

# **DSA3 seminar assignment**

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# **1 Introduction**

## 2 BFS and DFS

### 2.1 Purpose and theoretical background

Both BFS (breadth-first search) and DFS (depth-first search) are tree-search algorithms. They are both used to determine whether a graph is connected.

#### Theorem

A graph is connected if and only if, you can not split it into two such that there are no edges between them.

An approach to check whether a graph is connected is by checking all the partitions (the problem is that there are  $2^{n-1}$  of them so it is very inefficient). Another way to check whether a graph is connected is by checking all pairs of vertices - if there is a path connecting them (problem here again is that there are  $n^2$  pairs of vertices and for each of them we have to check if there is a path which is also very inefficient on top of number of partitions that we would need to check).

#### Definition

A graph is connected if and only if it has a spanning tree.

Checking whether a graph is connected in such a way (by determining whether it has a spanning tree) is far and away the most efficient way to check graph connectedness.

Both DFS and BFS work in a similar way (they both produce spanning trees), but with differences that impact the priority of edges selected. Depending on the way we used to choose edges we get different trees.

A **tree-search** algorithm on  $G$ :

- > Start with a single root vertex  $r \in V(G)$ . This is our initial tree (with just one vertex).
- > Repeat (for as long as possible):
  - Do we have a spanning tree?
  - Is the tree edge cut empty?
  - If not, add one edge from the cut to the tree.

If we want to check *connectedness* of  $G$ , that is the whole algorithm. As noted above, depending on the way we use to choose edges, different spanning tree will result (if the graph is connected of course).

Define: edge cut, rooted tree (also called r-tree), levels (distance from root to a specific vertex), ancestor, descendant, parent or predecessor and children.

$$v \in V$$

Predecessor function:  $p(v)$ , for all  $\{r\}$

## 2.2 BFS

Adjacency lists of vertices are considered on a first-come-first serve basis. Implemented with a *queue*.

For a connected graph, the algorithm will return:

- A spanning tree given by its predecessor function,
- the level function,
- the time function (order in which vertices are added to the tree)

In BFS algorithm we will first consider the neighbors (one-by-one) before we look through the neighbours of any of them.

Therefore, first edges incident to  $r$  are selected, and only after that we are looking at neighbors of the neighbors of  $r$ .

This is called Breadth-first search algorithm. This approach expands (spreads) the tree as much as possible.

We start with just the root  $r$  in the queue and we repeatedly pop the head of the queue, and push all its new neighbors to the queue.

### 2.2.1 BFS algorithm

> INPUT: a connected graph  $G$ , a vertex  $r \in V(G)$

> OUTPUT: an  $r$ -tree  $T \subseteq G$ , its predecessor function  $p$ , its level function  $l$ , the time function  $t$

#### BFS algorithm

$Q := \emptyset$ ,  $Q \leftarrow r$ ,  $l(r) := 0$   $t(r) := 1$ , mark  $r$ ,  $i := 1$

**while**  $Q \neq \emptyset$

$x \leftarrow Q$

    if  $x$  has unmarked neighbor  $y$  **then**

$i++$

$Q \leftarrow y$ , mark  $y$ ,  $p(y) := x$ ,  $l(y) := l(x) + 1$ ,  $t(y) := i$

**else**

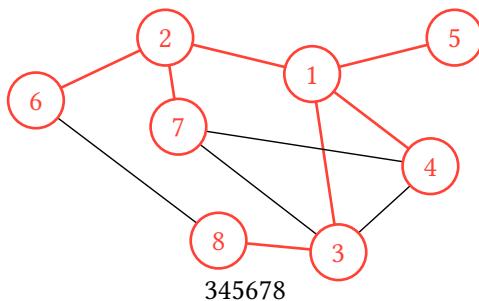
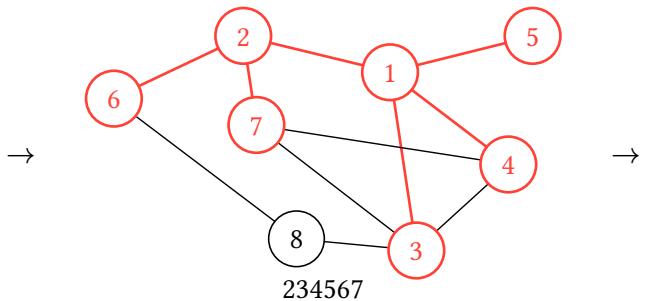
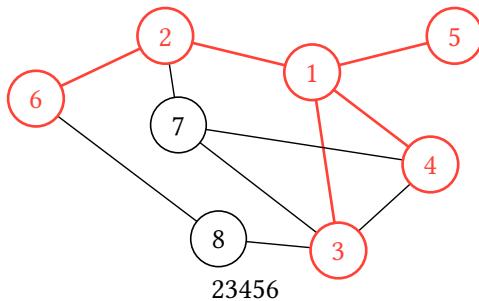
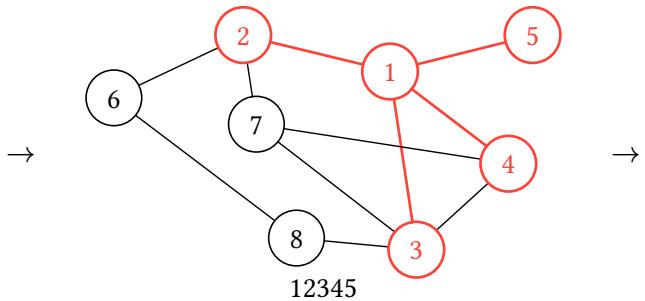
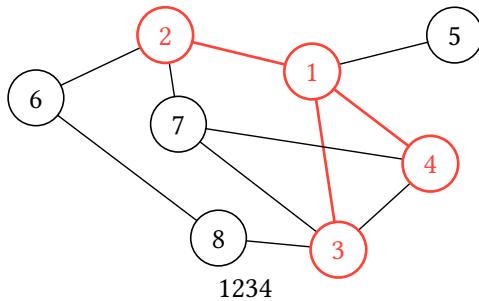
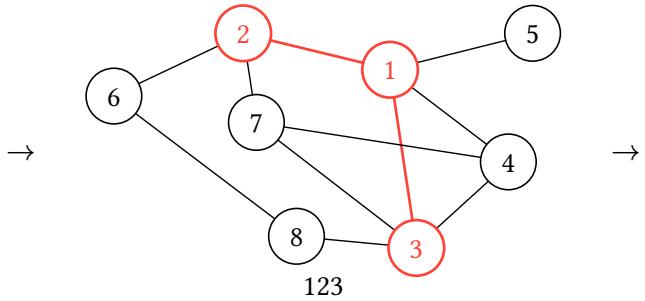
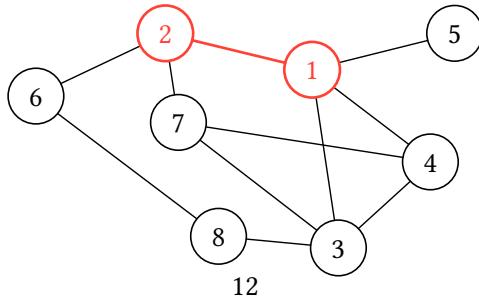
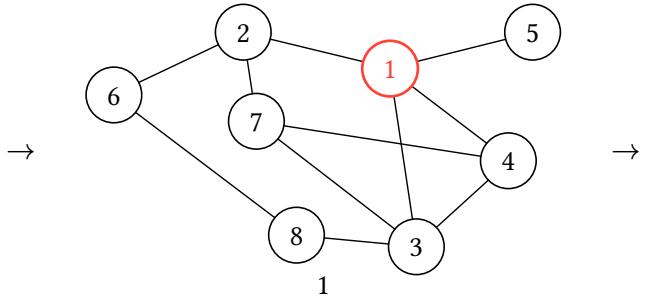
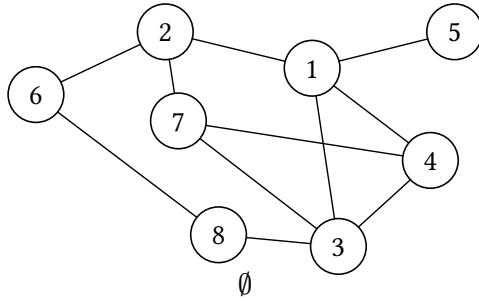
        remove head of  $Q$

**end if**

**end while**

**return everything**

### 2.2.2 BFS example



**detailed** (*queue is used*)

$\emptyset \rightarrow 1 \rightarrow 12 \rightarrow 123 \rightarrow$

$1234 \rightarrow 12345 \rightarrow 2345 \rightarrow$

$23456 \rightarrow 234567 \rightarrow$

$34567 \rightarrow 345678 \rightarrow$

$45678 \rightarrow 5678 \rightarrow 678 \rightarrow$

$78 \rightarrow 8 \rightarrow \emptyset$

### 2.2.3 BFS properties

**Theorem** Let  $T$  be a BFS tree of  $G$ , with root  $r$ .

- a.)  $l(v) = d_T(r, v)$ , , for every  $v \in V$ ,
- b.)  $|l(u) - l(v)| \leq 1$ , for every  $uv \in E(G)$ .

Level of  $v$  is exactly the distance from root  $r$  to  $v$ .

Every edge of the graph connects only vertices of the same level of the tree or difference by most 1.

**Theorem** Let  $T$  be a BFS tree of  $G$ , with root  $r$ . Then

$$l(v) = d_G(r, v), \text{ for every } v \in V$$

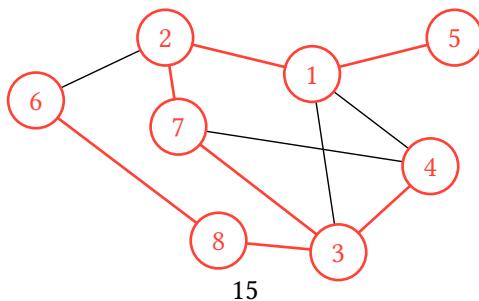
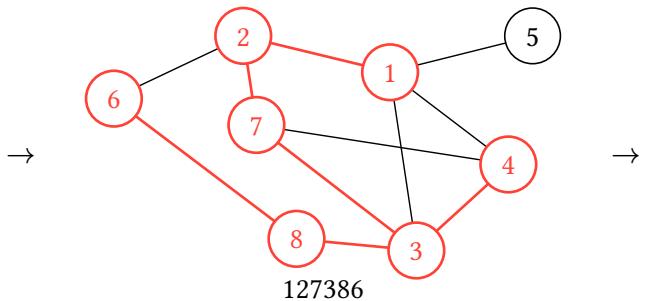
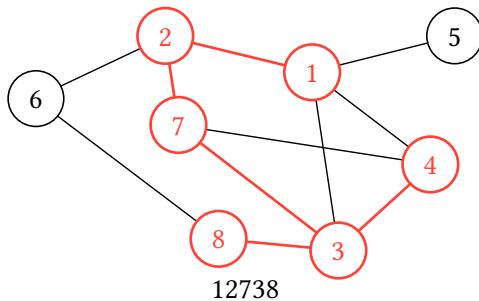
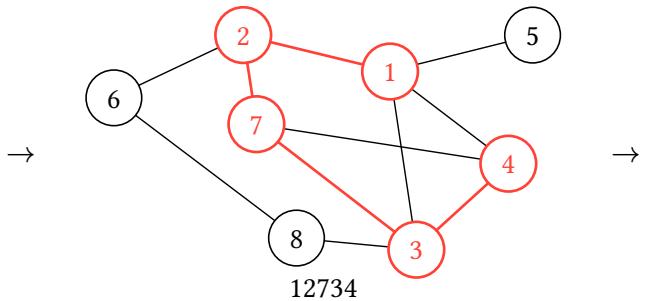
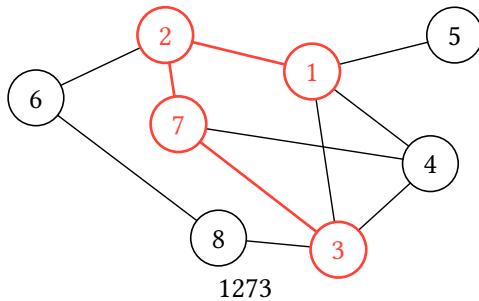
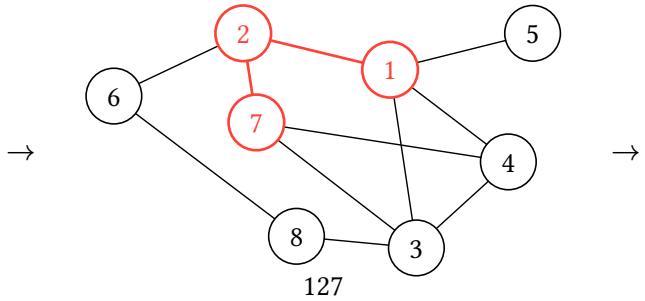
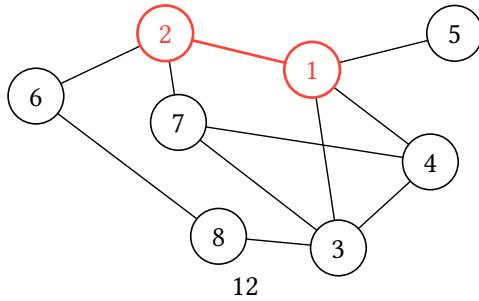
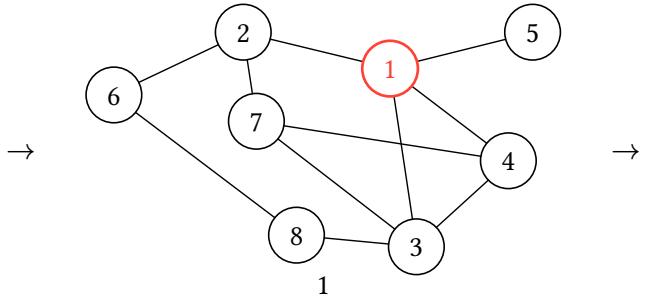
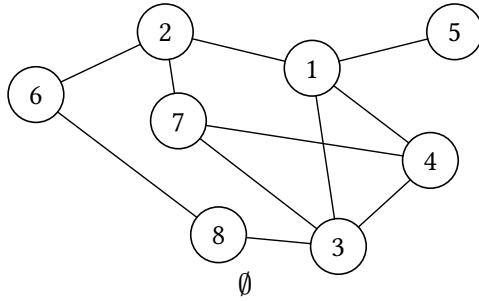
As seen from our example above.

## 2.3 DFS

Completely the same as BFS, except that we use a **stack** instead of a queue.

### 2.3.1 DFS algorithm

### 2.3.2 DFS example



**detailed (stack is used)**

$\emptyset \rightarrow 1 \rightarrow 12 \rightarrow 127 \rightarrow$   
 $1273 \rightarrow 12734 \rightarrow 1273 \rightarrow$   
 $12738 \rightarrow 127386 \rightarrow$   
 $12738 \rightarrow 1273 \rightarrow 127 \rightarrow$   
 $12 \rightarrow 1 \rightarrow 15 \rightarrow 1 \rightarrow \emptyset$

### **3 Applications**

## **4 Conculsion**