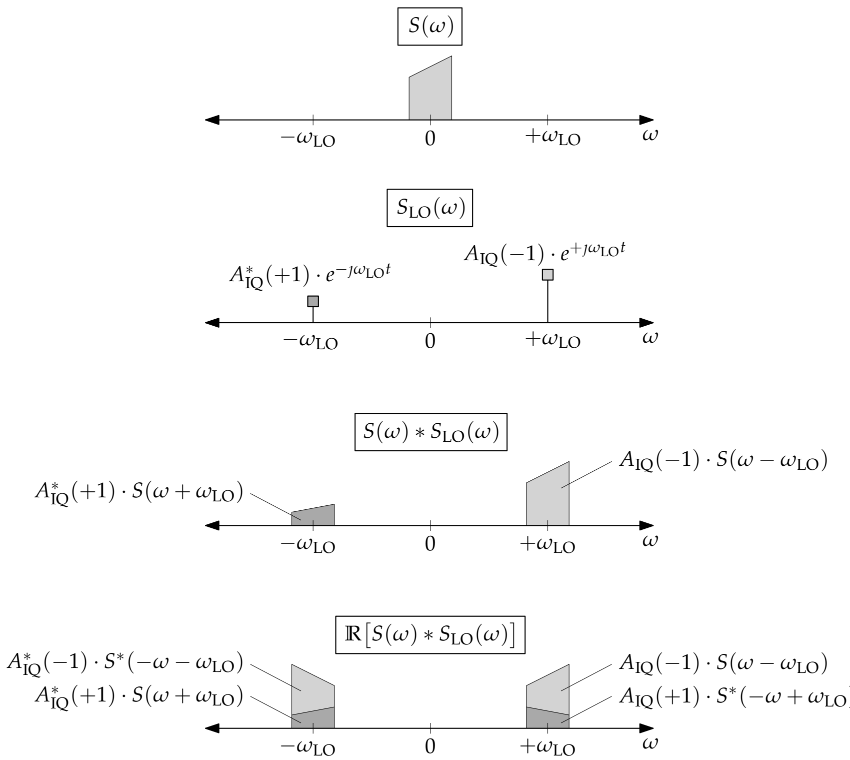

Figure 2: Frequency domain representation of the up-conversion process in a direct conversion transmitter in the presence of IQ mismatch. Mismatch is modeled in LO domain.

1.2 Receiver

A radio transmitter uses a negative frequency LO to down-convert the received RF signal to baseband. From (1), a negative frequency LO corresponds to $k = +1$, and, once again, we can substitute that in (4) to find the desired and image coefficients in the presence of IQ mismatch

$$\begin{aligned} A_{\mathrm{LO},desired} &= A^*_{\mathrm{IQ}}(-1) = \frac{1}{2}\Big((1+\Delta A_I)e^{-\jmath\Delta\phi_I} + (1+\Delta A_Q)e^{-\jmath\Delta\phi_Q}\Big) \\ A_{\mathrm{LO},image} &= A_{\mathrm{IQ}}(+1) = \frac{1}{2}\Big((1+\Delta A_I)e^{+\jmath\Delta\phi_I} - (1+\Delta A_Q)e^{+\jmath\Delta\phi_Q}\Big) \end{aligned} \tag{8}$$

and, given a complex baseband signal $s(t)$, the complex down-converted signal $s_{\mathrm{RX}}(t)$ is

$$s_{\mathrm{RX}}(t) = A^*_{\mathrm{IQ}}(-1) \cdot s(t) + A_{\mathrm{IQ}}(+1) \cdot s^*(t) \tag{9}$$

Figure 3 shows the down-conversion process in the frequency domain for a direct conversion receiver with IQ mismatch. Similar to its transmitter counterpart, the LO image in a receiver results in a signal image that degrades the SNR of the received signal. The image rejection ratio of the received, IRR_{RX} is given by

$$IRR_{\mathrm{RX}} = \left| \frac{A_{\mathrm{IQ}}(+1)}{A_{\mathrm{IQ}}(-1)} \right|^2 \tag{10}$$

which is the same expression found for the transmitter in (7).

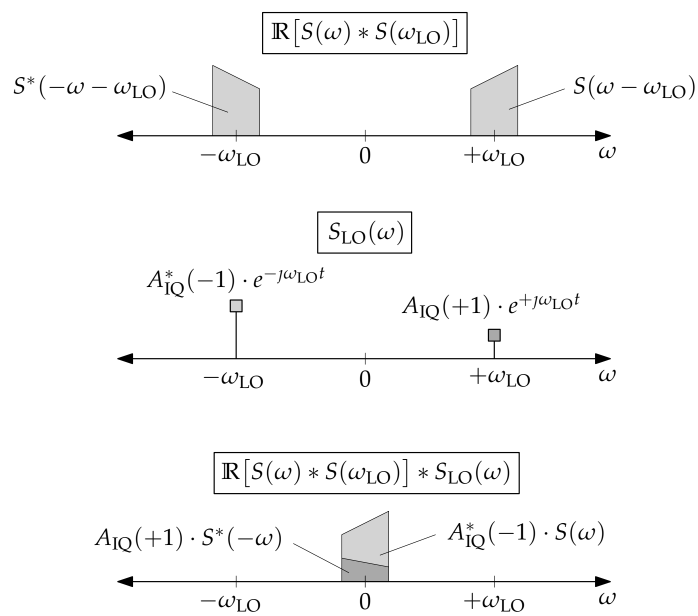

Figure 3: Frequency domain representation of the down-conversion process in a direct conversion receiver in the presence of IQ mismatch. Mismatch is modeled in LO domain.

2 Mismatch Model: Signal Domain

Alternatively, IQ mismatch can be modeled in the signal path instead of the LO path.

2.1 Transmitter

For a transmitter, the signal domain mismatch model can be derived graphically from Fig. 2 by moving the image component of the up-converted signal to the input baseband signal, and replacing the mismatched LO signal with an ideal complex sinusoid. The resulting up-conversion process is shown in Fig. 4. Note that the final up-converted signal is identical to that in Fig 2, and the equivalent baseband signal is now

$$s(t) = A_{\mathrm{IQ}}(-1) \cdot s(t) + A_{\mathrm{IQ}}(+1) \cdot s^*(t) \tag{11}$$

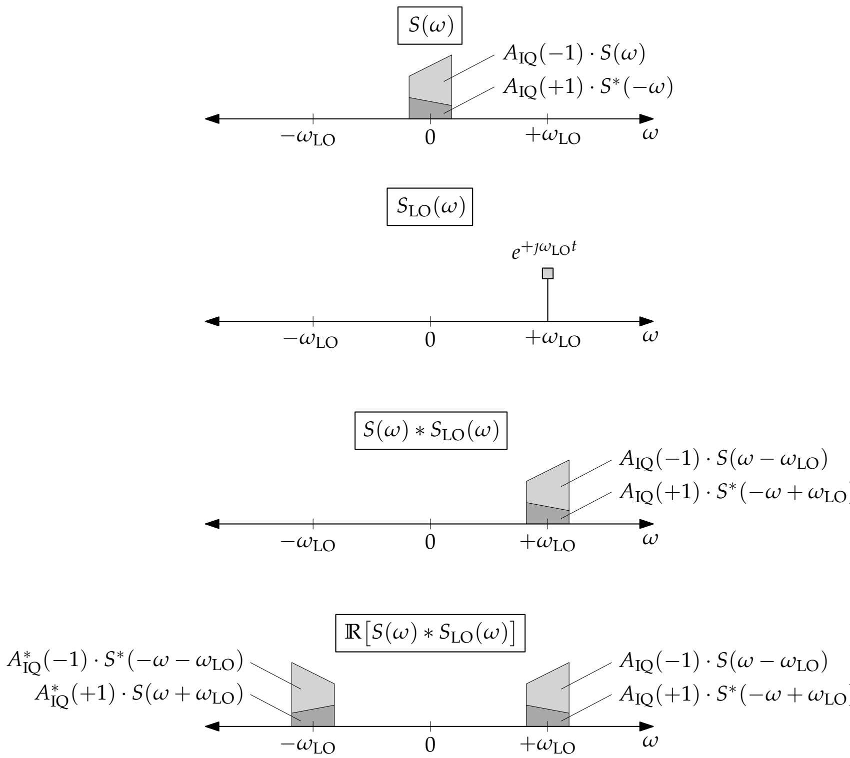

Figure 4: IQ mismatch modeled in the signal path of a direct conversion transmitter. The resulting up-converted signal is identical to that in Fig 2.

2.2 Receiver

Similarly, for a receiver, the signal domain mismatch model can be derived graphically from Fig. 2 by moving the image component of the down-converted signal to the input RF signal, and replacing the mismatched LO signal with an ideal complex sinusoid. The resulting down-conversion process is shown in Fig. 5. Note that the final down-converted signal is identical to that in Fig 3.

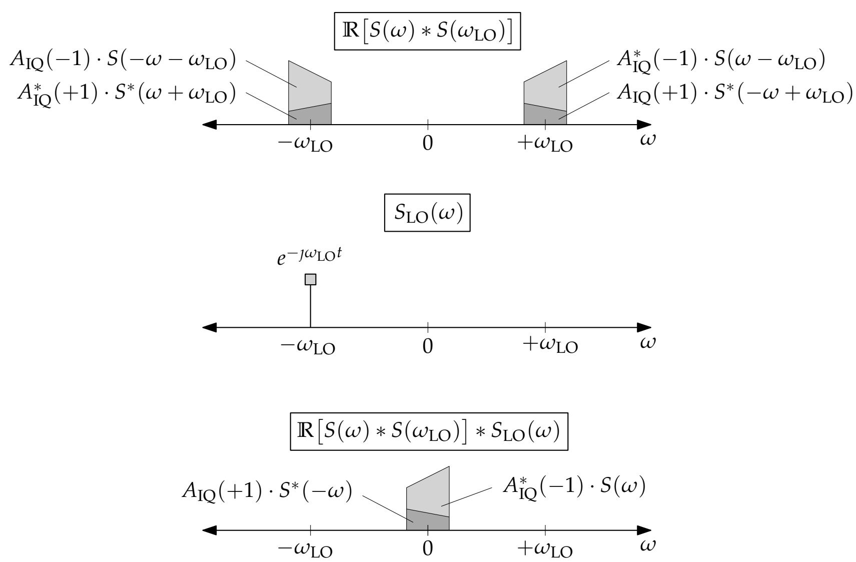

Figure 5: IQ mismatch modeled in the signal path of a direct conversion transmitter. The resulting down-converted signal is identical to that in Fig 3.