

1 Perpendicular drop point onto a line

line:

$$\mathbf{x} = \mathbf{m}t + \mathbf{c} \quad (1)$$

Given a point \mathbf{p} we want to know the value t_p such that $\mathbf{p} - \mathbf{x}_p$ is perpendicular to the line 1, ie

$$(\mathbf{p} - \mathbf{x}_p) \cdot \mathbf{m} = (\mathbf{p} - \mathbf{m}t_p - \mathbf{c}) \cdot \mathbf{m} = 0$$

Solving this equation for t_p yields:

$$t_p = \frac{(\mathbf{p} - \mathbf{c}) \cdot \mathbf{m}}{\|\mathbf{m}\|^2}$$

2 Intersection of two lines in \mathbb{R}^n ($n \geq 2$)

line 2:

$$\mathbf{x}_1 = \mathbf{m}_1 t_1 + \mathbf{c}_1 \quad (2)$$

line 3:

$$\mathbf{x}_2 = \mathbf{m}_2 t_2 + \mathbf{c}_2 \quad (3)$$

2.1 general ideas

Project lines onto a set of axes which are spanned by the projections of \mathbf{m}_1 and \mathbf{m}_2 . This will guarantee that if lines 2 and 3 intersect at all, then they will surely intersect in this axis plane. Then solve for t_1 and t_2 .

2.2 explicit steps

1. If $\mathbf{m}_1 = \mathbf{m}_2$ then the lines don't intersect, finish.
2. Choose a pair of axes in which the projections of \mathbf{m}_1 and \mathbf{m}_2 span the axis plane. Call these axes i and j , ie (m_{1i}, m_{1j}) and (m_{2i}, m_{2j}) are neither parallel or equal to $(0, 0)$.
3. There *will* be a solutions for t_1 and t_2 in the following pair of simultaneous equations

$$\begin{aligned} & \begin{cases} m_{1i}t_1 + c_{1i} = m_{2i}t_2 + c_{2i} \\ m_{1j}t_1 + c_{1j} = m_{2j}t_2 + c_{2j} \end{cases} \\ & \Rightarrow \begin{cases} t_1 = \frac{m_{2i}(c_{1j}-c_{2j}) - m_{2j}(c_{1i}-c_{2i})}{m_{1i}m_{2j} - m_{1j}m_{2i}} \\ t_2 = \frac{m_{1i}(c_{1j}-c_{2j}) - m_{1j}(c_{1i}-c_{2i})}{m_{1i}m_{2j} - m_{1j}m_{2i}} \end{cases} \end{aligned}$$

4. Put t_1 and t_2 back into equations 2 and 3 respectively, to solve for the intersection point.
5. Check that $\mathbf{x}_1 = \mathbf{x}_2$, if not then finish.
6. The intersection exists and we are finished.

3 Perpendicular drop point onto a plane

plane:

$$(\mathbf{x} - \mathbf{c}) \cdot \mathbf{n} = 0 \quad (4)$$

Given any point \mathbf{p} , we wish to find the point on plane 4, \mathbf{x}_p , such that $\mathbf{p} - \mathbf{x}_p$ is perpendicular to plane 4. $\mathbf{x}_p = \mathbf{p} - k\mathbf{n}$, for some number k . We substitute this into plane 4 to get

$$\begin{aligned} ((\mathbf{p} - k\mathbf{n}) - \mathbf{c}) \cdot \mathbf{n} &= 0 \\ \Rightarrow k &= \frac{(\mathbf{p} - \mathbf{c}) \cdot \mathbf{n}}{\|\mathbf{n}\|^2} \end{aligned}$$

4 Intersection of a line and a plane in \mathbb{R}^3

line:

$$\mathbf{x}_l = \mathbf{m}_l t_l + \mathbf{c}_l \quad (5)$$

plane:

$$(\mathbf{x}_p - \mathbf{c}_p) \cdot \mathbf{n}_p = 0 \quad (6)$$

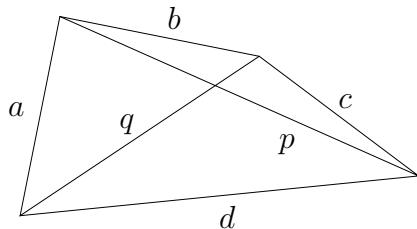
1. if $\mathbf{m}_l \cdot \mathbf{n}_p = 0$ then the line is parallel to the plane, and there is no intersection
2. substitute \mathbf{x}_l into the plane 6

$$\begin{aligned} ((\mathbf{m}_l t_l + \mathbf{c}_l) - \mathbf{c}_p) \cdot \mathbf{n}_p &= 0 \\ \Rightarrow t_l (\mathbf{m}_l \cdot \mathbf{n}_p) &= \mathbf{c}_p \cdot \mathbf{n}_p - \mathbf{c}_l \cdot \mathbf{n}_p \\ \Rightarrow t_l &= \frac{\mathbf{c}_p \cdot \mathbf{n}_p - \mathbf{c}_l \cdot \mathbf{n}_p}{\mathbf{m}_l \cdot \mathbf{n}_p} \end{aligned}$$

3. put t_l back into 5 to find the intercept point

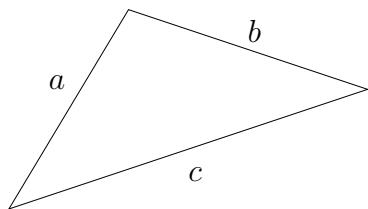
5 Useful geometric formulae

5.1 area of a quadrilateral in \mathbb{R}^2



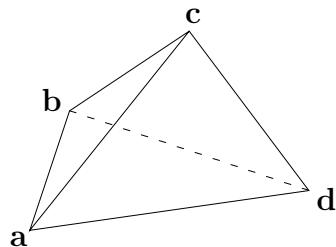
$$A = \frac{1}{4} \sqrt{4p^2q^2 - (b^2 + d^2 - a^2 - c^2)^2}$$

5.2 area of a triangle



$$A = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

5.3 volume of a tetrahedron



$$V = \frac{1}{6} \begin{vmatrix} (\mathbf{b} - \mathbf{a})_1 & (\mathbf{b} - \mathbf{a})_2 & (\mathbf{b} - \mathbf{a})_3 \\ (\mathbf{c} - \mathbf{a})_1 & (\mathbf{c} - \mathbf{a})_2 & (\mathbf{c} - \mathbf{a})_3 \\ (\mathbf{d} - \mathbf{a})_1 & (\mathbf{d} - \mathbf{a})_2 & (\mathbf{d} - \mathbf{a})_3 \end{vmatrix}$$