DSA3 seminar paper

Radoman Radoman 157m/21
University of Novi Sad, Faculty of Sciences
radoman88@gmail.com

1 Introduction

2 BFS and DFS

2.1 Purpose and theoretical background

Both BFS (breadth-first search) and DFS (depth-first search) are tree-search algorithms. They are both used to determine whether a graph is connected.

Theorem
A graph is connected if and only if, you can not split it into two such that there are no edges between them.

An approach to check wheter a graph is connected is by checking all the partitions (the problem is that there are 2^{n-1} of them so it is very inneficient). Another way to check whether a graph is connect is by checking all pairs of vertices - if there is a path connecting them (problem here again is that there are n^2 pairs of vertices and for each of them we have to check if there is a path which is also very inneficient on top of number of partitions that we would need to check).

Definition
A graph is connected if and only if it has a spanning tree.

Checking whether a graph is connected in such a way (by determining whether it has a spanning tree) is far and away the most efficient way to check graph connectedness.

Both DFS and BFS work in a similar way (they both produce spanning trees), but with differences that impact the priority of edges selected. Depending on the way we used to choose edges we get different trees.

A **tree-search** algorithm on G:
> Start with a single root vertex $r \ \varepsilon \ V(G)$. This is our initial tree (with just one vertex).
> Repeat (for as long as possible):
- Do we have a spanning tree?
- Is the tree edge cut empty?
- If not, add one edge from the cut to the tree.

If we want to check *connectedness* of G, that is the whole algorithm. As noted above, depending on the way we use to choose edges, different spanning tree will result (if the graph is connected of course).

Define: edge cut, rooted tree (also called r-tree), levels (distance from root to a specific vertex), ancestor, descendant, parent or predecessor and children.

Predecessor function: $p(v)$, for all $\frac{v \ \varepsilon V}{\{r\}}$

2.2 BFS

Adjacency lists of vertices are considered on a first-come-first serve basis.

Implemented with a *queue*:

Start with just the root in the queue.

Repeatedly pop the head of the queue, and push all its new neighbors to the queue.

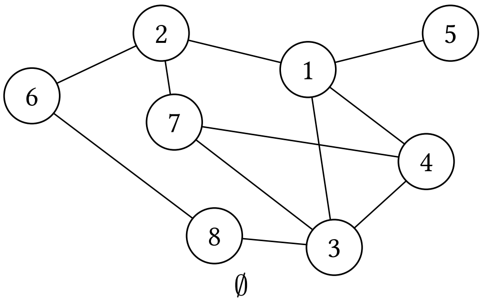
For a connected graph, the algorithm will return:
- A spanning tree given by its predecessor function,
- the level function,
- the time function (order in which vertices are added to the tree)

2.2.1 BFS algorithm

> INPUT: a connected graph G, a vertex $r\ \varepsilon V(G)$
> OUTPUT: an r-tree $T \in G$, its predecessor function p, its level function l, the time function t.

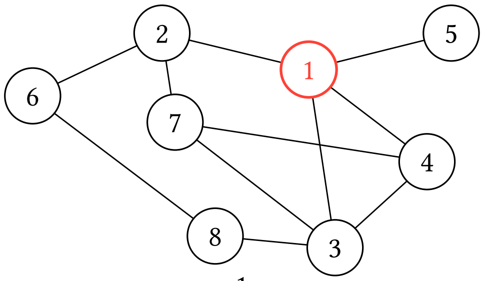
$Q\ \coloneqq \emptyset$, $Q\ \leftarrow r$, $l(r) \coloneqq 0$ $t(r) \coloneqq 1$, mark r, $i \coloneqq 1$
while $Q\ \boxed{?}\emptyset$

2.2.2 BFS example



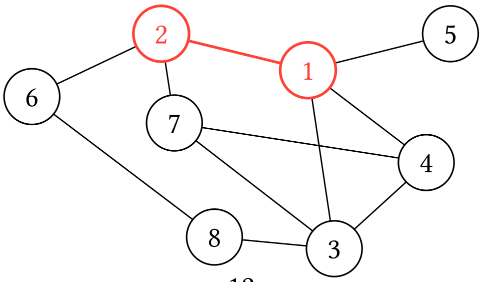
∅

→



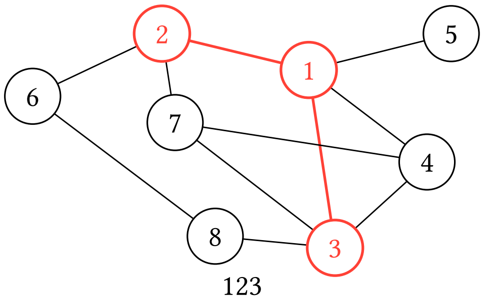
1

→



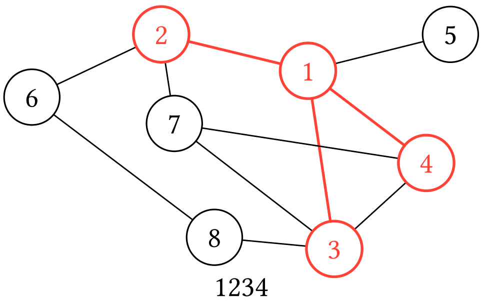
12

→



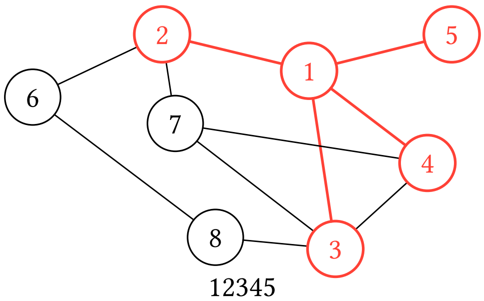
123

→



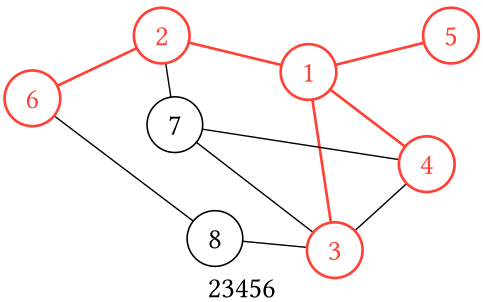
1234

→



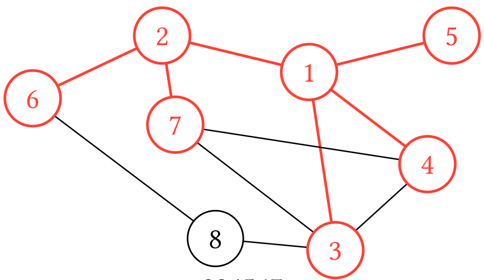
12345

→



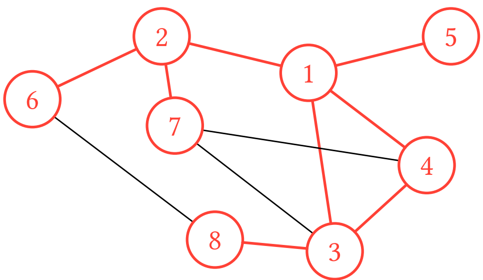
23456

→



234567

→

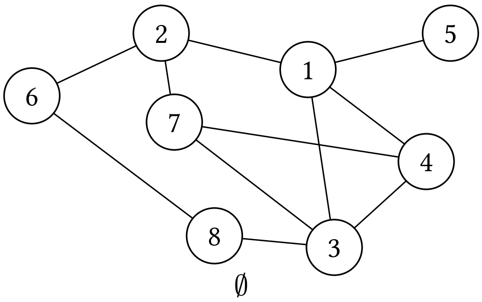


345678

2.3 DFS

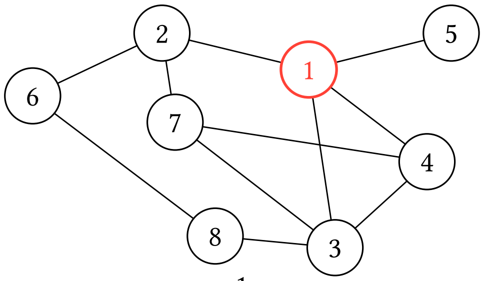
2.3.1 DFS algorithm

2.3.2 DFS example



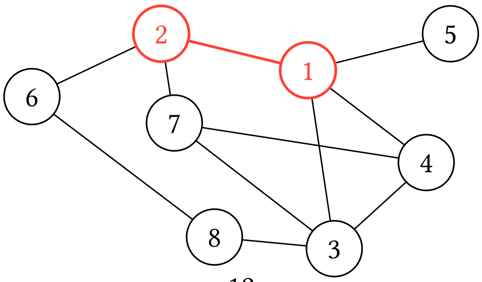
∅

→



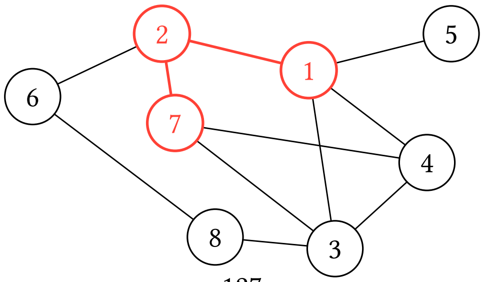
1

→



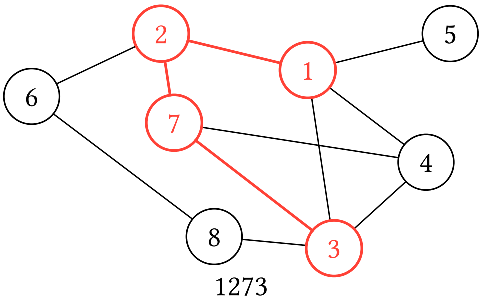
12

→



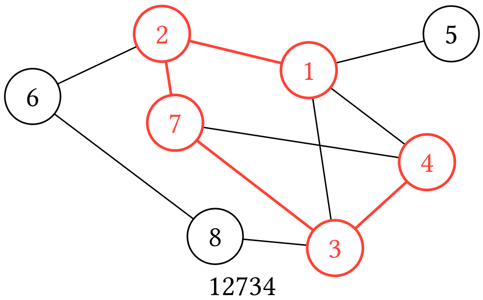
127

→



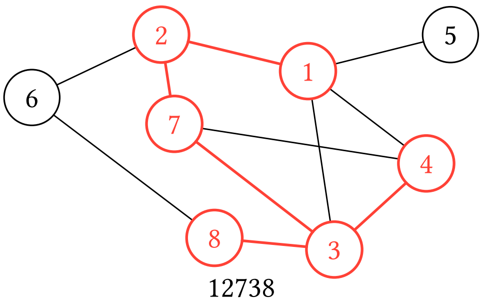
1273

→



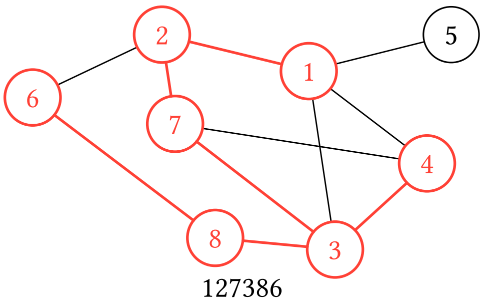
12734

→



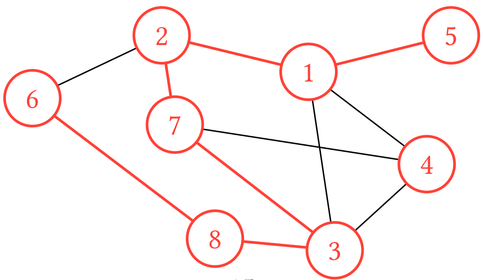
12738

→



127386

→



15

detailed (stack is used)
$\emptyset \to 1 \to 12 \to 127 \to$
$1273 \to$

3 Applications

4 Conculsion