Effect of Sampling Jitter on Modulated Signals

Shadi Youssef

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1 Recap

The SNR of a sinusoidal signal of frequency f_{CW} sampled with a jittery clock is given by

$$SNR_{\mathrm{CW}} = \frac{1}{(2\pi f_{\mathrm{CW}}\, t_{j,rms})^2} \tag{1}$$

where $t_{j,rms}$ is the rms jitter of the sampling clock and the CW subscript indicates a continuous wave signal.

2 Sampling Jitter of a Modulated Signal

We assume a modulated signal with a flat-top power spectral density (PSD), i.e. signal power is uniformly distributed across its bandwidth[1]. This simplifies the analysis, but the same procedure can still be carried out for other PSD distributions.

To analyze the effect of sampling jitter on such a signal, we first divide the signal spectrum into N equally spaced sub-bands as shown in Fig. 1. The center frequency f_n of the n-th sub-band is given by

$$f_n = f_0 + (n + \frac{1}{2})\frac{BW}{N} \tag{2}$$

where f_0 is the carrier (center) frequency of the signal and BW is the modulation bandwidth.

If sub-band spacing is small enough, we can apply (1) to find the n-th sub-band SNR

$$SNR_n = \frac{P_n}{N_n} = \frac{1}{(2\pi f_n\, t_{j,rms})^2} \tag{3}$$

[1]This is a good approximation for an OFDM signal for example

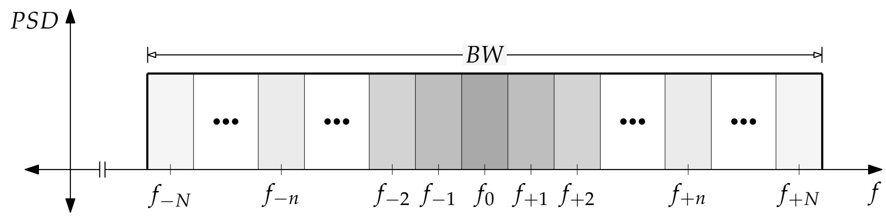

Figure 1: Flat-top PSD of a modulated signal divided into sub-bands for sampling jitter analysis.

where P_n and N_n are the sub-band power and noise, respectively. For a flat-top PSD, the power per sub-band is the total signal power P_T divided by the number of sub-bands N, and (3) can be re-arranged to find the jitter noise power per sub-band

$$N_n = 4\pi^2 \, t_{j,rms}^2 \frac{P_T}{N} \Big(f_0^2 + (\frac{BW}{N})^2 n^2 + 2 f_0 \frac{BW}{N} n \Big) \tag{4}$$

To total jitter noise floor is then the summation of (4) across all sub-bands

$$N_T = 4\pi^2 \, t_{j,rms}^2 \frac{P_T}{N} \sum_{n=-\frac{N-1}{2}}^{+\frac{N-1}{2}} \Big(f_0^2 + 2 f_0 \frac{BW}{N} n + (\frac{BW}{N})^2 n^2 \Big) \tag{5}$$

The second and third summation terms in (5) can be evaluated using Faulhaber's formula

$$\begin{aligned} \sum_{m=1}^{M} m &= \frac{1}{2} M(M+1) \\ \sum_{m=1}^{M} m^2 &= \frac{1}{6} M(M+1)(2M+1) \end{aligned} \tag{6}$$

So, by applying (6) to (5)

$$N_T = 4\pi^2 \, t_{j,rms}^2 \frac{P_T}{N} \Big(N f_0^2 + \frac{(N^2-1)}{N} \frac{BW^2}{12} \Big) \tag{7}$$

and SNR_T, the total SNR, becomes readily available from (7)

$$SNR_T = \frac{P_T}{N_T} = 4\pi^2 \, t_{j,rms}^2 \frac{P_T}{N} \Big(N f_0^2 + \frac{(N^2-1)}{N} \frac{BW^2}{12} \Big) \tag{8}$$

For an infinite number of sidebands ($N \to \infty$), the total SNR in (8) converges to an exact solution

$$SNR_T = \frac{1}{(2\pi t_{j,rms})^2 \Big(f_0^2 + \frac{BW^2}{12} \Big)} \tag{9}$$

For a narrowband modulated carrier, $f_0 \gg BW$, and (9) reduces to

$$SNR_T = \frac{1}{(2\pi f_0 \, t_{j,rms})^2} \tag{10}$$

which is similar to (1). For giga-hertz range carriers, however, direct sampling is usually not practical, and sampling is done after downconversion. Assuming a direct conversion receiver, $f_0 = 0$, and (9) reduces to

$$SNR_T = \frac{3}{(2\pi \, \frac{BW}{2} \, t_{j,rms})^2} \tag{11}$$

Equation (11) tells us that the jitter SNR of a modulated signal is a factor of 3 (4.7dB) better than a sine wave located at band edge ($BW/2$). This makes sense because jitter error is inversely proportional to frequency, so a band edge tone represents a worst-case error.