Design and Analysis of Algorithms
Graphs 3: Depth-First Search

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4.2.2

Does the BFS tree always give you the shortest path from the starting node to every other node? How can we modify the BFS pseudocode to keep track of what the shortest paths actually are? Prove or give a counterexample.

Claim: The BFS tree always will give the shortest path tree.

Proof: *by induction...* Say we have node u on the queue Q with a distance of $k = 0$ from the origin, u. All connected nodes v have a distance of $k = 1$ from u. *Induction:* When we push the neighbors of v, v', onto Q, v' is $k + 1$ distance from u. So each additional node is the shortest path from $u[k]$ to $u[k + 1]$.

We can modify the pseudo code by adding a distance property to each node as we push it to the queue.