DSA3 seminar assignment

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1 Introduction

2 BFS and DFS

2.1 Purpose and theoretical background

Both BFS (breadth-first search) and DFS (depth-first search) are tree-search algorithms. They are both used to determine whether a graph is connected.

Theorem
A graph is connected if and only if, you can not split it into two such that there are no edges between them.

An approach to check wheter a graph is connected is by checking all the partitions (the problem is that there are 2^{n-1} of them so it is very ineficient). Another way to check whether a graph is connect is by checking all pairs of vertices - if there is a path connecting them (problem here again is that there are n^2 pairs of vertices and for each of them we have to check if there is a path which is also very ineficient on top of number of partitions that we would need to check).

Definition
A graph is connected if and only if it has a spanning tree.

Checking whether a graph is connected in such a way (by determining whether it has a spanning tree) is far and away the most efficient way to check graph connectedness.

Both DFS and BFS work in a similar way (they both produce spanning trees), but with differences that impact the priority of edges selected. Depending on the way we used to choose edges we get different trees.

A **tree-search** algorithm on G:
> Start with a single root vertex $r \ \varepsilon \ V(G)$. This is our initial tree (with just one vertex).
> Repeat (for as long as possible):
- Do we have a spanning tree?
- Is the tree edge cut empty?
- If not, add one edge from the cut to the tree.

If we want to check *connectedness* of G, that is the whole algorithm. As noted above, depending on the way we use to choose edges, different spanning tree will result (if the graph is connected of course).

Define: edge cut, rooted tree (also called r-tree), levels (distance from root to a specific vertex), ancestor, descendant, parent or predecessor and children.

Predecessor function: $p(v)$, for all $\frac{v \ \varepsilon V}{\{r\}}$

2.2 BFS

Adjacency lists of vertices are considered on a first-come-first serve basis. Implemented with a *queue*.

For a connected graph, the algorithm will return:

- A spanning tree given by its predecessor function,
- the level function,
- the time function (order in which vertices are added to the tree)

In BFS algorithm we will first consider the neighbors (one-by-one) before we look through the neighbours of any of them.

Therefore, first edges incident to r are selected, and only after that we are looking at neighbors of the neighbors of r.

This is called Breadth-first search algorithm. This approach expands (spreads) the tree as much as possible.

We start with just the root r in the queue and we repeatedly pop the head of the queue, and push all its new neighbors to the queue.

2.2.1 BFS algorithm

> INPUT: a connected graph G, a vertex $r \ \varepsilon V(G)$
> OUTPUT: an r-tree $T \subseteq G$, its predecessor function p, its level function l, the time function t

BFS algorithm
$Q := \emptyset, \quad Q \leftarrow r, \quad l(r) := 0 \quad t(r) := 1, \quad \text{mark } r, i := 1$
while $Q \neq \emptyset$

consider the head x of Q
if x has unmarked neighbor y **then**

$i++$
$Q \leftarrow y, \quad \text{mark } y, \quad p(y) := x, \quad l(y) := l(x) + 1, \quad t(y) := i$

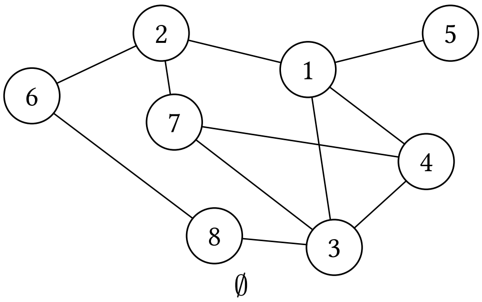
else

remove head of Q

end if

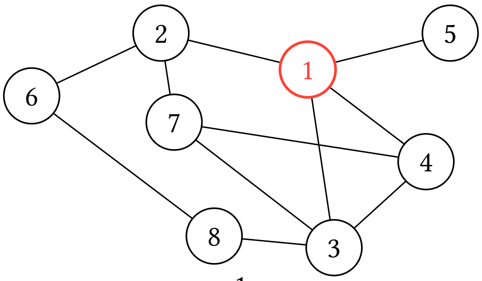
end while
return everything

2.2.2 BFS example



∅

→



1

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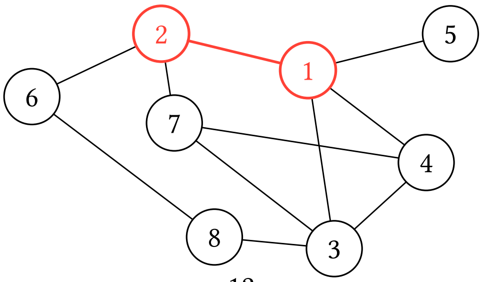
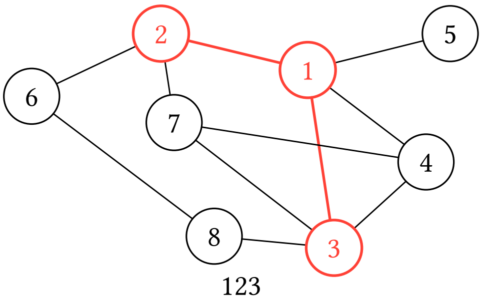


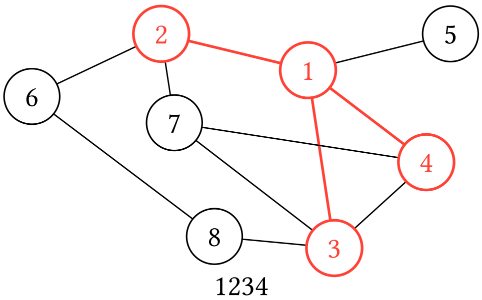
12

→



123

→



1234

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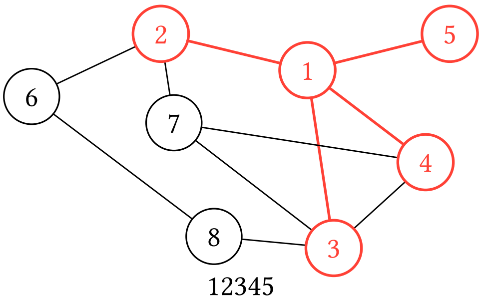

12345

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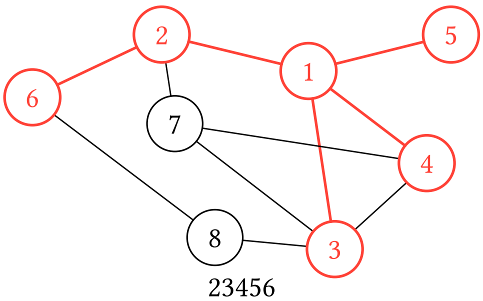
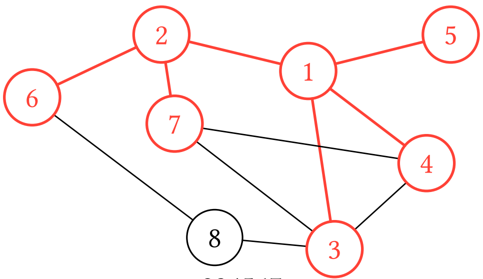


23456

→



234567

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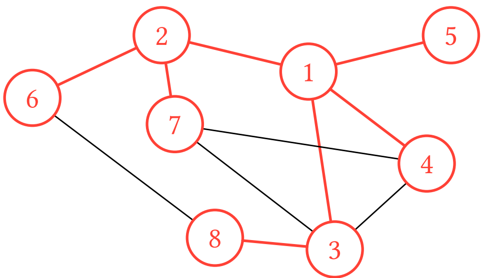


345678

detailed *(queue is used)*

∅ → 1 → 12 → 123 →
1234 → 12345 → 2345 →
23456 → 234567 →
34567 → 345678 →
45678 → 5678 → 678 →
78 → 8 → ∅

2.2.3 BFS properties

Theorem Let T be a BFS tree of G, with root r.

a.) $l(v) = d_T(r, v),$, for every $v \in V$,

b.) $|l(u) - l(v)| \leqslant 1$, for every $uv \in E(G)$.

Level of v is exactly the distance from root r to v.

Every edge of the graph connects only vertices of the same level of the tree or difference by most 1.

Theorem Let T be a BFS tree of G, with root r. Then

$l(v) = d_G(r, v)$, for every $v \in V$

As seen from our example above.

2.3 DFS

2.3.1 DFS algorithm

Completely the same as BFS, except that we use a **stack** instead of a queue.

> INPUT: a connected graph G, a vertex $r \ \varepsilon V(G)$
> OUTPUT: an r-tree $T \subseteq G$, its predecessor function p, its level function l, the time function t

DFS algorithm
$S := \emptyset, \quad S \leftarrow r, \quad l(r) := 0 \quad t(r) := 1, \quad \text{mark } r, i := 1$
while $S \neq \emptyset$

consider the top vertex x of S
if x has unmarked neighbor y **then**

$i++$
move y to the top of S, $\quad$ mark y, $\quad p(y) := x, \quad l(y) := l(x) + 1, \quad t(y) := i$

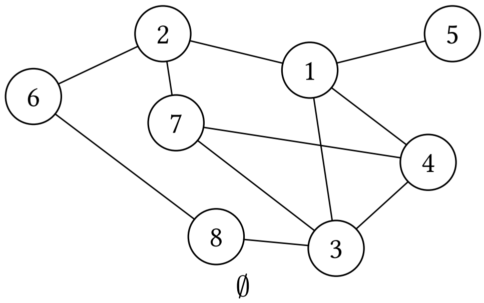
else

remove x from S

end if

end while
return everything

2.3.2 DFS example



∅

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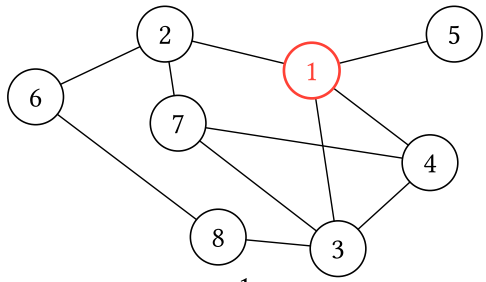
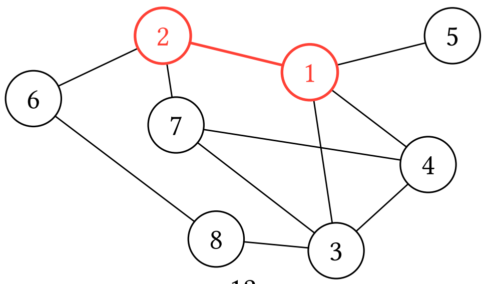


1

→



12

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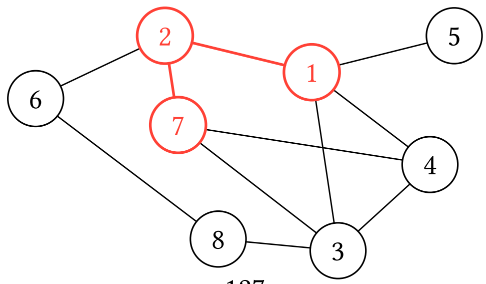
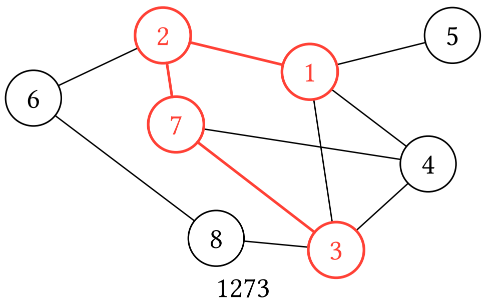


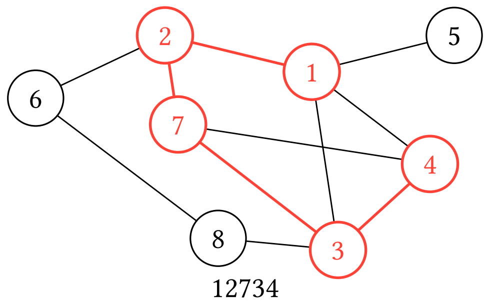
127

→



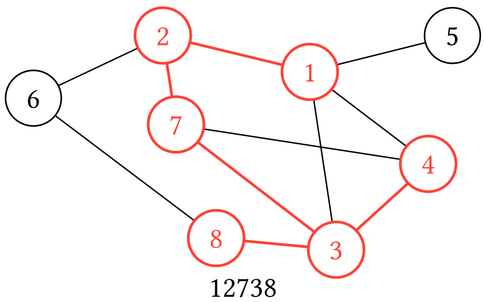
1273

→



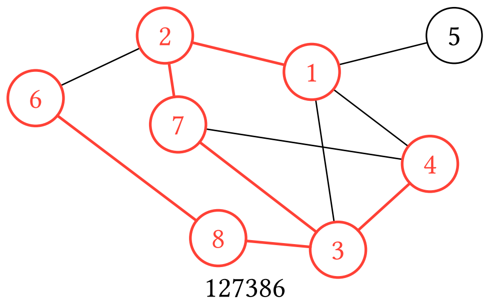
12734

→



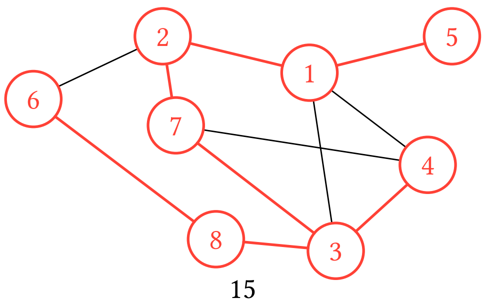
12738

→



127386

→



15

detailed *(stack is used)*

$\emptyset \to 1 \to 12 \to 127 \to$
$1273 \to 12734 \to 1273 \to$
$12738 \to 127386 \to$
$12738 \to 1273 \to 127 \to$
$12 \to 1 \to 15 \to 1 \to \emptyset$

3 Applications

4 Conculsion