test shapes

1 simple shapes

Four simple shapes were employed, based on standard measures on Euclidean space:

scalar $f(\mathbf{x}) = 1$

diamond $f(\mathbf{x}) = \begin{cases} 1, & \|\mathbf{x}\|_1 = |x_1| + |x_2| + \ldots + |x_n| \leq 1 \\ 0, & \text{otherwise} \end{cases}$

sphere $f(\mathbf{x}) = \begin{cases} 1, & \|\mathbf{x}\|_2 = (x_1^2 + x_2^2 + \ldots + x_n^2)^{\frac{1}{2}} \leq 1 \\ 0, & \text{otherwise} \end{cases}$

cube $f(\mathbf{x}) = \begin{cases} 1, & \|\mathbf{x}\|_\infty = \max\{x_1, x_2, \ldots, x_n\} \leq 1 \\ 0, & \text{otherwise} \end{cases}$

2 bell

A broad, smooth bell-shaped object was also employed. This function peaks at the origin, and tapers to zero at the farthest corner from the origin. If $\mathbf{c}$ is any corner of the grid, then we describe the intensity by the function $\cos^2\left(\frac{\pi}{2}\frac{\|\mathbf{x}\|}{\|\mathbf{c}\|}\right)$.

3 Shepp-Logan and Shepp-Logan-bold

From [2].

3.1 2D

On a $[-1,1] \times [-1,1]$ grid, where the background intensity is 0, and these values are additive. The ellipses are formed by taking the unit circle and dilating, rotating and shifting. This can be described as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r_x & 0 \\ 0 & r_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \end{bmatrix}$$

ellipse	θ	r_x	r_y	(c_x, c_y)	level 1	level 2
a	$0°$	0.69	0.92	$(0, 0)$	2	1
b	$0°$	0.6624	0.874	$(0, -0.0184)$	-0.98	-0.8
c	$-18°$	0.11	0.31	$(0.22, 0)$	-0.02	-0.2
d	$18°$	0.16	0.41	$(-0.22, 0)$	-0.02	-0.2
e	$0°$	0.21	0.25	$(0, 0.35)$	0.01	0.1
f	$0°$	0.046	0.046	$(0, 0.1)$	0.01	0.1
g	$0°$	0.046	0.046	$(0, -0.1)$	0.02	0.2
h	$0°$	0.046	0.023	$(-0.08, -0.605)$	0.01	0.1
i	$0°$	0.023	0.023	$(0, -0.605)$	0.01	0.1
j	$0°$	0.023	0.046	$(0.06, -0.605)$	0.01	0.1

3.2 3D

On a $[-1, 1] \times [-1, 1] \times [-1, 1]$ grid, again the values are additive. The ellipsoids are formed by taking the unit sphere and dilating, rotating, and shifting. This can be described as follows:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} r_x & 0 & 0 \\ 0 & r_y & 0 \\ 0 & 0 & r_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}$$

ellipse	θ	r_x	r_y	r_z	(c_x, c_y, c_z)	level 1	level 2
a	$0°$	0.69	0.9	0.92	$(0, 0, 0)$	2	1
b	$0°$	0.6624	0.88	0.874	$(0, 0, -0.0184)$	-0.98	-0.8
c	$-18°$	0.11	0.22	0.31	$(0.22, -0.25, 0)$	-0.02	-0.2
d	$18°$	0.16	0.21	0.41	$(-0.22, -0.25, 0)$	-0.02	-0.2
e	$0°$	0.21	0.35	0.25	$(0, -0.25, 0.35)$	0.01	0.1
f	$0°$	0.046	0.046	0.046	$(0, -0.25, 0.1)$	0.01	0.1
g	$0°$	0.046	0.046	0.046	$(0, -0.25, -0.1)$	0.02	0.2
h	$0°$	0.046	0.02	0.023	$(-0.08, -0.25, -0.605)$	0.01	0.1
i	$0°$	0.023	0.023	0.023	$(0, -0.25, -0.605)$	0.01	0.1
j	$0°$	0.023	0.02	0.046	$(0.06, -0.25, -0.605)$	0.01	0.1
k	$0°$	0.04	0.1	0.056	$(0.06, 0.625, -0.105)$	0.02	0.2
l	$0°$	0.056	0.1	0.056	$(0, 0.625, 0.1)$	-0.02	-0.2

4 stark

From [3].
This test object, on a 100×100 grid with background intensity of 0%, can be

completely defined as follows:
triangle (has an intensity of 100%)

vertex	x-coordinate	y-coordinate
1	15	20
2	20	20
3	20	15

circles

circle	(c_x, c_y)	radius	intensity
1	50,50	37.5	75%
2	31.25,50	18.75	60%
3	21.875,50	9.375	45%
4	17.1875,50	4.6875	30%
5	14.84375,50	2.34375	15%

5 dorn_house

This "house" shape is taken from Dorn [1].

References

[1] Oliver Dorn, Eric L. Miller, and Carey M. Rappaport. Shape reconstruction in 2D from limited-view multifrequency electromagnetic data. In Eric Todd Quinto, Leon Ehrenpreis, Adel Faridani, Fulton Gonzalez, and Eric Grinberg, editors, *Radon Transforms and Tomography*, volume 278 of *Contemporary Mathematics*, pages 97–122. American Mathematical Society, Rhode Island, 2001.

[2] L. A. Shepp and B. F. Logan. Reconstructing interior head tissue from x-ray transmissions. *IEEE Transactions on Nuclear Science*, NS 21:21–43, 1974.

[3] Henry Stark, John W. Woods, Indraneel Paul, and Rajesh Hingorani. Direct Fourier reconstruction in computer tomography. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, ASSP-29(2):237–245, April 1981.