Stabilization Control of a Cart-Inverted Pendulum System by Pole Placement Controller (PPC)

Raditya Bagus Wardhana
School of Electrical Engineering and Informatics
Bandung Institute of Technology
Bandung, Indonesia
raditya.baguswardhana@gmail.com

Noor Shafira
School of Electrical Engineering and Informatics
Bandung Institute of Technology
Bandung, Indonesia
noorshafiraa@gmail.com

***Abstract*— The design of control systems for the inverted pendulum system often involves two types of control problems: swing-up control problem and stabilization control problem. Objective of the control is to stabilize the plant at the unstable equilibrium point. To accomplish this, the method of Pole Placement Control (PPC) is employed. Pole Placement method places the eigenvalues of the close-loop system at the desired place by designing a full-state feedback controller in order to satisfy the transient and steady-state performance requirements of the system. Closed-loop control designed to achieve settling less than 2 s dan overshoot less than 20%. The system achieved settling time 1.96 s and Overshoot 28.6% at 0.78 s. When impulse disturbances are given to the pendulum, it is able to overcome the disturbances given on both positive and negative directions. The system responses to the disturbances are kept below ±10 cm and ±0.2 rad by the controller for the cart and pendulum respectively and is good enough considering that the cart limit is ±15 cm.**

Keywords—inverted pendulum, pole placement, control

I. Introduction

Inverted pendulum is a fascinating system that has been the subject of extensive research and development in the field of control. This system consists of a pendulum with initial position upside down on a moving cart. The goal of the control system for the inverted pendulum is to balance it by applying forces to the cart where the pendulum is at unstable equilibrium point. This balance requires continuous adjustment to maintain making the control system a challenging and rewarding task. Examples of current applications or implementations of the inverted pendulum concept include rocket launches, bipedal robots, Segways, and cranes.

The design of control systems for the inverted pendulum often involves two types of control problems: swing-up control problem and stabilization control problem. Swing-up control is used to bring the pendulum to the unstable equilibrium position from its stable equilibrium position. The swing-up problem is highly non-linear in nature and involves swinging up a pendulum from its stable equilibrium point to an unstable equilibrium point by the application of control input to the cart. When the pendulum approaches the unstable equilibrium configuration, the nonlinear swing-up controller is switched to a stabilizing controller, which is linear in nature. The stabilizing controller helps the pendulum to retain its position at the unstable equilibrium point.

The focus of this experiment is on the stabilization control problem. The aim is to design a control system that can maintain the inverted pendulum in its upright position. To accomplish this, the method of Pole Placement Control (PPC) is employed. PPC is a technique predominantly used in the design of state-feedback controllers for linear systems. This methodology enables the specification of the placement of the poles of the system, which subsequently influences the system stability and transient response. By strategically selecting the pole locations, a controller can be constructed that effectively stabilizes the inverted pendulum, ensuring its upright position even in the face of disturbances.

II. Problem Statement

A. Cart and Pendulum Modelling

The system consisting of a pole and cart that have two degrees of freedom (DOF) total. The mathematical model of the system is determined based on the free body diagram of the cart and pendulum.

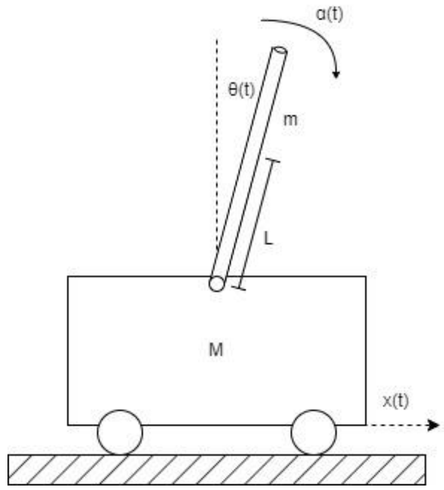

Fig. 1. Cart and pendulum physical model

Fig. 1 shows a visual representation of the forces acting on the system which is crucial to understand the dynamics and derivied the mathematical model of the system. The movement of the cart is constrained in horizontal direction only. $x(t)$ represent cart horizontal position from initial position where right side of vertical axis considered as positive position. The pole can move in rotational motion where $\theta(t)$ represent pendulum angular position from the vertical axis. Movement in clockwise direction is considered as positive direction. In this case, air fraction are negligible for simplicity. Therefore, there are only coulomb frictions such as cart friction and pendulum friction.

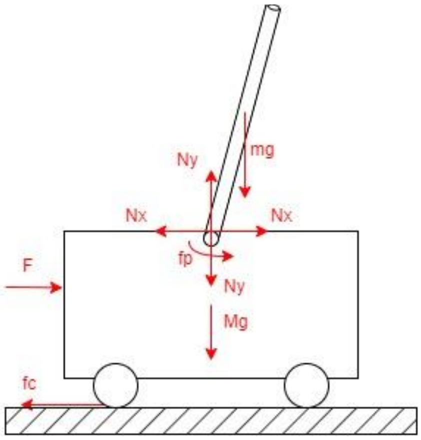

Fig. 2. Cart and pendulum free body diagram

The non-linear modelling of cart and pendulum system determined by applying Newton's law that shows in Fig. 2 free body diagram. The cart forces along the x-axis contained the equation as follows.[1]

$$F - f_c - N_x = M\frac{d^2x(t)}{dt^2} \tag{1}$$

The pole of the inverted pendulum system have both translational and rotational movements. The equations of motion for the pole's translational and rotational movements are determined as follows.

Pole translation:

$$N_x = m\frac{d^2(L\sin q\,(t) + x(t))}{dt^2} \tag{2}$$

$$mg - N_y = m\frac{d^2(L - L\cos q\,(t))}{dt^2} \tag{3}$$

Pole rotation :

$$N_y L \sin \theta(t) - f_p = I\frac{d^2q\,(t)}{dt^2} \tag{4}$$

Linear approximation performed on equations (2), (3), and (4) by assuming the angular position (θ) is very small ($\sin\theta \approx \theta$ and $\cos\theta \approx 1$).

$$N_x = m\frac{d^2(Lq\,(t) + x(t))}{dt^2} \tag{5}$$

$$mg - N_y = 0 \tag{6}$$

$$N_y L\theta(t) - f_p = I\frac{d^2q\,(t)}{dt^2} \tag{7}$$

Pole modelling generated by substituting equation (5) and (6) to (7) as follows.

$$(I+mL^2)\frac{d^2q\,(t)}{dt^2} + mL\frac{d^2x(t)}{dt^2} - mg\theta(t)L + f_p = 0 \tag{8}$$

Cart linear modelling generated by substituting equation equation (5) to (1) as follows.

$$(M+m)\frac{d^2x(t)}{dt^2} + mL\,\frac{d^2q\,(t)}{dt^2} + f_c - F = 0 \tag{9}$$

Laplace transform is applied to both linear modelling (8) and (9) where cart fraction $f_c = b_1\frac{dx}{dt}$ and pendulum fraction $f_p = b_2\frac{dq}{dt}$.

$$(M+m)s^2X(s) + mLs^2\theta(s) + b_1sX(s) = F(s) \tag{10}$$

$$\{(I+mL^2)s^2 - mgL + b_2s\}\theta(s) = -mLs^2X(s) \tag{11}$$

The linear inverted pendulum system transfer function generated a SIMO (Single Input Multiple Output) system. Giving force will produce change in angular position and horizontal position of the system.

$$\frac{q(s)}{F(s)} = \frac{-mLs}{\{(M+m)(I+mL^2)-m^2L^2\}s^3+\{b_1(I+mL^2)+b_2(M+m)\}s^2-\{(M+m)mgL-b_1b_2\}s-b_1mgL} \tag{12}$$

$$\frac{X(s)}{F(s)} = \frac{(I+mL^2)s^2+b_2s-mgL}{\{(M+m)(I+mL^2)-m^2L^2\}s^4+\{b_1(I+mL^2)+b_2(M+m)\}s^3-\{(M+m)mgL-b_1b_2\}s^2-b_1mgLs} \tag{13}$$

B. DC Motor Modelling

Assumed the input of the system is voltage (V_m) that applied to the armature while the output is shaft position (θ_m). The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in the following figure.[2]

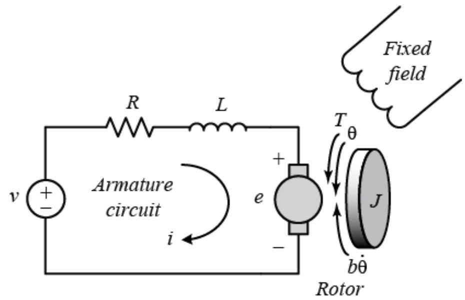

Fig. 3. Motor DC physical model [2]

As shown in Fig 3. by applying Kirchhoff's voltage law dervied equation as follows.

$$V_m = (R_m + L_ms)I_m(s) + V_b(s) \tag{14}$$

The motor inductance (L_m) is negligible. V_b represent back electromotive force voltage that is proportional to the motor angular speed (ω_m).

$$V_b = K_e\omega_m \tag{15}$$

τ_m represent motor torque that is proportional to motor current (I_m).

$$\tau_m = K_tI_m \tag{16}$$

By neglecting disturbance, motor angular position is proportional to position. Motor voltage is proportional to the control voltage.

$$\theta_m = \frac{x}{Gr_w} \tag{17}$$

$$V_m = K_cV_c \tag{18}$$

Traction force determined by combining all of the motor equation which result as follows.

$$F(t) = \frac{K_t K_c}{R_m G r_w} V_c - \frac{K_t K_e}{R_m (G r_w)^2} \frac{dx(t)}{dt} \tag{19}$$

Final system modelling generated by substitute equation (19) to cart-pendulum transfer functions (12) and (13).

C. *Inverted-pendulum system modelling*

Due to information limitation, the physical parameters of DC motor were defined by using MATLAB Toolbox Parameter Estimation.

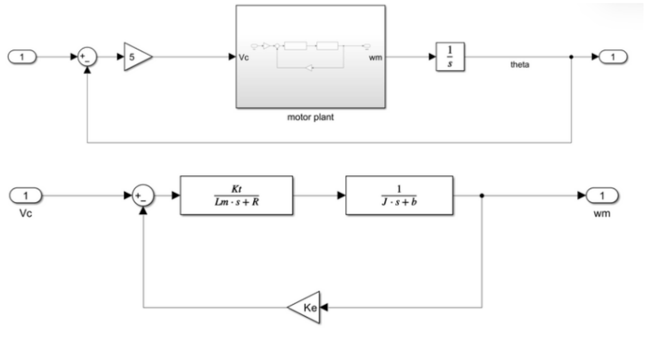

Fig. 4. Simulink model for parameter estimation

Fig 4 shows DC motor position model blocks in Simulink. The DC Motor block models both the electrical and mechanical characteristics of the motor. Closed loop position control with proportional gain, $K_p = 5$, was used to obtain the data for parameter estimation with varied input references. Parameter Estimation Toolbox was used to do the estimation process. The result of the simulation using parameter estimation compared to the collected data are as follows.

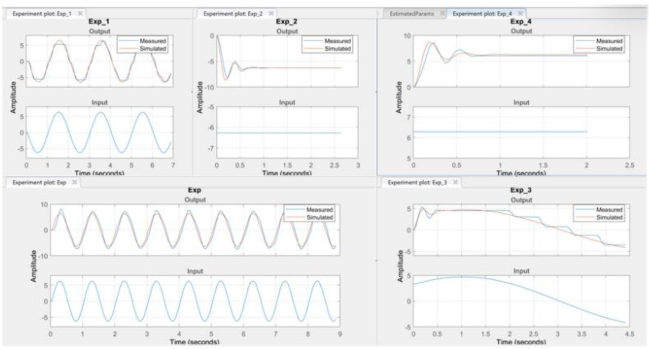

Fig. 5. Motor DC parameter estimation

TABLE I. PARAMETERS OF MOTOR DC BY PARAMETER ESTIMATION

Parameter	Symbol	Unit	Value
Motor torque-current constant	Kt	Nm/A	0.076
Motor-control voltage constant	Kc	-	1
Motor voltage speed constant	Ke	Vs/rad	0.1726
Electric resistance	R	Ω	1.3852
Gear ratio	G	-	1
Radius of cart wheel	r_w	m	0.015
Electric inductance	L_m	H	$2\ 10^{-6}$
Moment of inertia of the rotor	J	Kg m^2	$9.5132\ 10^{-4}$
Motor viscous friction constant	b	Nms	$2\ 10^{-6}$

TABLE II. PARAMETERS OF CART-PENDULUM SYSTEM

Parameter	Symbol	Unit	Value
Cart			
Cart mass	M	Kg	1.96
Cart coefficient of friction	b1	Kg/s	16.3
Pendulum			
Pendulum mass	m	Kg	0.045
Pendulum coefficient of friction	b2	Kg m^2/s	0.000402
Pendulum length from center of mass	L	m	0.125
Pendulum inertia	I	Kg m^2	0.000447

From experiment, it is also known that the DC motor has saturation voltage of 12V and dead zone voltage of 4V.

With the parameters provided in Table I and II, continuous state-space and transfer function representation of the system are obtained as follows. ($g = 9.8\ m/s^2$)

State space :

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -29.53 & -0.1363 & 0.0009942 \\ 0 & 0 & 0 & 1 \\ 0 & 144.4 & 48.6 & -0.3544 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ q \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1.85 \\ 0 \\ -9.046 \end{bmatrix} V \tag{20}$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ q \\ \dot{\theta} \end{bmatrix} \tag{21}$$

Transfer function :

$$\frac{X(s)}{V(s)} = \frac{1.85s^2 + 0.6465s - 88.65}{s^4 + 29.88s^3 - 38.28s^2 - 1415s} \tag{22}$$

$$\frac{q(s)}{V(s)} = \frac{-9.046s + 3.906\ 10^{-15}}{s^3 + 29.88s^2 - 38.28s - 1415} \tag{23}$$

Sampling time considered by referring to rule of thumb by Ogata which is 8 to 10 sample each cycle for underdamped case.[3] Minimum sampling time 10 sample each cycle determined with settling time (Ts) less than 2 seconds and overshoot (%OS) less than 20% as follows.

$$\%OS = 100e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \tag{24}$$

$$T_s = \frac{4}{\zeta\omega_n} \tag{25}$$

Damping ratio (ζ) to achieve overshoot less than 20% is 0,45594 and natural frequency to achieve settling time less than 2 seconds is 4.38654 rad/s.

$$Sample\ each\ cylce = \frac{\omega_r}{\omega_d} \tag{26}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 3.904 \tag{27}$$

$$\omega_r = \frac{2\pi}{T} \tag{28}$$

Sampling time of the system must be chosen as follows, $T_s \leq 0.16094$ s. Sampling time 0.06s is selected considering the limitation from the microcontroller used. By selecting a sampling time of 0.06 seconds, the transfer function and the discrete state space of the system are obtained as follows.

State space :

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0.02811 & -0.0001487 & -2.254\ 10^{-6} \\ 0 & 0.1698 & -0.003941 & -0.0001199 \\ 0 & 0.1575 & 1.088 & 0.06111 \\ 0 & 4.175 & 2.948 & 1.066 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ q \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0.001998 \\ 0.05201 \\ -0.009866 \\ -0.2615 \end{bmatrix} V \quad (29)$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ q \\ \dot{\theta} \end{bmatrix} \quad (30)$$

Transfer function :

$$\frac{X(s)}{V(s)} = \frac{0.001998z^3 - 0.003179z^2 - 0.0004578z + 0.001098}{z^4 - 3.324z^3 + 3.67z^2 - 1.513z + 0.1665} \quad (31)$$

$$\frac{q(s)}{V(s)} = \frac{-0.009866z^2 + 0.004403z + 0.005463}{z^3 - 2.324z^2 + 1.346z - 0.1665} \quad (32)$$

D. *Control Design Specifications*

Objective of the control is to stabilize the plant at the unstable equilibrium point, the up position. Pole Placement method places the eigenvalues of the close-loop system at the desired position by designing a full-state feedback controller to satisfy the performance specifications of the system. Design specifications of settling time less than 2 s dan overshoot less than 20% is chosen. Selected settling time allows for quick recovery to prevent the pole from falling. It also facilitates a more responsive control system, enabling the system to adapt swiftly to changes in the operating environment. Overshoot less than 20% selected by ensuring the system's response remains within acceptable bounds.

III. RESEARCH METHOD

A. *Controller Design*

The eigenvalues matrix A of the open-loop system are [0, -29.5558, -7.0841, 6.7591]. One of the eigenvalues lies in the right half of the plane. Therefore, the open-loop system is unstable. In order to stabilize the system, controller needs to be design so the eigenvalues all in the left half plane. Controllability test of the system to ensure system is controllable given as

$$C = \begin{bmatrix} 0 & 0.0000 & -0.0005 & 0.0161 \\ 0.0000 & -0.0005 & 0.0161 & -0.4771 \\ 0 & -0.0001 & 0.0027 & -0.0842 \\ -0.0001 & 0.0027 & -0.0842 & 2.4924 \end{bmatrix} 10^5 \quad (33)$$

The rank of the matrix result is 4. Hence, cart-pendulum is controllable.

Dominant poles of -2±3.9040j are obtained from the performance specifications. Two more poles are required for the system. These poles are to be placed far from the dominant poles as to not changes the system performances. Non-dominant poles of –100±j are chosen. The desired closed-loop for Pole in s-domain Placement Controller are obtained as [-100+j, -100–j, -2+3.9040j, -2-3.9040j]. Controller poles converted in z-domain are [0.8627+j0.2057, 0.8627-j0.2057, 0.0025-j0.0001, 0.0025+j0.0001].

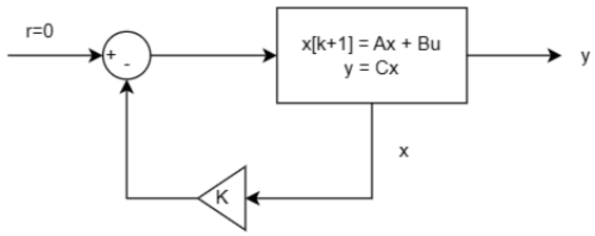

Fig. 6. Full-state feedback

Using pole placement, gain K for each states is obtained K = [-112.5611 -49.5610 -114.2773 -12.4969].

Compensated system eigenvalues are obtained as [0.8627+j0.2057, 0.8627-j0.2057, 0.0025-j0.0001, 0.0025+j0.0001] which are equal to the desired poles. Also notice that all the eigenvalues lie inside the unit circle. Therefore, the system has been stabilized.

B. *Simulation*

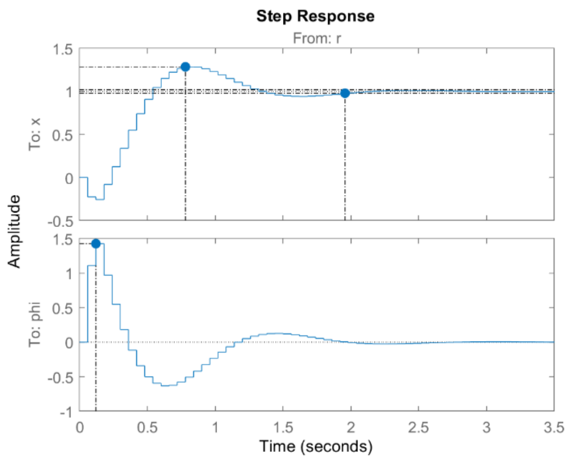

Fig. 7. Step Response of the System

Fig. 7 shows the desired poles is justified. Observe that the system achieved settling time 1.96 s and Overshoot (%) 28.6% at time 0.78 s.

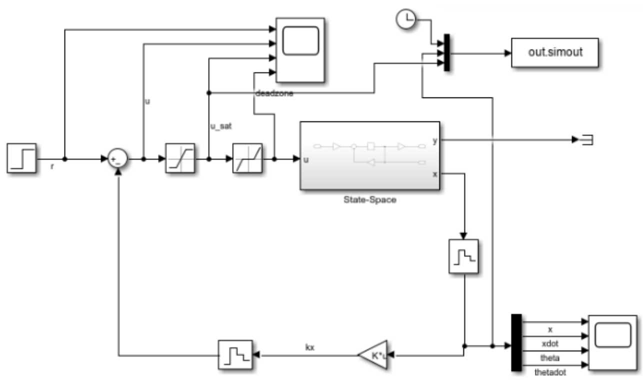

Fig. 8. Simulink model

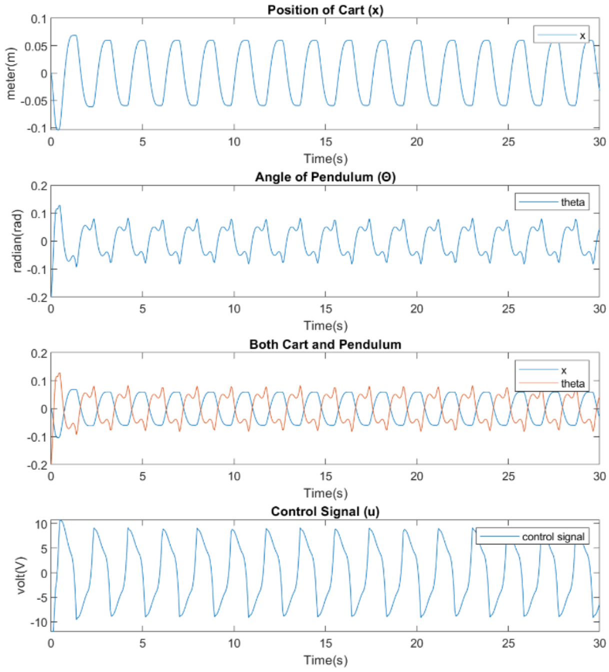

Fig. 9. Inverted pendulum simulation Simulink

Fig. 8 and Fig. 9 show the simulation model and simulation results of the inverted pendulum with motor output voltage saturation and motor dead zone into consideration. The result of compensated inverted pendulum on Simulink shows the pole angle and cart position is able to stabilize itself near its up position with oscillations occurring on both the cart and the pole. The oscillations on both the cart and the pole are acceptable because the values are still within the accepted region.

IV. Implementation

A. Circuit Design

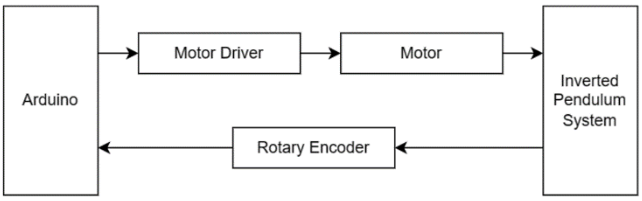

Fig. 10. System interface

Arduino MEGA provides input in the form of a Pulse Width Modulation (PWM) signal to the motor driver. The motor driver controls the speed and direction of the motor connected to the inverted-pendulum system. The movement of the motor translates into a shift in the cart's position and a change in the pole's angular position. The rotary encoder of the pole and cart provides feedback to the Arduino. These signals are then fed back to the Arduino, which uses them to adjust the PWM signal it sends to the motor driver, thereby controlling the motor's speed and direction more accurately. The pin connections among all these components are shown in Table III.

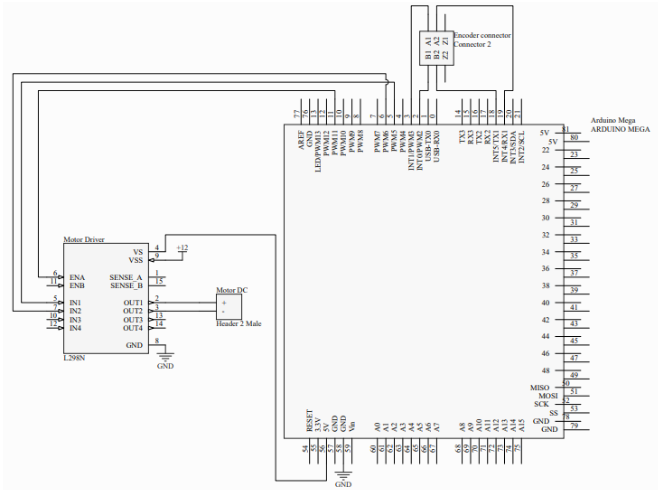

Fig. 11. Schematic

TABLE III. Pin Connection

Arduino MEGA	Component	Component Pin
11	Motor driver	ENA
5		IN1
6		IN2
5 V		VS
GND		GND
3	Encoder connector	A1M
2		B1M
19		A2M
18		B2M
Motor Driver	**Component**	**Component Pin**
VSS	Power supply	12 V
GND		GND
OUT1	Motor DC	+
OUT2		-

B. Flowchart Program

In this subsection, the implementation of the control scheme on the embedded system is discussed. During the start of the program, the necessary variables and timer are initialized. Variables include the system's states, I/O pins, etc. Timer is used to get the time difference to calculate the velocity of the cart and the angular velocity of pendulum. The program is coded to loop indefinitely even when the pendulum and cart have reached the wanted condition, 0 m for the cart and 0 rad for the pendulum. The program will output control signal only when the angle of the pendulum is below ±0.2 rad, this is to keep the linearization of the system valid. Once the angle of the pendulum is on the allowable range, the program will start calculating the control signal. First, the position of the cart and the angle of the pendulum are acquired from the rotary encoder. The values of velocity of cart and angular velocity of the pendulum are then calculated by utilizing the difference of position of cart over time and the difference of angle of pendulum over time respectively. Control signal is calculated by multiplying the negative values of K with the current corresponding states value. Saturation of the motor is taken into consideration by limiting the control signal to ±12 V. Finally, the control signal is converted to pwm value in the range of 0 to 254 and the motor is driven. The program loops back for the next iteration. Figure 12 shows the flowchart program of the implementation of the control scheme.

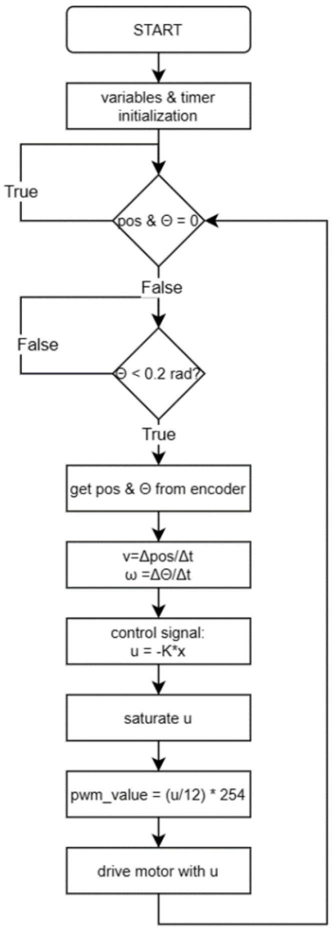

Fig. 12. Implementation of Control Scheme

V. Result and Analysis

In this section, the experimental results of the system are discussed. Feedback controller using pole placement method as mentioned in section III with gain, K = [-112.56,-49.56,-114.27,-12.49], is used and implemented on Arduino Mega board. The pendulum was placed initially at 0 rad, the up position. During the experiment, impulse disturbances were given to the pendulum to check whether the controller designed is able to overcome disturbances.

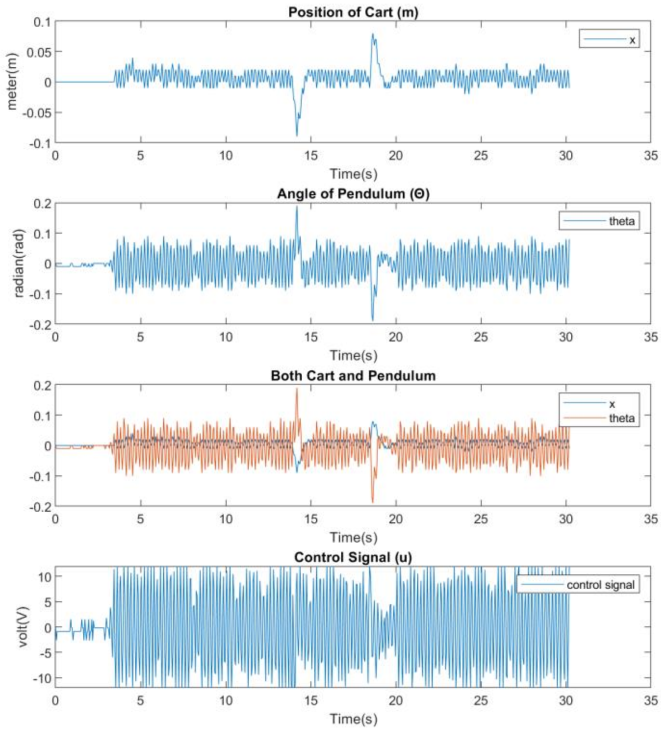

Fig. 13. Experimental Results

Figure 13 shows the experimental results for the position of cart, the angle of pendulum, and the control signal. From the experimental results, the pendulum is able to stabilize itself near the up position, 0 rad, using the designed controller. Oscillations occurred on both the cart and pendulum during the stabilization period. For the cart and the pendulum, oscillations are kept below ±5 cm and ±0.1 rad respectively. These oscillations are expected and are caused by the motor dead zone, which is around ±4V. When impulse disturbances are given to the pendulum, it is able to overcome the disturbances given on both positive and negative directions. The system responses to the disturbances are kept below ±10 cm and ±0.2 rad by the controller for the cart and pendulum and is good enough considering that the cart limit is ±15 cm.

VI. Conclusion

An approach to stabilize inverted-pendulum in unstable equilibrium position has been achieved by using pole placement controller. The compensated system approach desired settling time and overshoot that is settling time 1.96 s and Overshoot 28.6%.

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