Derivative Rules

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Constant Function Rule
If f is a constant function, $f(x) = c$, where c is a constant.

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = f'(x) = 0$$

Problem 1
Find the derivative of $f(x) = 3$

$$\Rightarrow f'(x) = 0$$

Problem 2
Find the derivative of $f(x) = \pi$

$$\Rightarrow \frac{\mathrm{d}f(x)}{\mathrm{d}x} = 0$$

Identity Function Rule
If f is the identity function, $f(x) = x$,

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = f'(x) = 1$$

Constant Multiple Rule
A constant multiplier c can be passed across the differentiation operator. If $g(x) = c \cdot f(x)$,

$$\frac{\mathrm{d}g(x)}{\mathrm{d}x} = g'(x) = c \cdot f'(x)$$

Problem 3
Find the derivative of $f(x) = \sqrt{2}x$

$$\Rightarrow f'(x) = \sqrt{2}$$

Problem 4
Find the derivative of $f(x) = \pi x$

$$\Rightarrow \frac{\mathrm{d}f(x)}{\mathrm{d}x} = \pi$$

Power Rule
For some natural number n, where $f(x) = x^n$,

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = f'(x) = n \cdot x^{n-1}$$

Problem 5
Find the derivative of $f(x) = x^3$

$$f'(x) = 3 \cdot x^{3-1}$$

$$\Rightarrow 3x^2$$

Problem 6
Find the derivative of $f(x) = 2x^3$

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = 2 \cdot 3 \cdot x^{3-1}$$

$$\Rightarrow 6x^2$$

Sum Rule
The derivative of a sum is the sum of the derivatives. Given $(f+g)(x)$,

$$\frac{\mathrm{d}[f(x)+g(x)]}{\mathrm{d}x} = (f+g)'(x) = f'(x)+g'(x)$$

Difference Rule
The derivative of a difference is the difference of the derivatives. Given $(f-g)(x)$,

$$\frac{\mathrm{d}[f(x)-g(x)]}{\mathrm{d}x} = (f-g)'(x) = f'(x)-g'(x)$$

Problem 7
Find the derivative of $(f+g)(x) = 5x^2+7x-6$

$$(f+g)'(x) = 5\cdot 2\cdot x^{2-1} + 7\cdot x^{1-1} - 0$$

$$\Rightarrow 10x+7$$

Problem 8
Find the derivative of $(f-g)(x) = 4x^6 - 3x^5 - 10x^2 + 5x + 16$

$$\frac{\mathrm{d}[f(x)-g(x)]}{\mathrm{d}x} = 4\cdot 6\cdot x^{6-1} - 3\cdot 5\cdot x^{5-1} - 10\cdot 2\cdot x^{2-1} + 5\cdot x^{1-1} - 0$$

$$\Rightarrow 24x^5 - 15x^4 - 20x + 5$$

Product Rule
Assuming f and g are differentiable functions, $(f \cdot g)(x)$,

$$\frac{\mathrm{d}[f(x) \cdot g(x)]}{\mathrm{d}x} = (f \cdot g)'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

Problem 9
Find the derivative of $(f \cdot g)(x) = (x^4 - 1)(x^2 + 1)$

$$(f \cdot g)'(x) = (x^4 - 1)(2 \cdot x^{2-1}) + (4 \cdot x^{4-1})(x^2 + 1)$$

$$\Rightarrow (x^4 - 1)(2x) + (4x^3)(x^2 + 1)$$

Problem 10
Find the derivative of $(f \cdot g)(x) = (x^2 + 17)(x^3 - 3x + 1)$

$$\frac{\mathrm{d}[f(x) \cdot g(x)]}{\mathrm{d}x} = (x^2{+}17)(3{\cdot}x^{3-1}{-}3{\cdot}x^{1-1}){+}(2{\cdot}x^{2-1})(x^3{-}3x{+}1)$$

$$\Rightarrow (x^2 + 17)(3x^2 - 3) + (2x)(x^3 - 3x + 1)$$

Quotient Rule

Assuming f and g are differentiable functions, $\left(\frac{f}{g}\right)(x)$,

$$\frac{\mathrm{d}\left(\frac{f(x)}{g(x)}\right)}{\mathrm{d}x} = \left(\frac{f}{g}\right)'(x) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g^2(x)}$$

Problem 11

Find the derivative of $\left(\frac{f}{g}\right)(x) = \frac{2x^2-3x+1}{2x+1}$

$$\left(\frac{f}{g}\right)'(x) = \frac{(2x+1)(2\cdot 2x^{2-1} - 3\cdot x^{1-1}) - (2x^2-3x+1)(2\cdot x^{1-1})}{(2x+1)^2}$$

$$\Rightarrow \frac{(2x+1)(4x-3) - (2x^2-3x+1)(2)}{(2x+1)^2}$$

Problem 12

Find the derivative of $\left(\frac{f}{g}\right)(x) = \frac{2x^2-1}{3x+5}$

$$\frac{\mathrm{d}\left(\frac{f(x)}{g(x)}\right)}{\mathrm{d}x} = \frac{(3x+5)(2\cdot 2x^{2-1}) - (2x^2-1)(3\cdot x^{1-1})}{(3x+5)^2}$$

$$\Rightarrow \frac{(3x+5)(4x) - (2x^2-1)(3)}{(3x+5)^2}$$