FIGURE 3.5 Some useful derived rules

Conjunction $\dfrac{\varphi_1 \qquad \varphi_2}{\varphi_1 \wedge \varphi_2}$

Simplification (1) $\dfrac{\varphi_1 \wedge \varphi_2}{\varphi_1}$ *Simplification (2)* $\dfrac{\varphi_1 \wedge \varphi_2}{\varphi_2}$

Disjunction (1) $\dfrac{\varphi_1}{\varphi_1 \vee \varphi_2}$ *Disjunction (2)* $\dfrac{\varphi_2}{\varphi_1 \vee \varphi_2}$

Modus Tollens $\dfrac{\varphi_1 \supset \varphi_2 \qquad \neg\varphi_2}{\neg\varphi_1}$ *Double negation* $\dfrac{\neg\neg\varphi}{\varphi}$

Disjunctive Syllogism $\dfrac{\varphi_1 \vee \varphi_2 \qquad \neg\varphi_1}{\varphi_2}$ *Hypothetical Syllogism* $\dfrac{\varphi_1 \supset \varphi_2 \qquad \varphi_2 \supset \varphi_3}{\varphi_1 \supset \varphi_3}$

Controls $\dfrac{P \textsf{ controls } \varphi \qquad P \textsf{ says } \varphi}{\varphi}$

Derived Speaks For $\dfrac{P \Rightarrow Q \qquad P \textsf{ says } \varphi}{Q \textsf{ says } \varphi}$ *Derived Controls* $\dfrac{P \Rightarrow Q \qquad Q \textsf{ controls } \varphi}{P \textsf{ controls } \varphi}$

Says Simplification (1) $\dfrac{P \textsf{ says } (\varphi_1 \wedge \varphi_2)}{P \textsf{ says } \varphi_1}$ *Says Simplification (2)* $\dfrac{P \textsf{ says } (\varphi_1 \wedge \varphi_2)}{P \textsf{ says } \varphi_2}$

FIGURE 3.6 A formal proof of *Conjunction*

1. φ_1	Assumption
2. φ_2	Assumption
3. $\varphi_1 \supset (\varphi_2 \supset (\varphi_1 \wedge \varphi_2))$	Taut
4. $\varphi_2 \supset (\varphi_1 \wedge \varphi_2)$	1,3 Modus Ponens
5. $\varphi_1 \wedge \varphi_2$	2,4 Modus Ponens