Homework 1

1) Write out a truth table for the following logical expressions:

a) [p∧(p →q)]→q

[p∧(¬p∨q)] →q Definition of implication

¬ [p∧(¬p∨q)] ∨ q Definition of implication

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| P | q | ¬p∨q | p∧(¬p∨q) | ¬ [p∧(¬p∨q)] ∨ q |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

b) [(p →q)∧(q →r)]→(p →r)

[(¬p∨q)∧(q →r)]→(p →r) Definition of implication

[(¬p∨q)∧( ¬q∨r)]→(p →r) Definition of implication

[(¬p∨q)∧( ¬q∨r)]→( ¬p∨r) Definition of implication

¬ [(¬p∨q)∧( ¬q∨r)] ∨ ( ¬p∨r) Definition of implication

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | q | r | ¬p∨q | ¬q∨r | ¬p∨r | ¬ [(¬p∨q)∧( ¬q∨r)] | ¬ [(¬p∨q)∧( ¬q∨r)] ∨ ( ¬p∨r) |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |

c) p∧¬(q∧(¬p∨r))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | r | (¬p∨r) | ¬(q∧(¬p∨r)) | p∧¬(q∧(¬p∨r)) |
| 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |

2) Determine all truth value assignments for the primitive statements p, q, r, s, t that make the following expression false: [(p∧q)∧r]→(s∨t).

[(p∧q)∧r]→(s∨t) premise

¬ [(p∧q)∧r] ∨ (s∨t) Definition of implication

[¬ (p∧q) ∨ ¬ r] ∨ (s∨t) de Morgan’s law

[(¬ p∨¬q) ∨ ¬ r] ∨ (s∨t) de Morgan’s law

¬ p ∨ ¬q ∨ ¬ r ∨ s ∨ t associativity

The statement is only false when:

p = T, q = T, r = T, s = F, t = F

3) Negate the following boolean expression and simplify the result as much as possible. Please show each step and name the rule you are using at each step: p∨q∨(¬p∧¬q∧r)

p∨q∨(¬p∧¬q∧r) premise

¬ (p∨q∨(¬p∧¬q∧r)) negation

(¬ p∧¬q∧ ¬(¬p∧¬q∧r)) de Morgan’s law

(¬ p∧¬q∧ (p∨q∨¬r)) de Morgan’s law

(¬ p∧¬q∧ p)∨( p∧¬q∧ q)∨( p∧¬q∧ ¬r) Distributive law

(F∧¬q)∨( p∧F)∨( p∧¬q∧ ¬r) inverse law

F∨F∨( p∧¬q∧ ¬r) Domination law

(p∧¬q∧ ¬r) Identity law

4) Show that the following two logical expressions are equivalent using the laws of logic:

(p →q)∧[¬q∧(r ∨¬q)] and ¬(q∨ p)

(p →q)∧[¬q∧(r ∨¬q)] premise

(¬p∨q)∧[¬q∧(r ∨¬q)] Definition of implication

(¬p∨q)∧[(¬q∧r) ∨(¬q∧¬q)] Distributive law

(¬p∨q)∧[(¬q∧r) ∨¬q] Idempotency

(¬p∨q)∧ ¬q Absorption law

(¬p∧ ¬q)∨(q∧ ¬q) Distributive law

(¬p∧ ¬q)∨F Inverse law

(¬p∧ ¬q) Identity law

¬(p∨q) de Morgan’s law

¬(q∨p) Commutative law

5) Prove the following logical argument using the rules of implication. Please show each step and state which rule you use.

(¬p∨q) →r

r →(s∨t)

¬s ∧¬u

¬u → ¬t

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∴ p

1. ¬s ∧¬u premise
2. ¬u Conjunctive Simplification(1)
3. ¬s Conjunctive Simplification(1)
4. (¬p∨q) →r premise
5. r →(s∨t) premise
6. (¬p∨q) →(s∨t) Law of Syllogism(4,5)
7. ¬u → ¬t premise
8. ¬t Modus Ponens(2,7)
9. ¬ (¬p∨q) Modus Tollens(3,6,8)
10. p∧¬q de Morgan’s law
11. p Rule of Conjunctive Simplification(10)

6) Create simple statements for p, q, r, s, t and u for problem number 5 above that make reasonable sense in real life.

p = it is daytime

q = it is cloudy

r = it is dark outside

s = it is shady

t = it is cool outside

u= it is winter

7) Let ? be an unknown boolean logical operator. The logical statement [(¬p ∧ q) ∨ r] ⇒ (q ? r) is equivalent to (p ∨ ¬q ∨ r). Given this information, there are 2 possible truth tables for the boolean logical operator ?. List, with proof, both of these truth tables.

[(¬p ∧ q) ∨ r] ⇒ (q ? r) premise

¬ [(¬p ∧ q) ∨ r] ∨ (q ? r) Definition of implication

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | (¬p ∧ q) | (¬p ∧ q) ∨ r | ¬ [(¬p ∧ q) ∨ r] | (q ∧ r) | (q ∨ r) |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | r | ¬ [(¬p ∧ q) ∨ r] ∨ (q ∧ r) | ¬ [(¬p ∧ q) ∨ r] ∨ (q ∨ r) | (p ∨ ¬q ∨ r) |
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

I don’t know