

# **DSA3 seminar assignment**

Radoman Radoman 157m/21

University of Novi Sad, Faculty of Sciences

radoman88@gmail.com

# 1 Introduction

## 2 BFS and DFS

### 2.1 Purpose and theoretical background

Both BFS (breadth-first search) and DFS (depth-first search) are tree-search algorithms. They are both used to determine whether a graph is connected.

#### Theorem

A graph is connected if and only if, you can not split it into two such that there are no edges between them.

An approach to check whether a graph is connected is by checking all the partitions (the problem is that there are  $2^{n-1}$  of them so it is very inefficient). Another way to check whether a graph is connected is by checking all pairs of vertices - if there is a path connecting them (problem here again is that there are  $n^2$  pairs of vertices and for each of them we have to check if there is a path which is also very inefficient on top of number of partitions that we would need to check).

#### Definition

A graph is connected if and only if it has a spanning tree.

Checking whether a graph is connected in such a way (by determining whether it has a spanning tree) is far and away the most efficient way to check graph connectedness.

Both DFS and BFS work in a similar way (they both produce spanning trees), but with differences that impact the priority of edges selected. Depending on the way we used to choose edges we get different trees.

A **tree-search** algorithm on  $G$ :

- > Start with a single root vertex  $r \in V(G)$ . This is our initial tree (with just one vertex).
- > Repeat (for as long as possible):
  - Do we have a spanning tree?
  - Is the tree edge cut empty?
  - If not, add one edge from the cut to the tree.

If we want to check *connectedness* of  $G$ , that is the whole algorithm. As noted above, depending on the way we use to choose edges, different spanning tree will result (if the graph is connected of course).

Define: edge cut, rooted tree (also called r-tree), levels (distance from root to a specific vertex), ancestor, descendant, parent or predecessor and children.

$$v \in V$$

Predecessor function:  $p(v)$ , for all  $\{r\}$

## 2.2 BFS

Adjacency lists of vertices are considered on a first-come-first serve basis. Implemented with a *queue*.

For a connected graph, the algorithm will return:

- A spanning tree given by its predecessor function,
- the level function,
- the time function (order in which vertices are added to the tree)

In BFS algorithm we will first consider the neighbors (one-by-one) before we look through the neighbours of any of them.

Therefore, first edges incident to  $r$  are selected, and only after that we are looking at neighbors of the neighbors of  $r$ .

This is called Breadth-first search algorithm. This approach expands (spreads) the tree as much as possible.

We start with just the root  $r$  in the queue and we repeatedly pop the head of the queue, and push all its new neighbors to the queue.

### 2.2.1 BFS algorithm

> INPUT: a connected graph  $G$ , a vertex  $r \in V(G)$

> OUTPUT: an  $r$ -tree  $T \subseteq G$ , its predecessor function  $p$ , its level function  $l$ , the time function  $t$

#### BFS algorithm

$Q := \emptyset$ ,  $Q \leftarrow r$ ,  $l(r) := 0$   $t(r) := 1$ , mark  $r$ ,  $i := 1$

**while**  $Q \neq \emptyset$

$x \leftarrow Q$

**if**  $x$  has unmarked neighbor  $y$  **then**

$i++$

$Q \leftarrow y$ , mark  $y$ ,  $p(y) := x$ ,  $l(y) := l(x) + 1$ ,  $t(y) := i$

**else**

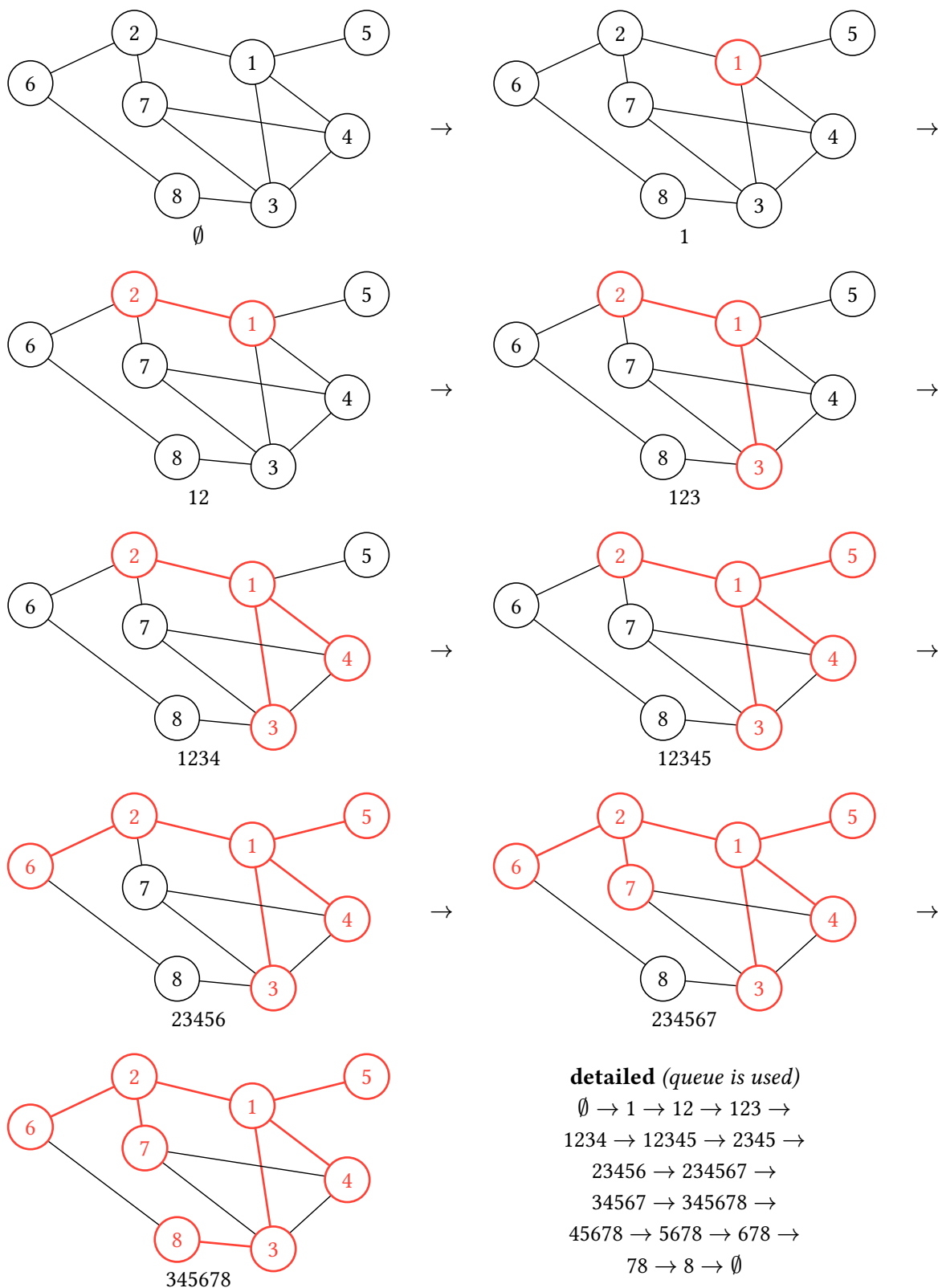
        remove head of  $Q$

**end if**

**end while**

**return everything**

### 2.2.2 BFS example



### 2.2.3 BFS properties

**Theorem** Let  $T$  be a BFS tree of  $G$ , with root  $r$ .

a.)  $l(v) = d_T(r, v)$ , for every  $v \in V$ ,

b.)  $|l(u) - l(v)| \leq 1$ , for every  $uv \in E(G)$ .

Level of  $v$  is exactly the distance from root  $r$  to  $v$ .

Every edge of the graph connects only vertices of the same level of the tree or difference by most 1.

**Theorem** Let  $T$  be a BFS tree of  $G$ , with root  $r$ . Then

$l(v) = d_G(r, v)$ , for every  $v \in V$

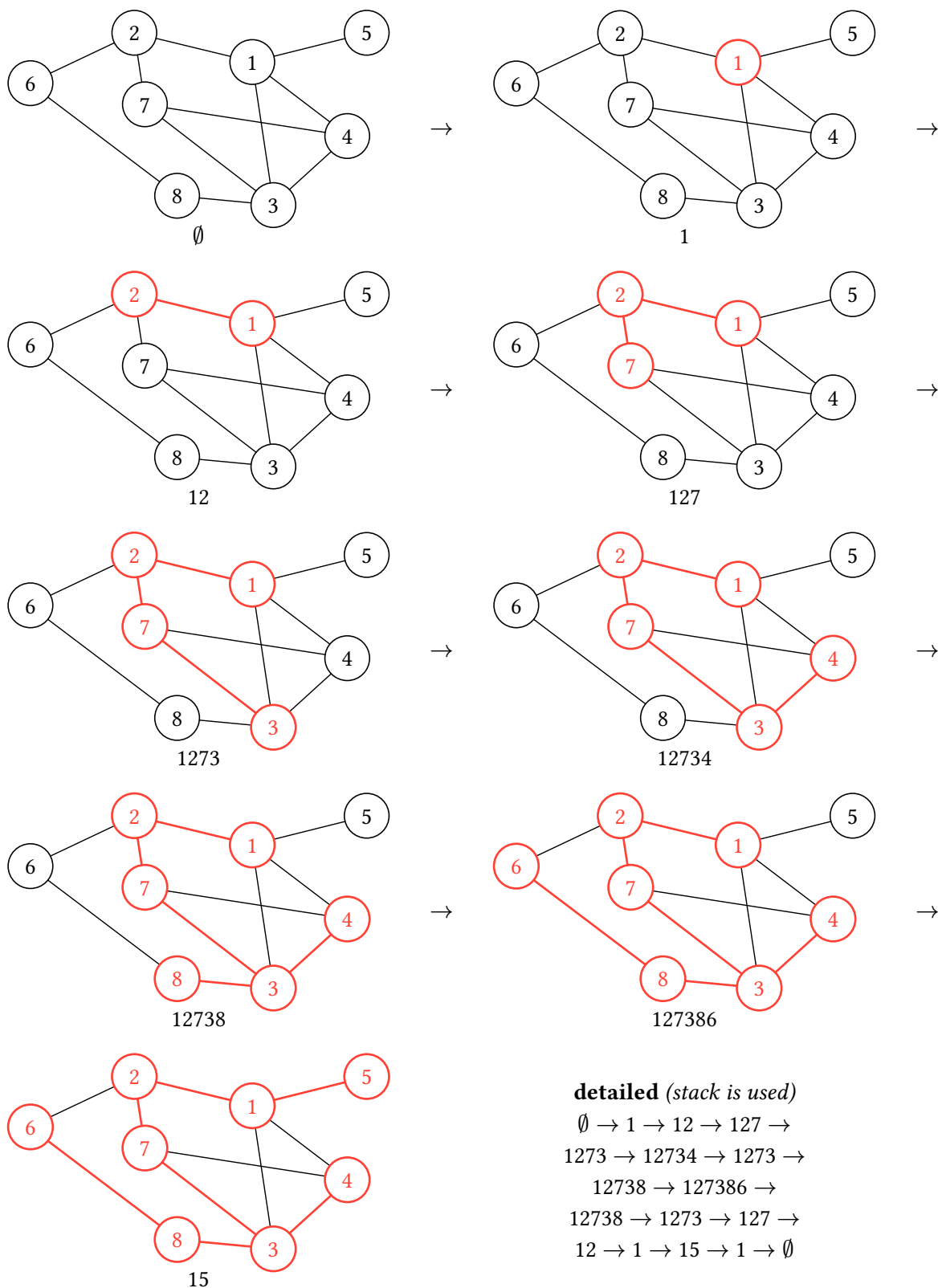
As seen from our example above.

## 2.3 DFS

Completely the same as BFS, except that we use a **stack** instead of a queue.

### 2.3.1 DFS algorithm

### 2.3.2 DFS example



### **3 Applications**

## **4 Conculsion**