LOGIC: STATEMENTS, NEGATIONS, QUANTIFIERS, TRUTH TABLES

STATEMENTS

A statement is a declarative sentence having truth value.

Examples of statements:
Today is Saturday.
Today I have math class.
$1 + 1 = 2$
$3 < 1$
What's your sign?
Some cats have fleas.
All lawyers are dishonest.
Today I have math class and today is Saturday.
$1 + 1 = 2$ or $3 < 1$

For each of the sentences listed above (except the one that is stricken out) you should be able to determine its **truth value** (that is, you should be able to decide whether the statement is TRUE or FALSE).

Questions and commands are not statements.

SYMBOLS FOR STATEMENTS

It is conventional to use lower case letters such as p, q, r, s to represent logic statements.
Referring to the statements listed above, let
p: Today is Saturday.
q: Today I have math class.
r: $1 + 1 = 2$
s: $3 < 1$
u: Some cats have fleas.
v: All lawyers are dishonest.
Note: In our discussion of logic, when we encounter a subjective or value-laden term (an *opinion*) such as "dishonest," we will assume for the sake of the discussion that that term has been precisely defined.

QUANTIFIED STATEMENTS

The words "all" "some" and "none" are examples of ***quantifiers.***
A statement containing one or more of these words is a ***quantified*** *statement.*
Note: the word "some" means "at least one."

NEGATIONS

If *p* is a statement, the **negation** of *p* is another statement that is exactly the opposite of *p.*

The negation of a statement p is denoted **~p** ("**not p**").

A statement p and its negation ~p will always have opposite truth values; it is impossible to conceive of a situation in which a statement and its negation will have the same truth value.

EXAMPLE

Let p be the statement "Today is Saturday."

Then ~p is the statement "Today is not Saturday."

On any given day, if p is true then ~p will be false; if p is false, then ~p will be true.

It is impossible to conceive of a situation in which p and ~p are simultaneously true.

It is impossible to conceive of a situation in which p and ~p are simultaneously false.

NEGATIONS OF QUANTIFIED STATEMENTS

Fact: "None" is the opposite of "at least one."

For example: The negation of "Some dogs are poodles" is "No dogs are poodles."

Notice that "Some dogs are poodles" is a statement that is true according to our everyday experience, and "No dogs are poodles" is a statement that is false according to our everyday experience.

In general:

The negation of "Some A are B" is "No A are (is) B."

(Note: this can also be phrased "All A are the opposite of B," although this construction sometimes sounds ambiguous.)

Fact: "Some aren't" is the opposite of "all are."

For example, the negation of "All goats are mammals" is "Some goats aren't mammals."

Notice that "All goats are mammals" is a statement that is true according to our everyday experience, while "Some goats aren't mammals" is a statement that is false according to our everyday experience.

In fact, it is logically impossible to imagine a situation in which those two statements have the same truth value.

In general, the negation of "All A are B" is "Some A aren't B."

LOGICAL CONNECTIVES

The words "and" "or" "but" "if...then" are examples of **logical connectives**. *They are words that can be used to connect two or more simple statements to form a more complicated* **compound statement.**

Examples of compound statements:

"I am taking a math class **but** I'm not a math major."

"**If** I pass MGF1106 **and** I pass MGF1107 **then** my liberal studies math requirement will
be fulfilled."

EQUIVALENT STATEMENTS

Any two statements *p* and *q* are **logically equivalent** if they have exactly the same meaning. This means that *p* and *q* will always have the same truth value, in any conceivable situation.

If *p* and *q* are equivalent statements, then it is logically impossible to imagine a situation
in which the two statements would have differing truth values.

Examples:

"Today I have math class and today is Saturday" is equivalent to "Today is Saturday and
today I have math class."

This equivalency follows simply from our everyday understanding of the meaning ot the
word "and."

"This and that" means the same as "That and this."

Likewise, "I have a dog or I have a cat" is equivalent to "I have a cat or I have a dog"

This equivalency follows simply from our everyday understanding of the meaning ot the
word "or."

"This or that" means the same as "That or this."

Logical equivalence is denoted by this symbol: $\equiv$

"Some A are B" ***is not equivalent to*** *"Some A aren't B."*

THE CONJUNCTION AND THE DISJUNCTION

THE CONJUNCTION

If p, q are statements, their **conjunction** is the statement "p and q."

It is denoted: $p \wedge q$

For example, let p be the statement "I have a dime" and let q be the statement "I have a
nickel." Then $p \wedge q$ is the statement "I have a dime **and** I have a nickel."

***In general, in order for any statement of the form "p ∧ q" to be true, both p and q must
be true.***

Example: "Tallahassee is in Florida and Orlando is in Georgia" is a false statement.

MORE ON THE CONJUNCTION

The word **but** is also conjunction; it is sometimes used to precede a negative phrase.
Example: "I've fallen **and** I can't get up" means the same as "I've fallen **but** I can't get
up."
In either case, if p is "I've fallen" and q is "I *can* get up" the conjunction above is symbolized as $p \wedge \sim q$.

THE DISJUNCTION

If p, q are statements, their ***disjunction*** *is the statement "p or q."*
It is denoted: $p \vee q$.
For example, let p be the statement "Today is Tuesday" and let q be the statement "1 + 1 = 2." In that case, $p \vee q$ is the statement
"Today is Tuesday **or** 1 + 1 = 2."
In general, in order for a statement of the form p ∨ q to be true, at least one of its two parts must be true. The only time a disjunction is false is when both parts (both "components") are false.
The statement "Today is Tuesday **or** 1 + 1 = 2" is TRUE.

EQUIVALENCIES FOR THE CONJUNCTION ("AND") AND THE DISJUNCTION ("OR")

As we observed earlier, according to our everyday usage of the words "and" and "or" we
have the following equivalencies:
1. "p and q" is equivalent to "q and p"
$p \wedge q \equiv q \wedge p$
2. "p or q" is equivalent to "q or p"
$p \vee q \equiv p \vee q$
For example, "I have a dime or I have a nickel" equivalent to "I have a nickel or I have a
dime."
Likewise, "It is raining and it isn't snowing" is equivalent to "It isn't snowing and it is
raining."

TRUTH TABLES

A ***truth table*** *is a device that allows us to analyze and compare compound logic statements.*
Consider the symbolic statement $p \vee \sim q$.
Whether this statement is true or false depends upon whether its variable parts are true or
false, as well as on the behavior of the "or" connective and the "negation" operator.
Later, we will make a truth table for this statement.

A truth table for this statement will take into account every possible combination of the
variables being true or false, and show the truth value of the compound statement in each
case

EXAMPLE 2.1.7
As an introduction, we will make truth tables for these two statements

1. $p \wedge q$
2. $p \vee q$

Solution to EXAMPLE 2.1.7 #1

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Note that in this truth table there is only one row in which the statement $p \wedge q$ is true. This the row where p is true and q is true. This conforms to our earlier observation that the only situation in which is conjunction is true is the case in which both of its component statements are true.

Solution to EXAMPLE 2.1.7 #2

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

THE BASIC RULES FOR CONSTRUCTING A TRUTH TABLE FOR A COMPOUND STATEMENT

1. The number of **rows** in the truth table depends upon the number of basic variables in
the compound statement. To determine the number of rows required, count the number of

basic variables in the statement, but **don't** re-count multiple occurrences of a variable.
1 variable---2 rows
2 variables--4 rows
3 variables--8 rows
4 variables--16 rows and so forth.
2. The number of **columns** in a truth table depends upon the number of logical connectives in the statement. The following guidelines are usually reliable.
A. There will be one column for each basic variable; and
B. To determine the number of other columns, count the number of logical connectives in
the statement; **do** re-count multiple occurrences of the same connective. The "~" symbol
counts as a logical connective.
In addition to the columns for each basic variable, there will usually be one column for
each occurrence of a logical connective.
3. The beginning columns are filled in so as to take into account every possible combination of the basic variables being true or false. Each **row** represents one of the
possible combinations.
4. In order to fill in any other column in the truth table, you must **refer to a previous
column or columns.**

TAUTOLOGIES

Referring to the truth table for the statement q∨~ (~p∧q) in the previous example: notice
that the column for that statement shows only "true." This means that it is never possible
for a statement of the form q∨~ (~p∧q) to be false.
A **tautology** is a statement that cannot possibly be false, due to its logical structure (its
syntax).
The statement q∨~ (~p∧q) is an example of a tautology.

EXAMPLE 2.1.9

Is the statement (p∨r)∨~(p∧q) a tautology?

We can answer this question by making a truth table. See the solution on the right.

EXAMPLE 2.1.9

p	q	r	p∧q	~(p∧q)	p∨r	(p∨r)∨~(p∧q)
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	F	T

The truth table shows that the statement (p∨r)∨~(p∧q) is a tautology.

EXAMPLE 2.1.9A The statement (p∨~q)∨(~p∧q) is not a tautology.