RC Calibration

Shadi Youssef

November 30, 2019

1 RC Variation

Ideally, the corner frequency ω_0 associated with a series or parallel RC combination is[1]

$$\omega_0 = \frac{1}{RC} \tag{1}$$

where R and C are the nominal values of resistance and capacitance, respectively. In practice, however, process spread, random mismatches and temperature variation are all factors that cause both R and C to deviate from their nominal values. Assuming a maximum deviation of ΔR for R and ΔC for C, the actual corner frequency ω_{RC} is

$$\begin{aligned}
\omega_{RC} &= \frac{1}{(R \pm \Delta R)(C \pm \Delta C)} \\
&= \frac{1}{RC} \cdot \frac{1}{(1 \pm \frac{\Delta R}{R})(1 \pm \frac{\Delta C}{C})} \\
&= \frac{1}{RC} \cdot \frac{1}{1 \pm \frac{\Delta R}{R} \pm \frac{\Delta C}{C} \pm \frac{\Delta R}{R}\frac{\Delta C}{C}} \\
&\approx \frac{1}{RC} \cdot \frac{1}{1 \pm \frac{\Delta R}{R} \pm \frac{\Delta C}{C}} \qquad (2)
\end{aligned}$$

where the terms $\frac{\Delta R}{R}$ and $\frac{\Delta C}{C}$ are recognized as the maximum relative variation of R and C, respectively, and, given that both are usually $\ll 1$, the second order term $\frac{\Delta R}{R}\frac{\Delta C}{C}$ has been neglected. Thus, the RC product variation is approximated as the sum of variations in R and C, and (2) can be written as

$$\omega_{RC} = \frac{1}{RC} \cdot \frac{1}{1 \pm \Delta_{RC}} \tag{3}$$

$$\Delta_{RC} = \frac{\Delta R}{R} + \frac{\Delta C}{C} \tag{4}$$

where Δ_{RC} is the maximum relative variation of the RC product.

[1] alternatively, ω_0 is the pole or zero associated with an RC combination

2 RC Calibration

A programmable capacitor can be used to calibrate both R and C variations. From (3), the capacitance range required for calibration $[C_{\text{min}}, C_{\text{max}}]$ is given by

$$\begin{aligned} C_{\text{min}} = & \quad C(1 - \Delta_{RC}) \\ C_{\text{max}} = & \quad C(1 + \Delta_{RC}) \end{aligned} \tag{5}$$

Implementing the programmable capacitor as an N-bit capacitor bank, calibration step size C_{LSB} is

$$C_{\text{LSB}} = \frac{C_{\text{max}} - C_{\text{min}}}{2^N - 1} \approx \frac{C_{\text{max}} - C_{\text{min}}}{2^N} \tag{6}$$

where a high resolution capacitor bank ($N \gg 1$) is assumed. Consequently, from (5) in (6)

$$\frac{C_{\text{LSB}}}{C} = \frac{\Delta_{RC}}{2^{N-1}} \tag{7}$$

and the minimum and maximum capacitance values can be expressed in terms of the capacitance bank resolution by substituting (7) in (5)

$$\begin{aligned} C_{\text{min}} = & \quad C - C_{\text{LSB}} \cdot 2^{N-1} \\ C_{\text{max}} = & \quad C + C_{\text{LSB}} \cdot 2^{N-1} \end{aligned} \tag{8}$$

2.1 Single-Path Calibration

For a single RC path, we can define the relative RC calibration accuracy ϵ

$$\varepsilon = \frac{\omega_{0,\text{cal}} - \omega_0}{\omega_0} \tag{9}$$

where ω_0 nominal corner frequency and $\omega_{0,\text{cal}}$ is the calibrated corner frequency. Post calibration, maximum error corresponds to a ± 0.5LSB capacitance error

$$\omega_{0,\text{cal}} = \frac{1}{R(C \pm \frac{C_{\text{LSB}}}{2})} = \frac{1}{RC} \cdot \frac{1}{1 \pm \frac{1}{2} \cdot \frac{C_{\text{LSB}}}{C}} \tag{10}$$

and from (1) and (10) in (9)

$$\varepsilon = \pm \frac{1}{2} \frac{C_{\text{LSB}}}{C} \cdot \frac{1}{1 \pm \frac{1}{2} \cdot \frac{C_{\text{LSB}}}{C}} \approx \pm \frac{1}{2} \frac{C_{\text{LSB}}}{C} \tag{11}$$

where, once again, we assume high resolution calibration ($\frac{C_{\text{LSB}}}{C} \ll 1$). Substituting (7) in (11) results in

$$\frac{\varepsilon}{\Delta_{RC}} = \pm \frac{1}{2^N} \tag{12}$$

The expression in (12) provides the relation between the maximum expected RC variation and the desired calibration accuracy in terms of the number of bits necessary to achieve that accuracy.

2.2 Two-Path Calibration

In some cases, multiple RC paths requires calibration relative to each other. One example is IQ calibration in a wireless transceiver. In this case, we define the relative RC calibration accuracy $\varepsilon_{\mathrm{IQ}}$

$$\varepsilon_{\mathrm{IQ}} = \frac{\omega_{0\mathrm{I,cal}} - \omega_{0\mathrm{Q,cal}}}{\omega_0} \tag{13}$$

where $\omega_{0\mathrm{I,cal}}$ and $\omega_{0\mathrm{Q,cal}}$ are the post calibration RC corner frequencies, for the I and Q paths, respectively. Using (9), we can then rewrite the IQ calibration error in (13) as the sum of I and Q calibration errors relative to the nominal corner frequency ω_0

$$\begin{aligned}
\varepsilon_{\mathrm{IQ}} &= \frac{\omega_{0\mathrm{I,cal}} - \omega_0 + \omega_0 - \omega_{0\mathrm{Q,cal}}}{\omega_o} \\
&= \frac{\omega_{0\mathrm{I,cal}} - \omega_0}{\omega_0} - \frac{\omega_{0\mathrm{Q,cal}} - \omega_0}{\omega_0} \\
&= \varepsilon_{\mathrm{I}} - \varepsilon_{\mathrm{Q}}
\end{aligned} \tag{14}$$

For worst-case post calibration error, ε_{I} and ε_{Q} have opposite polarities. That is, the I-path error would correspond to $+0.5$LSB, and the Q-path error would correspond to -0.5LSB, or vice versa, for a total error of ± 1LSB. Thus, by substituting (12) in (14)

$$\frac{\varepsilon_{\mathrm{IQ}}}{\Delta_{RC}} = \pm \frac{2}{2^N} \tag{15}$$

Comparing (12) and (15) reveals that, for the same RC variation and calibration resolution, the IQ calibration error is twice that of either path.

2.3 Example

Consider a simple RC combination with a 10MHz corner frequency and 10kΩ resistor. The maximum expected variation per component is 20%, and the required RC matching accuracy between I and Q paths is 2%.

First, the nominal capacitance value can be calculated from (1)

$$C = \frac{1}{\omega_0 R} = \frac{1}{2\pi \cdot 10 \cdot 10^6 \times 10 \cdot 10^3} = 1.6\mathrm{pF} \tag{16}$$

Next, from (15), the capacitor bank resolution is

$$N = \log_2\left(\frac{\Delta_{RC}}{\varepsilon_{\mathrm{IQ}}}\right) + 1 = \log_2\left(\frac{0.2 + 0.2}{0.02}\right) + 1 = 5.3 \tag{17}$$

That is, a 6-bit capacitor bank is required. Using (7), the corresponding capacitance step size can be found

$$C_{\mathrm{LSB}} = C \cdot \frac{\Delta_{RC}}{2^{N-1}} = 1.6 \cdot 10^{-12} \times \frac{0.2 + 0.2}{2^{6-1}} = 20\mathrm{fF} \tag{18}$$

which, from (8), means the capacitor bank range is

$$
\begin{aligned}
C_{\text{min}} &= C - C_{\text{LSB}} \cdot 2^{N-1} \\
&= 1.6 \cdot 10^{-12} - 40 \cdot 10^{-15} \times 2^{6-1} = 320\text{fF}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
C_{\text{max}} &= C + C_{\text{LSB}} \cdot 2^{N-1} \\
&= 1.6 \cdot 10^{-12} + 40 \cdot 10^{-15} \times 2^{6-1} = 2.88\text{pF}
\end{aligned}
\tag{20}
$$