Design and Analysis of Algorithms

5.3 Linear Programming

Daniel Shannon

May 5th, 2022

5.3.2

- You have a store that makes and sells calculators.
- Demand tells you to produce at least 100 scientific calculators and 80 graphing calculators per day.
- You can make at most 200 scientific calculators and 170 graphing calculators each day.
- Because of a contract, you must produce at least 200 calculators per day.
- Each scientific calculator gives you a $2 loss, and each graphing calculator gives you a $5 profit.

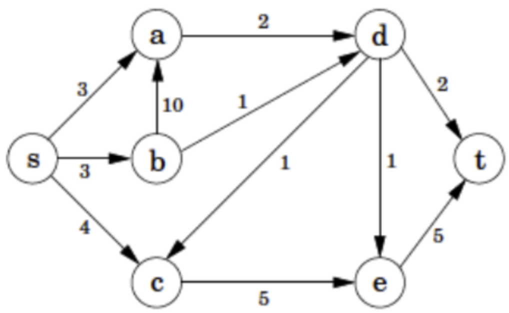
Formulate this as a linear programming problem

- x_0 scientific calculator
- x_1 graphing calculator
- p profit

$$100 \leq x_0 \leq 200$$
$$80 \leq x_1 \leq 170$$
$$x_0 + x_1 \geq 200$$
$$p = -2x_0 + 5x_1$$

5.3.5

Find the maximum amount of flow from s to t.



$$MAX(sa + ad + dt + sc + ce + et + 5sb + 3ba + 2bd + 2de + 2dt + 2et)$$

$$sa + ad + dt \leq 2$$

$$sc + ce + et \leq 4$$

$$sb + ba + ad + dt \leq 1$$

$$sb + ba + bd + dt \leq 1$$

$$sb + ba + ad + de + et \leq 1$$

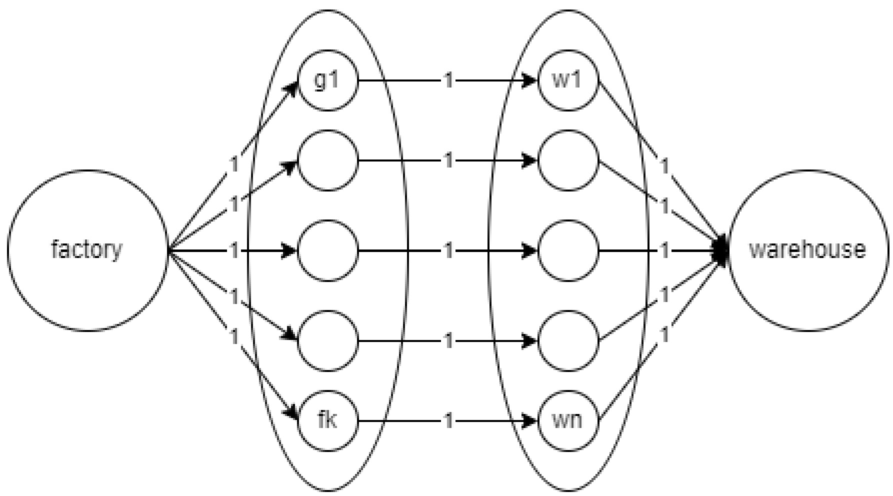
$$sb + bd + de + et \leq 1$$

$$sb + bd + dc + ce + et \leq 1$$

Max flow is 6.

5.3.7

- There are k factories that produce up to g_1, ..., g_k
- There are n warehouses that can store up to w_1, ..., w_n quantity of goods. (Assume integers.)
- A factory can supply a warehouse if they are within 100 miles of each other.
- What is the maximum number of goods that can be stored?



$$g_k - w_n \leq 100$$

$$g_k - w_n + s = 100$$

5.3.9

> Suppose that in addition to a maximum capacity on edges, each node has a demand: a certain amount of flow that it will use up. How do we find if there is a feasible flow?

Since the outflow is now dependent on the rate of inflow and consumption, I would take find the maximum of rate.

$$max[(r_{in} - c_{node}r_{node} \leq c_{out})\frac{d}{dr}]$$