CIS 623

Assured Programming with Formal Methods

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This supplement describes examples of pure functions, their mathematical definitions, and their Haskell implementations. They are used for the class CIS 623 for in classroom demonstration(s) and practice.

Mathematical functions

Functions defined by a formula

1. A polynomial function

$$f_1 : \mathbb{Z} \to \mathbb{Z}$$
$$f_1(x) = x^2 + 1$$

2. A function of several variables

$$f_2 : (\mathbb{Z}, \mathbb{Z}, \mathbb{Z}) \to \mathbb{Z}$$
$$f_2(x, y, z) = x + y + z + 1$$

3. A function defined by recursion

$$f_3 : \mathbb{Z} \to \mathbb{Z}$$
$$f_3(x) = \begin{cases} 0 & x \leq 0 \\ 2 & x = 1 \\ f_3(x-1) + 2 & x > 1 \end{cases}$$

4. A function defined using another function ([1])

$$f_4 : (\mathbb{Z}, \mathbb{Z}) \to \mathbb{Z}$$
$$f_4(x, y) = \begin{cases} 0 & y \leq 1 \\ \lfloor \frac{x}{y} \rfloor & \textit{otherwise} \end{cases}$$

5. A function which returns an *ordered tuple* of integers

$$f_5 : (\mathbb{Z}, \mathbb{Z}) \to (\mathbb{Z}, \mathbb{Z})$$
$$f_5(x, y) = \begin{cases} (0, 0) & y \leq 1 \\ (\lfloor \frac{x}{y} \rfloor, y - \lfloor \frac{x}{y} \rfloor) & \textit{otherwise} \end{cases}$$

[1] $\lfloor \cdot \rfloor$ denotes the *floor* function.

6. A trigonometric function

$$\begin{aligned} f_6 &: \mathbb{R} \to \mathbb{R} \\ f_6(r) &= sin\, r \end{aligned}$$

7. The (natural) logarithm function

   In this example, $\mathbb{R}^+$ denotes the set of positive real numbers.

$$\begin{aligned} f_7 &: \mathbb{R}^+ \to \mathbb{R} \\ f_7(r) &= log_e\, r \end{aligned}$$

8. Function composition

   Let A, B and C are sets, $\mathcal{F}$ is the set of functions defined from A to B, $\mathcal{G}$ is the set of functions defined from B to C, and $\mathcal{H}$ is the set of functions defined from A to C.

$$\begin{aligned} f_8 &: (\mathcal{F}, \mathcal{G}) \to \mathcal{H} \\ f_8(f, g) &= g \circ f \text{ where } \quad \forall x \in A, \;\; g \circ f(x) = g(f(x)) \end{aligned}$$