Why Power Match?

Shadi Youssef

March 28, 2020

1 Maximum Power from a Voltage source

Fig. 1 shows a load resistance R_L hooked up to a voltage source with equivalent internal resistance R_S. The load voltage and current are given by

$$v_L = v_S \frac{R_L}{R_S + R_L} \tag{1}$$

$$i_L = \frac{v_S}{R_S + R_L} \tag{2}$$

Equations (1) and (2), show that for a given source, increasing the load resistance increases the load voltage towards a maximum of v_S and decreases the load current towards zero (open circuit condition). Since the power delivered to the load is the product of its voltage and current, we expect to find a load resistance value that would maximize the amount of power it extracts from the source.

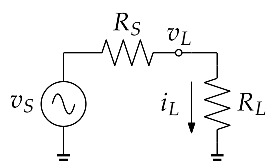

Figure 1: Sinusoidal source connected to a load.

To figure out the condition for maximum power transfer, we first calculate the power delivered to the load from (1) and (2) assuming a sinusoidal source

$$P_L = \frac{1}{2} v_S^2 \frac{R_L}{(R_S + R_L)^2} \tag{3}$$

We then differentiate (3) with respect to R_L and equate the result to zero

$$\frac{\partial P_L}{\partial R_L} = \frac{1}{2} v_S^2 \frac{(R_S + R_L)^2 - 2(R_S + R_L)R_L}{(R_S + R_L)^4} = 0 \tag{4}$$

$$(R_S + R_L) - 2R_L = 0 \rightarrow R_L = R_S \tag{5}$$

which shows the well known result, that maximum power transfer from the source to the load is achieved when the source and load are matched (power match).

It is worth noting that maximum power transfer is always achieved with 50% efficiency. This is because, under a matched condition, the source voltage is equally divided between the load and the internal source resistance, and, therefore, half the power is lost in the later.

Similar results can be derived for a current source.

2 Maximum Voltage from a Power Source

In case of a voltage source, maximizing the voltage across a given load is a matter of minimizing the internal resistance of the source. Conversely, in case of a current source, maximizing the current flowing to a given load is achieved by maximizing the internal resistance of the source. In some cases, a source is better modeled as a power source, for example, an antenna of a wireless receiver. In this case, we can show that maximizing the voltage across (or current through) a given load requires power match as well.

Applying the matching condition to (3), we get the power available from the source P_{AVS}, i.e. the maximum power we can extract from the source

$$P_{\text{AVS}} = \frac{1}{8}\frac{v_S^2}{R_S} \tag{6}$$

Using (6), we can represent the open circuit source voltage in terms of power available from the source

$$v_S = 2\sqrt{2P_{\text{AVS}}R_S} \tag{7}$$

Effectively, (7) allows us to do a source transformation from the power domain to the voltage domain(Fig. 2), and we can now recalculate the load voltage

$$v_L = 2\sqrt{2R_S P_{\text{AVS}}}\frac{R_L}{R_S + R_L} \tag{8}$$

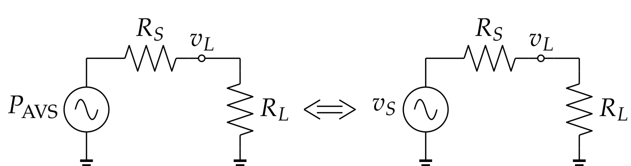

Figure 2: Power–voltage source transformation.

Differentiating (8) with respect to R_S and equating the result to zero gives the condition for maximizing the load voltage

$$\frac{\partial v_L}{\partial R_S} = 2\sqrt{2P_{\mathrm{AVS}}}R_L \frac{\frac{R_S+R_L}{2\sqrt{R_S}} - \sqrt{R_S}}{(R_S+R_L)^2} = 0 \tag{9}$$

$$\frac{R_S+R_L}{2\sqrt{R_S}} - \sqrt{R_S} = 0 \rightarrow R_S = R_L \tag{10}$$

which shows that power matching maximizes the load voltage.