Design and Analysis of Algorithms

6.6 General Functions

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7.6.2

- Let $f(k)$ be the running time of the k^th operation.
- Consider algorithms with the following running times:
  - $f(k) =$ the largest power of 2 that divides k
  - $f(k) = k$ if k is a power of 2, $f(k) = 1$
- Calculate amortized running time for each algorithm.

- $f(k) =$ the largest power of 2 that divides k

$$1, 2, 1, 4, 1, 2, 1, 8, 1, 2$$

$$\frac{n}{3} + \sum_{k=0}^{n} \frac{k}{2}$$

$$...?$$

n amortized runtime

- $f(k) = k$ if k is a power of 2, $f(k) = 1$

$$1, 2, 3, 4, 5, 6, 7, 8$$

$$1, 1, 3, 1, 5, 6, 7, 1$$

$$\frac{n}{1} + \sum_{k=0;k*=2}^{n} k$$

$$nlog(n)?$$

Solution

- $f(k) =$ the largest power of 2 that divides k

$$1\frac{n}{2} + 2\frac{n}{4} + 4\frac{n}{8} + 2\frac{n}{4} + ... + \frac{2^{log(n)}}{2} * \frac{n}{2^{log(n)}} + n$$

$$\frac{n}{2} + \frac{n}{2} + \frac{n}{2} + ... + \frac{n}{2} + n$$

$$\frac{n}{2}log(n) + n$$