Algorithmic game theory, 2011 sem 2

Problem Set 2
Due November 14, 2011 (Submit a pdf on blackboard)

Collaboration for solving these problems is very much encouraged.
However

- you should write down answers on your own
- you should mainly discuss the problems you have not solved yet

For this problem set you may have to read extra material from the two books we have been using (both available as pdfs online): Algorithmic game theory (by Nisan et al.) and Networks, crowds and markets (Easly and Kleinberg). Question 0 is eight marks, questions 1-12 are one mark each. Questions 1 - 12 are from Easley and Kleinberg, copied here for convenience.

0.
Write a brief summary for each presentation given in class (student presentations only) describing the main topics involved. The presentations were on the following topics: (a) Network formation games, (b) war games, (c) voting, (d) the secretary problem, (e) evolutionary game theory, (f) mechanism design, (g) auctions, (h) reputation systems, (i) security games, and (j) information networks. Each summary should be roughly half a page.

1. [crowds ch07ex02]
In the payoff matrix that follows, the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

		Player B	
		x	y
Player A	x	4, 4	3, 5
	y	5, 3	5, 5

Figure 7.9. A two-player game for Exercise 2.

(a) Find all pure-strategy Nash equilibria.
(b) Find all evolutionarily stable strategies. Give a brief explanation for your answer.
(c) Briefly explain how the answers to parts (a) and (b) relate to each other.

2. [crowds ch07ex03]
In this problem we consider the relationship between Nash equilibria and evolutionarily stable strategies for games with a strictly dominant strategy. First, let's define what we mean by *strictly dominant*. In a two-player game, strategy X is said to be a strictly dominant strategy for a player i if, no matter what strategy the other player j uses, player i's payoff from using strategy X is strictly greater than his payoff from any other strategy. Consider the following game in which a, b, c, and d are nonnegative numbers.

		Player B	
		X	Y
Player A	X	a, a	b, c
	Y	c, b	d, d

A two-player game for Exercise 3.

Suppose that strategy X is a strictly dominant strategy for each player; that is, $a > c$ and $b > d$.

(a) Find all of the pure-strategy Nash equilibria of this game.
(b) Find all of the evolutionarily stable strategies of this game.
(c) How would your answers to parts (a) and (b) change if we change the assumption on payoffs to $a > c$ and $b = d$?

Consider following the two-player, symmetric game where x can be 0, 1, or 2.

3. [crowds ch07ex04]
Consider following the two-player, symmetric game where x can be 0, 1, or 2.

Player B

Player A		X	Y
	X	1, 1	2, x
	Y	x, 2	3, 3

A two-player game for Exercise 4(a).

(a) For each of the possible values of x, find all (pure-strategy) Nash equilibria and all evolutionarily stable strategies.

(b) Your answers to part (a) should suggest that the difference between the predictions of evolutionary stability and Nash equilibrium arises when a Nash equilibrium uses a *weakly dominated strategy*. We say that a strategy s_i^* is weakly dominated if player i has another strategy s_i' with the following properties.

i. No matter what the other player does, player i's payoff from s_i' is at least as large as the payoff from s_i^*, and

ii. There is some strategy for the other player so that player i's payoff from s_i' is strictly greater than the payoff from s_i^*.

Now consider the following claim that makes a connection between evolutionarily stable strategies and weakly dominated strategies.

> *Claim:* Suppose that, in the following game below, (X, X) is a Nash equilibrium and that strategy X is weakly dominated. Then X is not an evolutionarily stable strategy.

Player B

Player A		X	Y
	X	a, a	b, c
	Y	c, b	d, d

A two-player game for Exercise 4(b).

Explain why this claim is true. (You do not have to write a formal proof; a careful explanation is fine.)

4. [crowds ch09ex02]

In this problem we ask how the number of bidders in a second-price, sealed-bid auction affects how much the seller can expect to receive for his object. Assume that there are two bidders who have independent, private values v_i that are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both $\frac{1}{2}$. (If there is a tie at a bid of x for the highest bid, the winner is selected at random from among the highest bidders and the price is x.)

(a) Show that the seller's expected revenue is $\frac{6}{4}$.

(b) Now let's suppose that there are three bidders who have independent, private values v_i that are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both $\frac{1}{2}$. What is the seller's expected revenue in this case?

(c) Briefly explain why changing the number of bidders affects the seller's expected revenue.

5. [crowds ch09ex04]

A seller intends to run a second-price, sealed-bid auction for an object. There are two bidders, a and b, who have independent, private values v_i that are either 0 or 1. For both bidders the probabilities of $v_i = 0$ and $v_i = 1$ are each $\frac{1}{2}$. Both bidders understand the auction, but bidder b sometimes makes a mistake about his value for the object. Half of the time his value is 1 and he is aware that it is 1; the other half of the time his value is 0 but occasionally he mistakenly believes that his value is 1. Let's suppose that when b's value is 0 he acts as if it is 1 with probability $\frac{1}{2}$ and as if it is 0 with probability $\frac{1}{2}$. So in effect bidder b sees value 0 with probability $\frac{1}{4}$ and value 1 with probability $\frac{3}{4}$. Bidder a never makes mistakes about his value for the object, but he is aware of the mistakes that bidder b makes. Both bidders bid optimally given their perceptions of the value of the object. Assume that if there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.

(a) Is bidding his true value still a dominant strategy for bidder a? Explain briefly.

(b) What is the seller's expected revenue? Explain briefly.

6. [crowds ch09ex06]

In this question we consider the effect of collusion between bidders in a second-price, sealed-bid auction. There is one seller who will sell one object using a second-price, sealed-bid auction. The bidders have independent, private values drawn from a distribution on $[0, 1]$. If a bidder with value v gets the object at price p, his payoff is $v - p$; if a bidder does not get the object his payoff is 0. We consider the possibility of collusion between two bidders who know each other's value for the object. Suppose that the objective of these two colluding bidders is to choose their two bids to maximize the sum of their payoffs. The bidders can submit any bids they like as long as the bids are in $[0, 1]$.

(a) Let's first consider the case in which there are only two bidders. What two bids should they submit? Explain.

(b) Now suppose that there is a third bidder who is not part of the collusion. Does the existence of this bidder change the optimal bids for the two bidders who are colluding? Explain.

7. [crowds ch09ex08]

In this problem we ask how irrational behavior on the part of one bidder affects optimal behavior for the other bidders in an auction. In this auction the seller has one unit of a good which will be sold using a second-price, sealed-bid auction. Assume that there are three bidders who have independent, private values for the good, v_1, v_2, and v_3, which are uniformly distributed on the interval $[0, 1]$.

(a) Suppose first that all bidders behave rationally; that is, they submit optimal bids. Which bidder (in terms of values) wins the auction and how much does this bidder pay (again in terms of the bidder's values)?

(b) Suppose now that bidder 3 irrationally bids more than his true value for the object; in particular, bidder 3's bid is $(v_3 + 1)/2$. All other bidders know that bidder 3 is irrational in this way, although they do not know bidder 3's actual value for the object. How does this affect the behavior of the other bidders?

(c) What effect does bidder 3's irrational behavior have on the expected payoffs of bidder 1? Here the expectation is over the values of v_2 and v_3, which bidder 1 does not know. You do not have to provide an explicit solution or write a proof for your answer; an intuitive explanation of the effect is fine. (Remember that a bidder's payoff is the bidder's value for the object minus the price, if the bidder wins the auction, or 0, if the bidder does not win the auction.)

8. [crowds ch09ex09]

In this problem we ask how much a seller can expect to receive for his object in a second-price, sealed-bid auction. Assume that there are two bidders who have independent, private values v_i, which are either 1 or 2. For each bidder, the probabilities of $v_i = 1$ and $v_i = 2$ are each $\frac{1}{2}$. Assume that if there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x. We also assume that the value of the object to the seller is 0.

(a) Show that the seller's expected revenue is $\frac{5}{4}$.

(b) Now let's suppose that the seller sets a reserve price of R, where $1 < R < 2$; that is, the object is sold to the highest bidder if her bid is at least R, and the price this bidder pays is the maximum of the second-highest bid and R. If no bid is at least R, then the object is not sold, and the seller receives zero revenue. Suppose that all bidders know R. What is the seller's expected revenue as a function of R?

(c) Using the previous part, show that a seller who wants to maximize expected revenue would never set a reserve price, R, that is more than 1 and less than 1.5.

9. [crowds ch15ex02]

Suppose a search engine has three ad slots that it can sell. Slot a has a clickthrough rate of 6, slot b has a clickthrough rate of 5, and slot c has a clickthrough rate of 1. There are three advertisers who are interested in these slots. Advertiser x values clicks at 4 per click, advertiser y values clicks at 2 per click, and advertiser z values clicks at 1 per click. Compute the socially optimal allocation and the VCG prices for it. Give a brief explanation for your answer.

10. [crowds ch15ex05]

Suppose a search engine has two ad slots that it can sell. Slot a has a clickthrough rate of 12 and slot b has a clickthrough rate of 5. There are two advertisers who are interested in these slots. Advertiser x values clicks at 5 per click and advertiser y values clicks at 4 per click.

(a) Compute the socially optimal allocation and the VCG prices for it.

(b) Suppose the search engine decides not to sell slot b. Instead, it sells only slot a using a sealed-bid, second-price auction. What bids will the advertisers submit for slot a, who will win, and what price will they pay?

(c) Which of these two possible procedures, (a) or (b), generates the greater revenue for the search engine? By how much?

(d) Now let's see if the result in part (c) is general or not. That is, does it depend on the clickthrough rates and values? Suppose there are two slots and two advertisers; the clickthrough rates are r_a for slot a and r_b for slot b, with $r_a > r_b > 0$; and the advertisers' values are v_x and v_y, with $v_x > v_y > 0$. Can you determine which of the two procedures generates the greater revenue for the search engine? Explain.

11. [crowds ch23ex01]

(a) Suppose there are four alternatives, named A, B, C, and D. There are three voters who have the following individual rankings:

$$B \succ_1 C \succ_1 D \succ_1 A,$$

$$C \succ_2 D \succ_2 A \succ_2 B,$$

$$D \succ_3 A \succ_3 B \succ_3 C.$$

You're in charge of designing an agenda for considering the alternatives in pairs and eliminating them using majority vote, via an elimination tournament in the style of the examples shown in Figure 23.3.

You would like alternative A to win. Can you design an agenda (i.e., an elimination tournament) in which A wins? If so, describe how you would structure it; if not, explain why it is not possible.

(b) Now, consider the same question but for a slightly different set of individual rankings in which the last two positions in voter 3's ranking have been swapped. That is, we have

$$B \succ_1 C \succ_1 D \succ_1 A,$$

$$C \succ_2 D \succ_2 A \succ_2 B,$$

$$D \succ_3 A \succ_3 C \succ_3 B.$$

We now ask the same question: Can you design an agenda in which A wins? If so, describe how you would structure it; if not, explain why it is not possible.

12. [crowds ch23ex02]

The Borda Count is susceptible to strategic misreporting of preferences. Here are some examples to practice how this works.

(a) Suppose you are one of three people voting on a set of four alternatives named A, B, C, and D. The Borda Count will be used as the voting system. The other two voters have the rankings

$$D \succ_1 C \succ_1 A \succ_1 B,$$

$$D \succ_2 B \succ_2 A \succ_2 C.$$

You are voter 3 and would like alternative A to appear first in the group ranking, as determined by the Borda Count. Can you construct an individual ranking for yourself so that this will be the result? If so, explain how you would choose your individual ranking; if not, explain why it is not possible.

(b) Let's consider the same question, but with different rankings for the other two voters, as follows:

$$D \succ_1 A \succ_1 C \succ_1 B,$$

$$B \succ_2 D \succ_2 A \succ_2 C.$$

Again, as voter 3, you would like alternative A to appear first in the group ranking determined by the Borda Count. Can you construct an individual ranking for yourself so that this will be the result? If so, explain how you would choose your individual ranking; if not, explain why it is not possible.