Design and Analysis of Algorithms

Graphs 3: Depth-First Search

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April 21st, 2022

3.3.3

> For all nodes u,v, either [pre[u], post[u]] is completely within [pre[v], post[v]], or the other way around, or there is no overlap. Why? pre[u] to post[u] is the time u is on the stack.

This is because each [pre[u],post[u]] is completely within [pre[v],post[v]] because only unexplored nodes are added to the stack. So, when a node is added to the stack (u), all of the nodes for (v) will be discovered before the stack is emptied and returns to u. This is because v cannot be added to the exploration of u. The name depth-first sort of implies this, too. If we start at a node (u), we search as deep as we can before returning to u, so any v searches will take less time than u.

3.3.5

> What is the running time of DFS?

The running time of DFS is O(N)

```
procedure explore(G,v)
Input: G(V,E) is a graph; v is in V
Output: visited(u) is set to true for all nodes u reachable from v
  previsit(v)                    //c1
  visisted(v)=true               //c2
  for each edge (v,u) in E:      //N(E)
    if not visited(u): explore(u) //c3
  postvisit(v)
```

It appears that the running time is $O(N)$, or or $O(E)$, since we call explore on depending on the size of the Edges. Each edge is explored 1 time!

3.3.7

> Suppose that we start the explore procedure at node u. Use induction to prove that it will find all nodes reachable from u. Induction refresher: Prove the base case, then state the induction hypothesis, then do the inductive step.

Claim: All nodes v that are $k \geq 0$ steps from u can be found with the explore procedure.

Base Case: For $k = 0$, $explore(k = 0)$ discovers $u = v$ immediately, and $k \geq 0$ nodes will be discovered, too.

Proof by Induction: For $k + 1$, $u[k + 1]$ will be discovered as $v[k + 1]$. Through induction all nodes for $k \geq 0$ will be discovered through explore.

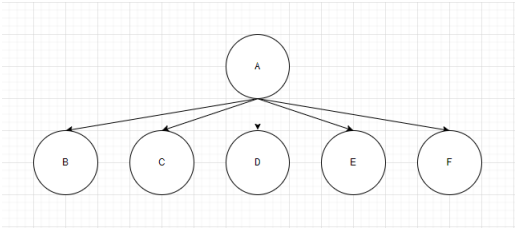
3.3.9

1. How many parents can a node have in the DFS tree?

   A node can have one parent because it can only be marked as visited coming from one other node.

2. How many children can a node have in the DFS tree?

   A node can have as many children as there are nodes coming from that node.



3. How many ancestors can a node have in the DFS tree?

   If there are k nodes in a graph, the DFS could have up to $k-2$ ancestors - the total number of nodes minus the parent and the bottom most node.

4. How many descendants can a node have in the DFS tree?

   A node can have $k-1$ nodes when there are k nodes in the graph. This is assuming the graph is essentially a linked list.