Homework 2

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1

The simplex algorithm forms feasible bases and works towards optimality from there. A non linearly independent condition is one which can be expressed as the sum and scale of others. Knowing this it is trivially easy to ignore linearly independent conditions by choosing the harshest one, for example:

c1: $3x+4y+2z+7w = 32$
c2: $2x+4y+ \quad 6w = 12$
c3: $1x+ \quad 2z+1w = 20$

Clearly c3 is c1-c2 and hence is linearly dependant, and can be ignored without affecting the optimal solution. In the slightly more complicated case where only the condition is linearly independent but the target isn't (c3: $x+2z+w = 18$) then we know already this is infeasible since either it is unreachable or c1 or c2 (or both) is.
We must assume linear independence before executing simplex as the algorithm is unable to detect this kind of infeasibility. Simplex works by only jumping from feasible basis to feasible basis. Since the basis is only infeasible when it contains c1, c2 and c3, it would be possible for simplex to 'hop' along bases which are not infeasible, and so miss this infeasibility constraint entirely.
As stated above linearly dependant constraints can be removed or flagged as causing infeasibility before simplex is executed if required.

2

The Klee-Minty cube is an n-dimensional distorted cube used to show the worst case running time for algorithms. Since the simplex algorithm deals with traversing the corners of a convex high-dimensional region the Klee-Minty cube is used to show that simplex makes 2^n 'hops' in the worst case, even though only n hops should be required (for hypercubes cubes). The cube is distorted so that adjacent corners are slightly more optimal than the last, and an optimal sorting of the cube's vertices would order all points from the 'start' node to the 'target' (most optimal) node. Since simplex only moves to another more optimal spot, the algorithm could potentially traverse all points, and there are exponentially many.
The Hirsch conjecture relates the number of facets and dimensionality of high-dimensional polygons (polytope). Specifically it proposes that the maximal number of traversals (diameter) between the two most distant vertices on any d dimensional polytope with n faces is n-d. The intuition being that in order to form a closed hull the polytope has to expend faces in different dimensions, excluding them from this maximal distance. Since simplex operates by 'hopping' from one vertex to the next this provides a lower bound on the minimal number of hops, and hence the best performance of possible algorithms (linear in the number of constraints). However as we know simplex performs significantly worse than linear time.