Phase Noise - Time Domain Model

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An ideal local oscillator (LO) signal, $v_{\mathrm{LO}}(t)$, is periodic, so it can be expressed as a Fourier series

$$v_{\mathrm{LO}}(t) = \sum_{k=-\infty}^{\infty} a_k e^{\jmath k \omega_{\mathrm{LO}} t} \tag{1}$$

In practice, however, an LO signal corrupted by noise exhibits variations in zero crossings, which can be modeled by introducing a time-dependent jitter term $t_j(t)$

$$v_{\mathrm{LO}}(t, t_j) = \sum_{k=-\infty}^{\infty} a_k e^{\jmath k \omega_{\mathrm{LO}} \left(t + t_j(t)\right)} = v_{\mathrm{LO}}(t) \cdot e^{\jmath k \omega_{\mathrm{LO}} t_j(t)} \tag{2}$$

Alternatively, this jitter effect can be expressed in terms of the corresponding phase variation $\theta_j(t)$

$$v_{\mathrm{LO}}(t, \theta_j) = v_{\mathrm{LO}}(t) \cdot e^{\jmath \theta_j(t)} \tag{3}$$

$$\theta_j(t) = k \omega_{\mathrm{LO}} t_j(t) \tag{4}$$

Usually, by design, $\theta_j(t) \ll 1$, and the effect of jitter can be reduced to an additive noise term $v_{nj}(t)$

$$v_{\mathrm{LO}}(t, \theta_j) \approx v_{\mathrm{LO}}(t)\big(1 + \jmath \theta_j(t)\big) \approx v_{\mathrm{LO}}(t) + v_{nj}(t) \tag{5}$$

$$v_{nj}(t) = \jmath v_{\mathrm{LO}}(t) \cdot \theta_j(t) \tag{6}$$

That is, jitter effectively introduces a quadrature copy of the LO signal, amplitude modulated by phase noise.

We can now evaluate the signal-to-noise ratio (SNR) of the LO signal, making use of the fact that LO and phase noise signals are statistically independent, and, therefore, the mean square of their product is the product of their mean squares

$$SNR_{\mathrm{LO}} = \frac{\overline{v_{\mathrm{LO}}^2(t)}}{\overline{v_{nj}^2(t)}} = \frac{\overline{v_{\mathrm{LO}}^2(t)}}{\overline{v_{\mathrm{LO}}^2(t)} \cdot \overline{\theta_j^2(t)}} = \frac{1}{\theta_{j,rms}^2} \tag{7}$$

The result in (7) tells us that the SNR is independent of the LO signal power, and is only a function of the rms phase noise $\theta_{j,rms}$.