Real-world algorithms Semester 1, 2011	Homework 1	J. Mestre

This assignment is **due on Mar 24** in class. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, must be *written individually* without consulting someone else's solutions.

Problem 1: Show how to express the following linear program

$$\begin{array}{ll} \text{minimize} & \mathbf{c} \cdot \mathbf{x} \\ \text{subject to} & \mathbf{Ax} \geq \mathbf{b} \\ & \mathbf{x} \ \textbf{free} \end{array}$$

as another program with the following form:

$$\begin{array}{ll} \text{minimize} & \tilde{\mathbf{c}} \cdot \tilde{\mathbf{x}} \\ \text{subject to} & \tilde{\mathbf{A}}\tilde{\mathbf{x}} = \tilde{\mathbf{b}} \\ & \tilde{\mathbf{x}} \geq \mathbf{0} \end{array}$$

Prove that the two program are equivalent by showing a cost-preserving linear mapping between the two feasible regions.

Problem 2: Derive the dual of the programs from above. Show that their duals are also equivalent by showing a cost-preserving linear mapping between the two dual feasible regions.

Problem 3: A linear program can either be infeasible, feasible but have unbounded objective value, or feasible and have bounded objective value. Each of these options imposes some restrictions on the nature of the dual. Fill out the following table summarizing the possible combinations of primal-dual program pairs.

DUAL $\rightarrow$ PRIMAL $\downarrow$	FEASIBLE BOUNDED	FEASIBLE UNBOUNDED	INFEASIBLE
FEASIBLE BOUNDED			
FEASIBLE UNBOUNDED			
INFEASIBLE			

For each combination state whether the it can occur or not. If your answer is 'yes', provide an example. If your answer is 'no', provide a short justification.