Coalitional Game Theory for Security Risk Management

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***Abstract*—Quantitative models for security risk management in organizations are recently receiving an increased attention in the research community. This paper investigates the possibility of cooperation among autonomous divisions of an organization with dependent security assets and/or vulnerabilities for reducing overall security risks. A coalitional game is formulated for modeling cooperation possibilities among these divisions based on their both positive (synergies) and negative (vulnerabilities) interdependencies. The proposed game constitutes a framework that allows to investigate how an organization can maximize its total utility through cooperation among its different divisions. The introduced utility accounts for the gains from cooperation, in terms of an improved synergy among the divisions, and the costs for cooperation that account for the friction among the divisions (e.g. due to social and human factors) as well as the difficulty of managing large-sized divisions. Using the proposed game model, the illustrative cases of two-coalition cooperation, two-division cooperation as well as a practical scenario when using an ideal cooperation protocol are analyzed.**

***Index Terms*—risk management, coalitional game theory, game theory, security.**

I. Introduction

Networked systems have become an integral and indispensable part of daily business in organizations. Hence, system failures and security compromises have direct consequences for organizations in multiple dimensions. For a telecommunication company, stolen customer data may turn into a public relation nightmare. The municipality of a whole city can be held hostage by a single disgruntled former system administrator. In a more simple and common scenario, a computer virus or worm infection may simply mean a free day for the whole office resulting in significant productivity loss. Such risks and security threats are the novel consequences of the information technology (IT) revolution, which at the same time has brought immense productivity gains and new business opportunities. As organizations and companies are becoming increasingly aware of these risks, their only option is to learn how to manage emerging IT and security risks. Risk management in this specific area is, consequently, a young and vibrant field with a lot of research challenges and opportunities.

Early IT and security risk management research has been mostly empirical and qualitative in nature. This is mainly due to the fact that IT risk management has its roots more in social and management sciences. Given the complexity of the problem and underlying networked systems, a risk management scheme that relies on too many manual processes cannot be expected to succeed. Hence, in recent years, there has been a surge of interest in developing quantitative and analytical frameworks for risk management. Such frameworks can formalize risk management processes as well as provide foundation for computer-assisted assessment and decision-making capabilities. For instance, in [1] and [2], using concepts from linear influence graphs and game theory, a framework for non-cooperative decision making between a number of organizations has been developed. Various properties such as the existence of a unique Nash equilibrium as well as the impact of free-riding have been studied. Further, the authors in [3], have proposed a quantitative scheme for prioritizing vulnerabilities to patch in computer networks. In a similar context, in [4], a framework for ranking the risks and analyzing security risk dependencies in organizations has been developed. The authors in [5] formulate a stochastic game model to study the interactions among an attacker and a defender, given correlated security assets and vulnerabilities between different network entities. In a nutshell, while there has been a recent interest in developing quantitative frameworks for security risk management, most of the research has been focused on decision-making among a number of *non-cooperative* entities (e.g., organizations or divisions). However, no work seems to have investigated how a number of organizations or divisions in an organization can cooperate in order to reduce their vulnerabilities, improve the synergy, and, consequently, reduce their potential security risks.

The main contribution of this paper is to propose a model for cooperation among a number of divisions in an organization while taking into account various risk management factors such as: interdependencies, vulnerabilities, resources, and friction. For this purpose, we propose a model based on coalitional game theory, whereby a number of organizations or divisions can form a cooperative group, i.e., a coalition, given the resulting benefits and costs from this cooperation. Using the proposed model, we analyze different cases and study the resulting coalitional structures as well as the conditions needed for cooperation. To the best of our knowledge, the use of coalitional game theory in a security risk management framework is quite novel and has not been exploited yet in the existing literature.

The rest of this paper is organized as follows: Section II presents the non-cooperative and cooperative security risk management models. In Section III, we study the ideal cooperation case and its impact on the cooperation of two divisions while in Section IV we present and analyze cooperation in a practical scenario. Finally, conclusions are drawn in Section V.

II. System Model

A. Non-Cooperative Model

Consider an organization having N divisions, and let $\mathcal{N}$ denote the set of these divisions. Each division can affect its neighbors both positively and negatively. The positive influence is due to the effect of security resources such as the budget, investments, and expertise. At the same time each division is under certain threats and has vulnerabilities that can affect neighboring divisions negatively. The divisions being "neighbors" often refers to their interdependencies due to their roles in the organization rather than geographical location. For the considered organization, we assume that the divisions possess the following properties:

- Each division $i \in \mathcal{N}$ has an amount $x_i > 0$ of security resources that describes the budget, investments, human personnel, expertise, and so on, that this division possesses. As mentioned, the amount of resources x_i that a division i possesses can affect positively the neighboring divisions.
- Each division $i \in \mathcal{N}$ is under a threat ν_i which can affect negatively the neighboring divisions.
- We consider *linear dependencies* for both resources and threats, and these dependencies will be described by *linear influence graphs* [1] as seen in the remainder of this section. This assumption of linear dependencies is common in existing risk management literature [1], [4], [5].

The security relationships between the divisions can be given by two linear influence graphs: (i)- a graph $G_p(\mathcal{N}, \mathcal{E}_p)$ with the set of vertices being the set of all divisions $\mathcal{N}$ and $\mathcal{E}_p$ being the set of edges, representing the *positive* influence of the resources of each division on other divisions and (ii)- a graph $G_n(\mathcal{N}, \mathcal{E}_n)$ representing the *negative* influence of the threats of each division on other divisions. The positive influence graph is captured by an $N \times N$ matrix $\boldsymbol{W}^p = [W^p_{ij}]$ with elements

$$W^p_{ij} = \begin{cases} 1, & \text{if } i = j, \\ \psi_{ij}, & \text{if } e_{ij} \in \mathcal{E}_p, \quad i,j = 1,\dots,N, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $0 < \psi_{ij} \leq 1$ quantifies the degree of positive influence from the resources of division i on division j and e_{ij} is the edge between those divisions. Similarly, the negative influence graph G_n is captured by an captured by an $N \times N$ influence matrix $\boldsymbol{W}^n = [W^n_{ij}]$ where each element is defined as

$$W^n_{ij} = \begin{cases} 1, & \text{if } i = j, \\ \eta_{ij}, & \text{if } e_{ij} \in \mathcal{E}_n, \quad i,j = 1,\dots,N. \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

where $0 < \eta_{ij} \leq 1$ quantifies how much the vulnerabilities of division i influence or threaten division j. Note that, each division's own vulnerabilities affect it fully and, thus, correspond to unit weight.

Let

$$\boldsymbol{x} := [x_1, x_2, \dots, x_N],$$

be the vector of *security resources* of all divisions that can be used to defend against security risks and

$$\boldsymbol{\nu} := [\nu_1, \nu_2, \dots, \nu_N],$$

be the vector of *threats* against *vulnerabilities* of respective divisions. Thus, the *total effective security resources* of a division $i \in \mathcal{N}$ is $(\boldsymbol{W}^{p\,T} \cdot \boldsymbol{x})_i$ and the *total effective threats* of division $i \in \mathcal{N}$ is $(\boldsymbol{W}^{n\,T} \cdot \boldsymbol{\nu})_i$.

For any division $i \in \mathcal{N}$, given the total effective security and threats, the utility of division i which it tries to maximize is given by

$$v(\{i\}) = b((\boldsymbol{W}^{p\,T} \cdot \boldsymbol{x})_i) - c((\boldsymbol{W}^{n\,T} \cdot \boldsymbol{\nu})_i). \tag{3}$$

The functions $b(\cdot)$ and $c(\cdot)$ are the actual resource benefit and threat cost of division i, which take its total effective resources and vulnerabilities as arguments, respectively.

B. Cooperative Model

For improving their effective security resources and reducing their effective threats, the divisions can cooperate by forming cooperative groups, i.e., *coalitions*. By forming a coalition $S \subseteq \mathcal{N}$, the divisions are able to

- Improve and strengthen the positive effects on each other, i.e., improve the weights $W^p_{ij}\ \forall i,j \in S$, $W^p_{ij} \geq 0$, for example by working together more effectively. It is further assumed that even if two divisions have no influence non-cooperatively, i.e., $W^p_{ij} = 0$, they might be able to create a positive influence by cooperating.
- Reduce the negative effect of threats on each others and decrease the weights $W^n_{ij}\ \forall i,j \in S$ by sharing information and collaboratively addressing vulnerabilities.

Given a coalitional structure (partition) $\mathcal{S} = \{S_1, \dots, S_M\}$, $S_k \subseteq \mathcal{N}$, $S_k \cap S_l = \emptyset\ \forall k \neq l$ and $\cup_{i=1}^M S_i = \mathcal{N}$ of the divisions set $\mathcal{N}$ the positive and negative effects are captured by modified matrices $\overline{\boldsymbol{W}}^p(\mathcal{S})$ and $\overline{\boldsymbol{W}}^n(\mathcal{S})$, respectively. For notational convenience, the arguments are dropped to obtain $\overline{\boldsymbol{W}}^p$ and $\overline{\boldsymbol{W}}^n$, when the coalition structure is clear from the context. For instance, given any two divisions $i, j \in \mathcal{N}$ and a coalition $S \subseteq \mathcal{N}$, the elements of the matrix $\overline{\boldsymbol{W}}^p$ are defined as

$$\overline{W}^p_{ij} := \begin{cases} 1, & \text{if } i = j, \\ W^p_{ij}, & \text{if } i \notin S \text{ or } j \notin S, \quad i,j = 1,\dots,N, \\ f(W^p_{ij}), & \text{if } i,j \in S. \end{cases} \tag{4}$$

where $f(W^p_{ij}) \geq W^p_{ij}$ is the cooperative improvement in the positive influence W^p_{ij} when the two divisions i and j belong to the same coalition. The main motivation behind (4) is that, whenever two divisions belong to the same coalition, given their resources and due to cooperation, the two divisions can improve the synergy among each other according to the function f.

Similarly, for the negative effects, the elements of the modified matrix $\overline{\boldsymbol{W}}^n$ are given by

$$\overline{W}^n_{ij} := \begin{cases} 1, & \text{if } i = j, \\ W^n_{ij}, & \text{if } i \notin S \text{ or } j \notin S, \quad i,j = 1,\dots,N, \\ g(W^n_{ij}), & \text{if } i,j \in S, \end{cases} \tag{5}$$

where $g(W^n_{ij}) \leq W^n_{ij}$ represents the cooperative reduction of the negative influence whenever the two divisions i and j join the same coalition. The underlying assumption here is that when two divisions are in the same coalition, they can operate cooperatively and reduce their threats on each other according to function g.

The formation of coalitions often entails, in addition to benefits, certain costs due to cultural, economical, or social reasons. For example, it may be quite costly for a well-organized division to cooperate with a badly organized one. There are usually natural frictions between divisions due to business culture or social differences that need to be overcome to establish a coalition. Furthermore, as the number of employees in a coalition increases, various challenges emerge, e.g., coordination or scaling of existing structures. These frictions and size effects can lead to non-negligible impediments to the potential cooperation between divisions as well as the whole organization.

In order to model this friction that can exist between various divisions, we introduce the concept of a *friction* graph, $Q(\mathcal{N}, \mathcal{E}_Q)$, defined over the set of divisions $\mathcal{N}$. The effects of the friction graph Q is captured by an $N \times N$ friction matrix $\boldsymbol{Q}$ where each element is given by

$$Q_{ij} = \begin{cases} \chi_{ij}, & \text{if } e_{ij} \in \mathcal{E}_Q \text{ and } i \neq j, \\ 0, & \text{otherwise,} \end{cases} \tag{6}$$

with $\chi_{ij} > 0$ a positive real number indicating the degree of friction between divisions i and j. In addition to the friction cost, for any coalition $S \subseteq \mathcal{N}$ the increase in its size $|S|$, defined as the cardinality of the set, yields an additional cost pertaining to the extra effort that the employees in S need to invest for coordination within their coalition. Taking into account the friction matrix $\boldsymbol{Q}$ and the size of coalition S, we define a cost function $p(\boldsymbol{Q}, S)$. It is natural to assume that the cost is an increasing function of the total friction induced by the graph $Q(\mathcal{N}, \mathcal{E}_Q)$ and of the coalition size $|S|$, which reflects the cost of coordination within large divisions. Subsequently, the value (i.e., *characteristic function* in coalitional game theory terms [6]) of any coalition $S \subseteq \mathcal{N}$ will be given by

$$v(S) = b\left((\overline{\boldsymbol{W}}^{p\,T} \cdot \boldsymbol{x})_S\right) - c\left((\overline{\boldsymbol{W}}^{n\,T} \cdot \boldsymbol{\nu})_S\right) - p(\boldsymbol{Q}, S) \tag{7}$$

where Q is the associated friction graph,

$$(\overline{\boldsymbol{W}}^{p\,T} \cdot \boldsymbol{x})_S := \sum_{i \in S} (\overline{\boldsymbol{W}}^{p\,T} \cdot \boldsymbol{x})_i$$

is the total cooperative effective *security resources* and

$$(\overline{\boldsymbol{W}}^{n\,T} \cdot \boldsymbol{\nu})_S := \sum_{i \in S} (\overline{\boldsymbol{W}}^{n\,T} \cdot \boldsymbol{\nu})_i$$

is the total cooperative effective *threats and vulnerabilities* for coalition S. For the simplicity of the analysis, it will be assumed in the remainder of this paper that both the "resource benefit" function $b(\cdot)$ and "threat cost" function $c(\cdot)$ are linear, such that

$$v(S) = (\overline{\boldsymbol{W}}^{p\,T} \cdot \boldsymbol{x})_S - (\overline{\boldsymbol{W}}^{n\,T} \cdot \boldsymbol{\nu})_S - p(\boldsymbol{Q}, S). \tag{8}$$

Consequently, the proposed problem can be modeled as a $(\mathcal{N}, v)$ coalitional game in characteristic form [6], [7] with the players being the divisions[1] and the characteristic function given by (7). Due to the presence of a cost for cooperation as per (7), traditional solution concepts for coalitional games, such as the core [6], may not be applicable. In fact, in order for the core to exist, as a solution concept, a coalitional game must ensure that the grand coalition, i.e., the coalition of all players will form. However, in the proposed game, the grand coalition may not always form due to the friction between the divisions. Instead, independent and disjoint coalitions may emerge in the organization. In this regard, the proposed game is classified as a *coalition formation game* [7], and the objective is to characterize the coalitional structure that will possibly form between the divisions.

III. Coalition Formation with Ideal Cooperation

The functions f and g in (4) and (5) depend strictly on the cooperative protocol of the divisions within a single coalition. The cooperative model proposed in Section II-B is quite generic and accommodates any kind of cooperative protocols. It is, nonetheless, useful to present an example to illustrate the properties of the proposed model. Hence, as a basic example, we introduce an *ideal cooperation protocol*. Under this protocol, any coalition S of divisions can maximize the positive effects of cooperation and totally eliminate the negative effects of the threats between its members. Thus, this cooperation protocol is specified by

$$f(W^p_{ij}) = 1, \quad \forall i, j \in S \tag{9}$$

and

$$g(W^n_{ij}) = 0, \quad \forall i, j \in S \text{ and } i \neq j. \tag{10}$$

The protocol describes an ideal case where by sharing expertise, skills, and resources, a group S of cooperating divisions can effectively eliminate the vulnerabilities and have a perfect synergy among themselves. Although in practical cases this assumption may not hold, but it will provide us with insights on the cooperation possibilities among different divisions.

As a first step of analyzing cooperation possibilities among the divisions, the merger of two coalitions is discussed next. In fact, cooperation between two coalitions constitutes a building block for the organization-wide cooperation. The following theorem states the necessary and sufficient condition for the merger of two coalitions.

Theorem 1: Consider two disjoint coalitions $S_1 \subseteq \mathcal{N}$, $S_2 \subseteq \mathcal{N}$, $S_1 \cap S_2 = \emptyset$ in the defined ideal cooperation environment and with value functions as per (8). The value of a merger between these two coalitions for the organization is larger than their aggregate value when being separate, i.e.,

$$v(S_1 \cup S_2) \geq v(S_1) + v(S_2),$$

if and only if, the following condition on the cost functions holds

$$p(\boldsymbol{Q}, S_1 \cup S_2) - (p(\boldsymbol{Q}, S_1) + p(\boldsymbol{Q}, S_2)) \leq \gamma, \tag{11}$$

where

$$\begin{aligned} \gamma \;\; := \;\; & \textstyle\sum_{j \in S_2} x_j \left(|S_1| - \sum_{i \in S_1} W^p_{ji}\right) \\ & + \textstyle\sum_{j \in S_1} x_j \left(|S_2| - \sum_{i \in S_2} W^p_{ji}\right) \\ & + \left(\textstyle\sum_{i \in S_2} \sum_{j \in S_1} W^n_{ji} \nu_j + \sum_{i \in S_1} \sum_{j \in S_2} W^n_{ji} \nu_j\right), \end{aligned}$$

is the *total effective benefit* of this merger for the organization.

Proof: Consider two disjoint coalitions $S_1 \subseteq \mathcal{N}$, $S_2 \subseteq \mathcal{N}$, $S_1 \cap S_2 = \emptyset$. The value of coalition S_1 is from (8),

$$v(S_1) = \sum_{i \in S_1} (\overline{\boldsymbol{W}}^{p\,T} \cdot \boldsymbol{x})_i - \sum_{i \in S_1} (\overline{\boldsymbol{W}}^{n\,T} \cdot \boldsymbol{\nu})_i - p(\boldsymbol{Q}, S_1),$$

$$\Rightarrow v(S_1) = \sum_{i \in S_1} \sum_{j \in \mathcal{N}} \overline{W}^{p\,T}_{ij} \cdot x_j - \sum_{i \in S_1} \sum_{j \in \mathcal{N}} \overline{W}^{n\,T}_{ij} \cdot \nu_j - p(\boldsymbol{Q}, S_1).$$

Using the definitions of f and g in (9) and (10), respectively, this value becomes

$$\begin{aligned} v(S_1) =& |S_1| \sum_{j \in S_1} x_j + \sum_{i \in S_1} \sum_{j \in \mathcal{N} \setminus S_1} W^p_{ji} x_j \\ & - \sum_{i \in S_1} \sum_{j \in \mathcal{N} \setminus S_1} W^n_{ji} \nu_j - \sum_{i \in S_1} \nu_i - p(\boldsymbol{Q}, S_1) \end{aligned}$$

[1] In the rest of this paper, the terms division and player are used interchangeably.

The value of coalition S_2,

$$v(S_2) = |S_2| \sum_{j\in S_2} x_j + \sum_{i\in S_2} \sum_{j\in \mathcal{N}\setminus S_2} W_{ji}^p x_j - \sum_{i\in S_2} \sum_{j\in \mathcal{N}\setminus S_2} W_{ji}^n \nu_j - \sum_{i\in S_2} \nu_i - p(\boldsymbol{Q}, S_2)$$

and the one of $S_1 \cup S_2$

$$v(S_1 \cup S_2) = |S_1 \cup S_2| \sum_{j\in S_1\cup S_2} x_j + \sum_{i\in S_1\cup S_2} \sum_{j\in \mathcal{N}\setminus S_1\cup S_2} W_{ji}^p x_j - \sum_{i\in S_1\cup S_2} \sum_{j\in \mathcal{N}\setminus S_1\cup S_2} W_{ji}^n \nu_j - \sum_{i\in S_1\cup S_2} \nu_i - p(\boldsymbol{Q}, S_1 \cup S_2)$$

are obtained similarly.

By substituting these equations into $v(S_1 \cup S_2) \geq v(S_1) + v(S_2)$ yields the necessary and sufficient condition in the theorem. ■

Theorem 1 provides a quantitative criterion in the form of a condition on the cost functions, which allows an organization (or divisions within) to assess when a merger of two potential coalitions is beneficial. This result can be further investigated by considering a special case with a concrete cost function. In many practical scenarios, the cost p of a coalition S can be defined as a linear function of the total friction and the coalition size,

$$p(\boldsymbol{Q}, S) = \begin{cases} \alpha \cdot \sum_{i\in S} \sum_{j\in S} Q_{ij} + \beta \cdot |S|, & \text{if } |S| > 1, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

where Q_{ij} is an element of the friction matrix $\boldsymbol{Q}$ related to the friction graph Q. Note that this matrix does not have to be symmetric. The parameters $\alpha \geq 0$ and $\beta \geq 0$ quantify the price of forming a coalition with $|S| > 1$ per unit friction and per unit size, respectively. The following result is a special case of Theorem 1 for the cost function defined.

Theorem 2: Consider two disjoint coalitions $S_1 \subseteq \mathcal{N}$, $S_2 \subseteq \mathcal{N}$, $S_1 \cap S_2 = \emptyset$ in the defined ideal cooperation environment with the value function (8), where the cost is defined in (12). If both coalitions have more than one division, $|S_1| > 1$ and $|S_2| > 1$, they cooperate to form the coalition $S_1 \cup S_2$ for the benefit of the organization, if and only if

$$\alpha \leq \frac{\gamma}{T(S_1 \cup S_2)},$$

where

$$T(S_1 \cup S_2) := \sum_{i\in S_1} \sum_{j\in S_2} (Q_{ij} + Q_{ji}) \tag{13}$$

is the total friction between the members of S_1 and the members of S_2.

Proof: Given any two disjoint coalitions S_1 and S_2, such that $|S_1| > 1$ and $|S_2| > 1$, the cost function for coalition $S_1 \cup S_2$ is given by (12) as follows

$$p(\boldsymbol{Q}, S_1 \cup S_2) = \alpha \cdot \sum_{i\in S_1\cup S_2} \sum_{j\in S_1\cup S_2} Q_{ij} + \beta |S_1 \cup S_2|,$$

$$\Rightarrow p(\boldsymbol{Q}, S_1 \cup S_2) = \alpha \cdot \sum_{i\in S_1} \sum_{j\in S_1} Q_{ij} + \alpha \cdot \sum_{i\in S_2} \sum_{j\in S_2} Q_{ij} + \alpha \cdot \sum_{i\in S_1} \sum_{j\in S_2} (Q_{ij} + Q_{ji}) + \beta|S_1| + \beta|S_2|. \tag{14}$$

By algebraic manipulation and by applying the definitions of cost p in (12) and total friction T in (13) to (14) yields

$$p(\boldsymbol{Q}, S_1 \cup S_2) = p(\boldsymbol{Q}, S_1) + p(Q_{S_2}, S_2) + \alpha T(S_1 \cup S_2) \tag{15}$$

By combining (15) with the result from Theorem 1, we can conclude that it is beneficial for the organization that two coalitions S_1 and S_2 with $|S_1| > 1$ and $|S_2| > 1$ would merge and form $S_1 \cup S_2$ if and only if $\alpha \leq \frac{\gamma}{T(S_1 \cup S_2)}$. ■

Interestingly, Theorem 2 verifies the intuition that the benefit of cooperation among two coalitions of divisions in a company mainly depends on the ratio between the total effective benefit γ to the total friction T between the members of the two coalitions. Furthermore, as per Theorem 2, the friction between the members of a single coalition does not affect whether this coalition will cooperate with another one or not, e.g., the friction between the members of S_1 has no impact on whether S_1 will cooperate with another coalition S_2. Finally, the theorem provides an upper bound on the price per unit of friction above which no cooperation is possible between any two coalitions of size larger than one.

The next example illustrates the results of Theorems 1 and 2 in the particular case of cooperation among two single divisions. Moreover, there are many generalizations and extensions to the presented results. One direction is to investigate nonlinear "resource benefit" $b(\cdot)$ and "threat cost" $c(\cdot)$ functions in (8). Another natural extension is to study the case of a generic non-ideal cooperation protocol where the improvement in the positive and negative weights is partial, i.e., f and g are different from those in (9) and (10), respectively.

Example - Merger of Two Divisions: As an illustrative example, consider the particular case of cooperation among two single divisions. Applying the result in Theorem 1, the condition for the two divisions to merge is

$$\alpha(Q_{12}+Q_{21})+2\beta \leq x_1(1-W_{12}^p)+x_2(1-W_{21}^p)+W_{12}^n\nu_1+W_{21}^n\nu_2, \tag{16}$$

Applying (16) to the following particular case yields interesting results. For instance,when the two divisions have no threats on each others, i.e., $\nu_1 = \nu_2 = 0$, β is very small, i.e., $\beta \approx 0$, and $q := Q_{12} = Q_{21}$, the condition in (16) reduces to

$$x_1 > \frac{2\alpha q - x_2(1 - W_{21}^p)}{(1 - W_{12}^p)}. \tag{17}$$

This condition has several interesting properties and interpretations:

- If there is no friction, $q = 0$, then cooperation is always beneficial.
- If there is already strong positive influence between divisions, $W_{12}^p \to 1$ and $W_{21}^p \to 1$, then they have almost no incentive to cooperate.
- If one division already benefits from strong positive influence, e.g., $W_{12}^p \to 1$, then the second division needs to have the sufficient resources x_2 to overcome the friction and cooperate.
- If the cost per friction unit α increases, then more resources are needed for the divisions to overcome the friction and cooperate.
- In the case of no prior positive influence, $W_{12}^p = W_{21}^p = 0$ (or even when W_{12}^p and W_{21}^p are very small) , it is sufficient

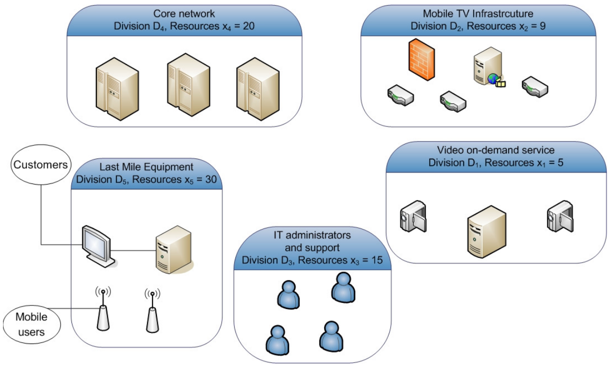

Fig. 1. Illustration of the various divisions in the proposed scenario along with their corresponding resources.

that the total resources of both divisions are greater than the friction, $x_1 + x_2 > 2\alpha q$.

- For fixed α, W_{12}^p and W_{21}^p, as one of the resources, e.g., x_2 gets bigger, less resources are needed from the other division, e.g., x_1 for cooperation to happen. Hence, large divisions and small divisions can, in general, benefit from forming a coalition for a given friction price.

Finally, the result in (16) allows an organization to easily assess whether two of its divisions can cooperate, given the different interdependency graphs.

IV. An Illustrative Scenario

Consider a large company which offers video on-demand services for subscribers. In order for these services to reach the consumers, various divisions of the company are involved in the process. In particular, we distinguish five key divisions involved in the delivery of the service:

1) The "Video on-demand service" division, referred to as D_1 hereafter, which is responsible for developing and updating the offered video on-demand solutions.
2) The "Mobile TV infrastructure" division, referred to as D_2 hereafter, which is responsible for the infrastructure that will run all sorts of multimedia applications (e.g., video on-demand or TV) over mobile TV.
3) The "IT support and administrators" division, referred to as D_3 hereafter, which is composed of the technicians and IT support staff that take care of maintaining a healthy infrastructure within the entire company.
4) The "Core network" division, referred to as D_4 hereafter, which is responsible for the company's backbone and main infrastructure. All services offered by the company need to go through the core network.
5) The "Last mile equipment" division, referred to as D_5 hereafter, which is the front end of the company and is responsible for the equipment that will deliver the services directly to the customers. These equipments include, for example, wireless base stations, wired fiber optics links, and so on. We also consider that the customer support and sales units are part of this division.

Using the proposed coalitional game model, we will investigate the coalitional structures that will form within this company, given the resources, influence, and friction among the different divisions. For the given scenario, the set of all players $\mathcal{N}$ is the set of all divisions, i.e., $\mathcal{N} = \{D_1, D_2, D_3, D_4, D_5\}$. On one hand, divisions D_1 and D_2 are quite small and restricted in terms of resources, such that $x_1 = 5$ and $x_2 = 9$. On the other hand, the divisions D_3 and D_4 are quite large with resources of $x_3 = 15$ and $x_4 = 20$, respectively. Finally, division D_5 is, generally, very large in most organizations, and, thus, $x_5 = 30$. Consequently, the vector of security resources is $\boldsymbol{x} = [5, 9, 15, 20, 30]$. An illustrative figure of these divisions is shown in Fig. 1.

Moreover, we consider that the last mile equipment division D_5 is the most exposed to threats, and, hence, its vulnerability is set to $\nu_5 = 20$. Further, the core network and IT support are also under significant threats and we set $\nu_3 = \nu_4 = 10$. The least vulnerable divisions are the services, i.e., the mobile TV infrastructure and the video on demand service, and, thus, their vulnerabilities are set to $\nu_2 = 8$ and $\nu_1 = 5$, respectively. Consequently, the vector of threats against vulnerabilities is $\boldsymbol{\nu} = [5, 8, 10, 10, 20]$. The linear influence between the five divisions is defined using the following positive and negative influence matrices:

$$\boldsymbol{W}^p = \begin{bmatrix} 1 & 0.4 & 0.2 & 0 & 0 \\ 0.8 & 1 & 0 & 0 & 0 \\ 0.2 & 0.1 & 1 & 0.8 & 0.9 \\ 0.7 & 0.95 & 0.6 & 1 & 0 \\ 0.4 & 0.7 & 0.8 & 0 & 1 \end{bmatrix} \tag{18}$$

$$\boldsymbol{W}^n = \begin{bmatrix} 1 & 0.7 & 0.1 & 0 & 0 \\ 0.95 & 1 & 0.6 & 0.2 & 0 \\ 0.2 & 0.7 & 1 & 0.8 & 0.7 \\ 0.5 & 0.95 & 1 & 1 & 0.7 \\ 0 & 0 & 0.8 & 0.6 & 1 \end{bmatrix} \tag{19}$$

The values for the influence levels selected in (18) and (19) are based on several real-life aspects of interactions among a company's divisions and are inspired from the discussions in [2]. For example, as the video on-demand service division D_1 is mainly highly dependent on the mobile TV infrastructure division D_2, the level of positive influence $W_{21}^p = 0.8$ of the resources of D_2 over D_1 is quite high. Further, due to the sensitivity of the data on the core network, the vulnerabilities of the core network division D_4 have a strong negative influence $W_{43}^n = 1$ on the IT division D_3. In contrast, threats on the last mile equipment division D_5 have almost no negative impact on the video on-demand service division D_1. Similar reasoning can be used to interpret all the selected influence levels in (18) and (19). Note that formal quantitative methods for defining the positive and linear influence matrices are still under thorough research in risk management literature (e.g., in [2], [5] and the references therein). Nonetheless, the proposed coalitional game model can handle any other positive and negative influence matrices.

For friction, we consider different friction levels that vary on a scale from "very low" friction to "very high" friction with each level having a corresponding value as per Table I. In this case, we propose the following friction matrix

$$\boldsymbol{Q} = \begin{bmatrix} 0 & 0.5 & 0.5 & 1.5 & 2 \\ 0.5 & 0 & 1 & 1 & 2 \\ 0 & 0.5 & 0 & 2 & 2.5 \\ 0 & 0 & 2.5 & 0 & 2.5 \\ 1 & 1 & 2.5 & 1.5 & 0 \end{bmatrix} \tag{20}$$

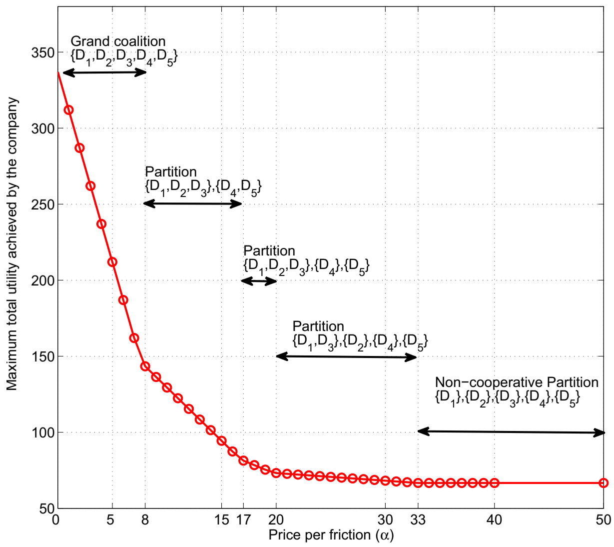

Fig. 2. The optimal utility-maximizing partitions and their corresponding maximum total utility values as the price per friction β increases.

The matrix $\boldsymbol{Q}$ in (20) captures the different friction levels that exist between the divisions. For example, it is well known that the IT division D_3 and the customer support and last mile equipment division D_5 have, in general, a very high friction among each other. In contrast, divisions D_1 and D_2 have very low friction between them. The rest of the friction levels in (20) are interpreted in a similar manner.

Given this proposed scenario, we normalize the price per unit size to $\beta = 1$ and we compute the optimal partition that maximizes the total utility of the company as the price per unit friction α varies. In Fig. 2, we show, as a function of the price per unit friction α, the optimal partition as well as the corresponding maximum total utility achieved by the company when this optimal partition is in place. First and foremost, Fig. 2 corroborates the fact that, as the price per friction α increases, the maximum total utility achieved as well as the size of the formed coalitions will decrease. For instance, we note that, as $\alpha < 8$ forming the grand coalition, i.e., the coalition of all divisions, is optimal and maximizes the total utility for the organization. However, in practice, the price per unit friction α is significant, and, thus, forming a single grand coalition seldom occurs. In this regard, for $8 \leq \alpha < 17$, the optimal partition implies that the two largest divisions D_4 and D_5 would group together into one coalition $\{D_4, D_5\}$, while the remaining relatively smaller divisions will form a second coalition $\{D_1, D_2, D_3\}$. The reason of this split can be seen using Theorem 2. For instance, by applying Theorem 2 on $\{D_1, D_2, D_3\}$ and $\{D_4, D_5\}$, we have $T(\{D_1, D_2, D_3\} \cup \{D_4, D_5\}) = 18$ and $\gamma = 137.6$ which consequently yields that, for the grand coalition to form and be more beneficial than $\{D_1, D_2, D_3\}$ and $\{D_4, D_5\}$, one must have $\alpha \leq \frac{137.6}{18} = 7.65$ which complies with the result shown in Fig. 2. For example, for $\alpha = 12$, the grand coalition yields a utility of only 37 while the optimal partition $\{\{D_1, D_2, D_3\}, \{D_4, D_5\}\}$ yields a utility of 115.4 which is around 212% of utility improvement relative to the grand coalition (at $\alpha = 12$).

Moreover, as α increases further, the optimal partition becomes composed of a larger number of small coalitions as the increased cost hinders further the formation of coalitions. For instance, for $17 \leq \alpha < 20$, coalition $\{D_4, D_5\}$ that was present for $\alpha < 17$ breaks into two non-cooperative divisions D_4 and D_5. This result can be seen using (16) which yields that, for D_4 and D_5 to cooperate we must have $\alpha \leq 16.75$. As α increases further, i.e., for $20 \leq \alpha < 33$, the optimal partition requires that only divisions D_1 and D_3 cooperate. Finally, as long as $\alpha < 33$, cooperation between certain divisions is beneficial for the company, however, as α goes beyond 33, the optimal partition is the case where all divisions remain non-cooperative. Hence, as demonstrated by Fig. 2, for the proposed scenario, depending on the price per friction paid by the company, different coalitional structure can emerge as optimal, from a utility-maximizing perspective.

TABLE I
FRICTION LEVELS AND THEIR VALUES.

Friction Level	Value
No friction	0
Very low	0.5
Low	1
Average	1.5
High	2
Very high	2.5

V. Conclusions and Future Work

In this paper, using coalitional game theory, we have proposed a novel model for cooperation among a number of divisions in an organization while taking into account various risk management factors such as: interdependencies, vulnerabilities, resources, and friction. Using the proposed model, we have showed that, for merging two coalitions or divisions using an ideal cooperation protocol, a condition on the price paid per friction exists. Further, we have analyzed a practical scenario and discussed the various coalitional structure that would emerge in an organization, as the price per friction varies. Future work can consider various additional aspects of coalition formation in a risk management environment, such as non-ideal cooperation, algorithms for distributed coalition formation decision making, and non-linear utility functions. In addition, the proposed model can easily extend to studying cooperation among multiple independent organizations rather than the divisions of a single organization as well as to other areas, e.g., network level security risk management.

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