

# **DSA3 seminar paper**

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# **1 Introduction**

## 2 BFS and DFS

### 2.1 Purpose and theoretical background

Both BFS (breadth-first search) and DFS (depth-first search) are tree-search algorithms. They are both used to determine whether a graph is connected.

#### Theorem

A graph is connected if and only if, you can not split it into two such that there are no edges between them.

An approach to check whether a graph is connected is by checking all the partitions (the problem is that there are  $2^{n-1}$  of them so it is very inefficient). Another way to check whether a graph is connected is by checking all pairs of vertices - if there is a path connecting them (problem here again is that there are  $n^2$  pairs of vertices and for each of them we have to check if there is a path which is also very inefficient on top of number of partitions that we would need to check).

#### Definition

A graph is connected if and only if it has a spanning tree.

Checking whether a graph is connected in such a way (by determining whether it has a spanning tree) is far and away the most efficient way to check graph connectedness.

Both DFS and BFS work in a similar way (they both produce spanning trees), but with differences that impact the priority of edges selected. Depending on the way we used to choose edges we get different trees.

A **tree-search** algorithm on  $G$ :

- > Start with a single root vertex  $r \in V(G)$ . This is our initial tree (with just one vertex).
- > Repeat (for as long as possible):
  - Do we have a spanning tree?
  - Is the tree edge cut empty?
  - If not, add one edge from the cut to the tree.

If we want to check *connectedness* of  $G$ , that is the whole algorithm. As noted above, depending on the way we use to choose edges, different spanning tree will result (if the graph is connected of course).

Define: edge cut, rooted tree (also called r-tree), levels (distance from root to a specific vertex), ancestor, descendant, parent or predecessor and children.

$$v \in V$$

Predecessor function:  $p(v)$ , for all  $\{r\}$

## 2.2 BFS

Adjacency lists of vertices are considered on a first-come-first serve basis.

Implemented with a *queue*:

Start with just the root in the queue.

Repeatedly pop the head of the queue, and push all its new neighbors to the queue.

For a connected graph, the algorithm will return:

- A spanning tree given by its predecessor function,
- the level function,
- the time function (order in which vertices are added to the tree)

### BFS Algorithm

> INPUT: a connected graph  $G$ , a vertex  $r \in V(G)$

> OUTPUT: an  $r$ -tree  $T \in G$ , its predecessor function  $p$ , its level function  $l$ , the time function  $t$ .

$Q := \emptyset, Q \leftarrow r, l(r) := 0, t(r) := 1, \text{mark } r, i := 1$

**while**  $Q \neq \emptyset$

### **3 Applications**

## **4 Conculsion**