IQ Calibration for Radio Transceivers

Shadi Youssef

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1 Recap

- In a radio transceiver, IQ mismatch introduces an image problem. In a direct conversion architecture, given an ideal complex signal $s(t)$

$$s(t) = s_I(t) + \jmath s_Q(t) \tag{1}$$

the baseband equivalent of the transmitted signal, $s_{\mathrm{TX}}(t)$, or the received signal, $s_{\mathrm{RX}}(t)$, becomes the sum of two terms

$$s_{\mathrm{TX}}(t) = A_{\mathrm{IQ}}(-1) \cdot s(t) + A_{\mathrm{IQ}}(+1) \cdot s^*(t) \tag{2}$$

$$s_{\mathrm{RX}}(t) = A^*_{\mathrm{IQ}}(-1) \cdot s(t) + A_{\mathrm{IQ}}(+1) \cdot s^*(t) \tag{3}$$

where the first term in each of (2) and (3) represents the signal and the second term represents its undesired image. The coefficients $A_{\mathrm{IQ}}(\pm 1)$ capture IQ mismatch

$$A_{\mathrm{IQ}}(k) = \frac{1}{2}\Big((1 + \Delta A_I)e^{+\jmath\Delta\phi_I} - k(1 + \Delta A_Q)e^{+\jmath\Delta\phi_Q}\Big) \tag{4}$$

where ΔA and $\Delta\phi$ are the amplitude and phase mismatches, respectively, and the I/Q subscripts denote the I/Q branches.

- For both transmitter and receiver, the image rejection ratio IRR is

$$IRR = \left|\frac{A_{\mathrm{IQ}}(+1)}{A_{\mathrm{IQ}}(-1)}\right|^2 \tag{5}$$

2 IQ Crosstalk

2.1 Transmitter

By using (1) and (4) in (2) and collecting phase mismatch terms

$$\begin{aligned} s_{\mathrm{TX}}(t) = &\frac{1}{2}\big((1 + \Delta A_I)e^{+\jmath\Delta\phi_I} + (1 + \Delta A_Q)e^{+\jmath\Delta\phi_Q}\big)\big(s_I(t) + \jmath s_Q(t)\big) \\ &+ \frac{1}{2}\big((1 + \Delta A_I)e^{-\jmath\Delta\phi_I} - (1 + \Delta A_Q)e^{-\jmath\Delta\phi_Q}\big)\big(s_I(t) - \jmath s_Q(t)\big) \end{aligned} \tag{6}$$

$$
\begin{aligned}
s_{\mathrm{TX}}(t) = &\big((1+\Delta A_I)\cos(\Delta\phi_I) + \jmath(1+\Delta A_Q)\sin(\Delta\phi_Q)\big)\, s_I(t) \\
&- \big((1+\Delta A_I)\sin(\Delta\phi_I) - \jmath(1+\Delta A_Q)\cos(\Delta\phi_Q)\big)\, s_Q(t)
\end{aligned} \tag{7}
$$

Assuming $\Delta A_I = -\Delta A_Q = \Delta A/2$ and $\Delta\phi_I = -\Delta\phi_Q = -\Delta\phi/2$ and rearranging terms into real and imaginary components

$$
\begin{aligned}
s_{\mathrm{TX}}(t) = &(1+\frac{\Delta A}{2})\cos(\frac{\Delta\phi}{2})\, s_I(t) + (1+\frac{\Delta A}{2})\sin(\frac{\Delta\phi}{2})\, s_Q(t) \\
&+\jmath\big((1-\frac{\Delta A}{2})\sin(\frac{\Delta\phi}{2})\, s_I(t) + (1-\frac{\Delta A}{2})\cos(\frac{\Delta\phi}{2})\, s_Q(t)\big)
\end{aligned} \tag{8}
$$

The real and imaginary terms in (8) represent a pair of wires carrying the transmitted I and Q components. In matrix form, (8) can be written as

$$
\begin{pmatrix} s_{\mathrm{TX},I} \\ s_{\mathrm{TX},Q} \end{pmatrix} = \begin{pmatrix} +(1+\frac{\Delta A}{2})\cos(\frac{\Delta\phi}{2}) & +(1+\frac{\Delta A}{2})\sin(\frac{\Delta\phi}{2}) \\ +(1-\frac{\Delta A}{2})\sin(\frac{\Delta\phi}{2}) & +(1-\frac{\Delta A}{2})\cos(\frac{\Delta\phi}{2}) \end{pmatrix} \cdot \begin{pmatrix} s_{\mathrm{I}} \\ s_{\mathrm{Q}} \end{pmatrix} \tag{9}
$$

where the explicit time dependence has been dropped for clarity. For small IQ mismatches, $\Delta A \ll 1$ and $\Delta\phi \ll \pi/2$, and (9) can be simplified to

$$
\begin{pmatrix} s_{\mathrm{TX},I} \\ s_{\mathrm{TX},Q} \end{pmatrix} \approx \begin{pmatrix} 1+\frac{\Delta A}{2} & +\frac{\Delta\phi}{2} \\ +\frac{\Delta\phi}{2} & 1-\frac{\Delta A}{2} \end{pmatrix} \cdot \begin{pmatrix} s_{\mathrm{I}} \\ s_{\mathrm{Q}} \end{pmatrix} \tag{10}
$$

$$
\mathbf{s_{TX}} \approx \mathbf{QME_{TX}} \cdot \mathbf{s} \tag{11}
$$

The quadrature modulator error matrix $\mathbf{QME_{TX}}$ maps the ideal IQ signal $\mathbf{s}$ to the mismatched IQ signal $\mathbf{s_{TX}}$. Graphically, this can be represented as shown in Fig. 1, which shows that the impact of IQ mismatch is to introduce cross talk between the I and Q branches of transmitter.

2.2 Receiver

In a similar manner, the receiver can be analyzed by using (1) and (4) in (3) and collecting phase mismatch terms

$$
\begin{aligned}
s_{\mathrm{RX}}(t) = &\frac{1}{2}\big((1+\Delta A_I)e^{-\jmath\Delta\phi_I} + (1+\Delta A_Q)e^{-\jmath\Delta\phi_Q}\big)\big(s_I(t) + \jmath s_Q(t)\big) \\
&+ \frac{1}{2}\big((1+\Delta A_I)e^{+\jmath\Delta\phi_I} - (1+\Delta A_Q)e^{+\jmath\Delta\phi_Q}\big)\big(s_I(t) - \jmath s_Q(t)\big)
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
s_{\mathrm{RX}}(t) = &\big((1+\Delta A_I)\cos(\Delta\phi_I) - \jmath(1+\Delta A_Q)\sin(\Delta\phi_Q)\big)\, s_I(t) \\
&+ \big((1+\Delta A_I)\sin(\Delta\phi_I) + \jmath(1+\Delta A_Q)\cos(\Delta\phi_Q)\big)\, s_Q(t)
\end{aligned} \tag{13}
$$

Assuming $\Delta A_I = -\Delta A_Q = \Delta A/2$ and $\Delta\phi_I = -\Delta\phi_Q = -\Delta\phi/2$ and rearranging terms into real and imaginary components

$$
\begin{aligned}
s_{\mathrm{RX}}(t) = &(1+\frac{\Delta A}{2})\cos(\frac{\Delta\phi}{2})\, s_I(t) - (1+\frac{\Delta A}{2})\sin(\frac{\Delta\phi}{2})\, s_Q(t) \\
&-\jmath\big((1-\frac{\Delta A}{2})\sin(\frac{\Delta\phi}{2})\, s_I(t) - (1-\frac{\Delta A}{2})\cos(\frac{\Delta\phi}{2})\, s_Q(t)\big)
\end{aligned} \tag{14}
$$

The real and imaginary terms in (14) represent a pair of wires carrying the received I and Q components. In matrix form, (14) can be written as

$$\begin{pmatrix} s_{\mathrm{RX},I} \\ s_{\mathrm{RX},Q} \end{pmatrix} = \begin{pmatrix} +(1+\frac{\Delta A}{2})\cos(\frac{\Delta\phi}{2}) & -(1+\frac{\Delta A}{2})\sin(\frac{\Delta\phi}{2}) \\ -(1-\frac{\Delta A}{2})\sin(\frac{\Delta\phi}{2}) & +(1-\frac{\Delta A}{2})\cos(\frac{\Delta\phi}{2}) \end{pmatrix} \cdot \begin{pmatrix} s_{\mathrm{I}} \\ s_{\mathrm{Q}} \end{pmatrix} \tag{15}$$

where the explicit time dependence has been dropped for clarity. For small IQ mismatches, $\Delta A \ll 1$ and $\Delta\phi \ll \pi/2$, and (15) can be simplified to

$$\begin{pmatrix} s_{\mathrm{RX},I} \\ s_{\mathrm{RX},Q} \end{pmatrix} \approx \begin{pmatrix} 1+\frac{\Delta A}{2} & -\frac{\Delta\phi}{2} \\ -\frac{\Delta\phi}{2} & 1-\frac{\Delta A}{2} \end{pmatrix} \cdot \begin{pmatrix} s_{\mathrm{I}} \\ s_{\mathrm{Q}} \end{pmatrix} \tag{16}$$

$$\mathbf{s_{RX}} \approx \mathbf{QME_{RX}} \cdot \mathbf{s} \tag{17}$$

Similar to its transmitter counterpart, the quadrature modulator error matrix $\mathbf{QME_{RX}}$ maps the ideal IQ signal $\mathbf{s}$ to the mismatched IQ signal $\mathbf{s_{RX}}$. Graphically, this can be represented as shown in Fig. 1, which shows that the impact of IQ mismatch is to introduce cross talk between the I and Q branches of receiver.

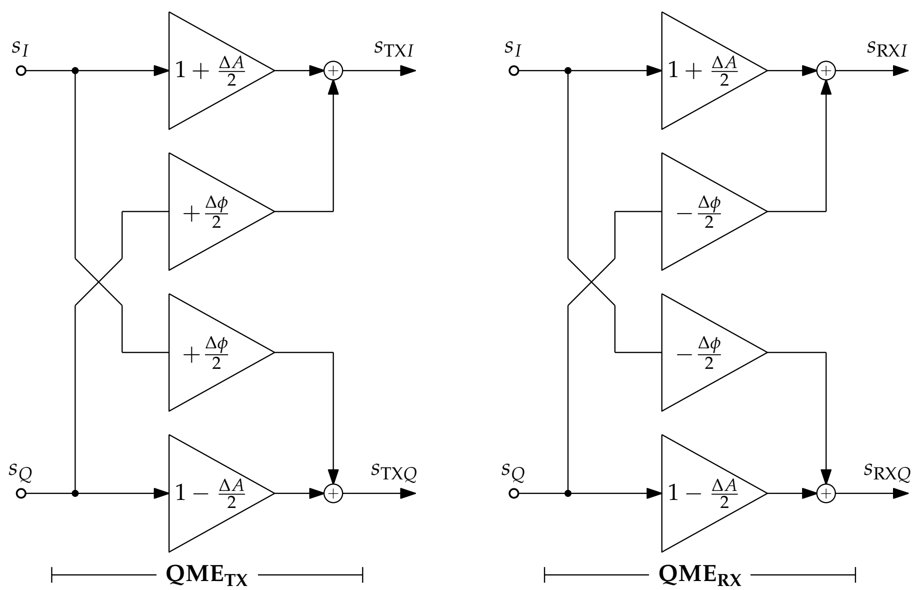

Figure 1: Quadrature modulator error model (QME) for a transmitter (left) and a receiver (right).

3 IQ Calibration

From (11) and (17), we can find a calibration matrix $\mathbf{QMC} = \mathbf{QME}^{-1}$ such that

$$\begin{aligned} \mathbf{s_{TX,cal}} &= \mathbf{QME_{TX}} \cdot \mathbf{QMC_{TX}} \cdot \mathbf{s} \\ \mathbf{s_{RX,cal}} &= \mathbf{QMC_{RX}} \cdot \mathbf{QME_{RX}} \cdot \mathbf{s} \end{aligned} \tag{18}$$

From (10) and (16), we can show that for small mismatches

$$\begin{aligned} \mathbf{QMC_{TX}} &\approx \begin{pmatrix} 1 - \frac{\Delta A}{2} & -\frac{\Delta \phi}{2} \\ -\frac{\Delta \phi}{2} & 1 + \frac{\Delta A}{2} \end{pmatrix} \\ \mathbf{QMC_{RX}} &\approx \begin{pmatrix} 1 - \frac{\Delta A}{2} & +\frac{\Delta \phi}{2} \\ +\frac{\Delta \phi}{2} & 1 + \frac{\Delta A}{2} \end{pmatrix} \end{aligned} \tag{19}$$

The cascade of the error and calibration models is shown in Fig. 2 and 3 for the transmitter and receiver, respectively.

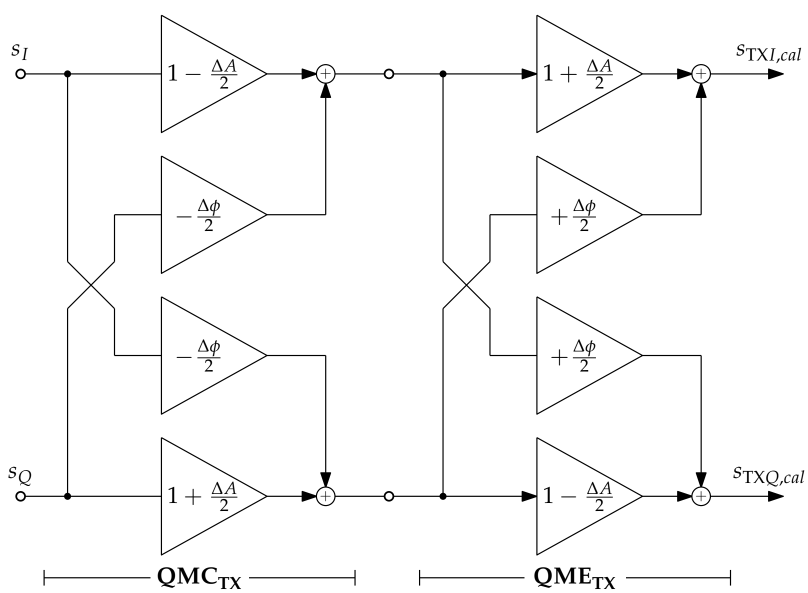

Figure 2: Quadrature modulator calibration model (QMC) model for a transmitter.

4 Mismatch Terms Estimation

The mismatch terms ΔA and $\Delta\phi$ can be estimated based on the statistical properties of the complex signal being transmitted or received.

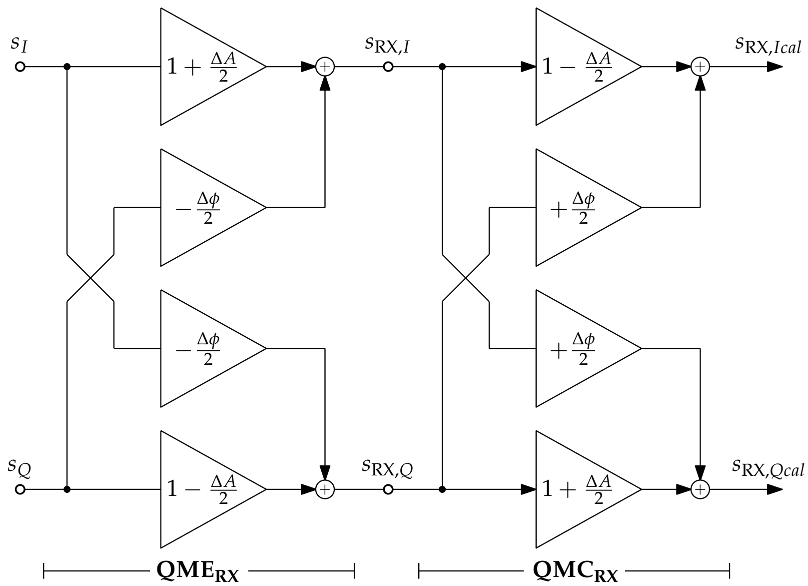

Figure 3: Quadrature modulator calibration model (QMC) for a receiver.

The ideal complex signal $s(t)$ is circular because (1) represents a vector that traces a circular path over time, and any IQ mismatch introduced destroys circularity. Evaluating circularity can be evaluated property is captured by the expected value of the square of the signal

$$\begin{aligned}\mathbb{E}\left[s^2(t)\right] &= \mathbb{E}\left[\left(s_I(t)+\jmath s_Q(t)\right)\cdot\left(s_I(t)+\jmath s_Q(t)\right)\right]\\ &= \mathbb{E}\left[s_I^2(t)-s_Q^2(t)\right]+\jmath\,2\,\mathbb{E}\left[s_I(t)\cdot s_Q(t)\right] \end{aligned}\tag{20}$$

The real term in (20) is the difference of IQ variances and is a measure of amplitude mismatch, while the imaginary term is the IQ co-variance and is a measure of phase mismatch. Thus, for the ideal complex signal $s(t)$

$$\begin{aligned}\mathbb{E}\left[s_I^2(t)-s_Q^2(t)\right] &= 0\\ \mathbb{E}\left[s_I(t)\cdot s_Q(t)\right] &= 0\end{aligned}\tag{21}$$

Thus, for a mismatched IQ signal, IQ variance and co-variance carry information about amplitude and phase mismatch, respectively.

4.1 Transmitter

Using (10), we calculate the IQ variance of the mismatched transmitter signal

$$\begin{aligned}\mathbb{E}\left[s_{\mathrm{TX},I}^2(t)-s_{\mathrm{TX},Q}^2(t)\right] &= (\Delta\phi^2/4)\,\mathbb{E}\left[s_I^2(t)-s_Q^2(t)\right] \\ &\quad +\Delta A\,\Delta\phi\,\mathbb{E}\left[s_I(t)\cdot s_Q(t)\right] \\ &\quad +\Delta A\,\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right] \qquad (22)\end{aligned}$$

and from (21) in (22), we can find an estimate of amplitude mismatch

$$\Delta A = \frac{\mathbb{E}\left[s_{\mathrm{TX},I}^2(t)-s_{\mathrm{TX},Q}^2(t)\right]}{\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right]} \approx \frac{\mathbb{E}\left[s_{\mathrm{TX},I}^2(t)-s_{\mathrm{TX},Q}^2(t)\right]}{\mathbb{E}\left[s_{\mathrm{TX},I}^2(t)+s_{\mathrm{TX},Q}^2(t)\right]} \qquad (23)$$

where we assume that the presence of IQ mismatch does not significantly change the total power of the signal so $\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right] \approx \mathbb{E}\left[s_{\mathrm{TX},I}^2(t)+s_{\mathrm{TX},Q}^2(t)\right]$.

Similarly, using (10), we calculate the IQ co-variance of the mismatched transmitter signal

$$\begin{aligned}\mathbb{E}\left[s_{\mathrm{TX},I}(t)\cdot s_{\mathrm{TX},Q}(t)\right] &= (1-\Delta A^2/4+\Delta\phi^2/4)\,\mathbb{E}\left[s_I(t)\cdot s_Q(t)\right] \\ &\quad +(\Delta\phi/2)\,\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right] \qquad (24)\end{aligned}$$

and from (21) in (24), we can find an estimate of phase mismatch

$$\Delta\phi = 2\,\frac{\mathbb{E}\left[s_{\mathrm{TX},I}(t)\cdot s_{\mathrm{TX},Q}(t)\right]}{\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right]} \approx 2\,\frac{\mathbb{E}\left[s_{\mathrm{TX},I}(t)\cdot s_{\mathrm{TX},Q}(t)\right]}{\mathbb{E}\left[s_{\mathrm{TX},I}^2(t)+s_{\mathrm{TX},Q}^2(t)\right]} \qquad (25)$$

4.2 Receiver

In a similar fashion, we use (16) to calculate the IQ variance of the mismatched receiver signal

$$\begin{aligned}\mathbb{E}\left[s_{\mathrm{RX},I}^2(t)-s_{\mathrm{RX},Q}^2(t)\right] &= (\Delta\phi^2/4)\,\mathbb{E}\left[s_I^2(t)-s_Q^2(t)\right] \\ &\quad -\Delta A\,\Delta\phi\,\mathbb{E}\left[s_I(t)\cdot s_Q(t)\right] \\ &\quad +\Delta A\,\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right] \qquad (26)\end{aligned}$$

and from (21) in (26), we can find an estimate of amplitude mismatch

$$\Delta A = \frac{\mathbb{E}\left[s_{\mathrm{RX},I}^2(t)-s_{\mathrm{RX},Q}^2(t)\right]}{\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right]} \approx \frac{\mathbb{E}\left[s_{\mathrm{RX},I}^2(t)-s_{\mathrm{RX},Q}^2(t)\right]}{\mathbb{E}\left[s_{\mathrm{RX},I}^2(t)+s_{\mathrm{RX},Q}^2(t)\right]} \qquad (27)$$

which is identical to the transmitter estimate in (27).

Similarly, using (16), we can estimate the IQ co-variance of the mismatched receiver signal

$$\begin{aligned}\mathbb{E}\left[s_{\mathrm{RX},I}(t)\cdot s_{\mathrm{RX},Q}(t)\right] &= (1-\Delta A^2/4+\Delta\phi^2/4)\,\mathbb{E}\left[s_I(t)\cdot s_Q(t)\right] \\ &\quad -(\Delta\phi/2)\,\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right]\end{aligned} \tag{28}$$

and from (21) in (28), we can find an estimate of phase mismatch

$$\Delta\phi = -2\,\frac{\mathbb{E}\left[s_{\mathrm{RX},I}(t)\cdot s_{\mathrm{RX},Q}(t)\right]}{\mathbb{E}\left[s_I^2(t)+s_Q^2(t)\right]} \approx -2\,\frac{\mathbb{E}\left[s_{\mathrm{RX},I}(t)\cdot s_{\mathrm{RX},Q}(t)\right]}{\mathbb{E}\left[s_{\mathrm{TX},I}^2(t)+s_{\mathrm{TX},Q}^2(t)\right]} \tag{29}$$

which is identical to the transmitter estimate in (29), except for a reversed polarity.