Homework 3

1. In general, you were told in class that for all integers a and b and positive integers n, if 𝑎 ≡ 𝑏 (𝑚𝑜𝑑 𝑛), then (𝑎) ≡ (𝑏) (𝑚𝑜𝑑 𝑛), where f is any function that operates on integers only. Using the definition of mod only, prove this specifically for the function (𝑎) = 𝑎2.

𝑎 ≡ 𝑏 (mod n) implies n|(a-b) meaning z\*n = (a-b) where z is an arbitrary integer

a2 – b2 = (a+b)\*(a-b)

(a+b)\*(a-b) = z\*n\*(a+b)

a, b, z are all integers therefore n|(a2 – b2), meaning that if 𝑎 ≡ 𝑏 (𝑚𝑜𝑑 𝑛), then a2 ≡ 𝑏2 (mod n)

2) Convert the following values from the bases indicated to base 10:

i) 36457

3\*7^3 + 6\*7^2 + 4\*7^1 +5\*7^0

1029 + 294 + 28 + 5

= 135610

ii) AAF716

10\*16^3 + 10\*16^2 + 15\*16^1 + 7\*16^0

40960 + 2560 + 240 + 7

= 4376710

iii) 123459

1\*9^4 + 2\*9^3 + 3\*9^2 + 4\*9^1 +5\*9^0

6561 + 1458 + 243 +36 +5

= 8303

iv) 300204

3\*4^4 + 0\*4^3 + 0\*4^2 + 2\*4^1 + 0\*4^0

768 + 0 + 0 + 8 + 0

= 77610

v) 1011010110012

1\*2^11 + 0\*2^10 +1\*2^9 + 1\*2^8 + 0\*2^7 + 1\*2^6 + 0\*2^5 + 1\*2^4 + 1\*2^3 +0\*2^2 + 0\*2^1 + 1\*2^0

2048 + 512 + 256 + 64 + 16 + 8 + 1

=2905

3) Convert the following base 10 values to the bases indicated:

i) 12435 to base 12

12435/12 = 1036 r 3

1036/12 = 86 r 4

86/12 = 7 r 2

7/12 = 0 r 7

1243510 = 724312

ii) 79770 to base 16

79770/16 = 4985 r 10

4985/16= 311 r 9

311/16 = 19 r 7

19/16 = 1 r3

1/16 = 0 r1

7977010 = 1379A16

iii) 691 to base 2

691/2 = 345r1

345/2=172r1

172/2=86r0

86/2=43r0

43/2=21r1

21/2=10r1

10/2=5r0

5/2=2r1

2/2=1r0

1 /2=0r1

69110 = 10101100112

iv) 4921 to base 7

4921/7= 703 r 0

703/7=100 r 3

100/7 = 14 r 2

14/7 = 2 r 0

2/7 = 0 r2

492110 = 202307

v) 88264 to base 8

88264/8=11033 r0

11033/8=1379r1

1379/8=172r3

172/8=21r4

21/8=2r5

2/8=0r2

8826410=2543108

4) Prove that if n is an integer, then n(3n+1) is an even integer.

Let us examine 3n+1

Any odd integer multiplied by 3 will be odd and any even integer multiplied by 3 will be even

Any odd number plus 1 will be even and any even number plus 1 will be odd

Therefor if n is even 3n+1 will be odd

If n is odd 3n+1 will be odd

Any odd number multiplied by an even number will be even

This means n(3n+1) will always be even

n(3n+1) is expressible as 2k where k ϵ Z

5) Prove that if n is an odd integer, then 𝑛4 ≡ 1 (𝑚𝑜𝑑 16). You may use the result from problem 4 to aid you in this proof. (Hint: At some point when you do your algebra, you should get an expression of the form a(3a + 1) where a is an integer. It is extremely helpful when you get to this point to use the result proved in question 4.)

n = (2m +1) where m ϵ Z

(2m +1)4 ≡ 1 (𝑚𝑜𝑑 16)

(2m+1) 4 = (4m2 + 4m + 1)2 = 16m4 + 32m3 +24m2 +8m +1

16 | (2m+1) 4 – 1 by definition of mod

16 | 16m4 + 32m3 +24m2 +8m +1 - 1

16 | 16m4 + 32m3 +24m2 +8m

16 | 8m(8m3+4m2+3m+1)

3m+1 = 2k where k ϵ Z. Derived from the answer to 4)

16 | 8m(8m3+4m2+2k)

16| 16m(4m3+2m2+k)

M and k are both integers therefore 16m(4m3+2m2+k) will always yield 16 multiplied my some integer

This implies that 16m(4m3+2m2+k), which is equal to (2m+1) 4 – 1 , will always be evenly divisible by 16

Therefore if n is an odd integer, then 𝑛4 ≡ 1 (𝑚𝑜𝑑 16)

6) Let x and y be integers such that 12 | (3x + 4y). Prove that 12 | (21x + 16y).

3x = 21x-24x

4y = 16y-12y

12 | (21x-24x + 16y-12y)

12|(21x+16y – (24x+12y))

12|( (21x+16y) – 12(2x+y) )

We know that 12(2x+y) is divisible by 12 therefore in order for (21x+16y) – 12(2x+y), which is equal to (3x + 4y), to be divisible by 12, (21x+16y) must be divisible by 12

7) Give a summary of the life and mathematical contributions of Evariste Galois. Please aim for a length of roughly 200 - 400 words. Your summary must be typed. Please state the sources you used in writing your summary.

Evariste Galois, a French mathematician, is best known for his method for determining a necessary and sufficient condition for a polynomial to be solvable by radicals. Galois displayed an aptitude for mathematics at an early age. When he was 14 he began reading math papers such as Adrien Marie Legendre's Éléments de Géométrie and the original papers of Joseph Louis Lagrange.

Group theory

Galois theory

Continued fractions