Homework 3

1. In general, you were told in class that for all integers a and b and positive integers n, if 𝑎 ≡ 𝑏 (𝑚𝑜𝑑 𝑛), then (𝑎) ≡ (𝑏) (𝑚𝑜𝑑 𝑛), where f is any function that operates on integers only. Using the definition of mod only, prove this specifically for the function (𝑎) = 𝑎2.

𝑎 ≡ 𝑏 (mod n) implies n|(a-b) meaning z\*n = (a-b) where z is an arbitrary integer

a2 – b2 = (a+b)\*(a-b)

(a+b)\*(a-b) = z\*n\*(a+b)

a, b, z are all integers therefore n|(a2 – b2), meaning that if 𝑎 ≡ 𝑏 (𝑚𝑜𝑑 𝑛), then a2 ≡ 𝑏2 (mod n)

2) Convert the following values from the bases indicated to base 10:

i) 36457

3\*7^3 + 6\*7^2 + 4\*7^1 +5\*7^0

1029 + 294 + 28 + 5

= 135610

ii) AAF716

10\*16^3 + 10\*16^2 + 15\*16^1 + 7\*16^0

40960 + 2560 + 240 + 7

= 4376710

iii) 123459

1\*9^4 + 2\*9^3 + 3\*9^2 + 4\*9^1 +5\*9^0

6561 + 1458 + 243 +36 +5

= 8303

iv) 300204

3\*4^4 + 0\*4^3 + 0\*4^2 + 2\*4^1 + 0\*4^0

768 + 0 + 0 + 8 + 0

= 77610

v) 1011010110012

1\*2^11 + 0\*2^10 +1\*2^9 + 1\*2^8 + 0\*2^7 + 1\*2^6 + 0\*2^5 + 1\*2^4 + 1\*2^3 +0\*2^2 + 0\*2^1 + 1\*2^0

2048 + 512 + 256 + 64 + 16 + 8 + 1

=2905

3) Convert the following base 10 values to the bases indicated:

i) 12435 to base 12

12435/12 = 1036 r 3

1036/12 = 86 r 4

86/12 = 7 r 2

7/12 = 0 r 7

1243510 = 724312

ii) 79770 to base 16

79770/16 = 4985 r 10

4985/16= 311 r 9

311/16 = 19 r 7

19/16 = 1 r3

1/16 = 0 r1

7977010 = 1379A16

iii) 691 to base 2

691/2 = 345r1

345/2=172r1

172/2=86r0

86/2=43r0

43/2=21r1

21/2=10r1

10/2=5r0

5/2=2r1

2/2=1r0

1 /2=0r1

69110 = 10101100112

iv) 4921 to base 7

4921/7= 703 r 0

703/7=100 r 3

100/7 = 14 r 2

14/7 = 2 r 0

2/7 = 0 r2

492110 = 202307

v) 88264 to base 8

88264/8=11033 r0

11033/8=1379r1

1379/8=172r3

172/8=21r4

21/8=2r5

2/8=0r2

8826410=2543108

4) Prove that if n is an integer, then n(3n+1) is an even integer.

Let us examine 3n+1

Any odd integer multiplied by 3 will be odd and any even integer multiplied by 3 will be even

Any odd number plus 1 will be even and any even number plus 1 will be odd

Therefor if n is even 3n+1 will be odd

If n is odd 3n+1 will be odd

Any odd number multiplied by an even number will be even

This means n(3n+1) will always be even

n(3n+1) is expressible as 2k where k ϵ Z

5) Prove that if n is an odd integer, then 𝑛4 ≡ 1 (𝑚𝑜𝑑 16). You may use the result from problem 4 to aid you in this proof. (Hint: At some point when you do your algebra, you should get an expression of the form a(3a + 1) where a is an integer. It is extremely helpful when you get to this point to use the result proved in question 4.)

n = (2m +1) where m ϵ Z

(2m +1)4 ≡ 1 (𝑚𝑜𝑑 16)

(2m+1) 4 = (4m2 + 4m + 1)2 = 16m4 + 32m3 +24m2 +8m +1

16 | (2m+1) 4 – 1 by definition of mod

16 | 16m4 + 32m3 +24m2 +8m +1 - 1

16 | 16m4 + 32m3 +24m2 +8m

16 | 8m(8m3+4m2+3m+1)

3m+1 = 2k where k ϵ Z. Derived from the answer to 4)

16 | 8m(8m3+4m2+2k)

16| 16m(4m3+2m2+k)

M and k are both integers therefore 16m(4m3+2m2+k) will always yield 16 multiplied my some integer

This implies that 16m(4m3+2m2+k), which is equal to (2m+1) 4 – 1 , will always be evenly divisible by 16

Therefore if n is an odd integer, then 𝑛4 ≡ 1 (𝑚𝑜𝑑 16)

6) Let x and y be integers such that 12 | (3x + 4y). Prove that 12 | (21x + 16y).

7) Give a summary of the life and mathematical contributions of Evariste Galois. Please aim for a length of roughly 200 - 400 words. Your summary must be typed. Please state the sources you used in writing your summary.

Evariste Galois, a French mathematician, is best known for his method for determining a necessary and sufficient condition for a polynomial to be solvable by radicals. Galois