Design and Analysis of Algorithms

Graphs 3: Depth-First Search

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3.4.2

Claim: A directed graph has a cycle *if and only if* its DFS reveals a back edge.
Proof:
Consider a directed graph with two nodes, A and B ($A \to B \to A$...), in a cycle. **Base Case:** *If we execute a DFS from node $N = 0$ (node A), we will explore node B. Since the only other option is A, which has been marked as explored, we will create a back edge to include the cycle.* If we start exploring at $N = 1$ (node B), A will be marked as explored, and by the same logic we will create a back edge to include the cycle from $B \Rightarrow A$. Here we can see the cyclic nature of back edges from $A \to B$ and $B \to A$ regardless of the starting node.

3.4.4

Claim: A directed graph has a cycle *if and only if* its DFS reveals a back edge. If a directed graph has a cycle, that meas the DFS tree will have a back edge.
Proof:
Consider the same graph $A \to B \to A$.... **Base Case:** A graph was explored from $N = 0$ (node A) to generate a DFS tree that has the structure $A \to B...A$ with a back edge from $B \to A$. This is cyclic in nature. Now consider a DFS tree generated from $N = 1$ (node B) with the structure $B \to A$ and a back edge to from $B \to A$. We can see that a DFS tree generated from a cyclic graph is reproducible regardless of the starting node.

3.4.6

Find the sources, sinks, and all possible linearizations.

1. **Sources** B
2. **Sinks** E,F
3. **Linearizations**
   $B \to A \to D \to C \to E$
   $B \to D \to A \to C \to E$

$B \rightarrow A \rightarrow D \rightarrow C \rightarrow F$
$B \rightarrow D \rightarrow A \rightarrow C \rightarrow F$