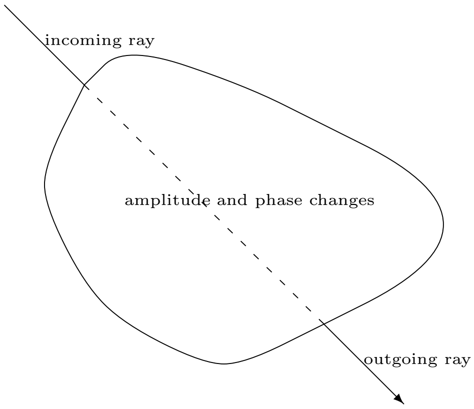
This formulation is wrong

1 Complex attenuation



As a ray passes through the medium in question in undergoes an amplitude attenuation and phase shift. At any given point, $\mathbf{x}$, this change can be represented by the attenuation kernel: $k(\mathbf{x})e^{i\phi(\mathbf{x})}$. Here, $k(\mathbf{x})$ is purely amplitude attenuation with the property $0 \leq k(\mathbf{x}) \leq 1$ (a value of 1 represents total transmition, and a value of 0 represents total attenuation). The term $\phi(\mathbf{x})$ represents the phase shift property of the medium, with $0 \leq \phi(\mathbf{x}) < 2\pi$. For all intents and purposes we may assume an inital amplitude of 1, and phase of 0.

From this formulation we wish to construct an intergral for determining the projected form of the outgoing ray. A nieve approach would be to simply integrate along the line of sight, $P_{\text{ray}} = \int_{\text{ray}} k(\mathbf{x})e^{i\phi(\mathbf{x})}\,dl$. However, in real world applications this formulation would not hold true if $k(\mathbf{x}) = 0$ for any point along the ray path (eg. an opaque object blocking the line of sight). So we introduce another function:

$$A\,(\text{ray}) = \begin{cases} 0 & \text{if } \exists \mathbf{x} \in \text{ray such that } k(\mathbf{x}) = 0, \\ 1 & \text{otherwise.} \end{cases}$$

We can implement this into our projection formula to give:

$$P_{\text{ray}} = A\,(\text{ray}) \int_{\text{ray}} k\,(\mathbf{x})\,e^{i\phi(\mathbf{x})}\,dl. \tag{1}$$

In real world problems we will measure $P(\text{ray}) = ae^{i\varphi}$, where a is the measured amplitude and φ is the phase. One way of measuring this is to take separate measurements of a and φ (eg. intensity maps and interferometry).