COT 3100 Fall 2017 Homework #2

1. Let n be an odd integer. Prove that n2 - 1 is divisible by 8. After you prove this formally, attempt to give an intuitive reason why this turns out to be true similar to the intuitive reason that integers of the form k(k+1), when k is an integer, are even.

Let n be an arbitrary odd integer

n = 2r + 1 where r is an arbitrary integer

n2 – 1 = (2r+1)2 -1

4r2 + 4r + 1 – 1

4r2 + 4r

4(r2 + r)

r2 + r is always even

Proof by contradiction

Assume r2 + r is odd

r2 + r = r(r + 1)

For a product of two integers to be odd both must be odd

No two consecutive integers are both odd

Therefore r(r + 1) is even

r(r + 1) = 2u where u is an arbitrary integer

4(r2 + r) = 4\*2u

8u

(8u)/8 = u

Therefore (n2 - 1)/8 will always be an integer

Intuitive reason

n2 - 1 is always either 0 or a factor of 8.

1. Prove the following statement using (a) direct proof and (b) proof by contradiction.If n is an even integer, then 3n+2 is even.

Direct proof:

Assume n is an arbitrary even integer

n = 2\*r where r is an arbitrary integer

3n+2 = 3(2r) + 2

3\*2\*r will always be a multiple of two and is therefore even for every integer r

Any even integer plus 2 is always even therefore 3\*(2\*r) + 2 is always even

3) Use direct proof to show that every odd integer can be expressed as the difference of two perfect squares. (Note: A perfect square is simply any integer of the form n2, where n is an integer.)

Consider the difference of two consecutive squares

(k+1)^2 – k^2 where k is an arbitrary integer

K^2 + 2k + 1 – k^2

2k+1

Therefore any odd integer can be expressed as the difference of two squares

4) Given a set of real numbers a1, a2, …, an, let their average be b. Prove that there exists

at least one number in the set that is greater than or equal to b. (Hint: use proof by contradiction!)

Proof by contradiction

Assume that there is no number greater than or equal to the b in the set and that the average of the set is equal to b.

(a1 + a2 +… an)/n = b

Sum b n times and divide the sum by n. This should be equal to b

(b + b +…)/n = b

If a1 through an are all less than be then

(b + b +…)/n > (a1 + a2 +… an)/n

However this yields b > b

A contradiction was reached therefore our initial assumption is false

5) Let S = {2, 3, 5} and T = {1, 2, 4, 6}. Explicitly list the members of the following sets:

𝑆 ∪ 𝑇, 𝑆 ∩ 𝑇. 𝑆 − 𝑇, 𝑆 × 𝑇, 𝑇 × 𝑆, ℘(𝑆) and ℘(𝑇).

𝑆 ∪ 𝑇 = {1, 2, 3, 4, 5, 6}

𝑆 ∩ 𝑇 = {2}

S-T = {3, 5}

𝑆 × 𝑇 = {(2, 1), (2, 2), (2, 4), (2, 6), (3, 1), (3, 2), (3, 4), (3, 6), (5, 1), (5, 2), (5, 4), (5, 6)},

𝑇 × 𝑆 = {(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (4, 2), (4, 3), (4, 5), (6, 2), (6, 3), (6, 5)}

℘(𝑆) = {{},{2},{3},{2,3},{5},{2,5},{3,5},{2,3,5}}

℘(𝑇)= {{},{6},{4},{4,6},{2},{2,6},{2,4},{2,4,6},{1},{1,6},{1,4},{1,4,6},{1,2},{1,2,6},{1,2,4},{1,2,4,6}}

6) Let *A*, *B*, and *C* be sets. Show that

(𝐴 − 𝐵) − 𝐶 = (𝐴 − 𝐶) − (𝐵 − 𝐶).

(A-B) = {x | x ϵ A ∧  ~~ϵ~~  B}

(𝐴 − 𝐵) – 𝐶 = {x | x ϵ (𝐴 − 𝐵) ∧  ~~ϵ~~  C}

= {x | x ϵ A ∧  ~~ϵ~~  B ∧  ~~ϵ~~  C}

(𝐴 − 𝐶) = {x | x ϵ A ∧  ~~ϵ~~  C}

(𝐵 − 𝐶) = {x | x ϵ B ∧  ~~ϵ~~  C}

(𝐴 − 𝐶) − (𝐵 − 𝐶) = {x | x ϵ (𝐴 − 𝐶) ∧  ~~ϵ~~  (𝐵 − 𝐶)}

= {x | x ϵ A ∧  ~~ϵ~~  B ∧  ~~ϵ~~  C}

7) Give a summary of the life and mathematical contributions of Srinivasa Ramanujan. Please aim for a length of roughly 200 - 400 words. ***Your summary must be typed.*** Please state the sources you used in writing your summary and please don't just copy the blurb from the textbook… (If you loved writing this summary, you should watch the movie, The Man Who Knew Infinity.)

Srinivasa Ramanujan, born in India on December 22, 1887, was an extremely gifted mathematician. Despite having a relatively short life of only 32 years he solved many problems in mathematics that were thought to be unsolvable at the time. He made contributions in mathematical analysis, number theory, infinite series, and continued fractions despite having almost no formal training in pure mathematics.

Even at an early age Ramanujan showed talent in academics. Just before his 10th birthday he passed his primary exams, which included English, Tamil, geography and arithmetic, with the best scores in the district (Kanigel). As he grew older his abilities became more apparent. At the age of 11 he exhausted the mathematical knowledge of two college students who lived with his family. He was lent a book on advanced trigonometry which he mastered by the age of 13 while also discovering sophisticated theorems on his own. Though he was clearly intelligent he have very little interest in fields outside of mathematics often failing classes simply because he was uninterested (Krishnamachari). Without a degree Ramanujan left college to research mathematics independently, living in poverty and often on the brink of starvation.

Looking for a job at the revenue department Ramanujan showed his mathematical notebooks to deputy collector V. Ramaswamy Aiyer (Kanigel). Not wanting to smother Ramanujan’s genius Aiyer sent Ramanujan, with letters of introduction, to his mathematician friends in Madras. Ramanujan's research was often doubted. Only after confirmation from other mathematicians was Ramanujan's academic integrity trusted. With Aiyer's help, Ramanujan had his work published in the Journal of the Indian Mathematical Society. He sent much of his work to British mathematicians who were often amazed by his work. Ramanujan wrote to G. H. Hardy who initially suspected Ramanujan to be a fraud. Hardy and many other Cambridge mathematicians expressed a desire to work with Ramanujan in Cambridge. At Cambridge Ramanujan’s gaps in education were filled. Ramanujan spent nearly five years at Cambridge collaborating and publishing with British mathematicians.

Kanigel, Robert (1991). The Man Who Knew Infinity: a Life of the Genius Ramanujan. New York: Charles Scribner's Sons. ISBN 0-684-19259-4.

Krishnamachari, Suganthi (27 June 2013). "Travails of a Genius". The Hindu. Retrieved 7 September 2016.