For the inductor *L*, assume the current through it is

$$i = I_m \cos(\omega t + \phi)$$

The voltage across the inductor is

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$

We can write the voltage as

$$v = \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

which transforms to the phasor

$$V = \omega L I_m e^{j(\phi+90^\circ)} = \omega L I_m e^{j\phi} e^{j90^\circ} = \omega L I_m \angle \phi + 90^\circ$$

As

$$e^{j90^\circ} = j$$

$$V = j\omega L I$$

In the preceding section, we obtained the voltage-current relations for

the three passive elements as

$$V = RI, \quad V = j\omega L I, \quad V = \frac{I}{j\omega C}$$

Z is a frequency-dependent quantity known as *impedance*, measured in ohms.

Table 5.2

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

As a complex quantity, the impedence may be expressed in rectangular form as

$$Z = R + jX$$

Or

$$Z = |Z| \angle \theta$$

where

$$|Z| = \sqrt{R^2 + X^2}, \quad \theta = \tan^{-1}\frac{X}{R}$$

Find *v*(*t*) and *i*(*t*) in the circuit shown in Figure 5.2.

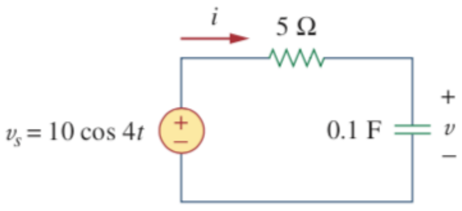

Figure 5.2

Solution:

From the voltage source,

$$\begin{aligned} &10\cos 4t, \omega = 4 \\ &V_s = 10\angle 0^\circ V \end{aligned}$$

The impedance is

$$Z = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\Omega$$

Hence the current

$$\begin{aligned} I &= \frac{V_s}{Z} = \frac{10\angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2} \\ &= 1.6 + j0.8 = 1.789\angle 26.57^\circ A \end{aligned}$$

The voltage across the capacitor is

$$\begin{aligned} V = IZ_C &= \frac{I}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4 \times 0.1} \\ &= \frac{1.789\angle 26.57^\circ}{0.4\angle 90^\circ} = 4.47\angle -63.43^\circ V \end{aligned}$$

Converting **I** and **V** to the time domain, we get

$$\begin{aligned} i(t) &= 1.789\cos(4t + 26.57^\circ)A \\ v(t) &= 4.47\cos(4t - 63.43^\circ)V \end{aligned}$$

Notice that *i*(*t*) leads *v*(*t*) by 90 as expected.