L-Matching Networks

Shadi Youssef

April 3, 2020

1 Series – Parallel Transformation

The two networks in Fig. 1 can be made to have the same equivalent impedance at an arbitrary frequency. Under this condition, we can transform one network to the other.

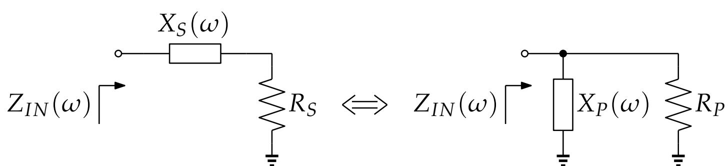

Figure 1: Series–parallel impedance transformation.

The impedance $Z_S(\omega)$ of the series network and the admittance $Y_P(\omega)$ of the parallel network are given by

$$Z_S(\omega) = R_S \pm jX_S(\omega) = R_S\Big(1 \pm j\frac{X_S(\omega)}{R_S}\Big) = R_S\Big(1 \pm jQ_S(\omega)\Big) \tag{1}$$

$$Y_P(\omega) = \frac{1}{R_P} \pm \frac{1}{jX_P(\omega)} = \frac{1}{R_P}\Big(1 \pm \frac{R_P}{jX_P(\omega)}\Big) = \frac{1}{R_P}\Big(1 \mp jQ_P(\omega)\Big) \tag{2}$$

where $Q_S(\omega)$ and $Q_P(\omega)$ are the quality factors of the series and parallel networks, respectively, and both are a function of frequency

$$Q_S(\omega) = \frac{X_S(\omega)}{R_S} \tag{3}$$

$$Q_P(\omega) = \frac{R_P}{X_P(\omega)} \tag{4}$$

Equating the impedance of both networks allows us to derive the desired transformation. Strictly speaking, the transformation is only valid at a single arbitrary frequency ω_0[1]

$$Z_S(\omega_0) = \frac{1}{Y_P(\omega_0)} \tag{5}$$

which, from (1) and (2), gives

$$R_S\Big(1 \pm \jmath Q_S(\omega_0)\Big)\Big(1 \mp \jmath Q_P(\omega_0)\Big) = R_P \tag{6}$$

By equating the real and imaginary parts on both sides of (6) we get

$$R_P = R_S\Big(1 + Q_S\, Q_P\Big) \tag{7}$$

$$Q_S - Q_P = 0 \tag{8}$$

where we dropped the dependence of the quality factor on frequency for clarity. Equation (8) tells us that the two networks have the same quality factor, which should be the case if they are equivalent, so we set $Q_S = Q_P = Q$

$$R_P = R_S(1 + Q^2) \tag{9}$$

And from (3) and (4) in (8)

$$\frac{X_S(\omega_0)}{R_S} = \frac{R_P}{X_P(\omega_0)} \tag{10}$$

Now, using (9), we can re-arrange (10)

$$\begin{aligned} X_P(\omega_0) &= \frac{R_S R_P}{X_S(\omega_0)} \\ &= \frac{R_S^2}{X_S(\omega_0)}(1 + Q^2) \\ &= X_S(\omega_0)\Big(\frac{R_S}{X_S(\omega_0)}\Big)^2(1 + Q^2) \end{aligned} \tag{11}$$

And finally, from (3) in (11)

$$X_P(\omega_0) = X_S(\omega_0)\Big(1 + \frac{1}{Q^2}\Big) \tag{12}$$

Equations (9) and (12) give us the means to transform the real and reactive parts, respectively.

2 Step-up L-Matching

Conversely, a step-up network matches a small resistance (load) to a large resistance (source). The L-network in Fig. 2 can achieve such load “amplification” provided that the reactive components $X_S(\omega)$ and $X_P(\omega)$ are of different types,

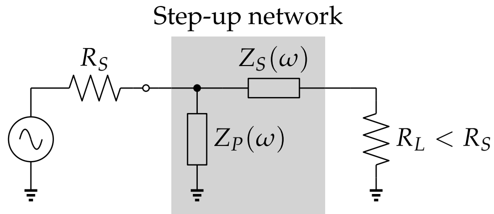

Figure 2: Step-up matching network.

i.e. if one is inductive, the other must be capacitive, and vice versa. To derive matching conditions, we first define the load quality factor Q_L

$$Q_L = \frac{X_S(\omega)}{R_L} \tag{13}$$

And by using (9) and (12), the series load network can be transformed into its parallel equivalent at the desired frequency ω_0

$$R_{S\to P} = R_L(1 + Q_L^2) \tag{14}$$

$$X_{S\to P}(\omega_0) = X_S(\omega_0)\Big(1 + \frac{1}{Q_L^2}\Big) \tag{15}$$

Now conjugate matching is simply a matter of enforcing shunt resonance at ω_0 and setting the real part equal to the source resistance

$$\jmath\Big(\frac{1}{X_P(\omega_0)} - \frac{1}{X_{S\to P}(\omega_0)}\Big) = 0 \tag{16}$$

$$R_S = R_L(1 + Q_L^2) \tag{17}$$

3 Step-down L-Matching

A step-down network matches a large resistance (load) to a small resistance (source). The L-network in Fig. 3 can achieve such load "attenuation" provided that the reactive components $X_S(\omega)$ and $X_P(\omega)$ are of different types, i.e. if one is inductive, the other must be capacitive, and vice versa. To derive matching conditions, we first define the load quality factor Q_L

$$Q_L = \frac{R_L}{X_P(\omega)} \tag{18}$$

[1]the transformation remains a fairly accurate and useful approximation over a narrow-band of frequencies around ω_0.

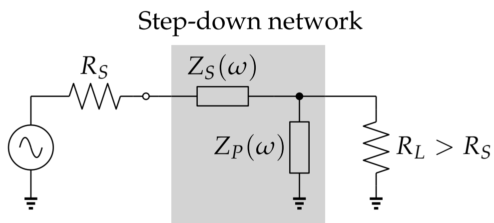

Figure 3: Step-down matching network.

And by using (9) and (12), the parallel load network can be transformed into an equivalent series network at the desired frequency ω_0

$$R_{P\to S} = \frac{R_L}{1+Q_L^2} \tag{19}$$

$$X_{P\to S}(\omega_0) = \frac{X_S(\omega_0)}{1+\frac{1}{Q_L^2}} \tag{20}$$

Now conjugate matching is simply a matter of enforcing series resonance at ω_0 and setting the real part equal to the source resistance

$$\jmath\Big(X_P(\omega_0) - X_{S\to P}(\omega_0)\Big) = 0 \tag{21}$$

$$R_S = \frac{R_L}{1+Q_L^2} \tag{22}$$