Due: 19th of May 2011 at 1pm.

COMP 5045 – Assignment 3

For full points you need to prove the correctness and complexity of your algorithms.

1. The Voronoi diagram and the trapezoidal map.

   (a) Let S be a set of six points with coordinates $(1,7),(3,1),(3,5),(7,2),(7,6)$ and $(9,3)$, see Fig. 1(a). Draw the Voronoi diagram of S. Explain how you calculate how to draw it by hand. [7 points]

   (b) Let L be a set of four line segments $\{e_1 = ((2,2),(6,3)), e_2 = ((1,5),(4,4)), e_3 = ((5,6),(1,9)), e_4 = ((3,8),(8,7))\}$ contained in a bounding square T with lower left corner at $(0,0)$ and upper right corner at $(10,10)$, see Fig. 1(b). Draw the trapezoidal map (decomposition) of L and T. [3 points]

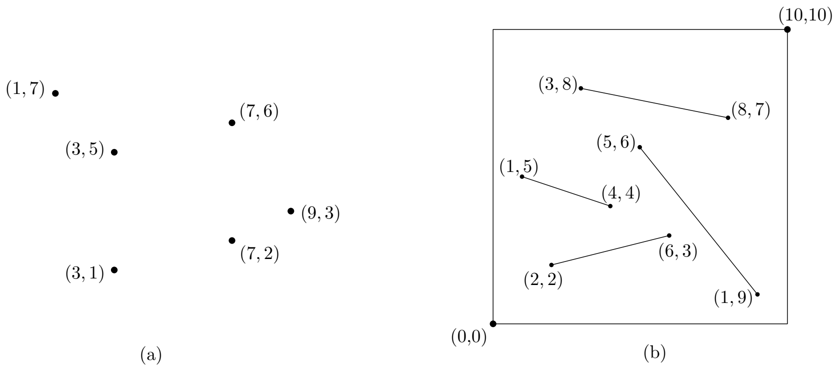

Figure 1: (a) The point set S. (b) The set L of lines contained in the bounding square T.

2. Let S be a set of n points in the plane. Give an $O(n^2)$ time algorithm to find the straight line containing the maximum number of points in S. [10 points]

3. Consider a set S of n points in the plane. Each point represents a city and contains its name, its two coordinates and its population. The aim is to preprocess S such that the following type of queries can be answered effciently: given a axis-parallel rectangle R, return the city with the largest population within R. For full points give a data structure using $O(n \log n)$ space and preprocessing time and $O(\log^2 n)$ query time. [10 points]

4. Use a plane sweep argument to prove that the trapezoidal map of n line segments in the plane in general position has at most $3n+1$ trapezoids. (That is, imagine a vertical line sweeping over the plane from left to right. Count the number of trapezoids that are encountered by the sweep line.) [10 points]

5. Let L be a set of n lines in the plane. Give an $O(n \log n)$ time algorithm to compute a minimal axis-parallel rectangle that contains all the vertices of the arrangement of L. [10 points]