FIGURE 2.1 Semantics of core logic, for each $\mathcal{M} = \langle W, I, J \rangle$

$$\begin{aligned}
\mathcal{E}_{\mathcal{M}}[\![p]\!] &= I(p) \\
\mathcal{E}_{\mathcal{M}}[\![\neg\varphi]\!] &= W - \mathcal{E}_{\mathcal{M}}[\![\varphi]\!] \\
\mathcal{E}_{\mathcal{M}}[\![\varphi_1 \wedge \varphi_2]\!] &= \mathcal{E}_{\mathcal{M}}[\![\varphi_1]\!] \cap \mathcal{E}_{\mathcal{M}}[\![\varphi_2]\!] \\
\mathcal{E}_{\mathcal{M}}[\![\varphi_1 \vee \varphi_2]\!] &= \mathcal{E}_{\mathcal{M}}[\![\varphi_1]\!] \cup \mathcal{E}_{\mathcal{M}}[\![\varphi_2]\!] \\
\mathcal{E}_{\mathcal{M}}[\![\varphi_1 \supset \varphi_2]\!] &= (W - \mathcal{E}_{\mathcal{M}}[\![\varphi_1]\!]) \cup \mathcal{E}_{\mathcal{M}}[\![\varphi_2]\!] \\
\mathcal{E}_{\mathcal{M}}[\![\varphi_1 \equiv \varphi_2]\!] &= \mathcal{E}_{\mathcal{M}}[\![\varphi_1 \supset \varphi_2]\!] \cap \mathcal{E}_{\mathcal{M}}[\![\varphi_2 \supset \varphi_1]\!] \\
\mathcal{E}_{\mathcal{M}}[\![P \Rightarrow Q]\!] &= \begin{cases} W, & \text{if } J(Q) \subseteq J(P) \\ \emptyset, & \text{otherwise} \end{cases} \\
\mathcal{E}_{\mathcal{M}}[\![P \textsf{ says } \varphi]\!] &= \{w | J(P)(w) \subseteq \mathcal{E}_{\mathcal{M}}[\![\varphi]\!]\} \\
\mathcal{E}_{\mathcal{M}}[\![P \textsf{ controls } \varphi]\!] &= \mathcal{E}_{\mathcal{M}}[\![(P \textsf{ says } \varphi) \supset \varphi]\!]
\end{aligned}$$

Propositional Variables: The truth of a propositional variable p is determined by the interpretation function I: a variable p is considered true in world w precisely when $w \in I(p)$. Thus, for all propositional variables p,

$$\mathcal{E}_{\mathcal{M}}[\![p]\!] = I(p).$$

For example, if $\mathcal{M}_0$ is the Kripke structure $\langle W_0, I_0, J_0 \rangle$ from Example 2.7, $\mathcal{E}_{\mathcal{M}_0}[\![g]\!] = I_0(g) = \{sw\}$.

Negation: A formula with form $\neg\varphi$ is true in precisely those worlds in which φ is *not* true. Because (by definition) $\mathcal{E}_{\mathcal{M}}[\![\varphi]\!]$ is the set of worlds in which φ is true, we define

$$\mathcal{E}_{\mathcal{M}}[\![\neg\varphi]\!] = W - \mathcal{E}_{\mathcal{M}}[\![\varphi]\!].$$

Thus, returning to Example 2.7,

$$\mathcal{E}_{\mathcal{M}_0}[\![\neg g]\!] = W_0 - \mathcal{E}_{\mathcal{M}_0}[\![g]\!] = \{sw, sc, ns\} - \{sw\} = \{sc, ns\}.$$

Notice that $\mathcal{E}_{\mathcal{M}_0}[\![\neg g]\!]$ is the set of worlds in which the children are *not* allowed to go outside.

Conjunction: A conjunctive formula $\varphi_1 \wedge \varphi_2$ is considered true in those worlds for which *both* φ_1 and φ_2 are true: that is, $\varphi_1 \wedge \varphi_2$ is true in those worlds w for which $w \in \mathcal{E}_{\mathcal{M}}[\![\varphi_1]\!]$ *and* $w \in \mathcal{E}_{\mathcal{M}}[\![\varphi_2]\!]$. Thus, we can define $\mathcal{E}_{\mathcal{M}}[\![\varphi_1 \wedge \varphi_2]\!]$ in terms of set intersection:

$$\mathcal{E}_{\mathcal{M}}[\![\varphi_1 \wedge \varphi_2]\!] = \mathcal{E}_{\mathcal{M}}[\![\varphi_1]\!] \cap \mathcal{E}_{\mathcal{M}}[\![\varphi_2]\!].$$