# Effect of Jitter on Radio Performance

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October 5, 2019

## 1 Recap

- A jittery LO signal $v_{\mathrm{LO}}(t, \theta_j)$ can be modeled as an ideal signal $v_{\mathrm{LO}}(t)$ and an additive noise term $v_{nj}(t)$

$$v_{\mathrm{LO}}(t, \theta_j) = v_{\mathrm{LO}}(t) + v_{nj}(t) \tag{1}$$

- The average power of the jitter component $\overline{v_{nj}^2(t)}$ is proportional to the LO signal power $\overline{v_{\mathrm{LO}}^2(t)}$

$$\overline{v_{nj}^2(t)} = \overline{v_{\mathrm{LO}}^2(t)} \cdot \theta_{j,rms}^2 \tag{2}$$

$$\theta_{j,rms} = \omega_{\mathrm{LO}}\, t_{j,rms} \tag{3}$$

  where $\theta_{j,rms}$ is the rms phase noise of the signal and $t_{j,rms}$ is the corresponding timing jitter.

- The signal-to-noise ratio of the LO signal $SNR_{\mathrm{LO}}$ is then given by

$$SNR_{\mathrm{LO}} = \frac{1}{\theta_{j,rms}^2} = \frac{1}{(\omega_{\mathrm{LO}}\, t_{j,rms})^2} \tag{4}$$

## 2 Frequency Translation

A linear time-variant (LTV) system performs linear frequency translation, and can simply be modeled as a multiplication process. Thus, the mixer output $v_{out}(t)$ in Fig. 1 is given by

$$v_{out}(t) = v_{in}(t) \cdot v_{\mathrm{LO}}(t, \theta_j) \tag{5}$$

where $v_{in}(t)$ is a baseband signal in case of a transmitter and a modulated carrier in case of a receiver.

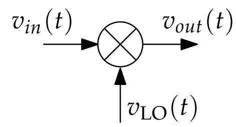


Figure 1: Mixer modeled as a multiplier.

To find the SNR at the mixer output due to LO jitter, we use (1) in (5) and take the mean square

$$\overline{v_{out}^2(t)} = \overline{v_{in}^2(t)\, v_{\mathrm{LO}}^2(t)} + \overline{v_{in}^2(t)\, v_{nj}^2(t)} \tag{6}$$

Since the LO signal is independent of both the input and jitter signals, the mean of products is the product of the means, and by using (2) we get

$$\overline{v_{out}^2(t)} = \overline{v_{in}^2(t)}\, \overline{v_{\mathrm{LO}}^2(t)} + \overline{v_{in}^2(t)}\, \overline{v_{\mathrm{LO}}^2(t)}\, \theta_{j,rms}^2 \tag{7}$$

The expression in (7) shows that the mixer output is the sum of two terms: the first is the noiseless frequency translated signal, and the second is the noise component due to jitter. Taking the ratio of the two terms provides the SNR at the output

$$SNR_{out} = \frac{1}{\theta_{j,rms}^2} = SNR_{\mathrm{LO}} \tag{8}$$

which from (4) is equal to the SNR of the LO signal. That is, when it comes to jitter, the amplitude of the input and LO signals is irrelevant, and the only way to improve the SNR at the output of the mixer is to improve the timing accuracy of the LO signal used.

# 3 Sampling

As shown in Fig. x, jitter causes a random shift in the sampling instance, resulting in an error voltage $v_e(t)$. The output voltage $v_{out}(nT_S)$ can then be expressed as

$$v_{out}(nTs) = v_{in}(nTs) + v_e(nT_s) \tag{9}$$

where $1/T_S$ is the sampling frequency and n is the discrete sample index. Assuming low jitter, we can use a linear approximation and express the resulting error term as a function of the signal slope at the time of sampling

$$v_{out}(nT_s) = v_{in}(nT_s) + \left.\frac{\partial v_{in}(t)}{\partial t}\right|_{t=nT_s} t_j(nT_s) \tag{10}$$

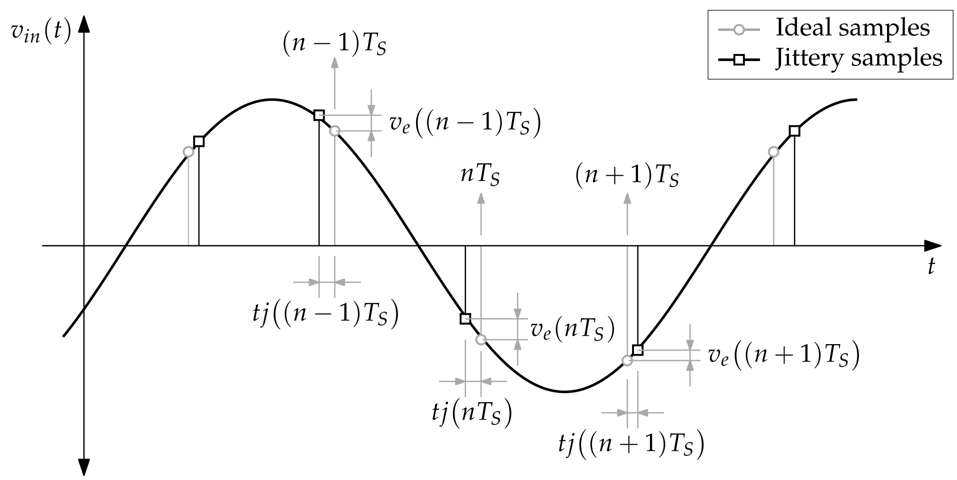


Figure 2: Sampling clock error for a sinusoidal signal.

For a sinusoidal input signal at maximum slope (worst case error), (10) translates to

$$v_{out}(nT_s) = A\sin(\omega_{in} \cdot nT_s) + A\omega_{in} \cdot t_j(nT_s) \tag{11}$$

where $tj(nT_S)$ is the $n$-th sample of the timing jitter. To find the SNR at the sampler output, we take the mean square of (11) and make use of the fact that the input and jitter signals are independent

$$\overline{v_{out}(nT_s)} = \frac{A^2}{2} + A^2\omega_{in}^2 \cdot t_{j,rms}^2 \tag{12}$$

The expression in (12) is the sum of two terms: the first is the noiseless sampled signal, and the second is the noise component due to sampling jitter. Taking the ratio of the two terms provides the SNR at the output of the sampler

$$SNR_{out} = \frac{1}{(\omega_{in}\, t_{j,rms})^2} \tag{13}$$

That is, for the same jitter, the SNR of the sampled signal is inversely proportional to the input signal frequency. This makes sense, since the higher the frequency, the larger the error as shown in Fig. x. We also observe, once again, that SNR due to jitter is independent of the signal amplitude.

Using (4), we can also rewrite (13) in terms of the sampling clock's SNR

$$SNR_{out} = (\frac{\omega_{LO}}{\omega_{in}})^2 \cdot SNR_{\mathrm{LO}} \tag{14}$$

The expression in (14) shows how the clock SNR translates to the sampler's output. At the Nyquist limit ($\omega_{in} = \omega_{\mathrm{LO}}/2$), the output SNR is 4 times (6dB) better than the clock SNR.