Real-world algorithms
Semester 1, 2011

Homework 4

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This assignment is **due on Apr 14** in class. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, must be *written individually* without consulting someone else's solutions.

Problem 1: Recall from class the two IP formulations for the facility location problem:

$$\begin{array}{lll} \text{minimize} & \sum_i f_i x_i + \sum_{i,j} c_{ij} y_{ij} & \\ \text{subject to} & \sum_j y_{ij} \leq N x_i & \forall\ i \\ & \sum_i y_{ij} \geq 1 & \forall\ j \\ & x_i, y_{ij} \in \{0,1\} & \forall\ i,j \end{array} \tag{IP1}$$

$$\begin{array}{lll} \text{minimize} & \sum_i f_i x_i + \sum_{i,j} c_{ij} y_{ij} & \\ \text{subject to} & y_{ij} \leq x_i & \forall\ i,j \\ & \sum_i y_{ij} \geq 1 & \forall\ j \\ & x_i, y_{ij} \in \{0,1\} & \forall\ i,j \end{array} \tag{IP2}$$

Where N denotes the number of clients. Let LP1 and LP2 be their corresponding linear relaxations. Prove that LP2 is strictly better than LP1 by showing that every feasible fractional solution to LP2 is feasible for LP1, but that the converse is not true.

Problem 2: Consider the following IP formulation for the maximum matching problem in a general undirected graph $G = (V, E)$:

$$\begin{array}{lll} \text{maximize} & \sum_{(u,v) \in E} x_{uv} & \\ & \sum_{v:(u,v) \in E} x_{uv} \leq 1 & \forall\ u \in V \\ & x_{uv} \in \{0,1\} & \forall\ (u,v) \in E \end{array} \tag{IP3}$$

Show that the following set of inequalities are valid for IP3[1]:

$$\sum_{(u,v) \in E : u,v \in S} x_{uv} \leq \left\lfloor \frac{|S|}{2} \right\rfloor \qquad \forall\ S \subseteq V$$

Show an instance where the linear relaxation of IP3 can be strengthened by adding the extra constraints.

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[1]That is, all feasible vectors of IP3 satisfy it.