

Design and Analysis of Algorithms

Graphs 3: Depth-First Search

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3.4.2

Claim: A directed graph has a cycle *if and only if* its DFS reveals a back edge.

Proof:

Consider a directed graph with two nodes, A and B ($A \rightarrow B \rightarrow A \dots$), in a cycle. **Base Case:** *If we execute a DFS from node $N = 0$ (node A), we will explore node B . Since the only other option is A , which has been marked as explored, we will create a back edge to include the cycle.* If we start exploring at $N = 1$ (node B), A will be marked as explored, and by the same logic we will create a back edge to include the cycle from $B \Rightarrow A$. Here we can see the cyclic nature of back edges from $A \rightarrow B$ and $B \rightarrow A$ regardless of the starting node.

3.4.4

Claim: A directed graph has a cycle *if and only if* its DFS reveals a back edge. If a directed graph has a cycle, that means the DFS tree will have a back edge.

Proof:

Consider the same graph $A \rightarrow B \rightarrow A \dots$. **Base Case:** A graph was explored from $N = 0$ (node A) to generate a DFS tree that has the structure $A \rightarrow B \dots A$ with a back edge from $B \rightarrow A$. This is cyclic in nature. Now consider a DFS tree generated from $N = 1$ (node B) with the structure $B \rightarrow A$ and a back edge to from $B \rightarrow A$. We can see that a DFS tree generated from a cyclic graph is reproducible regardless of the starting node.

3.4.6

Find the sources, sinks, and all possible linearizations.

1. **Sources** B

2. **Sinks** E,F

3. **Linearizations**

$B \rightarrow A \rightarrow D \rightarrow C \rightarrow E$

$B \rightarrow D \rightarrow A \rightarrow C \rightarrow E$

$$\begin{array}{l} B \rightarrow A \rightarrow D \rightarrow C \rightarrow F \\ B \rightarrow D \rightarrow A \rightarrow C \rightarrow F \end{array}$$

3.4.8

3.4.10

If the explore procedure is called on a node in a sink SCC, then the set of nodes visited is exactly that SCC. Why?

A sink SCC means that there are no outgoing edges from that SCC node. So if there are multiple SCCs then you can *enter* an sink SCC with explore, but you cannot leave.