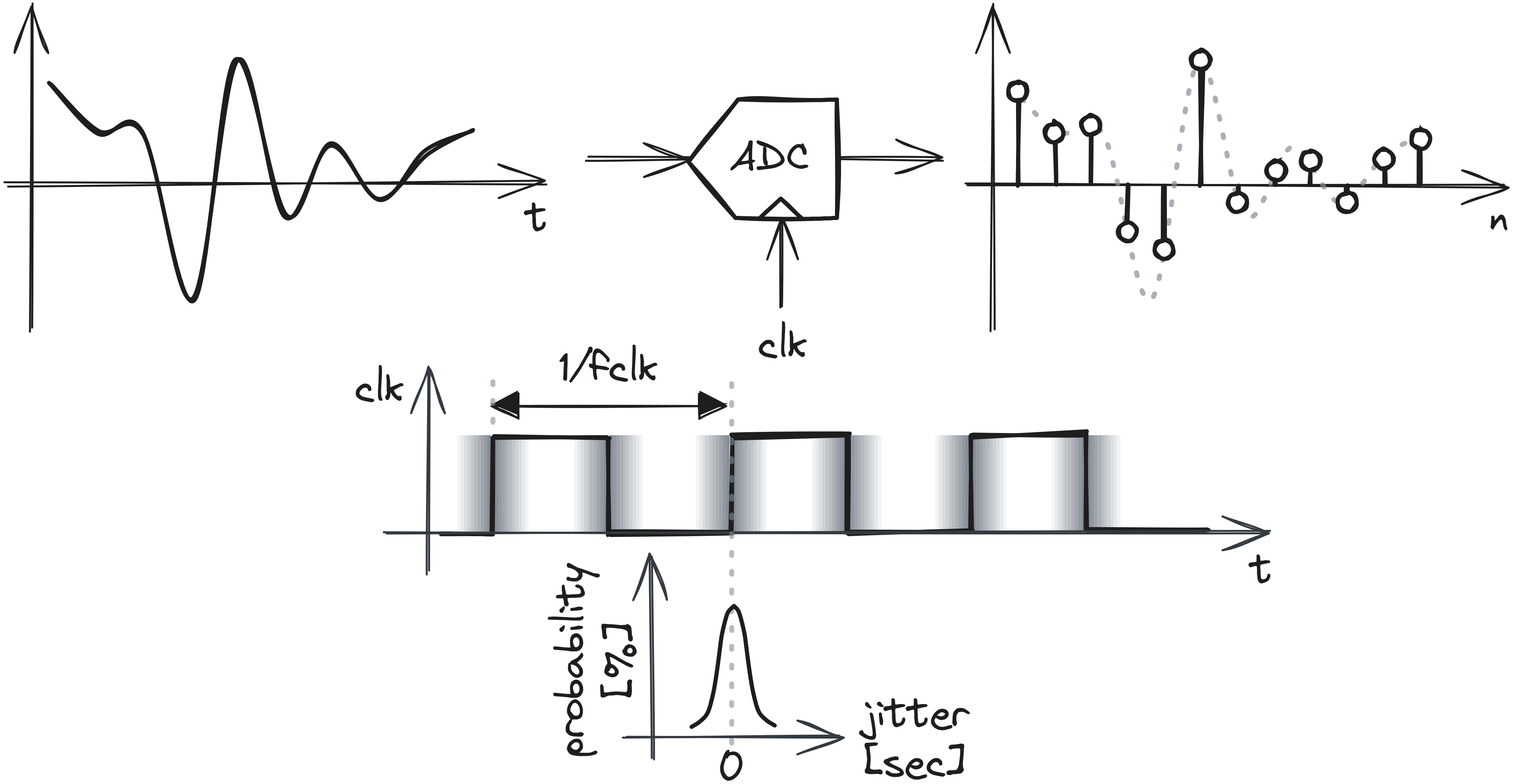
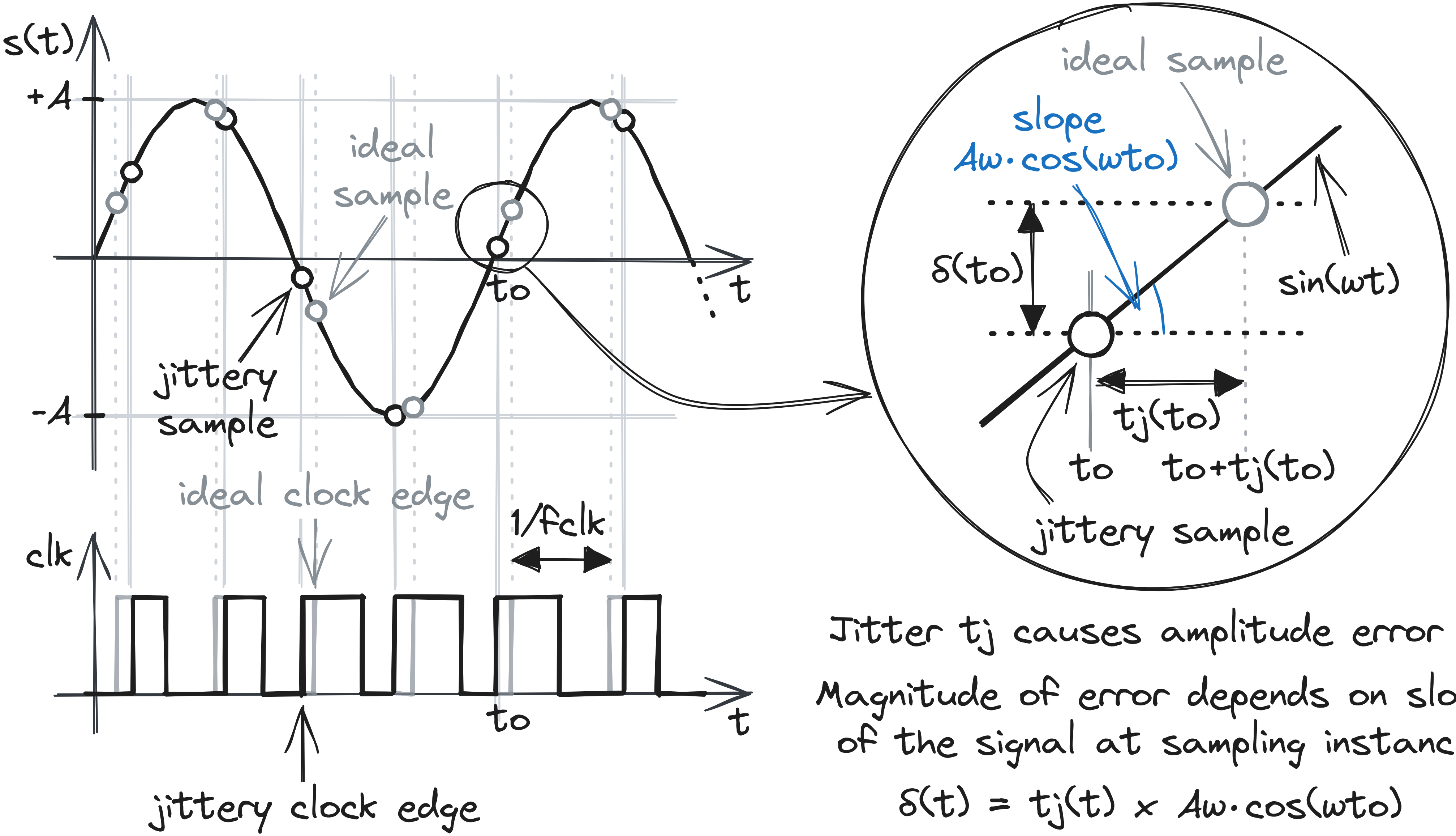
Impact of Sampling Clock Jitter on Modulated Signals

Shadi Youssef
@Radiohub

How does sampling a modulated signal with a jittery clock impact the signal quality?



Let's start with the simple case of sampling sine wave

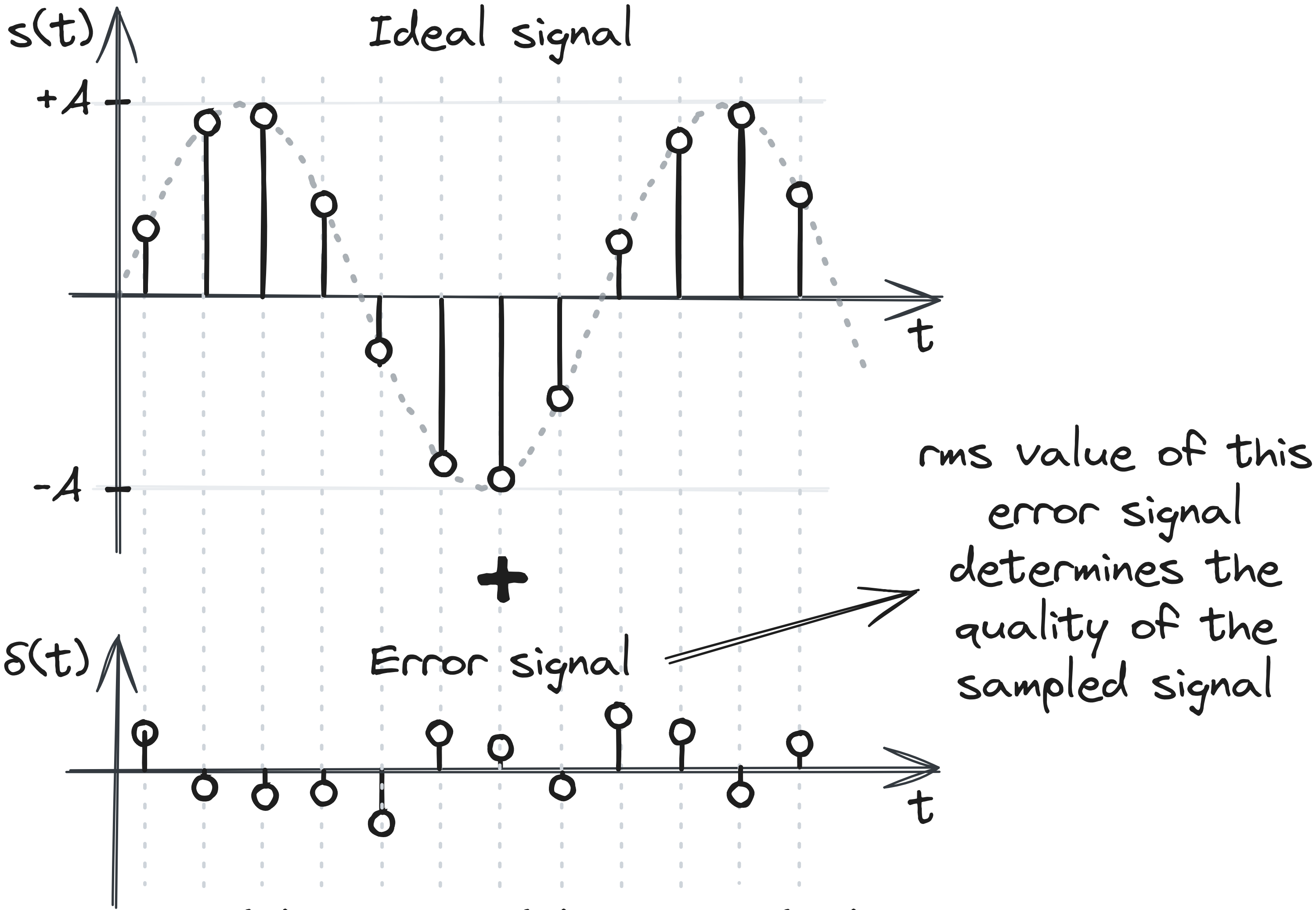


Jitter tj causes amplitude error δ

Magnitude of error depends on slope of the signal at sampling instance

$$\delta(t) = tj(t) \times A\omega \cdot \cos(\omega t_o)$$

The sampled sinusoid is the sum of an ideal sequence and error sequence



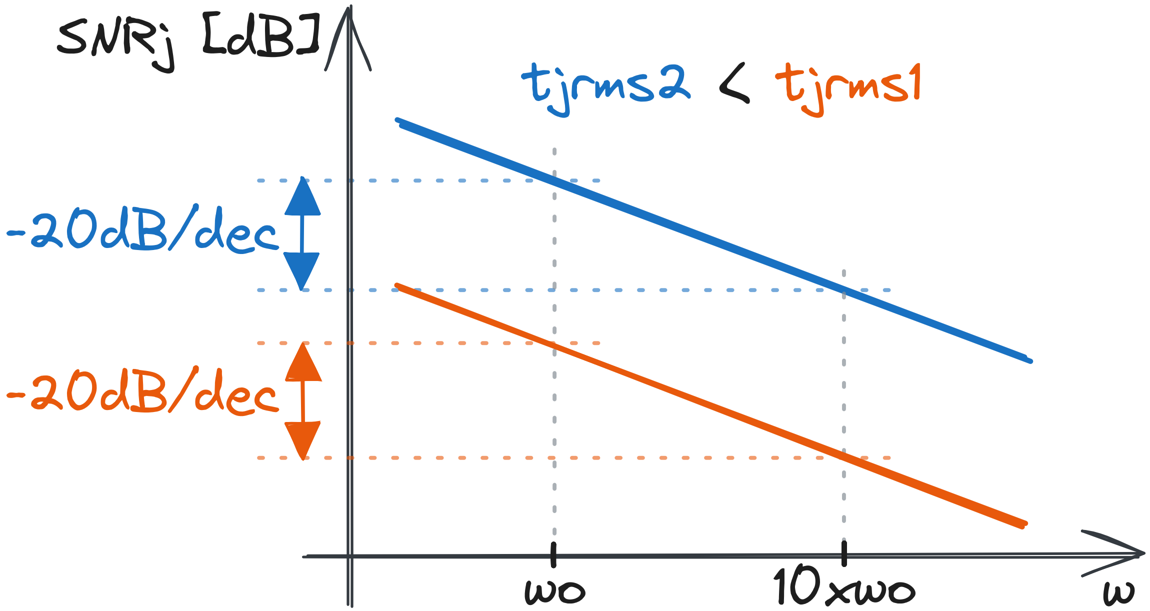
$\delta(t) = \omega \times tj(t) \times A \cos(\omega t)$

Because clock jitter $t_j(t)$ is uncorrelated to signal slope $\cos(\omega t)$

$$\delta_{rms} = \omega \times t_{jrms} \times A/\sqrt{2}$$

SNR due to jitter is the ratio of rms signal $(A/\sqrt{2})$ to rms error δ_{rms}

$$\boxed{SNR_j = 1/(\omega \times t_{jrms})^2}$$

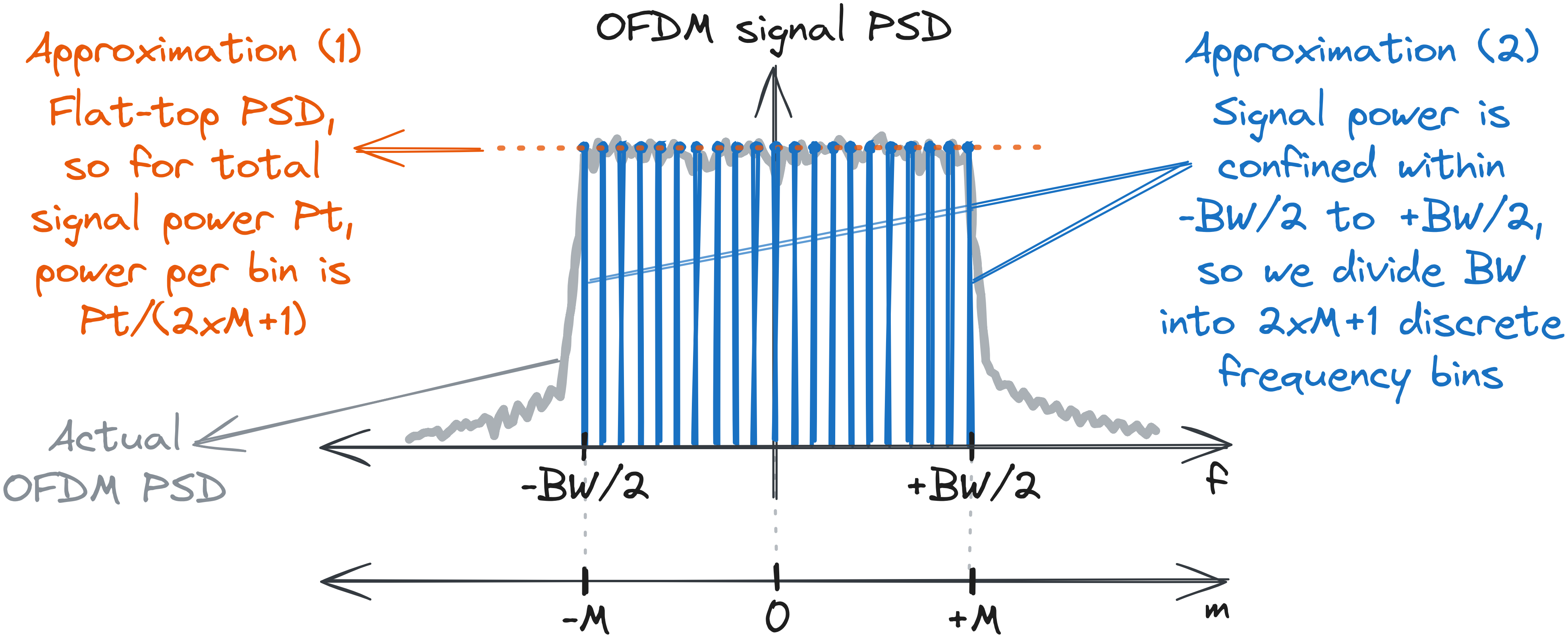


- SNR_j is independent of signal amplitude
- Higher frequency sinusoids are more sensitive to clock jitter
- For a given signal frequency, the only way to improve SNR_j is to lower clock jitter

Sampling OFDM Signals

Now, we can evaluate the impact of clock jitter modulated signals

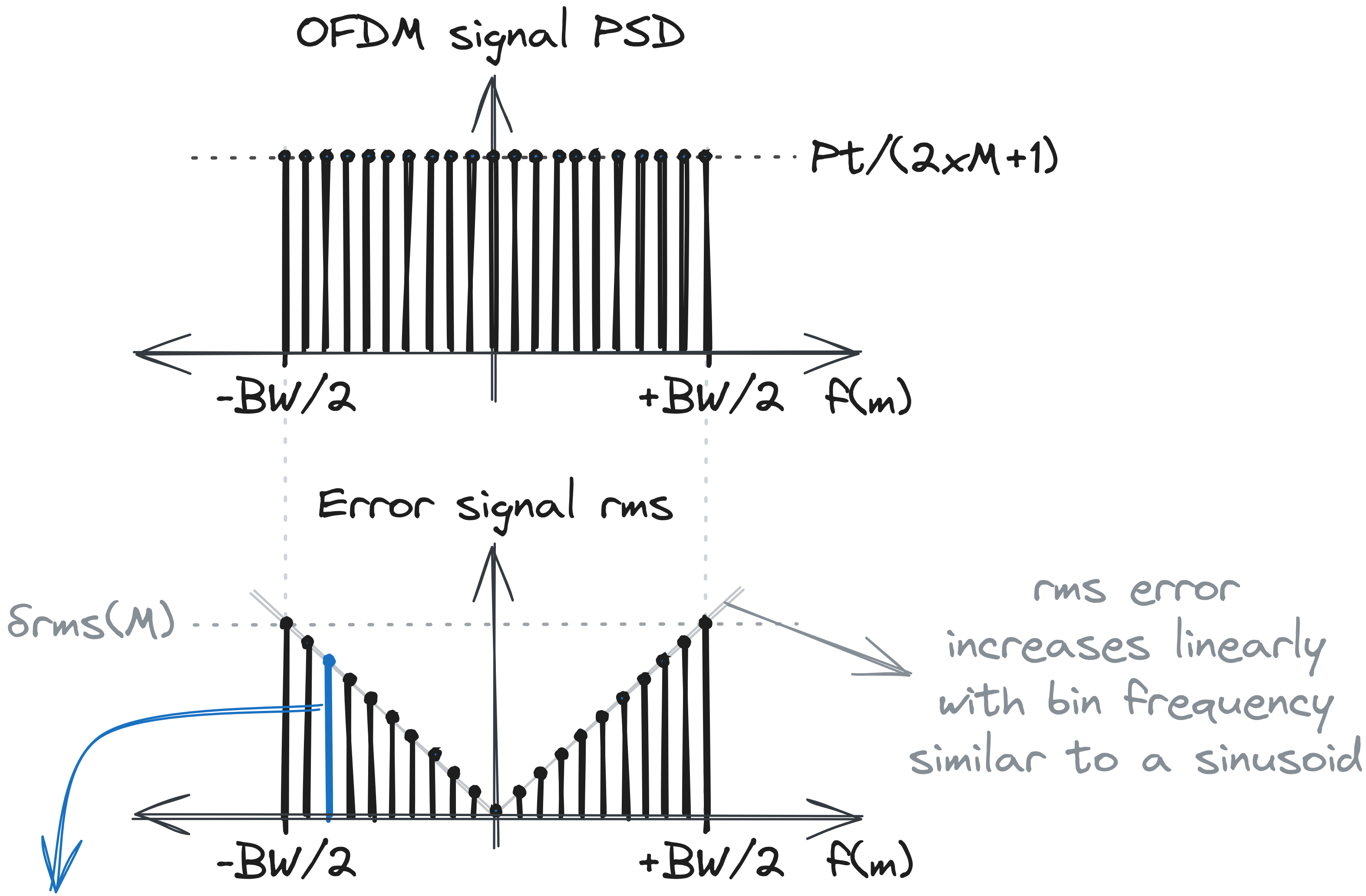
We start with OFDM signals and make 2 approximations



m-th frequency bin $f(m) = +m/M \times (BW/2)$

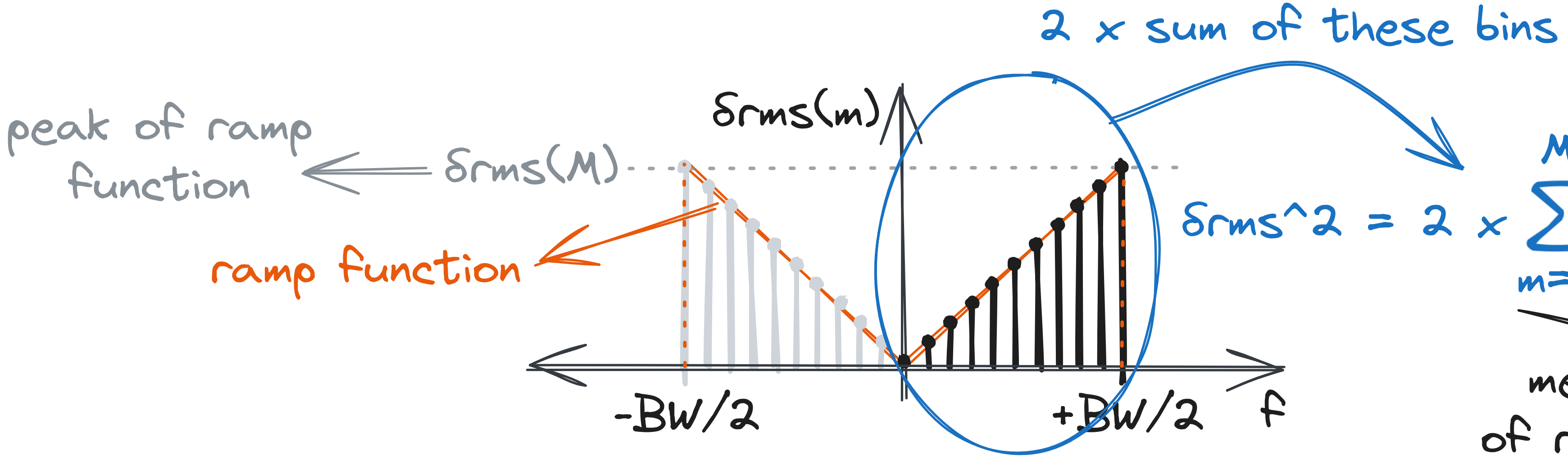
$m = -M, -(M-1), \ldots -2, -1, 0, 1, 2, \ldots M-1, M$

For each frequency bin, sampling jitter error is similar to that of a sinusoid



rms of m-th error bin $\delta rms(m) = \sqrt{Pt/(2xM+1)} \times 2\pi f(m) \times tjrms$

Total rms of error signal is the uncorrelated sum of all error bins



$$\delta rms\char`^2 = 2 \times \underbrace{\sum_{m=0}^{M} \delta rms(m)\char`^2}_{\text{mean square of ramp function} \,=\, \delta rms(M)/3}$$

For M -> ∞, $\delta rms = 3\sqrt{Pt} \times 2\pi \times BW/2 \times tjrms$

$$\boxed{SNRj = 3 \times 1/(2\pi \times BW/2 \times tjrms)\char`^2}$$

Average SNRj across the signal bandwidth is 3x higher than the SNR of the highest frequency component of the signal

For a multi-carrier signal like OFDM, highest frequency component ≈ highest frequency subcarrier

So, in general, for any meaningful definition of signal bandwidth
(for example, 3dB bandwidth, 90% power bandwidth .. etc.)

correction factor ≥ 1

$SNRj = CF \times SNRjmin$

Average jitter SNR across whole bandwidth

Worst-case jitter SNR corresponds to signal's baseband (BB) BW

For OFDM, we found that CF = 3

But OFDM is relatively easy to analyze

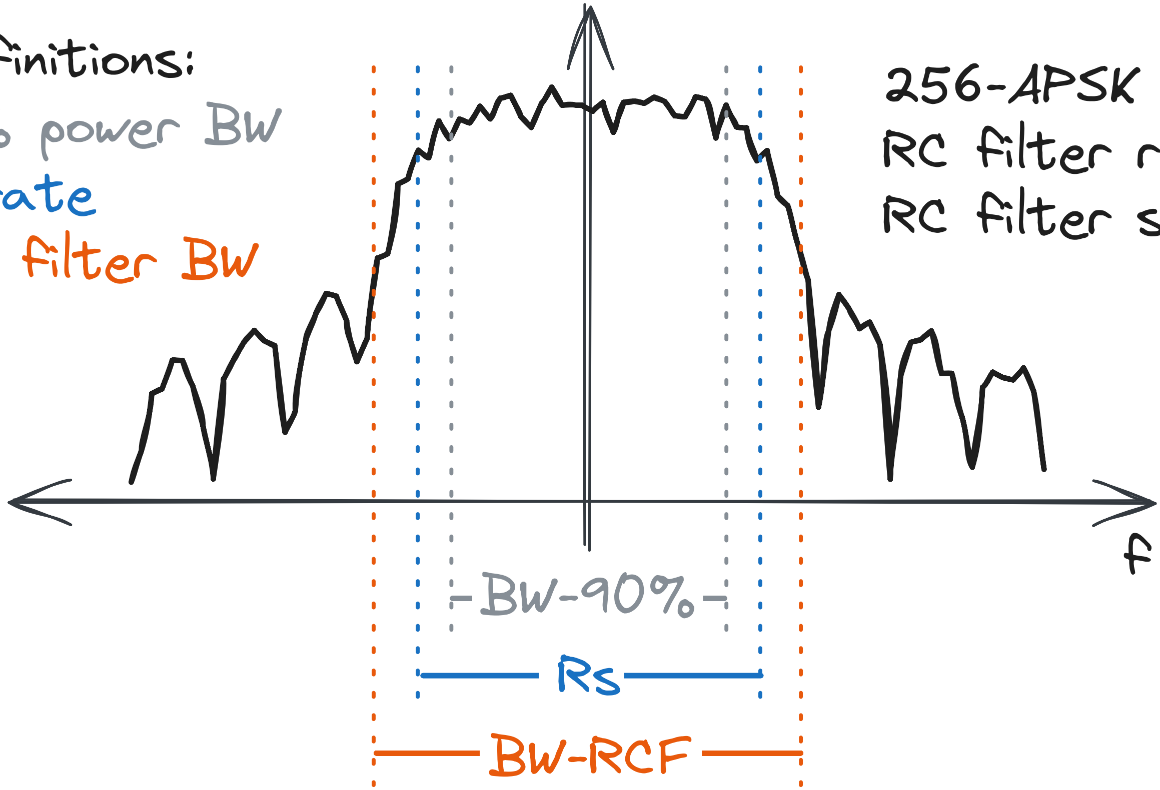
because it can be well approximated with a flat-top spectrum

and its bandwidth is well defined by the allocation of subcarriers

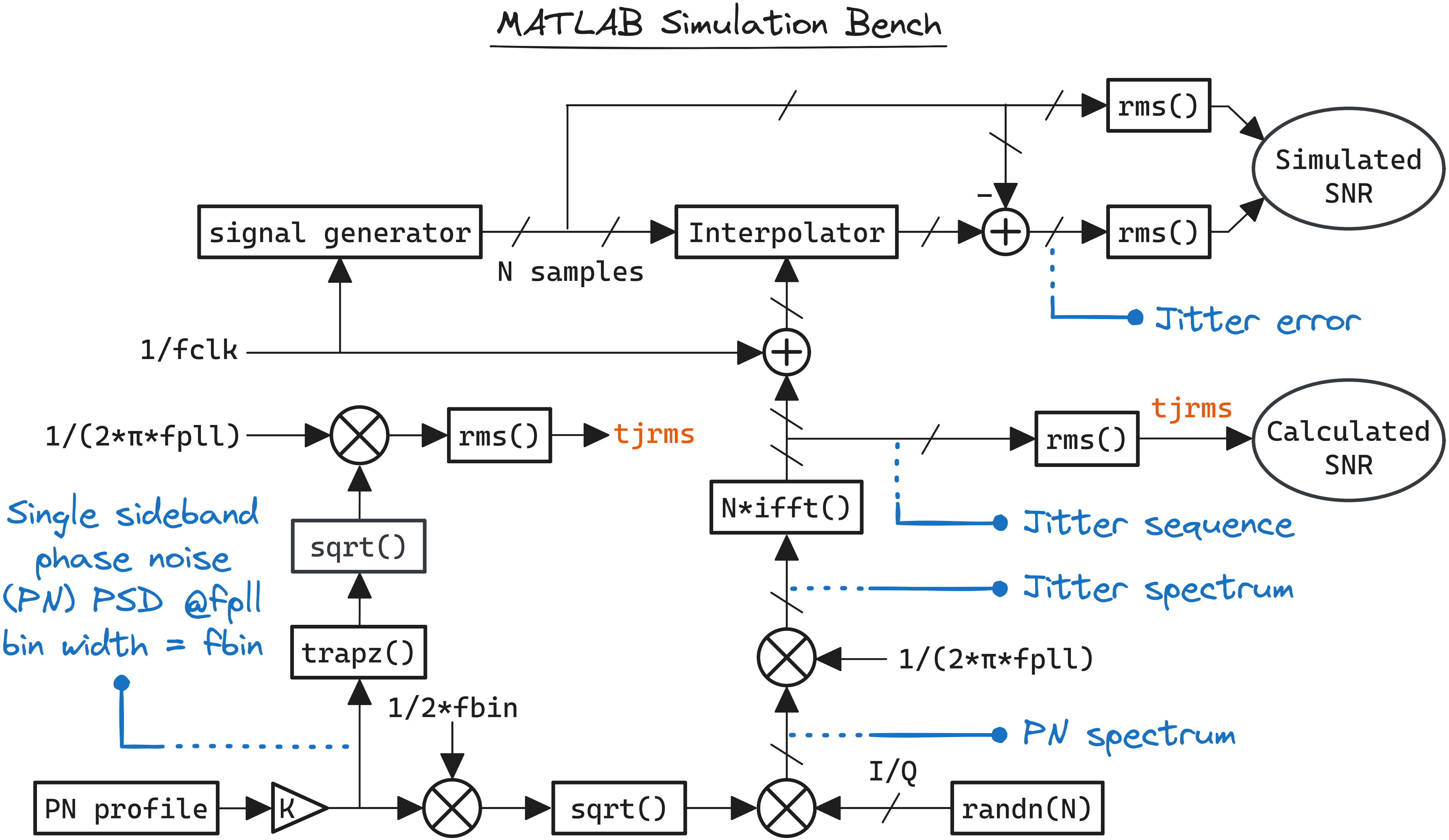
But not all signals are easy to analyze to evaluate jitter performance
For example, a DVB-S2X signal has a raised-cosine (RC) shaped spectrum
that gradually rolls off and its bandwidth is not very obvious
So, in general, the correction factor is easier to obtain via simulation

DVB-S2X signal PSD

Possible BW definitions:
- BW-90 = 90% power BW
- Rs = Symbol rate
- BW-RCF = RC filter BW



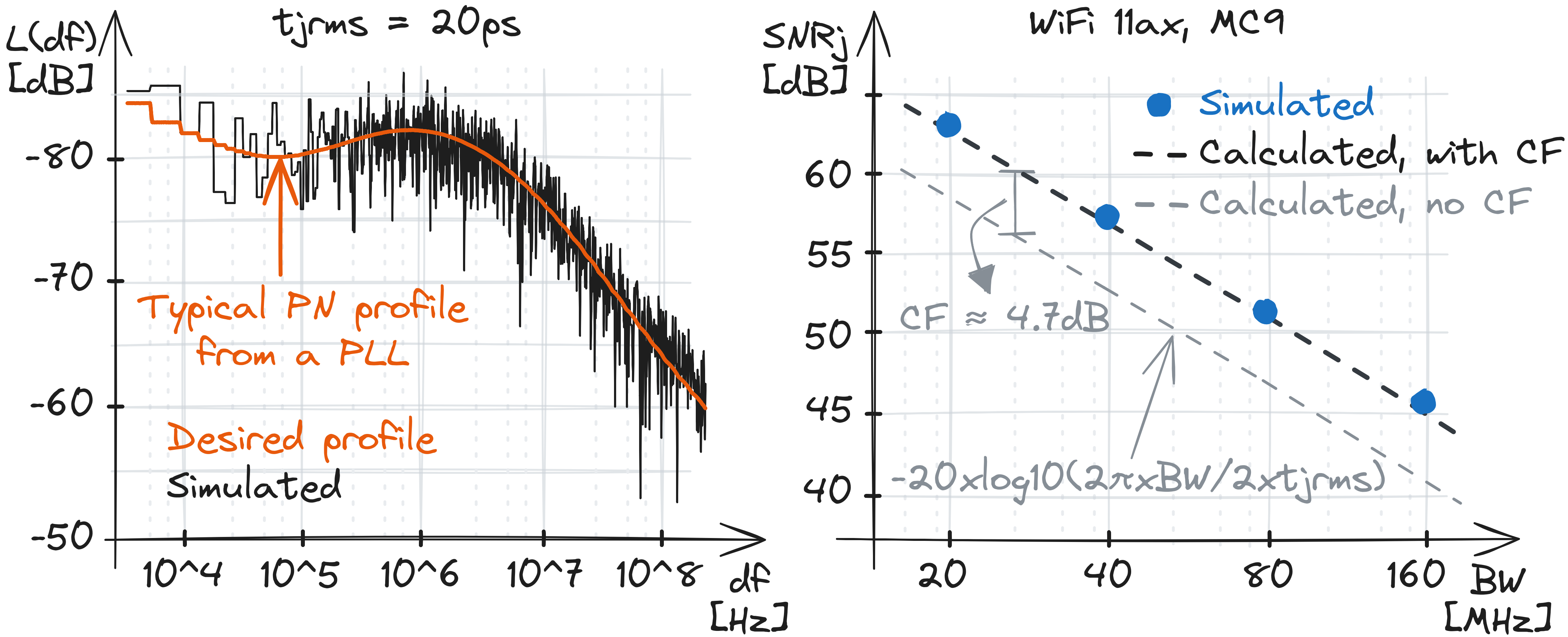
256-APSK
RC filter roll-off = 0.35
RC filter span = 4 symbols


MATLAB Simulation Bench
signal generator
N samples
Interpolator
rms()
rms()
Simulated SNR
Jitter error
1/fclk
1/(2*π*fpll)
rms()
tjrms
tjrms
rms()
Calculated SNR
N*ifft()
Jitter sequence
Single sideband phase noise (PN) PSD @fpll bin width = fbin
sqrt()
trapz()
Jitter spectrum
1/(2*π*fpll)
1/2*fbin
PN spectrum
I/Q
PN profile
K
sqrt()
randn(N)

MATLAB Simulation Results

OFDM signal + colored noise

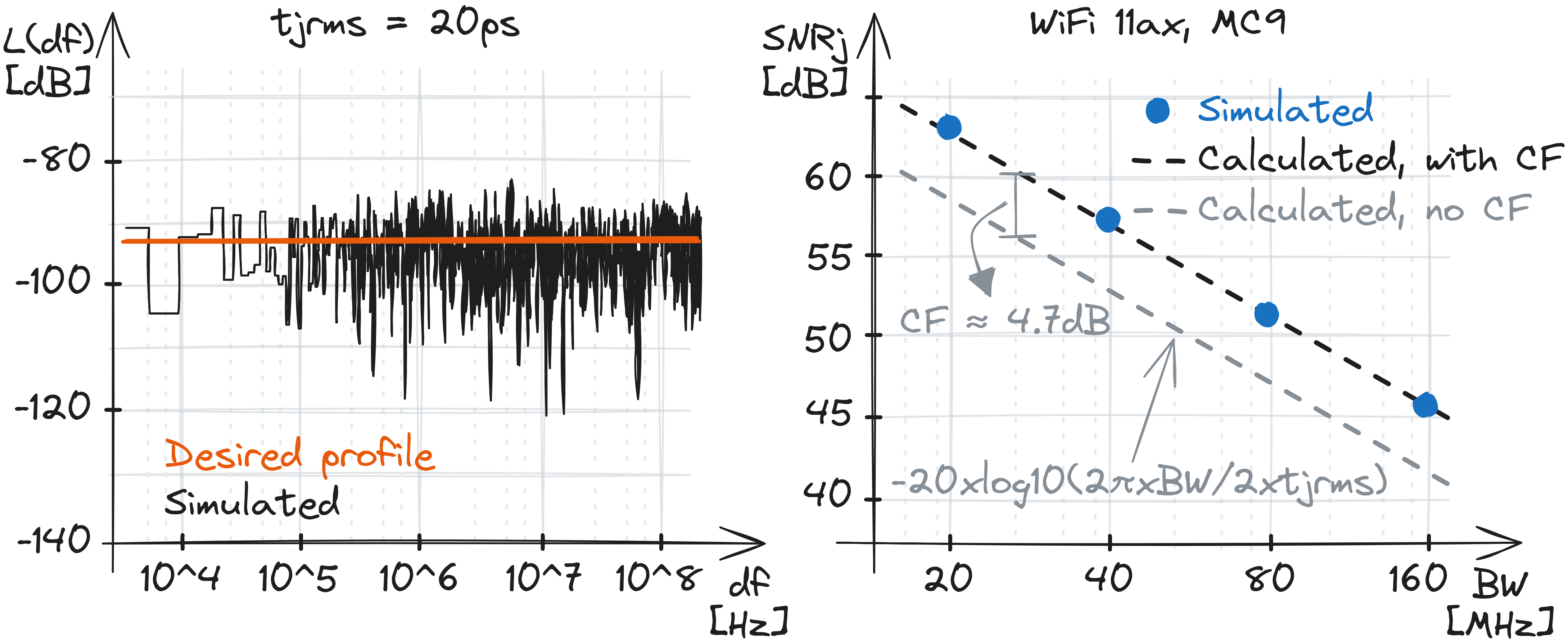
Simulated CF matches hand-calculations within +/- 0.1dB



MATLAB Simulation Results

OFDM signal + white noise

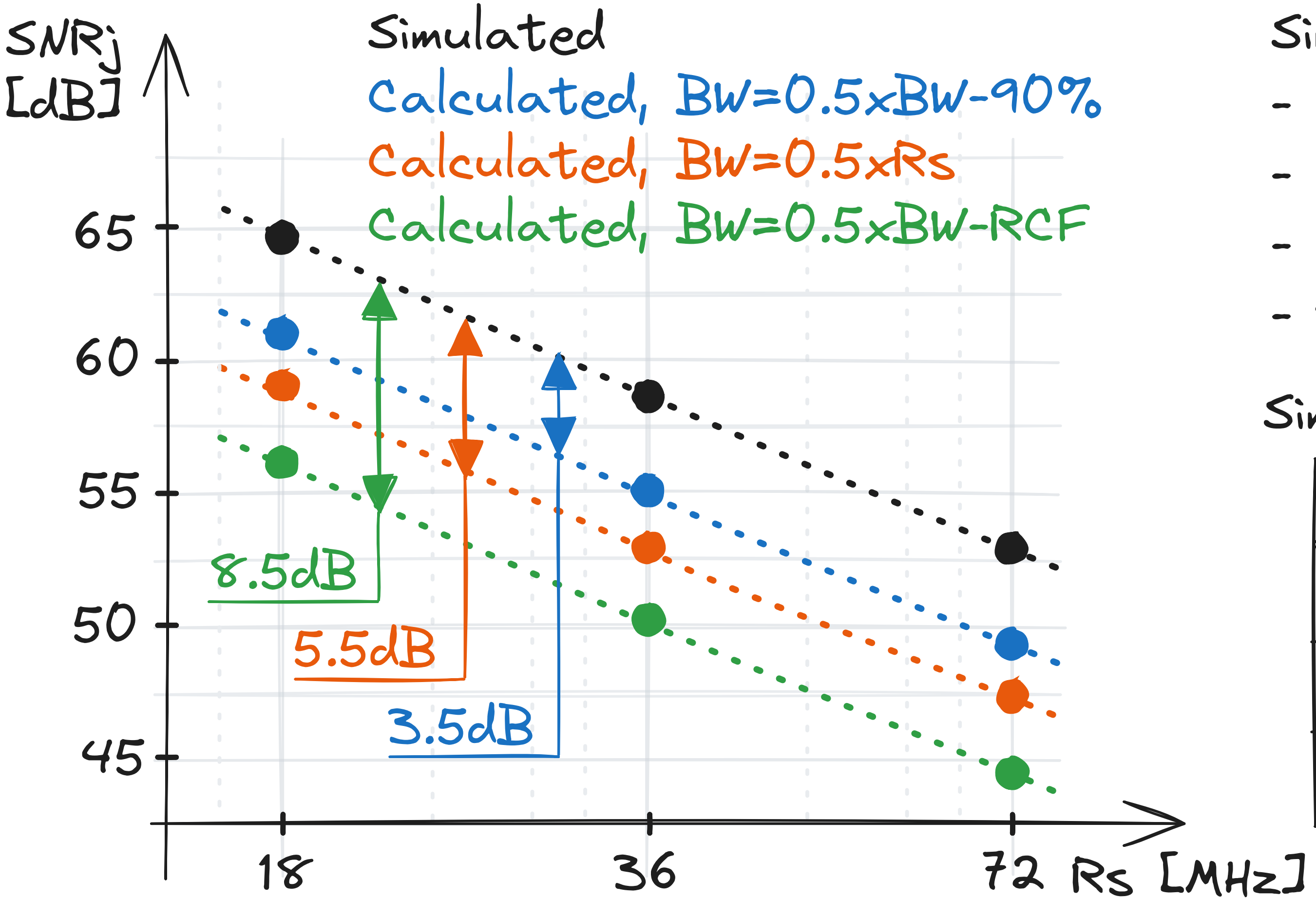
Results are independent of jitter profile



MATLAB Simulation Results

DVB-S2X + colored noise

CF is up to 8.5dB depending on the bandwidth definition used



Simulation parameters:

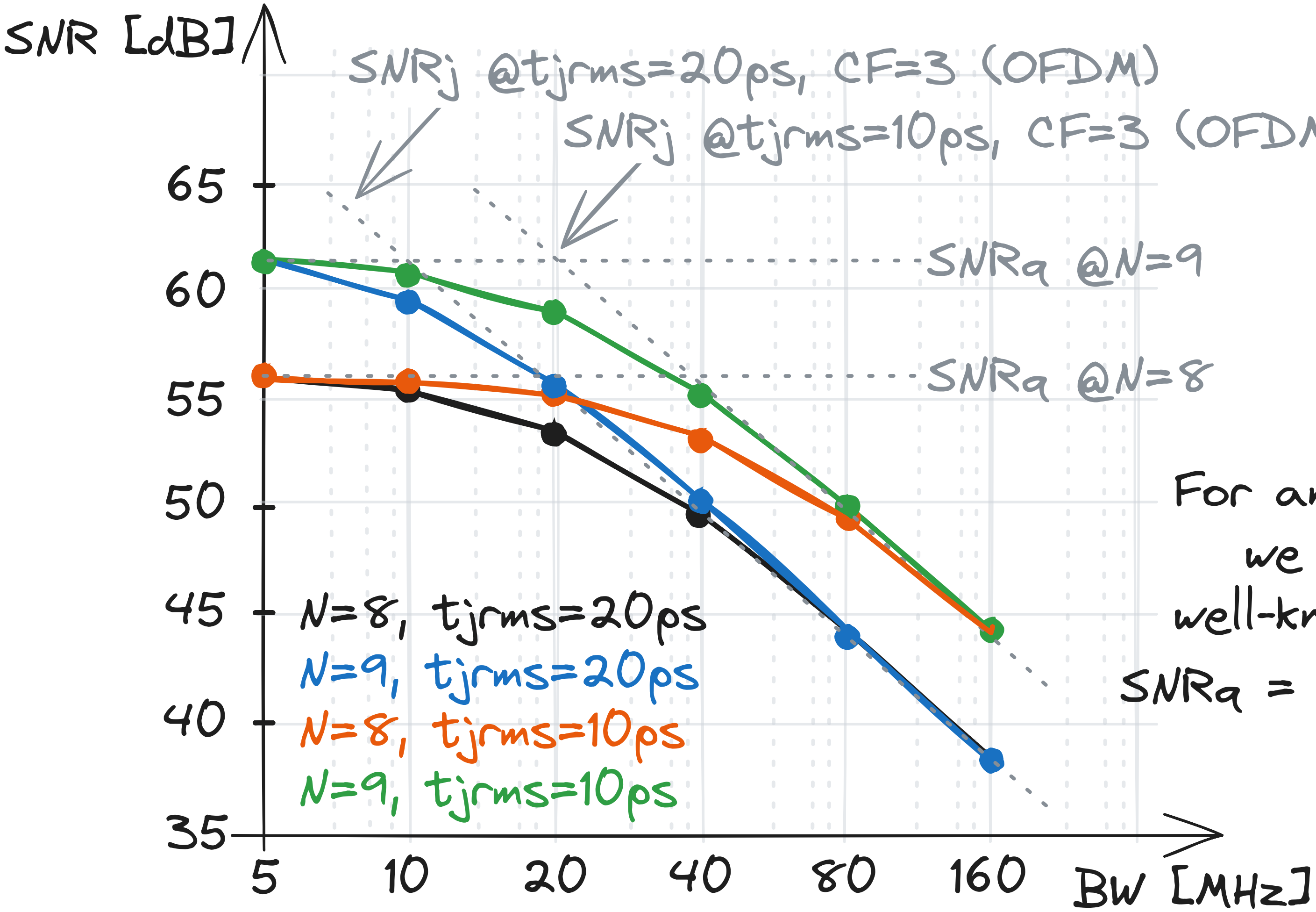
- 256-APSK
- RC filter roll-off = 0.35
- RC filter span = 4 symbols
- tjrms = 20ps

Simulated correction factor

BW	CF [dB]
0.5xBW-90%	3.5
0.5xRs	5.5
0.5xBW-RCF	8.5

Total SNR of the signal is the uncorrelated sum of quantization & jitter errors

$$1/SNR = 1/SNR_q + 1/SNR_j$$



For an N-bit ADC, we have the well-known relation

$$SNR_q = 6 \times N + 1.76 \text{ [dB]}$$

Design Example

Design requirements:

DVB-S2X, 256-APSK, $R_s = 36\text{MBps}$

Desired $SNR = 50\text{dB}$

Assigning equal noise budgets to quantization and jitter

$SNR_q = 53\text{dB}$

$SNR_j = 53\text{dB}$

Assuming ADC oversampling rate (osr) = 4

$SNR_q > 6 \times N + 1.76 + 10 \times \log10(osr)$

$N = 8\text{-bits}$

Taking BB BW as $R_s/2$ -> $CF = 5.5\text{dB}$ (from simulation)

$SNR_j > -20 \times \log10(2\pi \times R_s/2 \times t_{jrms}) + CF$

$t_{jrms} < 37.2\text{ps}$